Numerical Methods and Simulation
Techniques for Scientists And Engineers
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Lecture 13
Ordinary Differential equations

So, we want to discuss this solution of differential equations for this the coming maybe a week or a couple of weeks from now. And differential equations are all a very common to all of us in various branches of science and engineering. And we come across almost in every sphere every rather you know topic of science and engineering we come across differential equations that need to be solved to arrive at the solutions given certain conditions we will learn these conditions are either called as the initial value problems or they are called as a boundary value problems.

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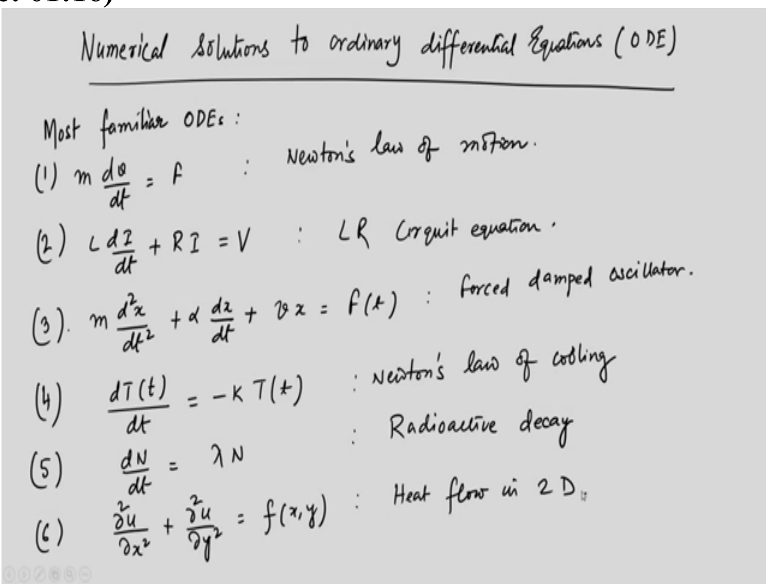

And so we are interested in talking about numerical approach or numerical solutions to, let us start with ordinary differential equations and then depending on the time that are that will be available to us at that time we will decide on the partial differential equations. And this ordinary differential equations are called as ODE and we will learn how to numerically solve these equations. So, we come across a number of them the most familiar of them let us list familiar ODE's are of course we are very familiar from our school days as this as the Newton's law of motion okay.

So, this mass into acceleration is nothing but the force and there is a differential equation that needs to be solved in order to arrive at v as a function of t and you may actually want to integrate it once more to get x as a function of time which means the displacement of the body of the particle as a function of time. Then we have seen in electrical circuits we have seen this differential equation which is called as the LR circuit equation okay.

Which is this L is the inductance and di dt is the change of current with respect to time RI is of course the voltage drop across the resistors resistance R and this is equal to the voltage applied so this is the LR circuit equation we are familiar with that as well. Then there are these md 2x dt 2 + alpha dx dt + v x is equal to f of t and this corresponds to the forced damped oscillator known to us without this alpha dx dt we would have had a you know of undamped oscillator just a forced oscillator and if f of t is equal to 0.

And these the damping term equal to 0 is a common equation for an oscillator simple harmonic oscillator but with f of t as a forcing term and these alpha dx dt which is the dissipative term or which arises because of the resistance or the viscosity of the fluid through which the body is moving we get an equation of this form and this equation has to be solved. And in order to find x as a function of t. So, that will tell us that how you know in a viscous medium or a dissipative medium how our body travels.

And what are the specific interesting situations that can arise which are you know some in the form of resonance and things like that. Then of course we are only listing a small subset of this we have a large number of equations that we come across. So, this is the Newton's law of cooling so this is the temperature as a function of time which is equal to sum this is the Newton's law of cooling this is the law according to a body cools T is the temperature and small t is the time.

So there is the change in rate of change of temperature is proportional to the temperature and with a constant which is given by K which of course depends upon the material. Another very interesting one is the law of radioactivity which can happen in other situation as well of course in Biosciences you can talk about the colony of bacteria or the colony of virus that are you know decaying according to this law.

So, this is the radioactive decay and one knows that these radioactive decays these equations are very important in determining the age of a very old sample of rock and one knows that what is the if one knows what is the number of atoms at t equal to 0 then one can actually predict the number of atoms at t equal to you know even after 1000 years. So, by looking at a piece of wood or a piece of rock one can actually say that what is the age of that and so on.

Then of course del square u del x square + del square u del y square so this is a 2 dimensional heat flow equations it is a heat flow in 2d okay. And there are number of equations which I do not want to repeat here Schrodinger equation that you know then there are Ficks law of diffusion and various things okay. So, these are all these examples that you see here they denote rate of change of a variable as a function of another variable and other parameters okay.

So, I think this summarizes that so here v is a function of p and there are these other quantities such as f etcetera are functions of other parameters like here in the second equation v can be a sinusoidal function or v can be a constant function which is independent of time. And here we are looking at the derivative of current with respect to time, so, how the current is changing and so on. So, these are the differential equations that we come across on a regular basis in various branches of science and engineering.

