

**Introduction to Statistical Mechanics**  
**Prof. Girish S Seltur**  
**Department of Physics**  
**Indian Institute of Technology – Guwahati**

**Module No # 01**  
**Lecture No # 01**  
**Prerequisites and Introduction**

Hello! welcome to this course on statistical mechanics. My name is Girish Setlur, I am Professor of Physics at Indian Institute of Technology Guwahati. It is my great pleasure to share with you the content of this course which is going to teach you the basics of statistical mechanics at the post graduate level based on the Indian syllabus. So let us get on with it shall we?

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### Prerequisites for the course

- A preparation in advanced calculus which includes multivariable integrations (Stokes, Gauss theorems), complex variables (residues, method of steepest descent), combinatorics (permutations combinations) and so on.
- Basic programming in some language.
- Knowledge of Classical Mechanics at the level of Goldstein, and knowledge of Quantum Mechanics at the level of Griffiths.
- Above all, a desire to learn about this important subject and a willingness to remedy gaps in one's prerequisites as and when it is felt.

So, the prerequisites for this course are you should have had a course in advanced calculus that includes multivariable integrations for instance. So you should be able to perform calculation with stokes theorem and Gauss theorem you know with specific examples involving cylinders and spheres and that sort of thing and then it is really helpful to know a little bit of complex variables, such as you know applications of the residue theorem and it is really also helpful to know the method of steepest descent or sometimes it is called the saddle point method of integrations.

And the other topic that is helpful to know beforehand is combinatorics, so in other words you know how to count permutations combinations but I am going to teach a lot of that myself in this course but it is nice to know that some of that beforehand. The other skill that I expect some of you to have that is really desirable to have this at least is basic knowledge of programming in some languages. I am not going to insist on which language but that is up to you.

So the other important prerequisite is that you should have some understanding of classical mechanics ,say at the level of Goldstein ,so you do not have to know everything but you should know , suppose I say Hamiltonian if you should know what I am talking about and so on hence so forth so phase space and so on. So the other thing that is really helpful to know is quantum mechanics at the level of Griffiths for example.

So you should be able to solve simple problems find the Eigenfunctions of particle in the box harmonic oscillator and so on and so forth and above all what is really important is for you to have a desire at to learn this important subject and a willingness to remedy gaps in your prerequisites if you feel that there are gaps that have to be filled. Okay so let us get on with the course itself but before I get into the technical aspects I feel that it is really nice to know some historical context.

Because many times what happens is that when physics is taught, far too often both teacher and student directly jump headlong into the technical aspects without pausing to think about why these notions are important or trying to place them in a historical context and ask themselves what is it that led to these concepts to be studied in the first place. So history is important also because if later on some of you feel that you are going to be able to contribute to the subject you should have a understanding of why a certain idea took root at a certain time in history and why it was abandoned later on and something better took its place.

So history is important for that reason. So let us find out what was the historical precedence which led to the development of modern statistical mechanics. So as you very well know that the subject of thermodynamics was the conceptual precursor or predecessor of statistical mechanics.

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## History

Historically, Thermodynamics was the conceptual precursor to Statistical Mechanics (source: Stephen Wolfram "A New Kind of Science")

- Antiquity - 1600 AD : Notions of heat and temperature widely accepted. It was believed at the time that heat was associated with the motion of microscopic constituents of matter.
- 1700 -1840 : Later the wrong notion that heat was instead a fluid like substance started becoming more popular.
- 1850 : Experiments of James Joule and others showed that heat is a form of energy.
- Sadi Carnot had explained the relation between heat and energy. This was important in the development of steam engines.
- In 1850 Rudolf Clausius and William Thomson (Lord Kelvin) formulated the First Law which is the idea that total energy is conserved. The Second Law of Thermodynamics which is the idea that heat cannot be completely converted to work was also formulated.
- In 1738 Daniel Bernoulli has pointed out that gases consist of molecules in motion. Clausius revived this idea in 1857.
- In 1860, James Clerk Maxwell derived the expected distribution of molecular speeds in a gas by taking into account molecular collisions.
- In 1872 Ludwig Boltzmann constructed an equation that he thought could describe the detailed time evolution of a gas regardless of whether it was in equilibrium or not.
- In the 1860s, Clausius had introduced entropy as a ratio of heat to temperature, and had stated the Second Law in terms of the increase of this quantity.
- Boltzmann then showed that his equation implied the so-called H Theorem, which states that a quantity equal to entropy in equilibrium must always increase with time. Since molecular collisions were assumed reversible, his derivation could be run in reverse, and would then imply the opposite of the Second Law. Boltzmann made the implicit assumption that the motion was uncorrelated before collision but not after which imposes irreversibility.
- In 1900 Gibbs introduced the notion of an ensemble - a collection of many possible states of a system, each assigned a certain probability. He argued that if the time evolution of a single state were to visit all other states in the ensemble - the so-called ergodic hypothesis - then averaged over a sufficiently long time a single state would behave in a way that was typical of the ensemble. Gibbs also gave qualitative arguments that entropy would increase if it were measured in a "coarse-grained" way in which nearby states were not distinguished.

