

Lecture 8

Experimental determination of Superconducting properties

So, let's take, a short recap of, the development of the field of, Superconductivity. It was in nineteen hundred and eight well just write as recap.

Refer slide time :(00:43)

Recap

1908 — SC is seen in ultra clean Hg.
1940 — Ginzburg Landau Theory
1957 — BCS Theory.
1986 — High- T_c Superconductor

So, 1908, the super conductivity is seen in, in writing in short for super conductivity as SC is seen, in ultra clean mercury and then on, a large number of superconductors, have been discovered and they have been debated upon many experiments have been done and it was around nineteen hundred and forty, thirty nine, forty, is the first phenomenological theory of superconductivity came up which is called as the, 'Ginsburg Landau Theory'. And of course after that 1957 was the celebrated BCS Theory which was the most successful microscopic theory that explained, all the phenomena, all the experimental observations, that are seen or that are made possible, in the context of superconductivity. So, this is a BCS theory and as we have seen that it's a non Perturbative theory it's a variational theory and it explains all the features, of superconductor conventional superconductor. What we mean by conventional will be made more clear, later when we talk about unconventional superconductors. And then of course, lots of superconductors were discovered including the organic superconductors and it was only, 1986 is when one, has seen this development of the high-temperature superconductivity. So, this classes of superconductors, have been, seen, which have T_c is ranging from 40 Kelvin to about, 200 Kelvin or even larger than that and these are all copper oxide ,based superconductors and in early 2000 they were discovery of iron based superconductors and so, on. So, various development that took place, regarding, the understanding of superconductivity, many of them were empirical and to say how they are let, us see some of the results that were proposed and were found to be true, at least in certain, superconductors.

Refer slide time :(03:30)

Empirical rules.

(i) Compounds with on an average 5-7 electrons show larger T_c

Ru Mo	which has	7	electrons	————	$T_c = 10.6 K$
Ru	" "	8	electrons	————	$T_c = 0.5 K$
Mo	" "	6	electrons	————	$T_c = 1 K$

(ii) Structural dependence.

So, some of these empirical rules, were found, that compounds, with on an average, average 5 to 7 electrons, show larger T_c Okay? And examples are the ruthenium molybdenum, which has, 5 electrons rather 7 electrons, of 7 electrons, this show T_c of, 10 point 6 Kelvin, whereas ruthenium, which has 8 electrons rather show T_c of 0.5 Kelvin, while molybdenum, which has T_c which has 6 electrons, show T_c of 1 Kelvin. So, when, when a compound is made, of ruthenium and molybdenum, which has on an average 7 electrons, which shows a T_c , which is about an order of magnitude higher. Okay? And then people, wanted to understand the structural dependence and when, these studies of the structural dependencies, were done and they found that there are structures such as a 15 crystal structures, just tell you what that is or which is also called as a, 'Beta Tungsten Structure', that has a higher T_c or all these structures have higher T_c 's let's see what a 15 or a beta tungsten looks like. Okay?

Refer slide time :(06:10)

A15 crystal structure of A_3B . (Also called as the β -tungsten structure)

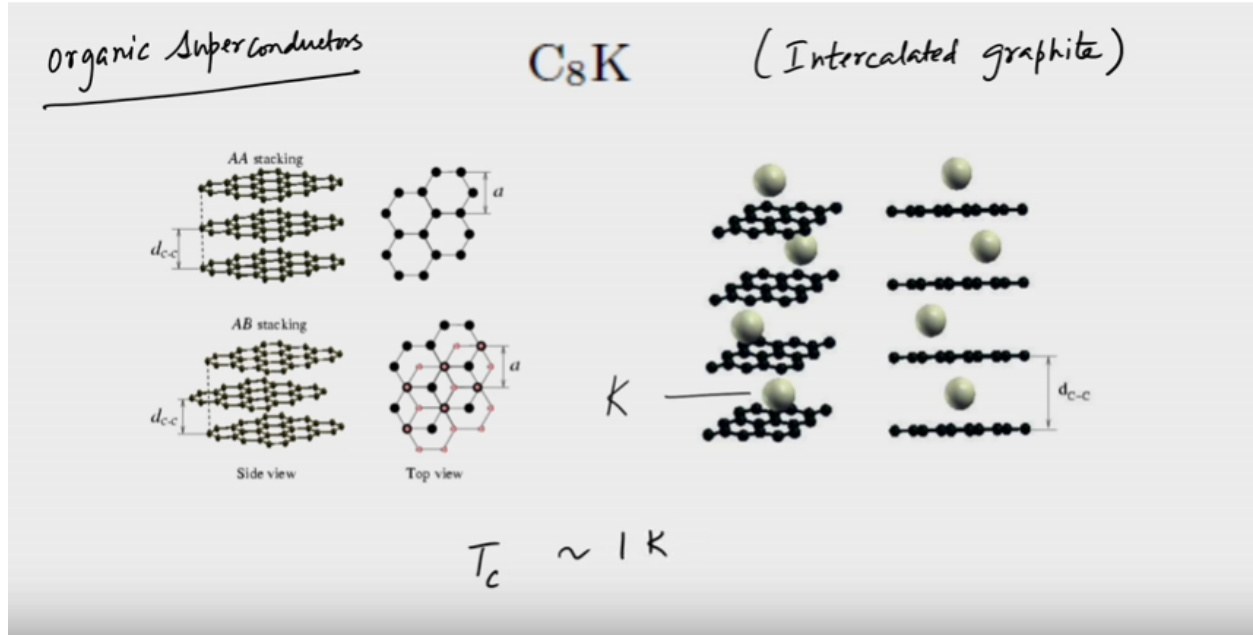
V_3Si — 17.1 K
 Nb_3Ge — 23.2 K
 ↑
 a maximum for quite a few years.