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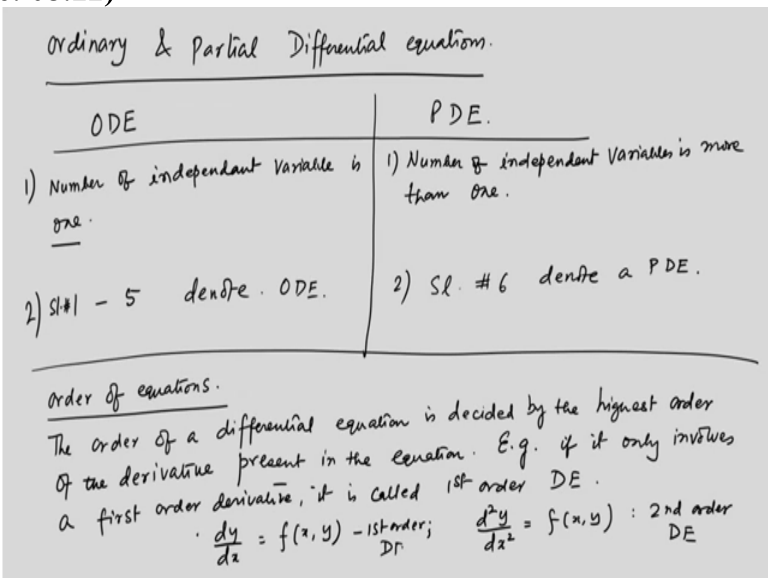

And we resolve to numerical methods because most of the time the analytic methods are not available for each of these equations. For example that we come across the harmonic oscillator the quantum harmonic oscillator, if you try to solve Schrodinger equation for oscillator potential which goes as half kx square you would find a solution which is an exact solution analytic solution but not without a lot of difficulty that is you have to cast that in the form that is available in the form of a special function and then one can get the results.

Finally we get the result as a function of or rather in terms of a hard might polynomial which has certain properties and these are multiplied by a Gaussian function and this is how one comes you know across the variety of solutions. But most of the time for any arbitrary potential Schrodinger equation would not be solvable like for example some of this example that I have given in the earlier slide they would not be solvable in presence of a very difficult driving parameter.

See the forcing term or the potential if they are functions of you know some complicated functions of time then you may not arrive at an analytic solution. So, numeric solutions are must having said that let us give a few definitions and we try to sort of see that what these differential equations they actually represent. So, let us just first distinguish between ordinary and partial differential equations.

Because you hear these names the partial differential equations or PDE and so on you need to know what they represent. So, let us call it as a ODE and a PDE here of course the number of independent variables is one, so that is called as ODE. And when the number of independent variables is more than one then this is of course called as a PDE okay. So, let us see that if you go to this last page all these one to 5 they will present ODE because the this dependent variable v depends upon only one independent variable that is t.

Similarly here the I depends only on t x depends only on t the temperature t depends only on time and end the number of atoms or the elements which remain after a time t that also depends upon time. Whereas if you see 6 then you see that these u is actually a function of x and y and that is why you have to take a partial derivative double partial double derivative and arrive at this heat flow equations.

So, this is so in the earlier sty slide1 to 5 denote, so if you want the serial number 1 to 5 denote ODE and serial number 6 denote a PDE. So, this is the difference between ODE and PDE that we make this thing is smaller. And let us now discuss the order of equations. So, the order of a differential equation, equation is decided by the highest degree or the highest order rather not degree highest order of the derivatives present in the equation.

So, for example if it only involves a first order derivative it is called as the first order differential equation. We will call it as de so we have say dy dx is nothing but f of x y and so on and whereas so this is first-order because the highest derivative is just first order derivative. And the d 2 y dx 2 is equal to again say f xy or some other parameter. So, this is actually a second order, second order d okay.

So, these are does this determine the order of the equation so we can say that we have a first order differential equation or we have a second order differential equation depending upon the highest order of derivative that is present in the equation. So, let us see these things so the first1is actually a first order differential equation, the second1is also a first order differential equation.

The third one is the second-order differential equation because the highest derivative is 2 order of derivative is 2, then it is again the first order differential equation, first order differential equation the last1is second order PDE the partial differential equation.

