Introduction to Statistical Mechanics
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Lecture no. #14
Landau Diamagnetisam

Okay, let us begin a new topic or new set of topics, which is going to be basically magnetism. So In other words, the statistical mechanics of magnetic materials. In magnetism as you probably know, there are many types of magnetism. For example, they are called diamagnets, para magnets and feromagnets. So we'll have to explain what these three different types of magnetism is and why we have three different types of magnetism.

So diamagnetism is a phenomenon that is seen in all materials, it is basically comes about as a result of Faraday's law. So, you know, if you apply a magnetic field to charged particles that are moving you will be setting up currents, and those currents will themselves produce a magnetic field which will oppose the applied magnetic field.

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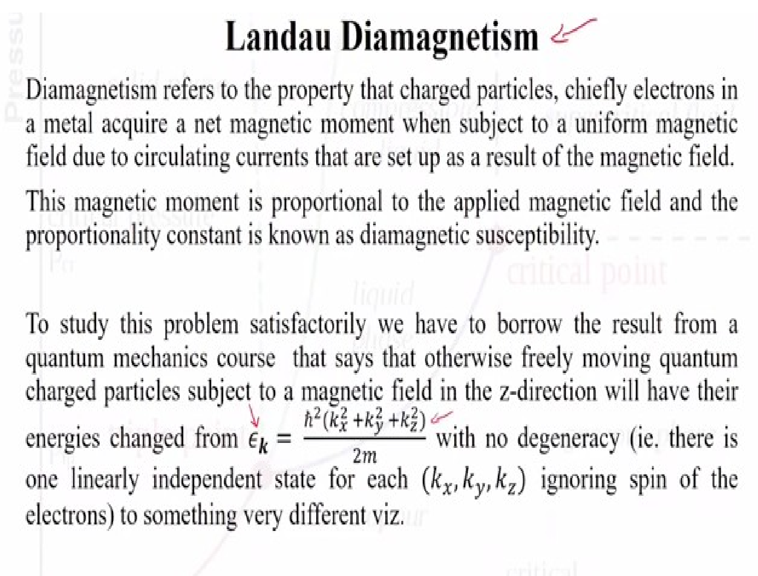

So in other words, So the phenomenon of diamagnetism is simply the idea that if you apply a magnetic field to a material, there is going to be an induced magnetic field.

Created by circulating charges in the material, which will oppose the applied magnetic field. So in other words, the magnetic moments, If you wish to call it that are not intrinsic to the material, they are induced by the applied magnetic field. So the stronger the applied magnetic field, the stronger the induced magnetic moment, and it is going to be in the opposite direction of the applied field. So this is called diamagnetism. So that is what i am going to be talking now.

But then there is going to be later on we will discuss paramagnetism where the charge carriers already have an intrinsic magnetic moment. So you do not have to apply a magnetic field to create a magnetic moment and they already have a magnetic moment for example, it can be because of the spin of the electron, which is an intrinsic magnetic moment or it could also be because of the nature of the orbitals of the atom.

So, if you have S orbital, then angular momentum is 0, but you can have a P orbital were L=1. So that will contribute to an intrinsic magnetic moment. So with the most electrons have a nonzero angular momentum, then it will contribute to a net magnetic moment of the atom. So, so spin and mag, The Angular and orbital angular momentum put together is responsible for an intrinsic magnetic moment, which leads to what is called paramagnetism.

So which will come to later. So let us start off by discussing diamagnetism. So the first correct theory of diamagnetism in real materials was given by the Russian physicist Landau. So that is why it is called Landau diamagnetism because it is his theory of diamagnetism in real materials. So, so, like I told you already, that magnetism refers to the property that charged particles namely, electrons in a metal acquire a net magnetic moment.

When subject to uniform magnetic field is due to circulating currents that are set up as a result of the magnetic field. So that this induced magnetic field is proportional to the applied field is and it is in, the proportionality consonants diamagnetic susceptibility, and it is negative implying that it is in the opposite direction. So, actually, in order to study this satisfactorily, We have to use quantum mechanics, This was realized by Landau long time back.

That the proper description of this phenomena and requires the use of quantum mechanics. So we’ll have to understand what happens to charged quantum particles, there is matter waves, which are also charged particles. That how do they are the wave function of those particles,how do they change it when you apply a uniform magnetic field? So It so happens that so this is this is a problem that you have to study, you know, in your quantum mechanics course.

Where you are to ask your teacher who teaches you quantum mechanics to explain to you to calculate the stationary states by solving time independent Schrodinger equation of charged particles in a uniform magnetic field. So when you do that, so i will just tell you the final answer. So it is like, you know, i will just do fast forward and tell you the final answer. But then, if you really want to know the details, you will have to consult a quantum mechanics textbook.