○ A ◐ B

10:02 / 49:29

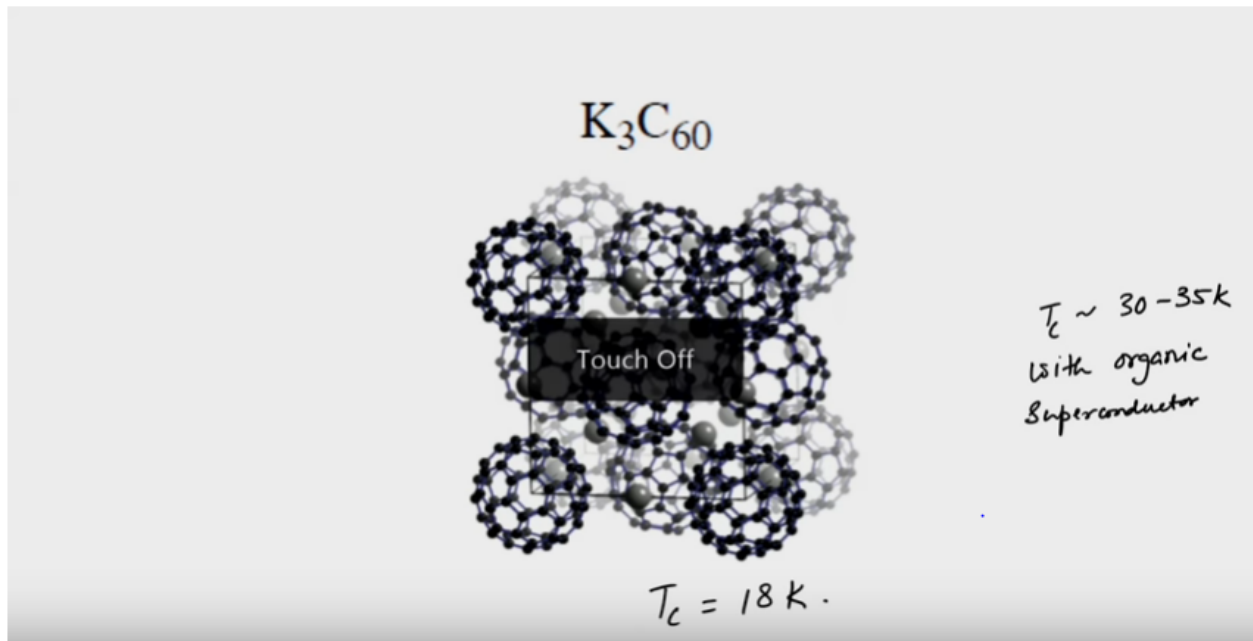
So, this is typically a 15 crystal structure, of a 15 crystal structure of a compound called a, $3b$ and so, these are the A atoms are actually at this or sorry a atoms are these blank ones that are not filled and the hatched ones are the B atoms. So, B atoms are actually at the body centered places, of this and if you form, a line connecting the mid sort, of mid of these sides and then you join, them and then they sit the, the A atoms sit at the at these symmetric positions, similarly at all the sides. So, all the sides will have these, A atoms two of them on each face and they are and the B atoms are which are shown in the thatch lines are at the a body centered places, now this has a larger T C. So, this is called it's also called as, 'Beta Tungsten Structure. Okay? And this has a TC of about so, they're two, compounds which were notable at that time, it was $v_3 si$ that is Vanadium silicon and niobium germanium. So, both these have this same structure a three be, a structure where a is equal to V for the $v_3 si$ and a equal to n be for the $NB, 3G$. So, this has a TC of about 17 point one Kelvin and this has a TC of about, n b3 GG as a TC of about twenty three point, two Kelvin. And so, these are considered as, a high T sees and in fact a large amount of research, really, had stopped off forty, CS to be, around 25 to 30 Kelvin and it has rarely in spite of all the efforts, it has really gone beyond that, in fact even with the discovery of the, the organic Superconductors, the TC only rose just by a few Kelvin, but it stayed pretty much around that till the discovery of the, high TC superconductors. So, this 23.2 was a maximum, for quite a few years and it is proposed, that with the electron phonon interaction, this is the maximum TC that one can have, as you understand that this temperature is very low, this temperature is nearly 250 degrees below, the freezing point and that is a freezing point of course water that we are talking about and so, for industrial applications or for any, sort of applications related to making of superconducting wires, etc this temperature is not a suitable one, if the temperatures the number that we show here are certainly not a viable to achieve, however this research continued and this organic superconductors also were, seen let me show you some organic superconductors.