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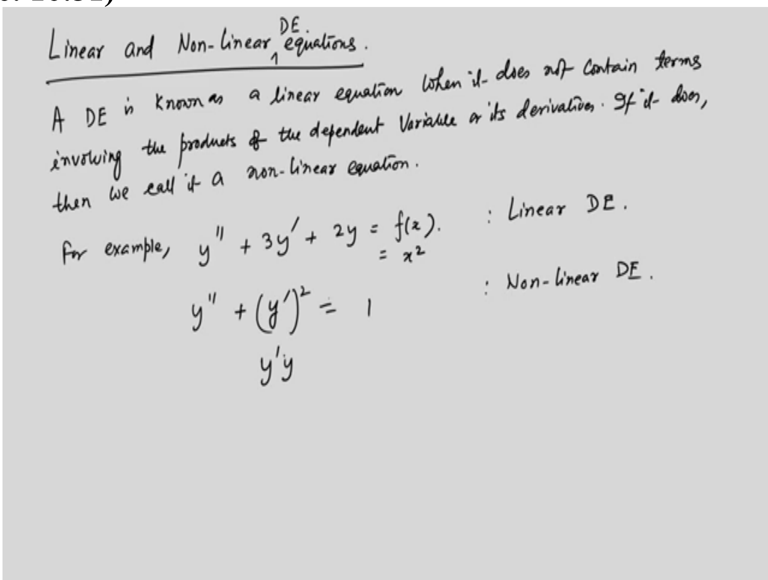

So, these are definitions of the order of equations, let us look at the linear and nonlinear equations okay. So, what is meant by a linear differential equation and nonlinear differential equation, so these are of course differential equations? So, a differential equation as a linear equation when it does not contain terms involving the products of the dependent variable or its derivatives okay.

If it does then we call it a nonlinear equation okay. So, say for example y double prime + 3y prime +2y equal to say f of x okay this is the example of a linear DE because there is no product term whereas a y so this fx could be x square or x cube that does not change it, say for example if x is equal to x square so that does not change it from linear to nonlinear because we are only talking about the dependent variable.

The dependent variable is y here, y depends on x so the independent variable is x and the dependent variable is y. So, that is why this is a linear equation. Whereas if you have a term such as y prime square equal to say 1 so this is the example of a nonlinear equation, it could be y prime y or it could be you know y double prime y and things like that but nevertheless I mean whenever you have a product term of the dependent variable then we call it a nonlinear differential equation.

And most of the time this linear differential equations one is able to arrive at an analytic solution whereas for the nonlinear differential equations most of the time such analytic solutions are not available. In fact we will give an example in some time let us then talk about the general and particular solution.

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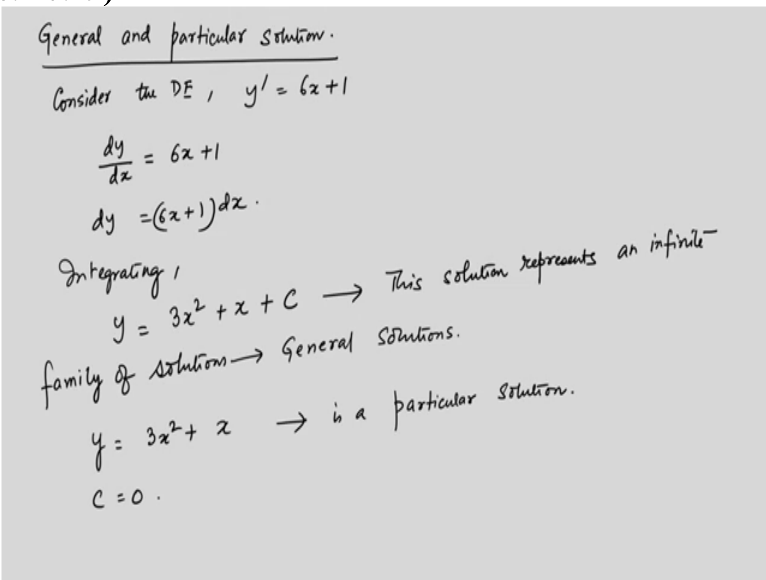

Consider the differential equation y prime equal to 6x + 1 okay, the solution is simple because you have a dy/dx is equal to 6x + 1 so a dy would be 6x + 1 into dx and if we integrate so we will say integrating y equal 2 so 6x dx is nothing but 3x square because 6x square by 2 and 1 dx will simply be equal to x but then since we are doing an indefinite integral we will have to add a constant of integration.

Now this solution represents an infinite family of solutions, so for each value of C it is a solution you take C equal 1 or C equal to 1/2 and C equal to 10 C equal to 100 they all denote valid solutions of this and you can put it back into this because you want to take a dy/dx and that would because this C is not specified here and it is just a number it does not depend upon x. So, when you take a derivative first derivative this vanishes and it always gives me 6x + 1.

And so this actually gives rise to an infinite family of solutions and this is called as the general solution. So, in general for these differential equations that we are talking about you will get an infinite family of solutions one for each value of the undetermined constant that we have in the solution ok. Once you specify the constant then the ambiguity goes away and one lands up with specific or a particular solution and this is what we say that we have y equal to 3x square + x is a particular solution okay.

So, this represents a solution for C equal to 0 so this is a particular solution that is that is there we can of course take a 3x square + x +1 is another particular solution which is for C equal to 1 and so on. So, this constant will have to be evaluated in order to arrive at a particular solution because for a given differential equations you want a particular solutions and most of the time you do not actually worry about a family of solutions.

