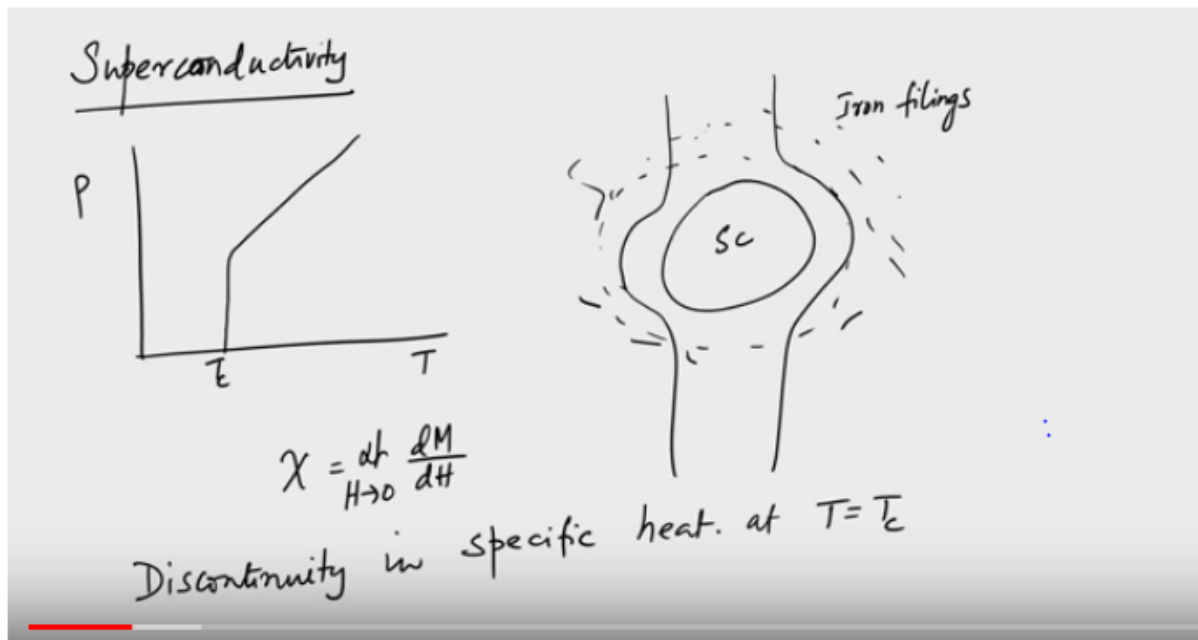


## **Lecture 7**

### **Thermodynamic properties of superconductors, specific heat**

So, let us recapitulate, some of the key features of superconductivity.

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We have seen that the resistivity. So, the resistivity goes as, this is as a function of temperature and it suddenly goes to zero within a very narrow energy or at a narrow temperature range at temperature which is called as a superconducting transition temperature. So, just by itself it talks about a phase transition but one doesn't know whether it's a superconducting phase transition even though the, the resistivity is going to zero but there has to be some other tests for a superconducting transition and one of them is the Meissner effect. So, basically it's an expulsion of the magnetic field. So, now this experiment is clear to most of you that how resistivity measurements are done. So, it's done in the lab with the help of a four probe measurement where you actually pass current through two probes and measure the voltage across the transverse probes. So, this is how the current or rather the current versus voltage characteristics and from there the resistance and resistivity is of course quantity which is independent of the material properties and it's independent of the geometric properties of the body or the dimensions of the body whereas resistance is a quantity which depends upon resistivity depends only on the internal or rather the, the properties of the sample whereas resistance depends also on geometric quantities.