Refer slide time :(10:08)



So, this is an organic superconductor, it's called as an, 'Intercalated', they are popularly known as intercalated graphite. So, these layers are what we know as grapheme. Now so, these are carbon atoms the black dots are the carbon atoms, which are arranged in the form of a honeycomb lattice. So, this is a side view being shown and this is the top view, being shown and then there are these two kinds of stacking x' that are usually taken when and now, this grapheme is actually a 2 sub lattice, it has a 2 sub lattice basis with, A and B so, if is above a if a sub lattice carbon is above the a sub lattice carbon in the successive layers, then it's called, 'AS stacking', and if a sub lattice is above the B sub lattice and it's only in the next, layer or the next to next layer, again the a sub lattices are aligned these called as, 'AB stacking'. So, here potassium is intercalated between, these layers of graphite and you can see that this balls are the these are the potassium, which is represented by K it's got a nice you know structure, which is like a serpentine structure. So, these are the intercalated graphite and this intercalated graphite actually showed superconducting, in below 1 Kelvin. So, this is organic superconductor. So, let's just write that these are organic superconductors and TC is about, just about one Kelvin so it's a small transition temperature, however there is another one let's show that,

Refer slide time :(12:16)



this is called as, 'K 360'. So, these huge buckyball structures or these full Aryans are occupying the, the body centered, positions of a cube and this has a larger TC, TC is about so, this called, 'K 360', and the TC is about 18 Kelvin. Okay? So, and by various you know using various organic compounds that TC could go all the way up to 30 to 35 Kelvin, I mean that's the maximum TC that one could achieve, with organic superconductors, these last two, the one that we have shown, this one the c8 k and this and this K 360 are called as the, 'Organic Superconductors'. That is pretty much the story, of this conventional super conductivity and many empirical, these such laws were proposed and most of them were found to be correct, within a certain restricted sense, I will also see later that there is another this phenomenological or empirical, plot that is presented which is called as a, 'Whim Or a Plot', in the context of when we come to the unconventional superconductivity. Right? Now let us look at some experimental, scenarios measuring the gap.

Refer slide time :(14:05)

Experiments for measuring the energy gap of a Superconductor

$$\Delta \sim k_B T_c$$

$$2\Delta = 3.5 k_B T_c$$

↑

↓

3 - 4.5 K.

(i) Absorption of EM radiation.