Which will tell us because remember, this is a statistical mechanics course and I have assumed some knowledge of quantum mechanics. So I cannot possibly derive the stationary states of charged particles in a uniform magnetic field. So i will tell you the final answer, which i am going to use, you please look it up, Just find out how to derive this result. So the idea is that if you do not have a magnetic field.

You know that the energy of the charged particles is going to be this and then the k_x,k_x,k_z that are the good quantum numbers. So the way functions are labeled by k vector. So however, if you have a magnetic field, this is no longer true. So If you are supposed to apply magnetic field in the z direction, so the only k survives as a good quantum number is k_z. See, the other two will no longer be good quantum numbers.

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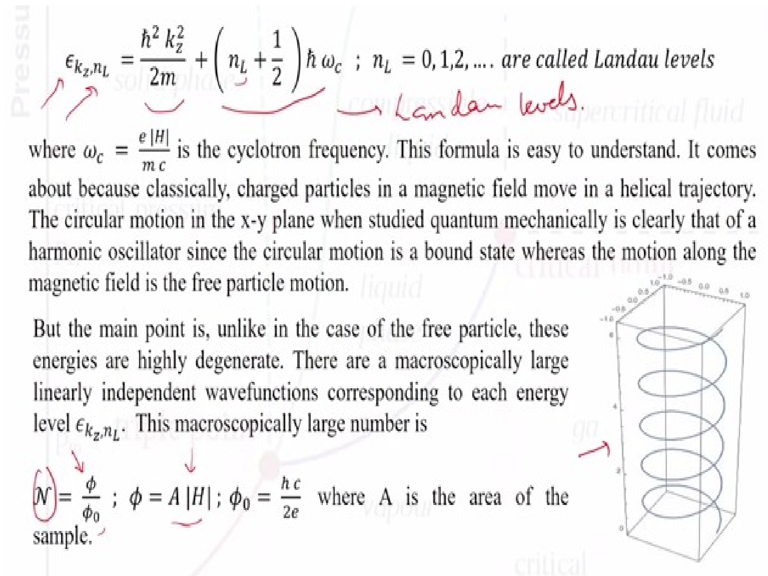

Rather, they get replaced by an integer called n_L. So, so instead of having a $\hbar^2 k_z^2/2m$, it gets replaced by harmonic oscillator energy levels, that is understandable, because you see, the z direction, that is a parallel to the applied magnetic field, The charged particle is free to move. So its energy continues to be free particle energy visualize first part is a $\hbar^2 k_z^2/2m$.

But then in the x y plane, you see that is when they see the magnetic field, they start going around in circles, basically, they will be circulating currents. So the circulating currents implies that there is a bound state. So bound state, you see that there is an analogy you can make with harmonic oscillators or harmonic oscillators are also bound states. I am not saying that that that is a proof, but you can get a sense that this is the reason why the energy level looks like that of a harmonic oscillator.

Because the in the xy direction, they start going around in circles, they cannot run away to infinity, they have to keep going round and round in circles, because of the magnetic field and as a result, their energies become quantized and they happen to have this harmonic oscillator form and the frequency with which they go round and round is basically the classical frequency that you can calculate using classical mechanics and that is called a cyclotron frequency.

That is the frequency of the path of the classical particle, But the point is, these are the energy levels. So k_z it is continuous, it can be –infinity +infinity. Alright, so like I was saying, the, the energy levels of the charged particles are described by these quantum numbers, called k_z, and n_L , and so the k_z , it can be anything it wants from minus infinity to plus infinity, because that corresponds to three particles moving in the z direction.

But n_L is basically the quantum numbers that correspond to the bone states in the xy plane. So I have told you already that the charged particles are forced to go round and round in circles, Because of the magnetic field in the xy plane. So basically, it is a bound state, so it resembles that of a harmonic oscillator. So, so you can rigorously prove also that it is actually the energy levels of a harmonic oscillator, and and nL are given by the integers 0,1,2,3 etc.

And the energy of the the quantized energies are $\hbar\omega_c$, but ω_c is basically, the cyclotron frequency, which is the classical frequency of the electron, of the charged particle. So if you treat the charged particles classically, you know, what is the frequency with which they go round and round. So that is basically the result of a detailed derivation that you will find in quantum mechanics textbooks also.

But then this is the story of the energy levels. But you have to ask yourself, what is the other half of the story, which is the story of the wave function? So what do the wave functions look like? I am not going to write them down. But I'll tell you that there is a humongous amount of degeneracy. In other words, see, normally what happens is that for a given energy, for example, for free particles if you think about it, that for a given energy, there is, of course, degeneracy. But if you decide to label the states, in terms of these k_x,k_y,k_z, and if you ignore spin, then there is no degeneracy, depends on what you use to label the states. So suppose you decide to label the stage not in terms of the energy, If you decide to label it in terms of the energy, then there is of course, a huge degeneracy.