So, how is this expulsion of magnetic field seen So, basically if you have a superconductor like this and you put iron filings here so basically scattered all over the place so this is the superconductor and these are iron filings which are like nails. So, if these nails are scattered all over and then you apply a magnetic field so, the magnetic field will not penetrate into the sample but rather will go around it and which will make these iron filings nicely lined around the superconducting specimen .Okay? so, they'll be nice sort of nicely arranged around the specimen and this shows that the magnetic flux density is actually is concentrated around the specimen and that's why they have lined up alRight but that's an experiment which is which is dependent on you know getting those iron filings and doing this rather it's there's a better experiment which talks about calculation of the susceptibility. And the susceptibility is defined as basically it's defined as  $\chi = \lim_{H \rightarrow 0} \frac{dM}{dH}$  in the limit H tending to 0 which means is the slope of the M versus H curve. And this from a value nearly equal to zero it goes to a value which is negative and maybe close to -1 because, superconductor is a perfect diamagnets. So, Chi is equal to minus 1 because M is in opposite in direction to H and that's why it is it goes to minus 1 so, that's the susceptibility data is another becoming negative is another test for a superconducting transition. Now there is another test which one often talks about which is called as the discontinuity in

the specific heat so, so specific heat shows a discontinuity across the transition. So, at  $T$  equal to  $T_C$  there is a discontinuity and one gets a sudden drop at transition temperature. And so, this is so this drop can be calculated and this drop can be sort of taken as a signature for a superconducting transition. So, far what we have done is that we have seen? that a superconductor is characterized by an energy gap which occurs or which appears due to the fact that these Cooper pairs they form a bound pair and there is certain amount of energy that is required to break the Cooper pair and that energy can be supplied in the form of magnetic energy or it can be supplied in the form of thermal energy. So, we usually talk about both depending on situations that we are interested in and we are going to talk about this discontinuity in specific heat but before that let us have a look at what does BCS Theory give about the temperature dependence of this gap that gap energy gap. That we are talking about which arises due to the formation of Cooper pairs.

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Determination of  $T_C$

$$E_k = \sqrt{\xi_k^2 + \Delta^2(T)} \quad \xi_k = \epsilon_k - \mu$$

At  $T = T_C$ ;  $\Delta \rightarrow 0$ .  $E_k = |\xi_k|$

$$\frac{1}{V} = \frac{1}{2} \sum_k \frac{\tanh(\beta E_k / 2)}{E_k}$$

$$= \frac{1}{2} \sum_k \frac{\tanh(\beta |\xi_k| / 2)}{|\xi_k|}$$

$$\frac{1}{V} = N(\epsilon_F) \int_0^{\beta \hbar \omega_D / 2} \frac{\tanh x}{x} dx$$

$x = \frac{\beta |\xi_k|}{2}$   $I = A \beta_c \hbar \omega_D$   
 $A = \frac{2e^2}{\pi} = 1.13$

$k_B T_C = \frac{1}{\beta_c} = 1.13 \hbar \omega_D e^{-1/N(\epsilon_F)V}$   
 for weak coupling superconductors  
 $N(\epsilon_F)V \ll 1$   
 $\sim 0.3$

$$\frac{\Delta(0)}{k_B T_C} = \frac{2}{1.13} \approx 1.764$$

$2\Delta(0) = 3.52 k_B T_C$   
 for most superconductors.  
 $3 - 4.5 k_B T_C$

16:59 / 55:09 Euler Constant  $\approx 0.577$

So, let's talk about either talk about determination of  $T_C$  which can be is also an important quantity the transition temperature .Okay? so, what happens across the transition from a superconductor to a normal metal a normal metal means that just the way you know about from your first course in solid state physics it contains a large number of electrons and they can be pretty much described by the Sommerfeld model which is a free electron model and most of the properties such as conductivity it such a specific heat etcetera. Can be described by a non interacting model or non interacting physics so, we want to know that what's the temperature at superconductor makes a transition to a metal or in the other way when the temperature is being lowered when a normal metal becomes a superconductor. And at what temperature does it happen and that temperature is what we call as a  $T_C$  or the transition temperature. So, just to remind you that we have computed the quasi particle energies which are given by this  $\xi_k$  square plus  $\Delta$  square  $\Delta$  is a function of  $T$  where  $\xi_k$  is nothing but  $\epsilon_k - \mu$