$$1 \text{ eV} = 11,600 \text{ K}$$

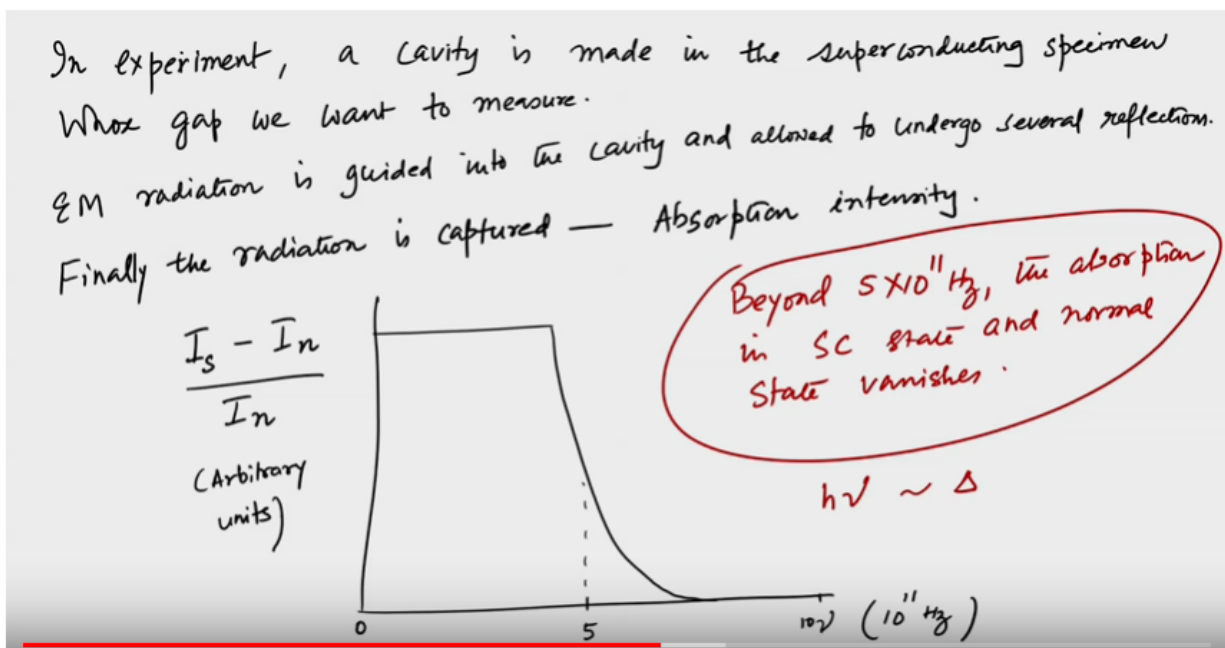
$$\Delta \sim 1-2 \text{ meV}$$

$$\nu \sim \Delta/h = 2.4 \times 10^{11} \text{ Hz}$$

Experiments measuring the energy gap, of a superconductor. So, just to remind you that these superconductors have a certain T_c which means that, below that T_c at any temperature, below T_c , the superconductivity is robust. So, there's an energy gap that forms because of the formation of Cooper pairs. So, the Cooper pairs are bound state of two electrons, of opposite spin and momentum and which gives rise to the fact that there is a certain amount of energy that is required, to break that pair and that energy is supplied usually in the form of for temperature, which is called as a 'Thermal Energy' or it could be in the form of magnetic energy, that is why a magnetic field, the most important thing about, a superconductor. Is the measurement of its energy gap? So, that we understand that how large is the gap because this gap is actually a measure of the transition temperature T_c , in fact it is found in the BCS Theory, that this gap is proportional to so, this gap is proportional to the $k_B T_c$ and in fact there is a factor that is associated with it in fact the factor is that the more correct expression is that 2Δ is equal to about $3.5 k_B T_c$. so, you can see that the Δ and $k_B T_c$ are of the same order in fact this a number 3.5 actually, ranges from 3 to 4.5 for most superconductors. So, the question is that how do we determine the gap. So, that we know what the T_c of that particular superconductor is and a variety of things our, measurement techniques are there, one of them is the absorption of electromagnetic radiation, I'm writing in short for electromagnetic EM radiation. So, the first question comes, that what is the radiation frequency that we should use in order to measure the gap, for that we should have an idea of what is the magnitude of the gap, this is of course we don't know a priori was the magnitude of the gap, because that's what we are trying to find out but at least, we should have an idea. So, that we know what the frequency of the, radiation is and this gap Δ , is of the order of a 1 to 2 milli electron volt. Now how do we say it's a 1 milli electron volt and your one electron volt, just for your knowledge it's 11,000 nearly 12 thousand Kelvin it's 11 thousand 600 Kelvin. So, if it's a 1 MeV then of course it's about 10 to 12 Kelvin. So, that's that transition temperature that we were talking about so, this gap must be of the order of a few million volt, may be of the order of 1 milli electron volt. So, if we want to use an electromagnetic radiation the frequency should be, this is that or let's just write it. So, it's by basically $h\nu$ has to match Δ in

order to break, the Cooper pair. So, this is by Delta over H and this is equal to about, if you use 1 milli electron volt, it comes to around 2.4 into 10 to the power 11 Hertz. So, it's basically in that regime, sorry we write Hertz like this. Okay? so, basically it's in this regime of, 10 to the power, 11 to 10 to the power, 12 Hertz which are more than, a gigahertz and around 1900 and 20s and 30s this kind of Electra frequencies or other magnetic radiation with this kind of frequencies were not available, it was only available probably in the early fifties 1950s where such things are, are possible or rather they are achieved and now of course, we have even micrometer range, electromagnetic radiation that are possible. So, let's see what how the experiments are done.

Refer slide time :(19:20)



So, in the experiment, a cavity, is made, in the superconducting specimen, whose gap we want to measure, then the EM radiation, is guided, into the cavity and allowed, to undergo, several reflection, finally, the radiation, is captured, after it has under undergone, multiple reflection and, and hence absorption. So, what we actually see is the absorption intensity. So, this is that absorption intensity, of this of this radiation and what happens is that this absorption intensity of the radiation is actually plotted as a function of the frequency. But before that what is plotted is actually I_s . So, assuming that the specimen is in the superconducting, regime and minus I_n . so, this means that the superconductivity is destroyed, while the experiment is carried on, keeping at the same temperature, but using a magnetic field. So, we know that magnetic field destroys, super conductivity give in reacts to a normal state. So, that normal

state intensity is also captured and divided by, basically that's the normalization, by the normalized intensity. So, basically you try to understand that there, is a kind of superconducting specimen and there's a small cavity being dug into, it the radiation electromagnetic radiation of that 10 to the power $11 - 10$ to the power 12 Hertz frequencies guided into it, it undergoes a lot of reflection and finally it's been captured by a detector, the absorption intensity, it's done for two cases one is when the sample is superconducting and when the sample is normal and the normal state is achieved, by keeping the same experimental setup, however I and at the same temperature, but however using a magnetic field, the superconductivity is destroyed giving rise to a normal state. And these are the intensities, the absorption intensities, of this the normalized of absorption intensities of this, superconducting at a normal state. And this is plotted as a function of, new and so, this is say new in, in units of, you know maybe 10 into 10 to the power 11 Hertz and so, on and then what is seen is that, that the, the plot is. So, this is around 5 and this is around maybe 10 and this is 0 and so on. So, this is that intensity, let's say it is in arbitrary units. Okay? So, what happens is that? Below a certain frequency, which is below 5 into 10 to the power 11 Hertz one actually gets, the superconducting state has a larger absorption and suddenly, what happens is that this goes to zero in the vicinity.