The family of solutions can exist but that does not specify the problem completely in order to specify the problem completely we need a particular solutions and to arrive at a particular solutions we need to know this C. And this C can be known if you apply certain conditions which it may be known as initial conditions okay. So, let us just give an example of that and these are called as the initial value problems when the initial conditions are specified okay.

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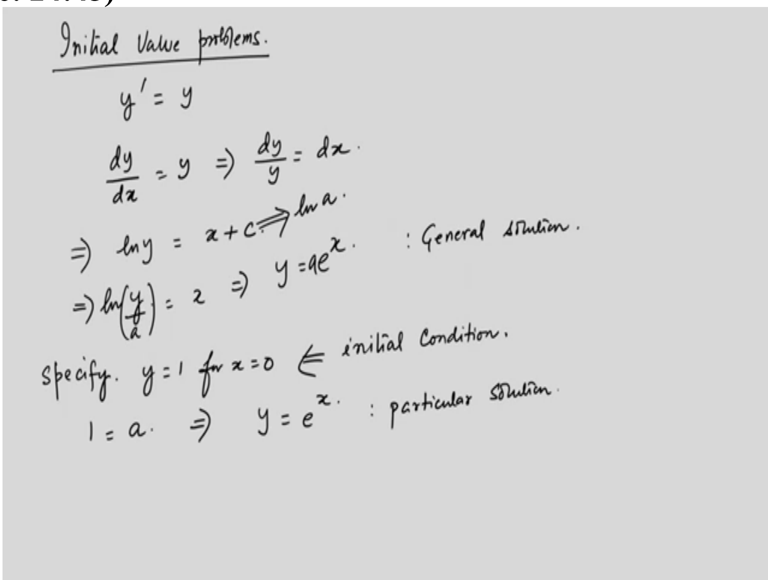

So consider one equation such as a y prime equal to y okay so this is a particular differential equation that you have to solve and the solution is simple you have a dy/dx equal to y. And this gives dy by y equal to dx and if you integrate it then it becomes equal to log of y equal to x + C and then you can actually get this y equal to we can write this C as nothing but this is the choice is up to you, you can write it as long a.

So that this becomes you know log of y by a equal to x because if the log comes on the other side it will become minus and then you can write it as a you know the 1 divided by the other so this will give me a y equal to a e to the power I am sorry a e to the power x. So, a e to the power x is a general solution and it has to be determined in order to appear or rather arrive at the particular solution. So, let us specify y equal to 1 for x equal to 0 okay.

If you do that then I so this is the initial condition and with this initial condition we can put x equal to 0, so y equal to 1 so this is equal to e so a becomes equal to 1 and this allows us to write y equal to e to the power x is actually a particular solution. So, what we were talking about in the earlier slide of the general solution and the particular solution so this is the general solution. So, we can reduce the general solution to a particular solution by using the initial condition.

Here the initial condition is arbitrarily specified that at x equal to 0 say y equal to 1 you can take other values of y that will change the value of the constant a but you will nevertheless get a particular solution for this given problem. And it is very important to understand that a lot of these physical problems that we come across in science and engineering they need to be initially solved by hand and then we would arrive at the solutions by numerical methods.

So, some of these solutions need some analytic skills for us to be able to write it in the form that the computer understands and the computer would be able to decode it and give you a result depending on the initial conditions or the boundary conditions. So, let us look at at least 1 or 2 this initial value problems.

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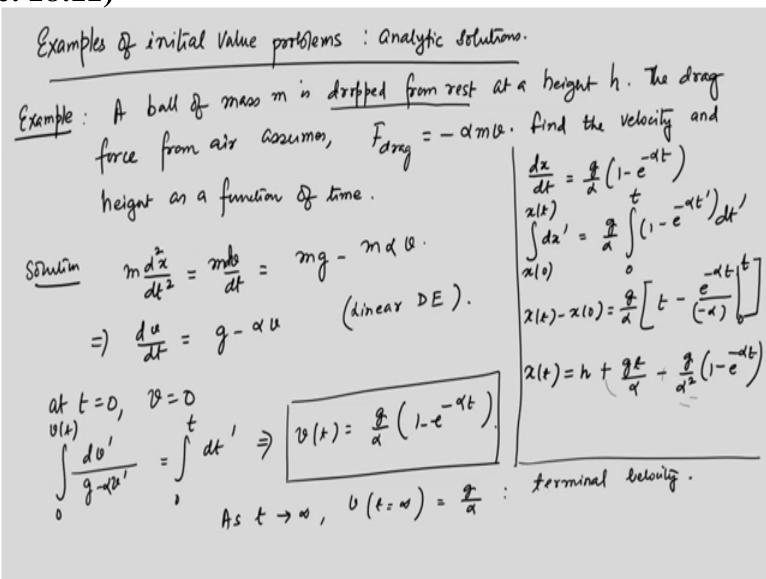

And these are analytic solutions the reasons that we are spending some time on the analytic solutions even though you put it in the computer and want to learn what are the numeric methods. In order to solve these equations but if you have a physical problem that physical problem has to be cast in the form of a differential equation which the computer understands and would solve it by the prescribed methods or given guidelines of solving or iteratively solving such equations.