So from antiquity up to 1600 , the notions of heat and temperature where quite familiar to many thinkers. So they kind of had a rough feel about what they were talking about ,so it was for instance believed at that time that heat was associated with the motion of microscopic constituents of matter. That was remarkable since you know at that time there was no discoveries where made about atoms, people did not know that matter is made up of subatomic particles called atoms and so on.

And yet people correctly surmised that heat was associated with the motion of subatomic constituents of matter. However unfortunately, later on the idea ,the wrong idea that heat was some kind of fluid in motion for some reason become popular. So you see an example of history kind of going in the wrong direction that subject takes a wrong turn and then finally it comes back to the right turn when experiments decide which is the correct point of view.

So for instance the experiments performed by James Joule in 1850 showed that heat is actually a form of energy. So later on Sadi Carnot had explained the relation between heat and energy in his famous treatise and this was important in the development of steam engines during the industrial revolution. In 1850 Rudolf Clausius and William Thompson also known as Lord Kelvin, they formulated the first law of thermodynamics which you all as basically stating that you know heat is converted to you know either it is converted to either internal energy or work is done.

So it is a statement of sort of conservation of energy, just tells you one form of energy gets converted to other forms of energy that you cannot really destroy or create energy out of nothing. So however that is fairly obvious but the less obvious second law of thermo dynamics which is the idea that you cannot convert heat completely to work in the absence of any other process was formulated so that is less obvious because first law of thermodynamics does not forbid heat from being completely converted to work.

So but second law of thermodynamics forbids that, so in 1738 Daniel Bernoulli pointed out that gases actually consist of molecules in motion. So you can see that these ideas kept on being reinvented independently by various thinkers ,so from antiquity people guessed correctly ,then in between people forgot about it that you know atoms are the basic constituents of matter and heat manifests itself because of motion of molecules.

So basically heat is just a manifestation of the kinetic motion of molecules that was an idea that was repeatedly rediscovered throughout history by various thinkers. So Clausius again revived this idea in 1857 and in 1860 James Clerk Maxwell of the electrostatics fame, so we all know about Maxwell equations of electromagnetism but also Maxwell made an important contribution to statistical mechanics because he used the idea of Clausius and his predecessors to derive the expected distribution of molecular speeds in a gas by taking into account molecular collisions.

So he derived what is now known as the Maxwell Boltzmann distribution of a classical gas ,okay. So in 1872 another towering figure of the subject Ludwig Boltzmann constructed a equation that he though could describe the detailed timer evolution of a gas regardless of whether it is in equilibrium or not. So in other words he thought he had an equation when solved explicitly explains how you know a gas that is starts off in a certain state evolves and eventually the idea was that it going to thermalize in other words the final state of a gas is that of a thermal equilibrium described by Maxwell Boltzmann distribution. Boltzmann thought that his equation will finally not only will he be able to derive Maxwell's contribution to the subject namely the molecular the distribution of molecular speeds but Boltzmann also thought that his equation will be able to explain how that finally equilibrium state is reached starting from any arbitrary initial state.

So in the 1860's Clausius had introduced a notion of entropy as the ratio of heat going into a system and it is absolute temperature and he had stated that the second law of thermodynamics in terms of an increase of this quantity with time. So Clausius had stated his second law of thermodynamics in terms of heat and temperature, so Boltzmann showed that his equation also could account for something analogous namely that he showed that there is a quantity called  $H$  in his equations which also increases with time, which he also identified with entropy.

But then later on his ideas Boltzmann's ideas were criticized by his compatriots and wise contemporaries rather and the main source of criticism was that Boltzmann equation could be run in reverse and then I would show that entropy is actually decreasing in time because Boltzmann equation was based on the laws of classical mechanics which are actually you know time reversal invariant. So whatever process in classical mechanics that can go forward in time can equally well go backward in time.