Because for a given energy, You can have different orientations of k which have the same energy. But suppose, I decided instead to label the states in terms of k itself, Then of course, there is no degeneracy assuming the charged particles do not have spin. But then here, you see, that is the point that even though I've decided to label the states in terms of the quantum numbers, k_z and n_L, See here.

If I decide to label the states in terms of k_x,k_y,k_z, which are the quantum numbers, then there is no degeneracy. there is for the free particles. But however here, Even if I decide to label the states in terms of the quantum numbers that are contained in the energy, namely k_z said and n_L, I will find that there is in fact, still a huge amount of degeneracy that is left over. And of course, do not ask me where this comes from.

This is as I told you, a result of a detailed analysis, a proper analysis of the quantum mechanics of charged particle and a uniform magnetic field. So these are called Landau levels, by the way, these are called Landau levels. And this these were first there are by Landau. So these are called Landau levels. But then, so now you are to see what is the degeneracy of this. So even if you decide to label the states by k_z,and n_L So What is the degeneracy?

That degeneracy is given by this ratio, which you will take my word for it? namely, it is proportional to the magnetic field and is proportional to the area of the sample. So This is something like a flux magnetic flux, magnetic field into area, the quantum of the magnetic flux. So it is, so it is as if you know, the degeneracy is basically counting the number of flux quanta that are penetrating your sample. Right? So the fundamental flux quantum is hc/2e**.**

And applied flux, which is magnetic field, which is applied times the area of the sample. So that divided by the flux quantum is a number of flux quanta that are penetrating the sample. So that is basically the degeneracy of these energies. So that huge number of very linearly independent wave functions corresponding to the same energy, $\varepsilon_{kz,nL}$. So have you followed what I was saying. So that is what degeneracy means.

So basically, for one k_z and one n_L, you have one energy, and you do not have one wave functions, you have this many wave functions and different wave functions, they have the same energy. So that is what degeneracy means. And this is basically a picture of how the classical trajectory would look like if you decided to treat the particle classically, it would go around and around in a helix like this. So as you very well know.

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The entropy of the system is given by $j = 1,2,\ldots,\mathcal{N};\ n_L = 0,1,2,\ldots$

$$e^{S(U,N)} = \sum\nolimits_{\{m_{j,k_z,n_L}=0,1\}} \delta_{U-\sum_{j,k_z,n_L} \epsilon_{k_z,n_L} m_{j,k_z,n_L},0}\ \delta_{N-\sum_{j,k_z,n_L} m_{j,k_z,n_L},0}$$

As usual we write,

$$e^{S(U,N)} = \sum\nolimits_{\{m_{j,k_z,n_L}=0,1\}} \delta_{U-\sum_{j,k_z,n_L} \epsilon_{k_z,n_L} m_{j,k_z,n_L},0}\ \delta_{N-\sum_{j,k_z,n_L} m_{j,k_z,n_L},0}$$

This means

$$e^{S(U,N)} = \sum_{\{m_{j,k_z,n_L}=0,1\}} \int_0^{2\pi}\frac{d\theta}{2\pi}\, e^{i\,\theta\,(U-\sum_{j,k_z,n_L}\epsilon_{k_z,n_L} m_{j,k_z,n_L})} \int_0^{2\pi}\frac{d\varphi}{2\pi}\, e^{i\,\varphi\,(N-\sum_{j,k_z,n_L} m_{j,k_z,n_L})}$$

So now I am going to use this, these final answers that I am going to assume this, and then I am going to go ahead and use my machinery of statistical mechanics, and try to calculate the entropy of a gas of fermions, subject to a uniform magnetic field. So now, how do I do that? So this is how I do it, And I have to conserve energy. So this is my energy. So notice that this is the total energy of the system. And that can also be calculated in this fashion. This is the energy of each particle.

And this is the number of particles having this energy. And there is this j label, which is basically a label, which tells you the degeneracy. So it is 1,2, up to that ration, so that is what I was saying here. So basically, this is the number of electrons or whatever electrons having energy kz, nL and being in that jth level of degeneracy, Okay. So this is how you label the states. Actually, if you want to completely specify a state of a set of charged particles in a uniform magnetic field, this is complete set of quantum numbers.

It is not just kz and nL, but also if you include this j, which goes from 123, up to this ratio N the script N and so that constitutes the complete set of quantum numbers. So because you have specified the quantum numbers completely, So the number of particles in a given quantum state can be either 0 or 1, Because they are fermions. So they are either 0 or 1. So This is either 0, or 1 okay. So that is why I decided to sum over only 0 or 1.