So that requires some analytic skills and we are going to give 2 examples for that here. So, a ball of mass m is dropped from rest at a height h the drag force from the air to this h assumes a form f drag equal to - alpha mv. It is important to note that most of the time the drag force of the dissipative force or the viscous force are written as proportional to velocity it does not have to be. But in the first approximation we take them to be proportional to velocity.

If the drag force becomes too large then of course the several you know other powers of v could appear and in real problems in rocket motion and all that the drag force could be larger than simply linear or fee but here we are going to consider to be to be linear. So, the question is that find the velocity and height as a function of time. So, the solution is simple we know about it so this is the thing that we have to write.

So this is equal to m dv dt this is equal to mg - m alpha v, so this is equal to so if we write down this this is a dv dt equal to g - alpha v, now importantly note that it is a linear equation because there is no term which involves v and v Square and v v Prime and things like that and it is so it is a linear equation, so it is a linear differential equation and the degree is of course one. so, it is given that is dropped from rest which means at t equal to 0, v equal to 0, it is at rest at t equal to 0 so it is initially when it is initially dropped it will be that.

So we have to solve for 0 to v as a function of t and now as we have said earlier we will use a dummy variable here and g - alpha v prime equal to 0 to t dt Prime and if you integrate it, it becomes equal to v of t equal to gover alpha 1 - exponential - alpha t ok. So, this is the solution of this equation and you see that I get v as a function of t and every quantity is known g is the acceleration due to gravity, alpha is the coefficient of that friction or the dissipative force of the viscous force that is there and this is exponential - alpha t.

So, this is the a velocity expression at any given time t so it clearly says that you know as t increases the velocity this component actually becomes smaller and smaller and at t large t this would the second term inside the bracket would go to 0 which would render a constant value which is G over alpha to the velocity and this is known as the terminal velocity. So, as t tending to infinity v at t equal to infinity this is equal to g over alpha.

And this is nothing but the terminal velocity which a body attains. We are still left with the task of finding height as a function of time or the distance as a function of time. So, let us see we it can do it here. So, once when we get v of t so we can write it as dx dt and this is equal to g over alpha 1 - exponential - alpha t so this goes from if you integrate dx Prime so this goes from x equal to g equal to 0 at x equal to x at t equal to x or h it does not matter I mean we have written it in terms of x so understanding that x is the vertical distance that the body travels under these gravitational field and the drag force of the air.

So, this is equal to a g over alpha which is a constant of 1 - exponential of alpha t Prime and dt Prime and then we are integrating over from some 0 to some t, so this t is of course variable t and then we can integrate this equation this becomes x t - x 0, so this is equal to g over alpha and1will give me a t and this will give me a exponential alpha t divided by - alpha so this and then at t equal to 0 and sorry this is not infinity but this is equal to t.

And so this becomes equal to x of t now x at 0 you can write it as h if you want because it is is dropped from a height h so light write it as h - gt over alpha square gt over alpha so this + and - g over alpha Square 1 - exponential - alpha t. Once again you see that all these quantities are known to you. So, we have arrived at a particular solution of the differential equation the second order differential equation that we have started with.

Of course the we have started with a second order differential equation in terms of x but we have converted into the first order by writing it in terms of v. So, this is the solution for this very interestingly you can see that there is a competition as t increases as t increases the height tends to increase because of this term and as t tends to be large, so this term the last term the exponential - alpha t will go to 0 as t becomes large.

So this negative term will appear so there is a positive term as t goes to infinity and there is a negative term as t goes to infinity so they would compete against each other. And any result that you get will have to be physically analyzed in order to arrive at conclusion. Let us see do another one which you might have seen in your electrodynamics course.

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Ex2. A particle of mass 'm' and charge 'q' moves in a magnetic field $\vec{B} = (0, 0, B)$. where B is constant. The initial conditions are at $t=0$ are (i) $\vec{v}_0 = \vec{u} = (u_1, 0, u_3)$ and (ii) $\vec{r}_0 = (0, u_1/\omega, 0)$ where $\omega = \frac{qB}{m}$. Solve the DE and find the trajectory. Further show that the speed of the particle is constant.

$$\left\{ \begin{array}{l} \vec{r}_0 = \left(0, \frac{u_1}{\omega}, 0\right) \\ \vec{v}_0 = (u_1, 0, u_3) \\ \vec{B} = (0, 0, B). \\ \vec{v}_0 = \vec{u} \end{array} \right.$$

Solution: $\vec{F} = q\vec{v} \times \vec{B} \Rightarrow m\frac{d\vec{v}}{dt} = q\frac{d\vec{r}}{dt} \times \vec{B}.$

$$m\int_{v_0}^{v(t)} d\vec{v} = q\left(\int_{r_0}^{r(t)} d\vec{r}\right) \times \vec{B}.$$

$$\boxed{\vec{v}(t) = \frac{q}{m}(\vec{r} - \vec{r}_0) \times \vec{B} + \vec{u}.}$$

And so particle of mass m and charge q moves in a magnetic field the magnetic field is only in the z direction. So, we will write it as 0 0 B there is one notation that you should get familiar with because many of these books especially the book by Griffiths on electrodynamics that uses such notation. These are written in terms of components xy and z so there is no x component of y component that is a z component is constant.