So, there's an absorption edge and in the vicinity of that 5 into 10 to the power 11 Hertz, it goes to zero probably a little more sharp, but this is what we show here, I mean this could be a little sharp depending on the on the material and so, basically what it see means is that beyond, these 5 into 10 to the power 11 Hertz this frequency, there is no difference between the superconducting, state absorption and the normal state absorption. Now the question is that why does it happen? And of course if you go to larger frequencies the, difference between them remains as 0 and so, this abrupt drop, of this absorption spectrum results, from the quantum, this theory of or rather the quantum energy of the radiation, is breaking or it's becoming equal to the, binding energy of the pair or the energy gap that we talked about and then the pairs are the Cooper pairs are dissociated, to summarize this plot, beyond a certain frequency, which is a here that, 5 into 10 to the power 11 Hertz frequency, the absorption, in superconducting state and normal state vanishes, which means that there is no distinction between, the superconducting state and the normal state, which means that it's basically, the superconductivity is gone. So, that tells you, that there is this frequency, which is equal to the gap and by the gap can be estimated, for this kind of from this kind of experiments, it is a notable to mention, that similar, absorption ages are also seen in semiconductors, however the gap in semiconductors is typically, three orders of magnitude more or ten thousand or a thousand times, more than that of the superconducting gap and in silicon it's about 1.4 electron volt in germanium it's about 0.8 electron volt and hence the frequencies or rather the absorption edges are seen at much smaller frequencies, because, the energy gap being larger. So, basically this conclusively tells us that, the magnitude of the gap can be determined from this absorption of the electromagnetic radiation.

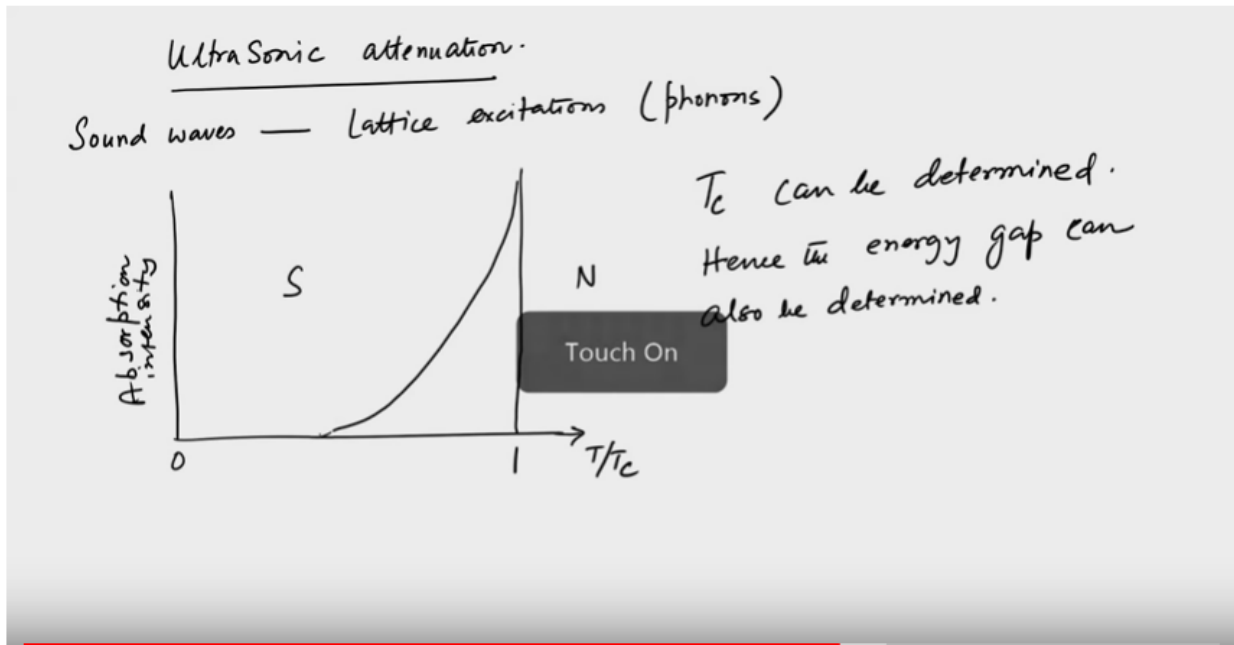
Refer slide time :(27:16)

Experimental data

Material	T_c (in K)	2Δ (meV)
Nb_3Sn	18	6.55
MgB_2	40	10
Rb_3C_{60}	30	12
$ErRh_4B_4$	9	3