And I can go ahead and calculate the entropy. As usual, I am going to use my trick that I can write this as, you know, a Kronecker delta in terms of integral over some angles. And I do the summation,

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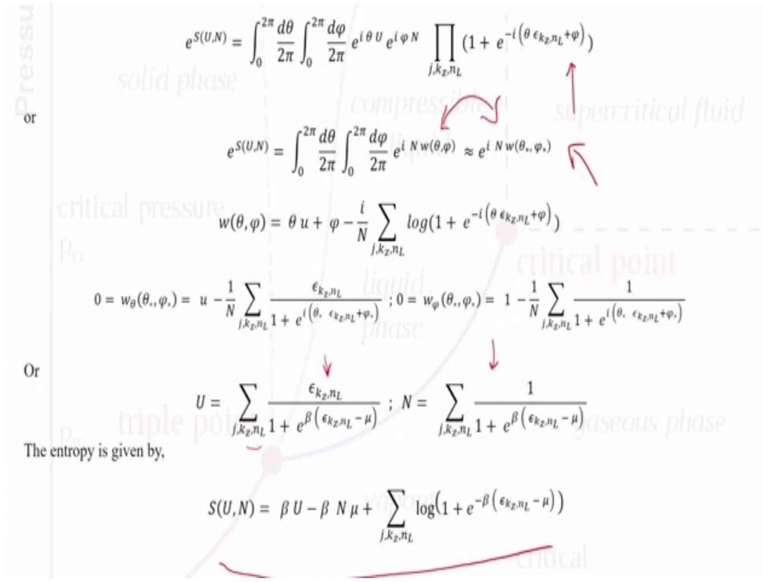

and I do the usual things here. So you should follow along with a piece of paper and a pen and try to work out all the steps. And you know why, or why I went from here to there. So that is basically the side of the point approximation.

Where I feel entitled to replace θ by θ* because that is the most probable value. So in the most probable value , as before, so there is a proper temperature and chemical potential by temperature and so on. So finally, I do all that and I get this answer for the total energy of the system. And the total number of particles at a temperature T and chemical potential μ and the entropy is derivable in this fashion.

So notice that there is a notice the sum, the sum is over kz and nL, which are the quantum numbers of the energy of this particles, but it is also a sum over j, which are these additional quantum numbers, which tell you the degeneracy of the various energy levels.

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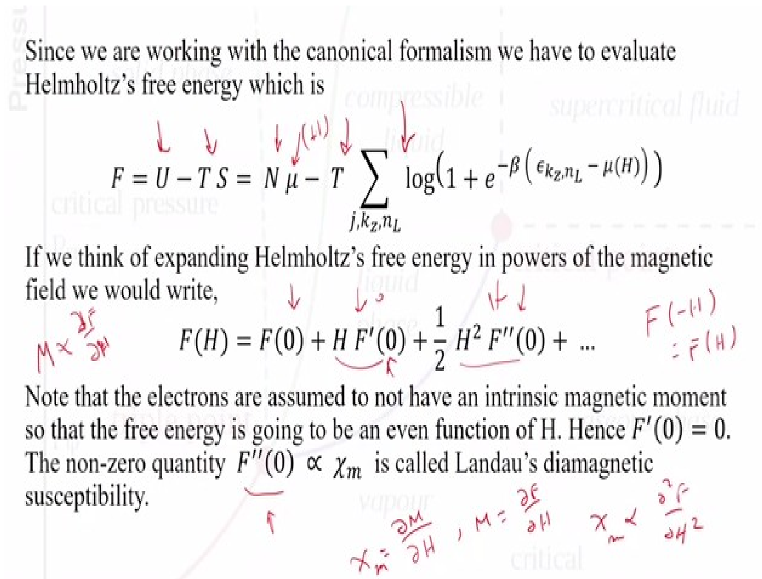

Okay, so now I have to go ahead and see if I can calculate the helmholtz free energy. So I am purposely trying to, you know, give emphasis in this course, on the micro canonical approach.

where in all the examples, I first tried to calculate entropy, And so that i am doing that on purpose, because most of the other textbooks and other lectures, they kind of quickly migrate from micro canonical ensemble, which is focused on entropy. And they directly jump to canonical ensemble, which is basically based on helmholtz free energy. And of course, that is justifiable, because, finally, those are the quantities that are of interest.