So there is a magnetic static magnetic field a constant magnetic field and where of course we can write B is constant. The initial conditions are at t equal to 0 that is time equal to 0 r 1 the v 0 that is a velocity at t equal to 0 call it u which has components in the x and z direction with magnitude u1 and u3 okay. And it is been released from the position r 0 which is equal to 0 u 1 over Omega and 0 where Omega equal to qB over m which is called as a cyclotron frequency.

Solve DE solve the relevant DE and find the trajectory farther show that the speed is constant speed of the particle of this charged particle okay. So, this is clear so there is a charge q mass m moving into a constant magnetic field with some velocity the magnetic field is only in the z direction the at t equal to 0 when the particle was released there was a velocity in the x and in the z direction.

And it is been released from y point that is where x and z coordinates are 0 but the y coordinate has a value u 1 over Omega where Omega is called as the low range sorry the frequency the cyclotron frequency which is qB over mq being the charge and m being the mass of the particle so you have to find the trajectory okay. So, it is a very clear that we have to solve the equation or rather this differential equation which is relevant for this problem which is the Lorentz force equation.

So the Lorentz force equation so let us write down the solution this analysis will have to be done before we give it to the computer. The computer will take a certain algorithm and we will keep you know iterating that algorithm to solve that or arrive at a given result but this has to be done prior to that. So, it is a qv cross B so this gives m dv/dt it is q dr/dt which is v sorry the this is cross B okay. So this has to be done very clearly r 0 is given a 0 u 1 Omega 0 v 0 is u 1 0 u 3 B equal to 0 0 B ok.

So, these are the conditions that are given and v 0 equal to so this is you know this is so v 0 is equal to u ok the components are like that so this is u equal to this that is given. So, I start integrating this expression so it is a v 0 and v as a function of t and then there is a dv so this is a q r 0 to r at t dr cross with B so what one gets is that one solves for this and one lands up with this r - r 0 cross B + u, so this is the solution of this equation. Now if you have to really know that trajectory this one vector equation is not really very helpful.

What you have to do is that the trajectory means you have to solve for x but if you simply solve for r that will not so very easily give you the feel of the trajectory that the particle is executing in this constant magnetic field. So, what one has to do is that split this equation into components that is x, y and z components and we can do that easily because you have B in certain direction r - r 0 in certain directions and so on.

We cross it and then add the u etcetera and then write it in terms of vx vy vz which are vx is equal to dx dt vy equal to dy dt and vz equal to dz dt and then we can write down this writing it component wise.

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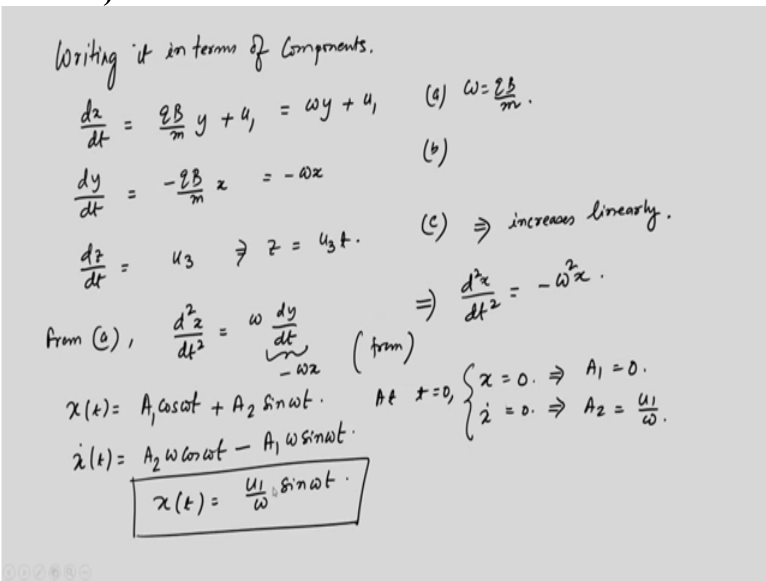

So this is a dx dt this is equal to q B over m y + u1 this is equal to Omega y because q B over m is equal to Omega u 1 given that Omega equal to qB over m or dy dt this you have to do it carefully - qB over m x equal to - Omega x, so let us call this as equation a this is equation b and the dz dt is nothing but u3 which is equal to which gives us z equal to u3 t. So, this C is particularly interesting it says that whatever may be your z directions that is usually the vertical direction the particle actually displaces uniformly over that a linear in function as a function of time.