But later much later it was realized that Boltzmann has made a hidden assumption namely that he had assumed implicitly that when collisions occur the speeds of the molecules are uncorrelated before collision and but then they become correlated after collision because they obey you know the conservation of energy momentum that sort of thing. So because of this Boltzmann unwittingly had introduced irreversibility in his equations and as a result his equations predicted that entropy increases with time.

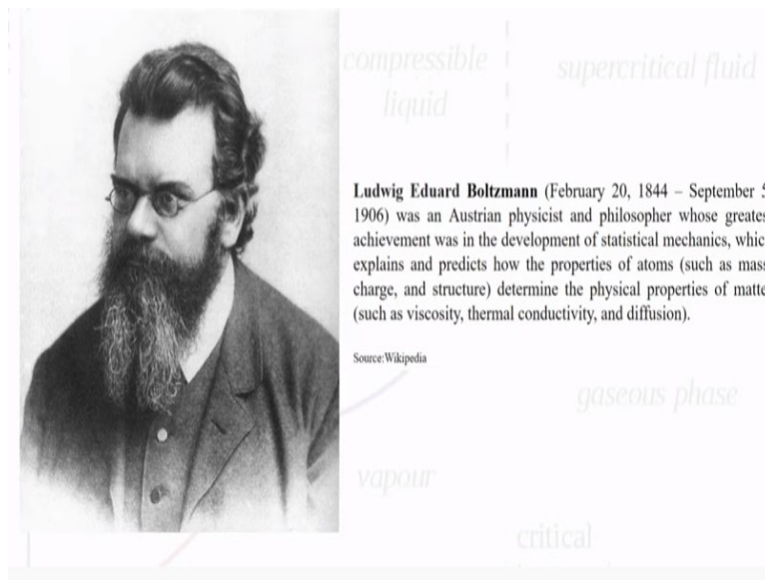
So in effect he was basically assuming whatever he was trying to prove and finally in 1900s another scientist name Gibbs introduced the notion of an ensemble. So the idea of the ensemble was that it is a collection of states of a system such that you are able to assign the probability of each of those states occurring. So that you identify set of closely related states and you call that as one kind of an entity and then you keep dividing all the possible states or reclassify all the states of the system into these ensembles.

So this is also called coarse graining so you assign probabilities to each of these cluster or states and what Gibbs showed that it is possible through this coarse graining to also show why entropy should increase with time. So he showed that the second law of thermodynamics which states

that entropy is non-decreasing function of time for isolated systems is simply a reflection of our unwillingness or inability to precisely pin down all the microstates.

So we tend to lump all different microstates into the same category and that fuzziness is what causes the second law of thermodynamics. So I realize that my explanations will probably not satisfy any of you because it is rather vague ,I have used words and sentences to describe it but you will have to bare with me because in the next couple of slides and lectures as we very precisely in quantitatively pinning down what I was talking about ,okay.

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So before I get into the actual nitty-gritty of this subject it is worthwhile to flash this nice photograph in front of you and this is the photo of none other than Mr.Boltzmann himself and you can see that he lived between 1844 and 1906 and he was philosopher and a physicist and he was instrumental in the developmental of statistical mechanics and he also showed how these ideas could be used to determine various properties of substances such as Viscosity, thermal conductivity, diffusion and so on ,okay.

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## Zeroth Law, First Law and Second Law

**Zeroth Law:** If A and B are in equilibrium with a third C, they are also in equilibrium with one another.

A system is said to be in equilibrium if its macro-properties do not change appreciably over time. A macroscopic system in equilibrium is characterized by a number of thermodynamic coordinates or state functions eg. pressure and volume for a fluid, tension and length for a wire, electric field and polarization for a dielectric.

So now let us get to the actual subject so instead of starting with statistical mechanics I start with its intellectual predecessors just to remind you of some of the prerequisite as it were and namely the zeroth law, the first law and second law of thermodynamics. So zeroth law is very simple it is says that if two systems A and B are in equilibrium with a third system C then A and B are also equilibrium with one another.

So this is a fairly obvious statement and intuitive one but it is worth stating because if we do not have this stated explicitly we would not be able to use it later. So but then keep in mind that I used a term called equilibrium without defining it and so this is unfortunately a common occurrence in physics, many times physicist use terms without properly defining what they are that is unlike in other subjects like mathematics where you know its practitioners are very careful in defining what they are talking about.