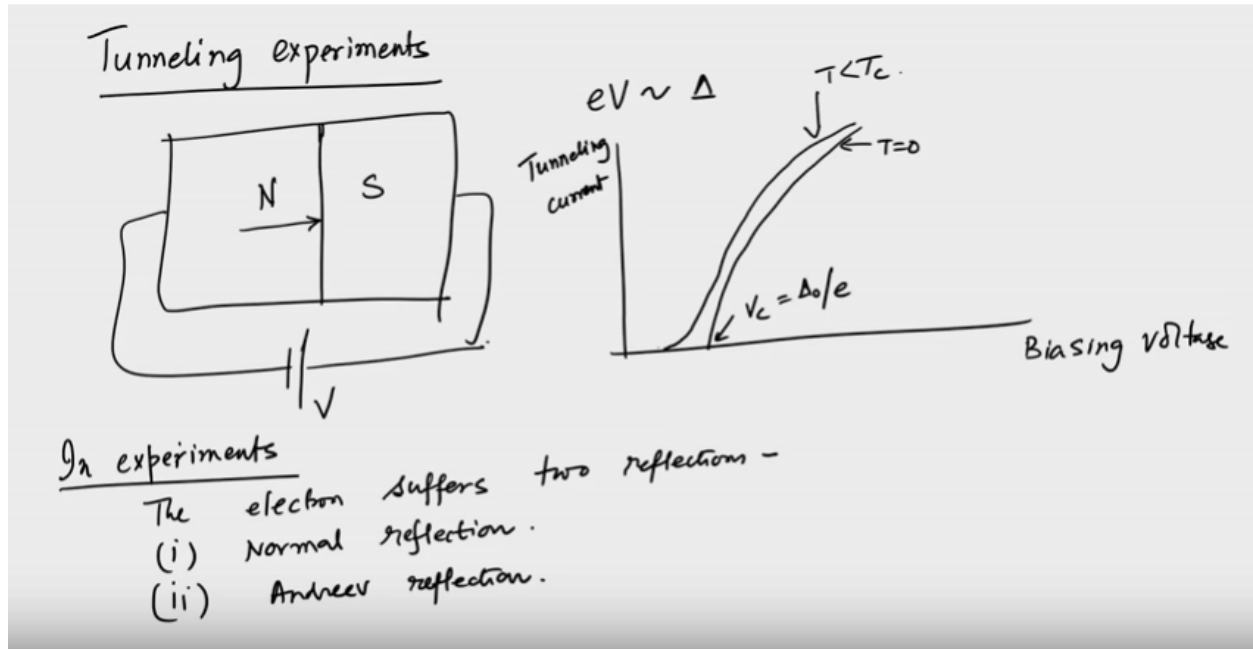
Let us see one more experiment, that shows also that measures gap, before that let us also, you know, sort of write down, the magnitude of the gap. So, this is an experimental data. So, for Nb_3Sn , which is again, a 15 crystal structure, which is called as the, the, 'Beta Tungsten Structure'. That T_c is about 18 Kelvin. So, T_c in Kelvin and this to Δ in MeV, I just and there's no particular reason that 2Δ is used, but since the 2Δ has a value, that's called as a magnitude of the gap, Δ is in this conventionally, because, the Fermi energy is actually placed at the middle. So, there's a gap minus Δ plus Δ . So, the magnitude of the gap is 2Δ . So, this is about six point five, five milli electron volt, this is a fairly newer superconductor boride superconductors, called, 'Magnesium Boride', it has 40 Kelvin and this has MeV 2Δ equal to 10 MeV, Rb_3C_{60} , these are these organic superconductors intercalated, organic superconductors, which has 30 Kelvin and the gap is about 12 MeV and there is an erbium, rotation, boron that's a 9 Kelvin and this is equal to 3 Kelvin. So, as you see that they are there is a nice, correlation or rather 2Δ , is actually very closely, correlates with T_c . so, once you have a magnitude of the gap available, to you through experiments T_c 's or at least an estimation of the T_c is also available. Let us see one more experiment here.

Refer slide time :(29:40)



Which is called as the, 'Ultra Sonic Attenuation'. So, basically, there are these sound waves and nothing, but the lattice excitations or phonons. So, these sound waves actually interact with the conduction, electrons. So, this is called as the, 'Electron Phonon Coupling'. And these so, if one sends a sound wave through a superconducting specimen. So, it could happen that there are still below, T_c there are some, unpaired electrons, conduction electrons, available and they would absorb the radiation and the absorption, intensity of this ultrasound, can be measured and what it shows is that, it shows a plot like this, shows a plot like, this so, this is T versus T_c and so, this is equal to one. So, let us write it so, this is equal to one and this is the absorption intensity. Okay? So, one gets a plot like this, what it means is the following? That below T_c so, this left side, is between zero and one we have a superconductor and this is a normal. So, in superconductor there are very few unpaired electrons, they are all mostly their Cooper pairs, which are with, which from the you know the constituents, of the superconducting state. And that's why the superconducting state has a gap, say there are few unpaired electrons, those will absorb, the sound and this as you go deeper, into the superconducting regime, by reducing the temperature, the number of such unpaired electrons, go down drastically and so, the ultrasound attenuation also, will go down drastically, giving going to zero. And so, the absorption intensity will go to 0, which means that through a superconducting specimen, if the ultrasound is sent or the sound waves are sent, they will propagate, without any significant absorption one, one would get the same, intensity of sound as, one would have gotten, as the incident sound. So, this gives the, the idea of the T_c , this is plotted of course T by T_c , but in actual experiments, one doesn't know what is T_c . So, it's plotted as a function of T and the T_c can be extracted, from this and the T_c as, I said a number of time T_c , is a measure of the gap. So, the gap can be determined since. So, T_c can be determined, hence the energy gap determines. Okay? So, this is the ultra sound, attenuation and then there are, also some.