You know, from a practical standpoint, but then I felt that in order to have a different flavor to the subject, different from what is readily available. I felt that it is nice to, you know, impress upon the audience that it is possible to do all this from, you know, from really fundamental first principles like micro canonical ensemble, but it is, it is a matter of taste, you can go ahead and do it using canonical ensemble, as I have told you, those two are interchangeable in the thermodynamic limit.

Alright, so now i am really going to go ahead and calculate the Helmholtz free energy because I am going to use it to find what is called a diamagnetic susceptibility, which is defined in terms of Helmholtz free energy. So as you very well know Helmholtz free energy is defined as internal energy - absolute temperature - times entropy. So that is how it is going to look like. And If I, if I use this, so there is my entropy. And so this is what it is going to look like.

So now, the idea is that if I expand the free energy, so notice that everything is going to N is fixed, but μ can depend on the magnetic field. So the number of particles is fixed, but them you can change depending upon the magnetic field. And temperature is fixed number of particles in the canonical and symbol what is all fixed number of particles is fixed, temperature is fixed and volume is fixed, so in that sense these things can change with the magnetic field and temperature, the chemical potential. So I can go ahead and expand the Helmholtz free energy in terms of the applied magnetic field, so I will get a Taylor series in this fashion. So if I decide to do this, and Taylor series, and it so happens that, in this example, the Helmholtz free energies and even function of H. So it is going to look like this. So this is going to be zero.

So that is also, that is understandable, in the sense that, you see the susceptibilities or the magnetization, is basically the rate of change of Helmholtz free energy with the magnetic field if you like, so it is like magnetic moment, induced magnetic moment is proportional to the dF/dH. But we also know that if you turn off the magnetic field, in a diamagnetic material, the magnetic moment should also go to 0, unlike in a paramagnet, where it does not go to 0.

So if this is nonzero, then the rate of change of the Helmholtz free energy with magnetic field does not go to 0 as the magnetic field tends to 0. So that is the reason why you do not, you should not be having this in a diamagnetic material. So that is going to be 0 in a diamagnetic material. So the series really starts from here. And this quantity is therefore proportional to what is called the diamagnetic susceptibility.

I am going to define this object more carefully later on, but basically the coefficient of the leading term in the Taylor series of the free energy with respect to the applied magnetic field, so you expand the Helmholtz free energy in terms of the applied magnetic field. And the coefficient that appears in the leading term is basically proportional to the magnetic susceptibility. that is understandable, because like I told you, so diamagnetic susceptibility is basically defined as the induced magnetic moment, change in the induced magnetic moment for unique change in the applied magnetic field, and the magnetic moment itself is defined as the change in the helmholtz free energy with the applied magnetic field. So as a result, ξ_m is proportional to d^2F / dH^2.

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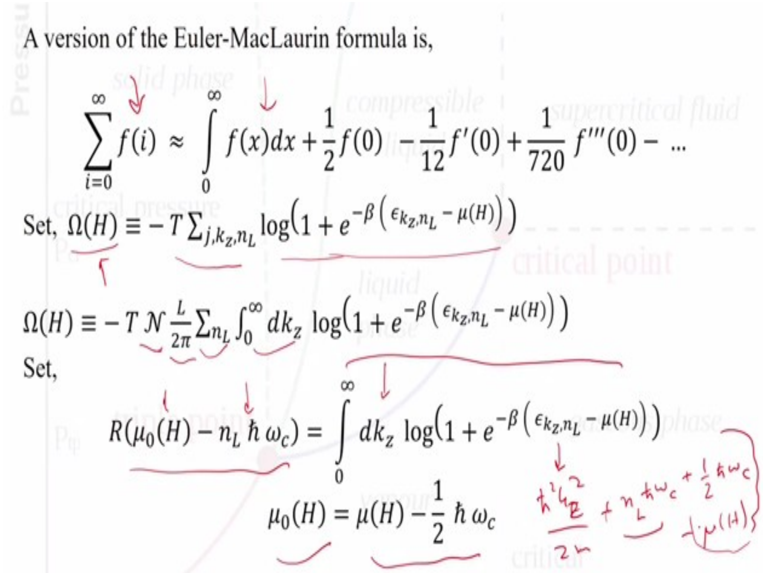

So now the question is, how do I, you know, do this summation, so it looks nasty, because firstly, doing the summation, over kz is not that difficult.

Because we know that kz is continuous, in the sense that I mean, of course, if it is particle in a box, then it is not continuous. It is discrete, but you know that we are in the thermodynamic limit in the z direction. So we know how to convert that into an integral. But however, these Landau levels are, by definition discrete, so there is nothing continuous about them. So we have to learn how to do discrete sums. But we are not good at that.