So there are many reasons for this in physics and partly it is historical and kind of training that kind of does not put proper emphasis on precision in use of proper terminology and the other reason I feel is also because unlike mathematics which is purely intellectual activity, physics is about understanding nature and concept of nature are necessarily vague because when they first present themselves they don't present themselves as very concrete mathematical notions.

So it is a big struggle for a physicist to start with a notion which comes from nature and try to make it very precise. So as it happens that many of these notions continue to be vague for a great

period of time. So however I will put in enough effort to make sure that I am able to define this notion as precisely as I can whenever it is possible okay. So I spoke of systems in equilibrium ,so let me define what that is?

So a system is said to be in equilibrium if its macro properties do not change appreciably with time. So in other words if properties that you can measure of that system do not change with time appreciably then obviously you say that system is in equilibrium, that is fairly obvious it is a intuitive. So but then you know if you have a macroscopic systems what are the sort of properties I am talking about so there are many properties that a macroscopic system can possess for example it could be talking about pressure volume if it is a gas I could be talking of pressure volume and or if it is a wire I am talking about thermal expansion of a wire it could be the tension applied to the wire , it could be length of the wire and so on.

Or if it is a dielectric I could be talking about the applied electric field , I could be talking about the induced polarization and so on. So if all these quantities do not change with time then I say the system is in equilibrium.

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**First Law:** The work required to change the state of an isolated system depends only on the initial and final states and not on the intermediate states through which the system passes.

The above process is called “adiabatic” which means - no heat flows in or out of the system and no mass enters or exits the system.

**Second Law:**

*Clausius statement:* No process is possible whose sole result is the transfer of heat from a colder to a hotter body.

*Kelvin statement:* No process is possible which involves converting heat completely into work.

So let me define what I am talking about here with respect to the first law, so what is the first law so there is several ways of stating the first law of thermodynamics. So one way of stating the first law of thermodynamics is to say that the work required to change the state of an isolated system



depends only on the initial and final states and not on the intermediate states through which the system passes.

So let me repeat that, that is lot of words to swallow so the first law of thermodynamics states that the work required to change the state of the isolated system depends only on the initial and final states and not on the intermediate states through which the system passes. So what does that mean? So basically this should ring a bell because in classical mechanics we all encounter what is known as a conservative force.

So if you are thinking about say work done due to a conservative force then you know that the work done is basically just a change in potential energy between the initial and final state and it does not depend upon the intermediate positions or speeds or whatever it is that the system or the particle passes through. So it simply depends upon the initial and final state and that is the change in potential energy.

So it is similar here, so the work required to change the state of the system if it is only depends on the initial final states, just like should remind you of a conservative system in classical mechanics. So that is precisely the first law so it is says that if it is an isolated system that is what happens so the work required does not depend on the intermediate states, so that is the first law ,so it seems like a very unusual way of stating the first law but we will come back to how this you know this constant with some other formulation of first law that you may be more familiar with.

So there is another terminology that I have to introduce and explain and that is called adiabatic, so the above process that I am talking about where there is an isolated system ,so by isolated means no heat flows in or out of the system and no mass enters or exits the system. So it kind of there is a hard boundary and the only thing that you can do is work but you cannot really push and or extract energy from it in any other way.

So in that case, so if the only thing you can do is do work on it so then that is called adiabatic. So when the process is adiabatic then the work required to change the state of the system does not depend on the intermediate states ,okay. So that is the first law of thermodynamics, whatabout

the second law of thermodynamics so this second law of thermodynamics there are two equivalent statements which describes the second law of thermodynamics.

So I will prove the equivalence subsequently, so Clausius stated the second law of thermodynamics in the following way. So he said that no process is possible whose sole result is the transfer of heat from a colder to a hotter body. So you can see that this is our intuitive formulation of the second law of thermodynamics. So we have experienced this a lot in our daily life.