minus  $\mu$  because, we decided to work on the grand canonical ensemble so, a chemical potential is needed to be introduced and so, these are the basically the single particle energies or the band energies and this  $\Delta$  is the gap. Now at  $T = T_C$  so  $\Delta$  goes to 0 and what happens is that your  $E_k$  becomes  $\pm \epsilon_k$  with a modulus sign. Okay? so, this is the so, that happens to the quasi particle energies which boil down to the single particle energies which should happen in a normal state. So, recall the gap equation the gap equation was simply equal to  $1 - \frac{1}{2} \sum_{\mathbf{k}} \frac{\tanh(\beta \epsilon_k / 2)}{\epsilon_k}$  And so, this is the Gap equation so, what is  $V$ ,  $V$  is the attractive electron-electron interaction which we have seen that it appears because, of interaction of electrons via phonons and so, that is the attractive part of it and then we are summing over this that it's a key sum it's a momentum sum that's done over this  $\tanh(\beta \epsilon_k / 2)$  divided by  $\epsilon_k$  where  $\epsilon_k$  is the just what is written above the is the quasi particle energies. Right. So, then we need to compute this in order to compute  $T_C$  or  $\Delta$  which is a function of the gap. And so, this is equal to  $1/2$  so, that is equal to this and then you have a  $\tanh(\beta \epsilon_k / 2)$  divided by  $\epsilon_k$  and so, this is now we have to compute this and in order to compute this we can convert the sum into an integral and this integral if you need to do that as has been told earlier or you may be exposed to that idea from statistical mechanics or from your earlier solid-state physics course. That in order to convert this into an integral we need a density of states now which density of states density of states is a function of energy. So, we should in principle include this into an integral and integrate over all I mean the density of states over the entire energy range that we are talking about but then we know that the I mean super conductivity is the phenomenon that is taking place at the Fermi level or at the Fermi surface or at the Fermi energy. So, if we replace the entire density of states  $n(\epsilon)$  just by  $N(\epsilon_F)$  that should be good enough for doing this problem. So, we'll let's just do that and that yields this is equal to  $N(\epsilon_F)$  and a little bit of care is needed in order to set the limit the limit is basically set by the phonon energies which are given by  $\hbar \omega_D$  where we have simply written this whole thing as  $X$  and divided by  $X$  and this is a  $DX$ . So, basically the  $X$  is taken as where  $X$  is equal to  $\beta \epsilon_k / 2$  and so on. So, you can just check that if you do that then of course your, your integral energy integral goes from the zero to  $\hbar \omega_D$  that's for the energy integral but then since you have done a variable transform with  $\beta$  etc coming so, that will be so, the  $X$  is equal to nothing but a  $\beta \hbar \omega_D / 2$ .

So, now this integral has to be evaluated in if we want to know what is the transition temperature and that can be evaluated or rather that is a standard integral and this integral has a value let's call that as  $I$  and this  $I$  has a value which is equal to so, this is  $e^{-\gamma}$  and a  $\beta \hbar \omega_D$  this is a standard integral where  $\gamma$  is nothing but  $2e^{-\gamma}$  to the power  $\gamma$  by  $\pi$  and which has a value which is 1 point 1 3. Okay? So, the whole thing now looks very simple we are not going into the details of how to compute the integral just know that this is a standard integral and can be computed either you compute it numerically but this has a standard form in terms of which are called as the Euler's constant this  $\gamma$  is called as the Euler constant which has a value 0.577. Okay? So, too much of information is given already but what is important here is that that our  $\beta C$  or which we can write it as  $\hbar \omega_D / T_C$  so, one can actually calculate the from  $I$  just simplify this in order to get the  $\hbar \omega_D / T_C$  which is nothing but equal to  $1 - \exp(-\beta C)$  and that comes out to be  $1 - \exp(-\hbar \omega_D / T_C)$  and exponential minus 1 over in  $\epsilon_F V$ . Okay? This has several implications as has already been told that one this form of  $T_C$  suggests that it's a non perturbative theory. So, basically in terms of the electron-electron interaction so, even if  $V$  is infinitesimal so, which means that there's an infinitesimal attraction between the electrons which can form a cooper pair however this whole theory or this expression of  $T_C$  that you are seeing is, it goes as an exponential minus 1 by 1 by  $n(\epsilon_F) V$  and for weak coupling superconductors  $n(\epsilon_F) V$  is much, much smaller than one or at least much smaller than one and in the BCS case it is taken as point three well which is smaller than one but not much smaller than one and so, this tells you that a  $\Delta$  at zero now we also have if you remember