Because we know, well, if it was a continuous integral. We know how to do that using saddle point and this and that. But if it is a discrete sum we are stuck. So it would be really nice if we could convert a discrete sum into an integral, which we know how to do, right, using saddle point, and so on. So Fortunately, there is a mathematics who has an identity, which is called Euler MacLaurin formula.

Which allows us to write a discrete summation, such as this in terms of an integration in terms of a continuous integration. So I won't again, derive this, I won't tell you the derivation, you will have to take my word for it, you will have to look up some maths books, or you know, just take my word for it. So it is just an identity from mathematics. And So I am going to try and use this to do the summation. So i am going to define this as Ω.

So This is also what is called the grand partition function. So I mean, I, maybe I will touch upon this a little later. So this, this also has a physical meaning. But right now, it is just a symbol. It just means this sum. So the question is, how do you do this? So you see the, notice that the quantity that sitting here is independent of j. So when I sum over j, this degeneracy, so it is just some over all the degenerate states, and whatever I am summing over doesn't depend on that index.

So finally, what I get is the number of degenerate energy levels, which is the, you know, the flux, that is threading the system, divided by the quantum of the flux. So then, whatever remains is basically the summation over kz, which I have converted to an integration, which is, of course, we know how to handle but what we do not know how to handle is this one, which is the summation, discrete summation over Landau levels.

So in order to do this, since I know how to do the integration of kz, and I am going to take the liberty of giving this a name. So I am going to call this quantity, this. The reason why I call it this is because you see, this energy is nothing but $\hbar^2 k_z^2/2m$. So it is going to look like $n_L \hbar \omega_c$, then it is going to look like plus $\frac{1}{2} \hbar \omega_c - \mu H$. So i am going to put these two together and call it mu not okay.

So the rest of it is this one. So mu not this this much. Okay, so that is the reason why i have given it that name okay.

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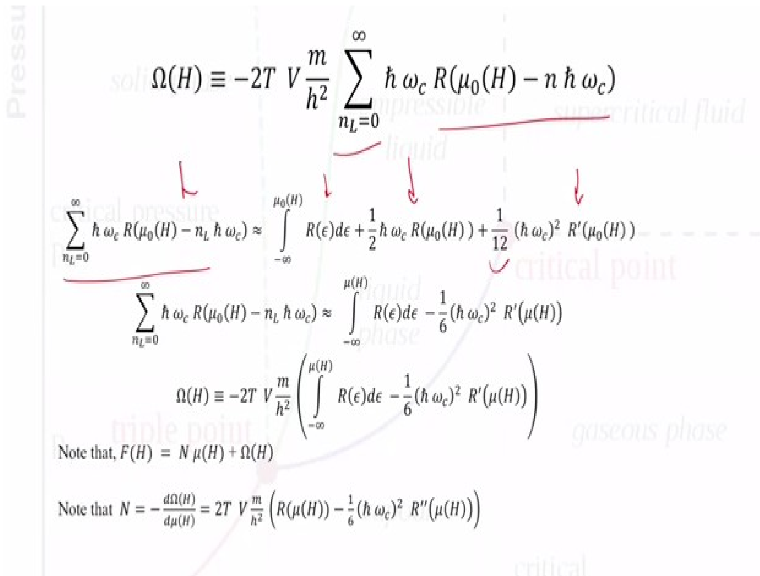

So whatever that is, I am now forced to evaluate this discrete sum in this fashion. So now I am going to use my Euler MacLaurin formula. And which allows me to rewrite, so you please verify all these steps by yourself. So this is using the earlier this result.

So i am going to use this result, which is Euler Maclaurin formula, I am going to stop right here, I am going to stop there. So I am going to use this. And so the first term is this one. So this term that I am looking at, so the summation becomes an integral so this summation has become this integral. But then there is f(0), 1 over 1/2 of f(0). And that is what this is, right? And then this is the first derivative, which is 1/12. So 1/12. And i am just, and of course, the second derivative.

I do not care about. So now I am going to rewrite in terms of so if I decide to rewrite in terms of μ and mu not this, this term cancels out.

So you please do this yourself, All I have done is, so instead of writing in terms of mu not, I have written in terms of mu. So when I do that, this term cancels out. So this term cancels out. So I will end up with this. And immediately this, And notice that there is a change in sign, mainly because of certain conspiracies, and then this 1/12 will become 1/6. So you please work this out yourself. I can't explain everything.

So this can be a useful exercise to you to explain how to go from here to there. Alright, so now I am going to substitute that, I am going to substitute this here, and I end up with this result. So notice that the helmholtz free energy is, so i am going back here. So this is this is your Ω. So this is your Ω(H). So basically, it is helmholtz free energy n times muH+omega(h). So which is what I have written here. So it is

$$F(H) = N\,\mu(H) + \Omega(H).$$

But then you see if I decide to keep N fixed, which is of course, what I am mandated to do, But then if I decide to keep N fixed and differentiate Ω with respect to μ. So why is this the case so let me explain that to you. I love to explain why this is. So this is my Ω. So now suppose I calculate the d Ω /dμ Suppose so there is a μ setting here. So what I get basically is, So I see if I calculate d Ω /dμ. So there is a μ sitting there.