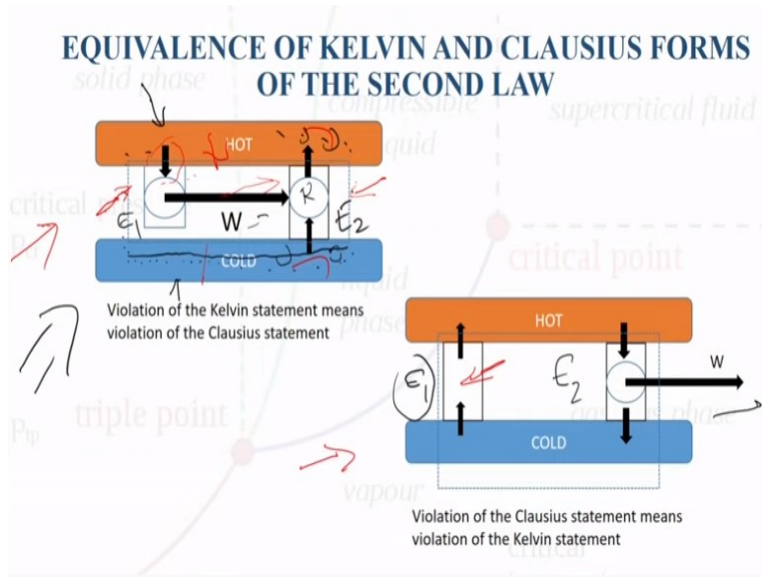
So if you have a cold object and hot object if you put them say in contact with each other, it's heat always flows from the hotter object through the colder object and then they become equally warm or whatever at the end. But it never happens that at the colder object spontaneously become even colder and transfers all its heat, whatever little heat it had to the hotter object and the hotter object becomes even hotter.

So that of course does not happens spontaneously, so if it did then it would mean that you can you know run your refrigerator without any electricity because after all that is what a refrigerator is, it takes heat from a colder object inside the refrigerator and throws it out of the refrigerator thereby cooling your vegetables or whatever it is in the fridge even to a lower temperature.

So if you could do that spontaneously without supplying any electricity to your refrigerator that would be a great thing but unfortunately that violates Clausius statement of the second law of thermodynamics ,so that is in violation of second law of thermodynamics so that is reason why that is not possible okay. So the other statement, the other version of the second law of thermodynamics is due to Kelvin and he said that no process is possible which involves converting heat completely into work.

So it is not possible to convert heat completely into work so notice that this idea of converting heat to work even if it were possible to do fully there is if you could completely convert all the heat to work it would not violate the first law of thermodynamics. So you can still conserve energy and still convert all the heat to work but however converting all the heat to work does violate the second law of thermodynamics, okay these two versions of second law of thermodynamics is seen to be very different from each other.

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So let me try and convince you that they are equivalent, so the way to convince you that they are equivalent is to examine these two set ups. So first let us focus on the left side up here so I am talking about a situation where you have a hot object and a cold object and so I put this hot and cold objects in touch with what are called engines. So I have an engine here called  $E_1$  where this engine takes heat from a hot object and completely converts it to work.

So notice the black arrows there is solid black arrow here so this solid black arrows says that heat is going in from the hot object and it is exiting as work from this engine. So this engine is the one of those illegal engines which violates Kelvin's statement of the second law of thermodynamics. So notice that what does Kelvin say Kelvin states that no process is possible which involves converting completely heat into work.

So you cannot convert heat completely into work however this engine does exactly that so I have assumed that let there be an engine that violates Kelvin statement of the second law of thermodynamics. So in which case there is an engine let us assume which converts heat completely into work. So what I want to convince you is that violation of the second law of thermodynamics according to kelvin means violation of the Clausius statement of second law of thermodynamics.

So if you violate the Kelvin statement you also end up violating Clausius statement so in the next example I will show you that if you violate Clausius statement conversely also is the same as violating Kelvin statement. So now let us focus on proving the first idea that is violating Kelvin statement is the same as violating Clausius statement. So now I have started with an engine that violates Kelvin statement because this engine takes heat from hot object and converts it completely to work.

Now I can use this work you see and use it run a second engine where this is a legitimate engine. The first E1 is the illegal engine this is E2 is a legal engine. So the E2 engine exist in nature so because what it does is it just takes work so think of it may be AC mains. So you are supplying some electricity and you are taking energy from a cold object and dumping it into a hot object it is like a refrigerator it is a refrigerator.

So it is like you are taking heat from your vegetables here and this is  $W$  is your this is your refrigerator in your home and you are supplying electricity called  $W$  and you are running this refrigerator and this refrigerator is taking heat from the cold vegetables and throwing it outside the refrigerator into the hot background. So thereby cooling the vegetables even further, so put together now I want to ask myself what is the combined effect of both these engines.