So as time increases it goes farther away from the origin in a linear fashion. So, there is in the z direction. So, from a you can do a double derivatives and can write it as d2x dt 2 equal to Omega dy dt since u is u1 is constant u1 does not have to do anything now from B this thing can be replaced by - Omega x from B. So, this gives that d2x dt2 it is equal to - Omega square x and needless to say that all of you recognize that this is the equation of a simple harmonic motion where the double derivative of the displacement is proportional to the displacement but with a negative sign here.

And the solution is a very well known x of t is a sinusoidal function on and a cosine function a cos Omega t + a let us write it A1 and A2 sine Omega t. Now it is important to note that we have 2 unknown constants which make the solution as general but we need particular solutions because we have been given those initial conditions. So, the initial condition is that at t equal to 0 x is equal to 0 because it is from the origin and then x dot is also equal to 0 if x equal to 0.

Now x equal to 0 if you put t equal to 0 A1 will not allow you to become 0 because for t equal to 0 cos becomes equal to 1 so you have a 0 on the left but these the first term with a cosine Omega t that prohibits it making the I mean this rather matching both sides of the equation and hence this tells that A1 has to be equal to 0. So, this is an important thing it comes over and over again that you have to use the physical condition that the cosine does not become 0 however the sine becomes 0.

If the cosine does not become 0 I cannot have 0 equal to1+ 0 okay that is a sort of bad equation and should be avoided we should not even write that so we will have to write the A1 is equal to 0. So, only A2 constant remains and if x dot is equal to 0 we can get x dot t as you know A2 Omega cos Omega t, now at t equal to 0 the velocity is also 0 in the x direction. So, which tells that my A2 for t equal to 0 so this is equal to 0 which says that A2 has to be u 1 over Omega right.

Because so this gives A2 equal to u1 over Omega because this is what it comes from this so this is so your A2 Omega should be at you know so this so if I do it I will have to do for the both so it is A1 Omega sine Omega t with a minus sign and now you put t equal to 0 you will get A2 equal to u1 over Omega. So, if I collate both these things I get a x of t which is equal to u1 over Omega sine Omega t. So, this is the particular solution where everything is on I mean known.

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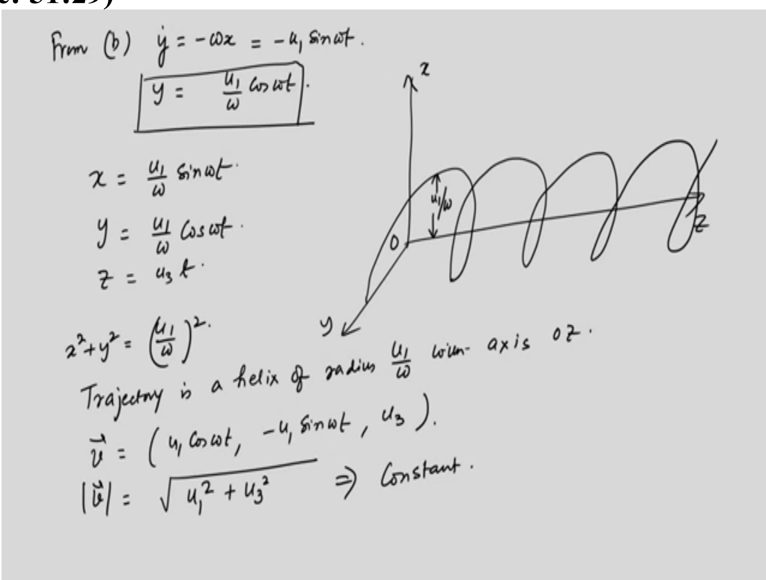

From B from the second equation we also get y dot equal to - Omega x so this is equal to - u 1 sine Omega t and so y becomes equal to u 1 over Omega cosine Omega t so that is the that is the equation because we integrate this equation in order to get this. So, this is the second so in the so in the x direction it has a sine Omega t with some amplitude and also in the y Direction is the same amplitude and cosine Omega t so you understand that if x and y they have the same amplitude one goes as sine the other goes as cosine it is going to be a circular motion okay.

In the xy plane so we can write down all these things because we have already gotten a solution this1was the for the z. So, we can write down this solution once again so x equal to u 1 over Omega sine Omega t y equal to u 1 over Omega cosine Omega t z equal to u 3 over u 3 into t so if I take x square + y square equal to u 1 by Omega whole square so the trajectory is a helix of radius u 1 over Omega with the axis with access you know like oz said let us draw this then I mean just for my convenience I am drawing this as the z direction and this is your o and this is x and this is y so it is like you know it is like this and so on okay.

So, it is just you know goes uniformly in the z direction as a function as the time increases and this thing is of course you u1 over Omega so how far it goes from the this axis the z axis it is equal to u 1 by Omega we are still left with one question it is it is a basically physics problem I because it involves differential equation have you brought it up and showed an analytic solution. So, the v that one gets is u 1 cosine Omega t - u 1 sine Omega t see we have all calculated the ux that is dx dt and dy dt and this is like u 3.