Refer slide time :(33:50)



Tunneling experiments so, what happens is that if an electron? So, make a junction of N and s and this is the and we are connecting, a battery voltage. So, an electron can actually be made to, appear at the junction, now this, the electron will not pass through because the electron, will see it as a barrier as the superconducting state, is say fully paired, which offers an energy gap, which is equal to the Delta. So, if this electron does, not have this then the there is no, current and that is what is seen in actually, this experiment. So, this is that the current or the conductance and so, it shows that it is like this. Okay? So, this is as a function of the voltage, the biasing voltage. So, this V_c is equal to like Δ_0/e and this is of course at T equal to zero or very low temperature and at T less than T_c this so, this is T less than T_c naught equal to zero. So, one has a form which is that so, it's a smooth curve let me just draw it once more. So, that's the properties of this junctions, that it doesn't allow, flowing of the current and one, one gets this the, the tunneling current versus the, voltage so, it's IV characteristics, for these NS Junction and one finds that, it happens it starts taking of, the current starts taking of only, at V_c the critical voltage, which is equal to Δ_0/e . So, just by noting V_c in an experimental setup, one could find out what is Δ_0/e , but it seems like, the thing is not, that straightforward, there are additional things that are happening which are called as the, 'Andreev Reflection'. And let us quickly, go through, what is on the rave reflection? So, in, in experiments, I mean this is also an experimental, but in other experiments, the

electron suffers two reflections, one of them is called as a, 'Normal Reflection'. And the second is called as the, 'Andreev Reflection'. So, the normal reflection is just the same as what one sees here, that the electron when it doesn't have the energy, to overcome the barrier, that is that exists in the superconducting side, it will simply turn back giving rise to a normal reflection and there's something called a, 'Sundry Reflection'. So, what happens is that at the normal superconductor interface? So, let us explain Andreev reflection.

Refer slide time :(38:15)

Andreev reflection.
An electron incidents at the interface of the N-S junction.

- (i) A Cooper pair is injected into the S region
- (ii) The incident is annihilated.
- (iii) A hole retraces the path of the incident electron.

Touch On

- (A) Energy is Conserved
- (B) Momentum is almost Conserved.

So, an electron, incidence, at the interface, of the, normal superconductor, will simply write his NS Junction, then what happens is that? The Cooper pair is injected, into the, superconductor region, which is the lead. So, is that a picture that we had shown? So, and then, the incident electron, is annihilated third, importantly because, there's a generation of a Cooper pair, a hole retraces the path, path of the incident electron. So, one A energy is conserved, in the process, will show that and momentum, is almost conserved, will tell you, what we mean by almost conserved. Okay?

Refer slide time :(40:15)

Andreev reflection.

Energy is Conserved.

- (i) Cooper pair has energy $2\epsilon_F$
- (ii) incident electron $-\epsilon_F + \epsilon$
- (iii) Reflected hole $\epsilon_F - \epsilon$

In this process a $2e$ charge is transferred from $N \rightarrow S$

Opposite motion.
 $\frac{1}{\hbar} \frac{\partial \epsilon}{\partial k}$ is oppositely directed.

Momentum is Conserved upto $\xi = \frac{\hbar v_F}{\pi \Delta_0}$

$$\hbar |k^{el} - k^h| \leq \frac{\hbar}{\xi}$$

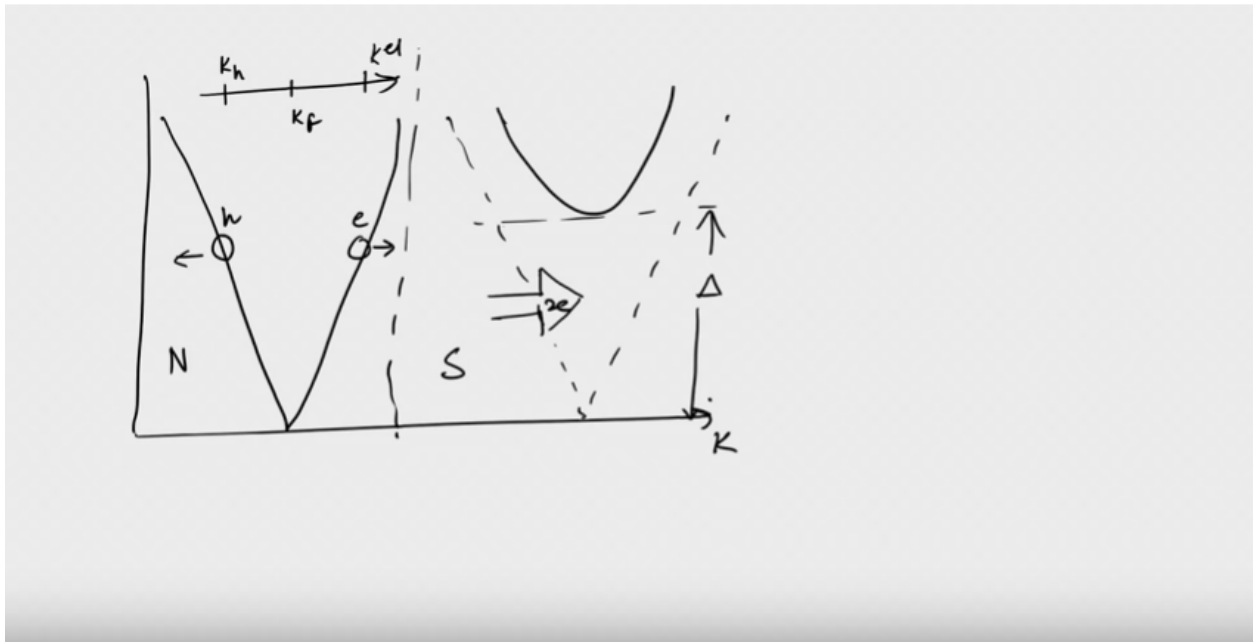
$$k^{el} = k_F + \frac{\epsilon}{\hbar v_F}$$