So in order to examine that I have drawn these dotted lines here perhaps you can see it, so there is this huge rectangle here that encloses both these engines and you see if you look at the boundary of this rectangle, so let us ask ourselves what is the effect of both these engines put together. So if you focus on this big rectangle dotted rectangle you can see that heat actually enters this big rectangle from here and then it also exits from here.

But then if you focus on the bottom part you see that heat exits the cold object and enter the rectangle but then finally therefore in order to be consistent with the first law of thermodynamics that heat should actually exit and enter the hot object. So heat cannot be destroyed so you see there is nothing happening to the cold object except that the only one thing is happening and that is heat is escaping from the cold object and entering this rectangle.

So after it enters the rectangle it gets dumped into the hot object so that is all that happening as far as the rectangle is concerned this work that is happening is internal to inside the rectangle. So

I am not worried about what is happening inside the rectangle think of this rectangle as a black box. So in which case if you think of this as a black box so heat from a cold object is entering this black box and then getting finally dumped into the hot object. So this is a violation of the Clausius statement of the second law.

So which says that no process is possible whose sole result in the transfer of heat from a colder to hotter object. So put together that is what is happening so we started with an illegal engine that violated Kelvin's form of second law of thermodynamics but then we also coupled it with the normal engine and together we are able to show that means we are also violating the Clausius statement of the second law of thermodynamics because put together it implies taking heat from a cold object and dumping it into hot object.

So we can do the reverse and we can prove that the violation of the Clausius statement means the violation of Kelvin statement. So how do you prove that, as usual we start with an illegal engine E1 now what does the illegal engine E1 do? It violates the Clausius statement of the second law of thermodynamics and what is the Clausius statement? So it says you cannot transfer heat from a colder to hotter object spontaneously.

So in that is what I am going to do I am going to assume let there be an engine called E1 which does exactly that it takes heat from a colder object and transmits to hotter objects and then no work is being done no energy is being supplied for this. So it is like a refrigerator that runs even if the mains are switched off. So this is an illegal engine that does not exist in nature so now imagine that I am able to do this in which case I take heat from a cold object and dump it in a hot object.

Notice that this does not violate the first law of thermodynamics because the energy is conserved I mean whatever energy was here is finally finding its way there, so it does not violate the first law of thermodynamics but it violates the Clausius statement of the second law. So now I've taken heat from cold object and dump it into the hot object spontaneously. Now let me use some of that heat that I have now dumped into the hot object to run another engine and normal engine now and this normal engine takes heat from a hot object and some of it gets converted to work and some of it gets dumped into the colder object.

So now put together as usual I am going to draw dotted line here so if draw a dotted line and you see that all the arrows that going in and out of the cold object are inside that black box which is the rectangular dotted line. So if I ask myself what are the arrows coming out and going into that rectangular dotted line you see that there is only one arrow that is coming out and that is work that is work that is coming out.

So if work is being done you can see that heat is being supplied and obviously heat is being supplied because otherwise it would violate the first law. So heat is being supplied and it is getting completely converted to work so no heat is now being dumped into the cold reservoir so because what is being dumped its internal to the black box. So if you look at it as on overall system as a black box.

So no heat is actually being dumped into the cold reservoir so as far as this black box is concerned so it is as if heat is being converted completely to work as result of one illegal engine and one legal engine. So as a result we are able to convince ourselves that a violation of the Clausius statement of the second law of thermodynamics means that we are also violating the Kelvin statement.

So this proves the equivalent of the Kelvin and Clausius form of the second law of thermodynamics. So now let me also try to describe what I mean by some of the other terms that I may have thrown about and those are called macrostates and microstates. And these terms I am going to use frequently so it is better to you know pin them down as best as I can because later on you might be confused okay.

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- Entropy is a function of the macrostate of the system. A macrostate is a collection of variables that describe the bulk system such as total internal energy, volume and number of particles if we are talking about a gas for example. It could also describe related concepts which we will encounter later such as temperature, pressure and chemical potential.
- A microstate is a huge collection of variables equal to the number of microscopic degrees of freedom in the system. Thus a given macrostate typically corresponds to a huge number of microstates. That is, different microstates can correspond to the same macrostate.

So a thermodynamics system you see is not some abstract mathematical construct but it is a concrete physical system such as gas, solid, liquid, Ferromagnet now this is not the impression you get if you pick up any text book on statistical mechanics you will see that it is full of equations and you get the idea that is all mathematics but we are actually describing a physical system here. So every thermodynamic system is made of subatomic constituents that participate in the dynamics and dictate the behavior of the bulk system.