So this v becomes a constant because of this sine square so this is like; so there is like a vx square + vy square + vz square and vx square + vy square gives a simple u 1 square + u 3 square and this is nothing but a constant ok. So, the magnitude or the velocity of the particle is constant this is something that we did purposefully knowing fully well that you know this really is an analytic method we are going to go to numeric methods.

But this analytic solutions are very helpful in understanding the intricacies or the physicality’s of a given problem.

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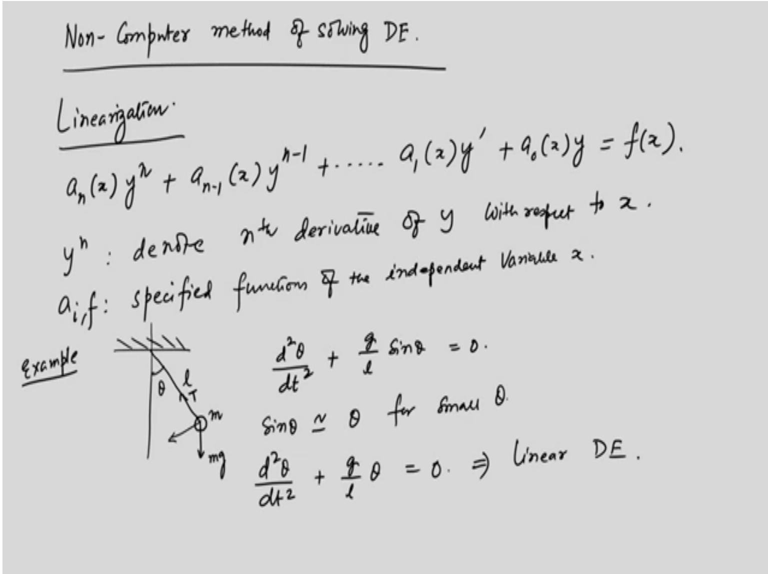

So, let us just spend maybe 5 minutes or maybe even less than that in trying to understand that what happened or how people used to solve both engineers applied mathematicians and other scientists they used to solve these differential equations when we do not have computers. So, it is a non computer method non-computer method of solving differential equation and this is they used to do linearization.

So, once again let me put this thing in perspective that we said that it is really required for us to go through certain you know computer I mean these numerical methods in order to arrive at the solution for most of the problems which are complex enough so that we do not have an analytic solution. But we are talking about the pre-computer era when computers were not so easily available or maybe computation was a big task.

And then these mathematicians and these engineers they followed some procedure called as a linearization procedure. So, which means that even if you have a nonlinear equation you try to linearize it that is you cast it in this form with a and x y to the power n + a n - 1 x y to the power n - 1 and so on and then a 1 x y prime + a 0 x y and equal to some f of x ok. So, this equation had to be solved where y n denote not y to the power n but it denote a net derivative of y with respect to x.

And this ai that is a n n - 1 a 1 etcetera this ai and f these are specified functions of the independent variable x all right. So, we could follow this for certain you know problems and1example that we all know is a simple pendulum. So, the simple pendulum is just a rigid support and there is a you know a length of string and a mass of the bob and this theta is the angular displacement from the mean position.

And there are just 2 equations which have to be written there is a vertical force which is mg and there is a force which will tend to take it towards this due to the tension of the string and so on. So, this is the tension and if you resolve these 2 forces one gets a t equal to you know something like mg sine theta and this other one will be rather cos theta and things like that and then one gets this equation as d2 theta dt 2 this is a + g by l sine theta this is there in any book on which discuss a simple pendulum.

This is equal to 0 and this equation as you see is a nonlinear equation because it involves sine theta and sine theta involves all powers of theta that is theta, theta cube theta to the power 5 and so on so it is a heavily non-linear equation. But if you observe that sine theta can be written as theta for small theta. Both sine theta and tan theta can be written as theta for small theta. Because tan theta is sine theta over cos theta but cos theta becomes equal to 1/4 theta small.

So, we retain the first term of this expansion of sine and understanding that the theta cube would be much smaller if theta itself is small typically like less than 4 degrees. If that is the equation or rather that is the simplification that we are allowed to have it then we write it as theta and of course it becomes a linear equation. So, this is a linearization linear DE and this is a linearization that has been imposed as an approximation and this is a good approximation.

Because if you take the pendulum too far away from the mean position it may no longer remain harmonic and you will not arrive at a solution that you are very familiar with and the reasons are you know very easy to see that the theta cube when theta to the power 5 terms will come which will inhibit for the equation to become give rise to a harmonic you know solution. And so that the pendulum will not you know smoothly oscillate back and forth and and the time period can be determined okay.

So, these are mostly you know we have discussed analytic methods of solution of these differential equations now we shall discuss the numeric methods and there are a number of them the mostly iterative methods and we will learn them one by one.