So If I differentiate this omega, okey let me do that. Here, for example. Okay, So

$$\Omega(H) = -T\ln(1+e^{-\beta(\varepsilon-\mu(H))})$$

okay. So that is what that was, so now, suppose I want to calculate d omega/d mu. So what is that going to look like? So it is gonna look like $1/e^{-\beta(\varepsilon-\mu(H))}$, times this one, ε-μ, then again, if I differentiate exponential chain rule, so if I differentiate the exponential with μ, i will get a β out there.

$$d\,\Omega/d\mu = \Sigma\ -T\beta\, e^{-\beta(\varepsilon-\mu)}/(1+e^{-\beta(\varepsilon-\mu)}) = -n$$

And this is nothing but you see, -n and so this is the number of particles there is the Fermi-Dirac distribution, -n and So that is the reason why this is n is negative derivative of chemical potential with a spectrum mu okay. So, now I am going to use that.

So n is the derivative of this with respect to medium. So if I differentiate with respect to μ here, So I get from so the derivative of this with respect to μ become this, and derivative of R' with respect to μ becomes R''. Okay, and then there is a minus sign where the minus sign goes away becomes a plus sign there. Alright, so it looks very nasty. But let us proceed, you will see that the final answer will look quite simple.

So now that I know what it is, I am going to use this later. So now i am going to go ahead and calculate Ω, the derivative of Ω with respect to H. So if I do that, I get this result. So basically, I just have to take this, I have to repeatedly manipulate this formula. So first, I differentiate this with respect to mu and I get formula for total number of particles. Now, I decided to differentiate with respect to H, which is magnetic field, then I get this formula.

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$$\Omega'(H) \equiv -\mu'(H)\, 2T\, V\frac{m}{h^2}\left(R(\mu(H)) - \frac{1}{6}(\hbar\,\omega_c)^2\, R''(\mu(H))\right) - 2T\, V\frac{m}{h^2}\left(-\frac{1}{3}(\hbar\,\omega_c)\hbar\frac{e}{m\,c}\, R'(\mu(H))\right)$$

$$N = 2T\, V\frac{m}{h^2}\left(R(\mu(H)) - \frac{1}{6}(\hbar\,\omega_c)^2\, R''(\mu(H))\right)$$

hence,

$$\Omega'(H) \equiv -\mu'(H)\, N - 2T\, V\frac{m}{h^2}\left(-\frac{1}{3}(\hbar\,\omega_c)\hbar\frac{e}{m\,c}\, R'(\mu(H))\right)$$

or

$$F'(H) = N\,\mu'(H) + \Omega'(H) = 2T\, V\frac{m}{h^2}\left(\frac{1}{3}(\hbar\,\omega_c)\hbar\frac{e}{m\,c}\, R'(\mu(H))\right)$$

$$F''(0) = 2T\, V\frac{m}{h^2}\left(\frac{1}{3}\left(\hbar\frac{e}{m\,c}\right)\hbar\frac{e}{m\,c}\, R'(\mu(0))\right) = \frac{2T\, V}{(2\pi)^2}\left(\frac{e^2}{3\,m\,c^2}\, R'(\mu(0))\right)$$

And then notice that I already have this. So I can put these two together and write down a derivative of omega the grand partition with respect to the applied magnetic field is going to look like this. Okay, so now the change of the rate of change of the helmholtz free energy with the magnetic field is given by this formula, because N is fixed, so I have just have to differentiate the chemical potential and the grind partition function. So when I put them together, you see that this which is nasty, which I would not have known.

It would have been hard for me to calculate mu dash, it drops out of my calculations, fortunately. So I end up with this nice, compact looking formula for the rate of change of the free energy with a magnetic field, which is of course, you should recognize that is proportional to the induced magnetization. Now, so that is understandable, because you see it, it is basically proportional to the cyclotron frequency which is proportional in turn to the applied magnetic field.

So, they induced magnetization is proportional to the applied magnetic field, which is as it should be. So, now, I differentiate again, with respect to the magnetic field, and I end up with this formula for the second derivative. So, recall that that was the, that coefficient in the Taylor series, which, which was supposed to be proportional to the diamagnetic susceptibility. So I end up with this result for that coefficient.

So, I'll to find out what this is in order to complete my calculation. So for that, I have to go back to that definition of this R so R was this. So when H is 0 μ_0 is the same as μ.