$$k^h = k_F - \frac{\epsilon}{\hbar v_F}$$

So, so basically what happens is the following, at this boundary. So, this is the NS boundary, so, this is N and s so, there's an electron that goes. Okay? A hole comes back. Okay? And a cooper pair, that transmits here, with charge $2e$. Okay? So, this is an electron and a hole, let me draw it, better. So, this is that Cooper pair and since Cooper pairs can travel, inside a superconductor. So, they'll that would travel and this electron gets, annihilated and there's a hole. So, this electron has spin up and the hole has spin down and basically, this can give rise to low energy, tunneling current. So, at very low energy, you don't need to actually, cross the gap you still can have tunneling current, even at zero bias, that is, very low energies. So, this is known as Andreev reflection. So, the, the how is momentum and energy conserved. So, k electron equal to k_F plus, ϵ by \hbar cross v_F and k whole it's equal to k_F minus, ϵ by \hbar cross v_F , they have opposite direction, opposite motion and which appears from the group velocity. So, $\frac{1}{\hbar} \frac{\partial \epsilon}{\partial k}$ is actually negative, as you can see from these expressions. So, the group velocity is oppositely directed, then of course energy is conserved. So, Cooper pair has energy to ϵ_F , that's a minimum energy. So, the these things the Cooper pairs are formed, at ϵ_F . So, the incident electron, electron has energy, $\epsilon_F + \epsilon$ and reflected hole, has energy, which is $\epsilon_F - \epsilon$. So, if you add both of them it becomes two ϵ_F and the momentum, let's do it here, momentum, is conserved, up to, up to a key, electron minus key hole, this should be, less than one oversee or if you want if it's \hbar cross and \hbar cross here. So, it's, it's up to this, where's Isaacs, is the coherence,

length which is equal to $\hbar v_F$, by $\pi \Delta$. So, in this process, to e charge, is to each charge is transferred, from, n to s. Okay? Let's just show it graphically.

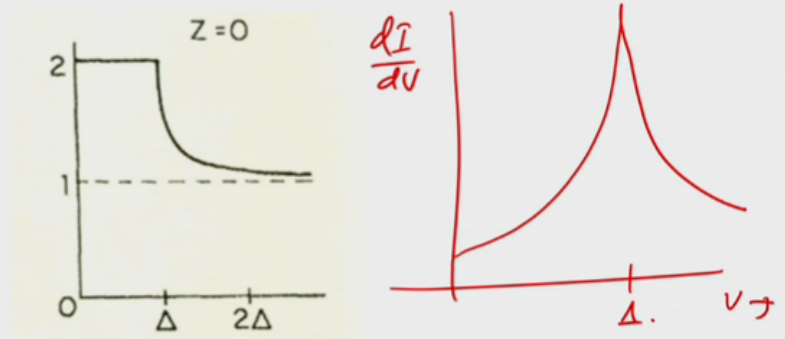
Refer slide time :(45:23)



So, it's this is the, e versus K, this is the so, this is your n and this is the S. so, this gap is Delta. Okay? So, there is a, Cooper pair that is of $2e$ that is that goes from here, so there is this K. So, there is a hole here which moves in this direction, there's an electron here, which moves in this direction, should have drawn at the same. So, this is that electron this is that hole. So, this is the momentum of the hole, this is the momentum of the, electron. And so, the somewhere, in between this k_F so, this is the superconducting state. So, this is normal N and this is s and so, that is the so, basically the structure. So, there's a Cooper pair that, propagates and so on. So, this called as and, 'Andreev Reflection'. And to see that we have current at almost zero.

Refer slide time :(46:51)

Andreev Reflection



So, this is the and andreev reflection. So, basically you see that below Delta, there is a current, which is pretty large and then of course add Delta, the current after beyond Delta the current, goes down for a for a reason that also can be explained simply. So, this is the feature, of this on reflection and it is there its present, in all normal superconductor junctions or even if one actually introduces, a barrier between the normal and superconducting Junction, just to say that the normal is usually a metal, such as aluminium etc and sometimes, the insulating barrier being introduced, between the normal and the superconducting Junction, one can do the same experiment, there are additional features, which we don't want to go into here, neither we want to go into the, detailed derivation, of these tunneling current, but that is quite possible, it requires, want to understand the boogaloo for design, formalism, in order to handle the superconducting, properties and so, there, there are so, so this insulating layer in between is formed by what is known as alumina Al_2O_3 the aluminium oxide, which is usually, a very good insulator and not only that it can be, very nicely you know, formed as an interfacial, regime with a desired thickness for a given experimental setup. So, these are some of these experiments. So, this edge would determine, that there is a there is a the, the Delta is there and if I take a differential conductance, then it will show that, there is a peak there. Okay? So, that this is I versus V whereas the $\frac{dI}{dV}$ versus V would show a P and so, where there is a like a discontinuity and so, this will be the position of Delta or the energy gap. So, these are some of the experiments, this list is of course not exists exhaustive, there are many other experiments, but one needs to understand at least a few, how experimentally one determines, the gap.