Now so what is statistical mechanics? It is an attempt to link the dynamics of the microscopic constituents of matter to the behavior of the bulk system through a function known as entropy. So this is a preamble it is important for me to say all this because you will see how this gets linked to what I am going to say next which is about macrostates and microstates. Now entropy is actually a function of what is known as the macrostate of the system.

So a macro state is really a collection of variables that describe the bulk of the system such as total internal energy , volume and number of particles if you are talking about a gas for example. So it could also describe related concepts which we are going to encounter later such as temperature, pressure and chemical potential. So basically entropy is a function of the macrostate of the system.

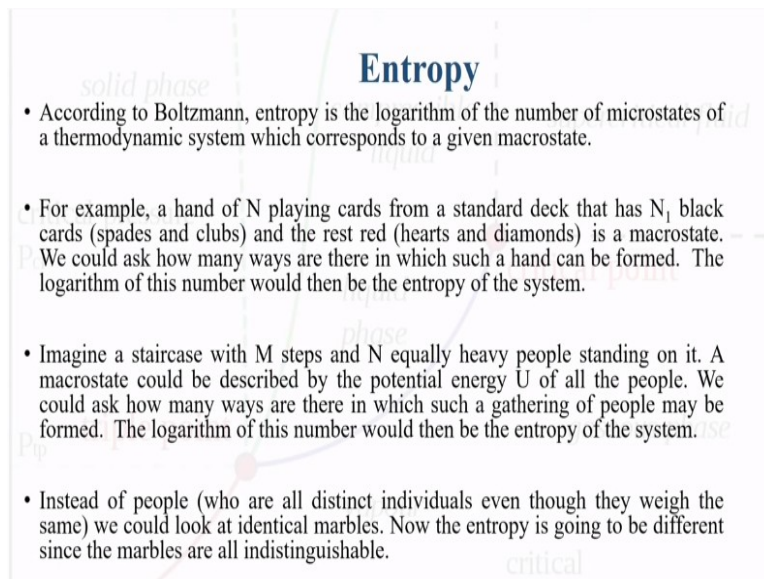
So macrostate is something where you do not really worry about what the system is made of at the microscopic level. So you just describe the system in terms of the overall properties such as

total internal energy volume etc. So by contrast a microstate is actually a huge collection of variables that describes the nitty-gritty of the microscopic constituents of that substance.

So actually a microstate actually keeps tracks of the precise description of the subatomic constituents of the system that you are looking at. So it obviously sensitively depends upon whether those microscopic constituents obey quantum mechanics or if they obey classical mechanics. So as a result you will get statistical mechanics of quantum system and classical systems. So however the macro states in both cases continue to be the same namely total internal energy, volume, pressure, temperature and so on.

But then the results for your thermodynamic quantity or later on we will see equation of state that sensitively depends upon what the basic fundamental laws governing the microscopic constituents are. So that is called a microstate so microstate pays careful attention to the fundamental laws governing the subatomic constituents and microstate is basically a collection of variables that describes in precise detail what are those microscopic constituents doing at any given time okay.

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### Entropy

- According to Boltzmann, entropy is the logarithm of the number of microstates of a thermodynamic system which corresponds to a given macrostate.
- For example, a hand of  $N$  playing cards from a standard deck that has  $N_1$  black cards (spades and clubs) and the rest red (hearts and diamonds) is a macrostate. We could ask how many ways are there in which such a hand can be formed. The logarithm of this number would then be the entropy of the system.
- Imagine a staircase with  $M$  steps and  $N$  equally heavy people standing on it. A macrostate could be described by the potential energy  $U$  of all the people. We could ask how many ways are there in which such a gathering of people may be formed. The logarithm of this number would then be the entropy of the system.
- Instead of people (who are all distinct individuals even though they weigh the same) we could look at identical marbles. Now the entropy is going to be different since the marbles are all indistinguishable.

So let me get to this notion , the important notion of entropy so according to a Boltzmann one of the founding fathers of this subject, the entropy is nothing but the logarithm of the number of microstates of the number of microstates of a thermodynamics system which corresponds to



given macrostate. So let me explain what that means so the idea is that if you have a thermodynamics system and somebody tells you that it is in a given macrostate namely it has a total internal energy which is fixed and total number of particle which is fixed and say it has a volume that is fixed obviously I am thinking of a gas.