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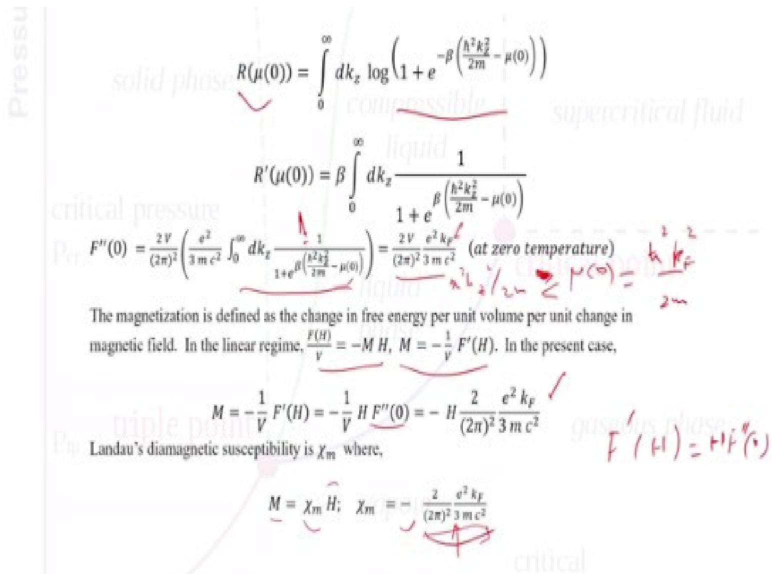

So this is proportionate to the magnetic field. So μ_0 is same as $\mu(0)$ because this is proportional to the applied magnetic field. The cyclotron frequencies proportional apply magnetic field.

So now, as a result, I can write down this, okay, so this is from this by definition, then if I differentiate with respect to μ_0 , I end up with this formula. And then I substitute that back into this expression. And I get this result. So now finally, I take the zero temperature limit. So at zero temperature, this becomes basically a step function. So at 0 temperature, beta tends to infinity. So when beta tends to infinity, this whole thing is 0.

If you know $\hbar^2 k_z^2/2m > \mu_0$ it is 0. So basically, I have to integrate only in situations where it is less than mu not, so in other words, have to integrate. So this is nothing but . So I am going to write this in terms of my for me momentum, so I have to integrate from kz from 0 to kF. So that is the Fermi vector. And so this becomes one in that region. So this is just for me statistics. And I end up with this result, okay.

So that is what it is. So notice that case, it is from 0 to infinity - sorry, it is from 0 to Kf. So that is how it is been, I mean, we have defined it that way, remember, so it is 0 to Kf. So it is only Kf which comes in the formula. So, finally, we are ready to write down our formulas for the magnetic susceptibility. So, notice that in the linear regime, The helmholtz free energy per unit volume is basically induced magnetic moment times the applied magnetic field.

So, so this would be in the linear regime. So in general, in the nonlinear regime, The induced magnetization is defined as the rate of change of the helmholtz free energy per unit volume with respect to the magnetic field. So this is going to be my induced magnetic moment. So I further know that this is also proportional to so F'(H) is basically HF''(0). So that is because the Taylor series of the first term is 0.

As I told you, that induce magnetic moment is proportional applied magnetic field. So this, we have taken a lot of trouble to calculate just now. And this is done. So finally, I am able to write the induced magnetic moment as being proportional to the applied magnetic field. And the coefficient is called the magnetic Landau's diamagnetic susceptibility, It comes with a - sign, signifying that it is, it is it opposes the applied magnetic field.

And the final answer is, this classic result is one third. So without the one third is basically what is called Pauli's paramagnetic susceptibility. So which I am going to discuss next. But right now, this is the result which Landau derived long ago in the 1930s. And As you can see, this calculation is not very easy. And as a result, it is not frequently covered in many of the stat mech courses that we teach, but I feel that it is important to discuss this.

Because we will very quickly discuss Pauli paramagnetism is mathematically easier. And then I stopped by that they do not teach Landau diamagnetism. So that is because it involves reckoning with Landau levels and their associated degeneracy and we know this Euler Maclaurin formula, which is a very unusual and unfamiliar mathematical tool, which appears to be necessary in this calculation. So, so given all these facts that it is not surprising that many courses kind of gloss over this result.

So I find that it is important to include this to make this course a bit different from what is usually out there. Okay, so I hope you enjoyed this particular portion. So If you really, I can understand if you did not understand everything, So you should work out all the steps on your own. just pause and rewind and work out all the steps on your own. And we will meet for the next class which is something slightly simpler which is Pauli's paramagnetism. Thank you.