So now I have to ask myself what are the number of ways in which i can rearrange these sub atomic constituents of this system of this gas in such a way that my total internal energy does not change and volume does not change number of particle does not change and yet the states that I am going to get as a result of this rearrangement are all different? So if am able to count how many such rearrangements are possible and I take the logarithm of that, that really corresponds to according to Boltzmann the entropy of the system and so that is defined to be the entropy of the system.

So let me give you some concrete example from everyday experience so not related to physics per se but from it also applies to everyday examples like I am going to give you right now. So for example we could think of a hand of  $N$  playing cards so imagine you have a standard deck of playing cards on your desk and you select  $N$  playing cards and from that  $N$  playing cards you are also know that you are selected  $N_1$  black cards which is basically spades and clubs and the remaining  $N - N_1$  are clearly the red playing cards so that i mean in standard deck you only have either red cards or black cards.

So you either have spades and clubs or hearts and diamonds so imagine that you are have  $N_1$  black cards and  $N - N_1$  red cards. So now these two numbers are examples of macro states  $N$  and  $N_1$  these are two parameters that describes your micro states of this system. Now what is the microstate the microstate is the precise which particular set of cards you have in your hand that would describe a microstate.

For example you can have you know 2 of spades, 3 of clubs, 4 of diamonds, ace of hearts so all these particular set of cards would describe a microstates. So the idea that you have  $N_1$  black cards and  $N - N_1$  red cards is actually a macrostate. Now if you are able to fix the macrostate namely you fix  $N_1$  and you fix  $N - N_1$  so you are able to fix so you have decide to fix that total

number of black cards you have in your hand and the total number of red cards you have in your hand.

And then you ask how many ways I can rearrange my hand so that is still maintained? and the number you get is basically according to Boltzmann related to entropy. So the logarithm of that number is according to Boltzmann the entropy of the system, so the other example that I can give you is a staircase say with  $M$  steps and  $N$  equally heavy people standing on it let us assume for simplicity I have  $N$  equally heavy people standing on a stair case made of  $M$  steps and this is a macrostate.

So this would corresponds to a macro states so as macro states is described by this two numbers  $M$  and  $N$  but then there is another parameter that I am going to fix which also belong to the macro state and that is the total energy  $U$ . So I am going to say that I also demand I also restrict that the total energy of all the people standing on that staircase should be  $U$ . So notice that a person when is standing on the first step will have say some energy let us call it 1.

So one standing on step number 1 will have energy 1 so if the same person climbs to step number 2 that person will have energy 2. So if two people are standing on step 1 they will have energy 2 so if one person is standing on step 1 and one person is standing on step 2 so the combined system as energy 3 and so on. So now what I restrict is that if I have  $N$  people and I want to make sure that the total energy of all those people is  $U$  is fixed to  $U$ .

Now with all these restriction so they have three restriction one is the number of steps is fixed to be  $M$  the number of people are fixed to be  $N$  and the total energy of all the people standing on the staircase is fixed to be  $U$ . Now with these three restrictions I can ask myself how many ways can I rearrange the people standing on the stair case you know I can persuade the person who is standing on top to come to the bottom may be the second the person is standing on step number 2 I can ask that person to go to the step 3 and so on.

So I have to rearrange their positions in such a way that I maintain their total energy to be  $U$ . So then I count how many ways in which I can do this how many ways in which I can rearrange all these people so that the total energy is  $U$  and then I take the logarithm of this number and that is according to Boltzmann the entropy of the system. So notice that I spoke of people and human

beings are all different individuals so they are equally heavy I have assumed but one person can be wearing a red shirt one person can be Aditya the other person can be Savitiri.

So they can be different they are all different people may be they are equally heavy but they are all different people. But now so you get an answer if there are all different people you get a certain result for the entropy so there is the rearrangements you know take into account the fact that they are all different .but instead of people I can imagine distributing marbles on the staircase.

So imagine that they are equally heavy identical marbles on the staircase so in this example I am not going to distinguish between the marbles they are absolutely identical. Now I can ask the same question how many ways are there of distributing the marbles on the staircase. So you will see that the answer is going to be different from what it was if I was talking about people because now the marbles are all identical.

So now even here I can find the entropy which would be logarithm of the numbers of ways of rearranging these marbles consistent with these examples. So I am going to stop here and in the next class we will continue and see what entropy is and if entropy is really a measure of disorder or not? okay thank you.