that we also have formed for the Delta at zero which is basically the, the magnitude of the binding energy of two fermions or two electrons measured from the Fermi surface the individual Fermi surfaces which also has a form which is very similar to that if you please go back and check that and that is with this really tells you that this is equal to 2 divided by 1 point 1 3 which is almost equal to 1 point 7 6 4. So, which means that Delta which is the gap at T equal to 0 is really of the order of TC it's likely higher than TC it's one point seven six times higher than TC and in fact this is a signature of BCS theory that that is a - Delta zero which is about 3.5 -KTC in fact for most of the superconductors this value is between three to four point five KTC. So, that's the transition temperature that's how the transition temperature is defined with regard to the value of the gap. So, the transition temperature of conventional BCS superconductor is twice the gap divided by three point five two and then the scale is KB which is a Boolean constant so, so this is for, for most conventional superconductors all Right. So, this is what one really needs for, for knowing the TC because that's a very important quantity in the context of superconductivity now let us looks at that.  
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Temperature dependence of the gap.

$$\frac{1}{N(\xi_F)V} = \int_0^{\hbar\omega_D} \frac{\tanh\left[\frac{1}{2}\beta(\xi^2 + \Delta^2)^{1/2}\right]}{(\xi^2 + \Delta^2)^{1/2}} d\xi.$$

In the regime  $T \rightarrow 0$

$$\Delta(T) = 1.74 \Delta(0) \left[1 - \frac{T}{T_c}\right]^{1/2}.$$

$-\Delta/k_B T \approx 0$

What is the form of the gap or the temperature dependence of the gap now this we cannot compute analytically but we have to do it numerically So, just to remind you of this expression that we have just derived this is equal to this is a H cross Omega D and a tan hyperbolic and I'm now writing it in terms of so, it's a betas I square plus Delta square this is the full gap and this is to the power half and divided by Y square plus Delta square this is to the power half and then DS I and this has to be computed numerically in order to get Delta as a function of T. So, in the vicinity of T equal to zero so, what happens is that the temperature variation is very small it's exponentially small which is so in the regime T going to 0 the this gap is exponentially small or rather this temperature variation is exponentially small and it goes as exponential Delta over KT and which, which almost equal to zero so basically the variation is the temperature variation is zero and we're basically this tan hyperbolic

term is very insensitive to the temperature variation and stays close to 1. Okay? And then I mean basically Delta remains a constant and so, basic until so, now you keep increasing the temperature until a sort of time comes when there are a significant number of quasi particles that are excited and in which case the gap starts going down. Okay? And as I said that I cannot do it without computing it numerically. So, one has a one can compute it numerically and one can find that the Delta P is equal to a delta at T equal to zero. So, we'll call it just Delta zero and there is a factor 1.74 as we have simply seen just earlier and this is equal to one minus T over TC whole to the power half or we can simply drop this let me write this a little more neatly. So, this is one point seven for Delta zero and so, this is the temperature dependence of the gap. Okay? And as I said that it can be computed numerically and plotted and from the plot one can actually get this nice variation so, this is as a function of T over TC and this is delta T over Delta zero and this value is equal to one and this value is equal to one what I said earlier is that at very small temperature much smaller than TC the variation is extremely small and Delta remains nearly, a constant equal to one and then as temperature increases the x-axis increases then there are sufficient number of quasi particles that are excited and the gap starts going down and at T equal to TC the gap goes down very sharply. So, that's the temperature dependence of the gap so these are all needed in order for us to understand that how you know these the specific discontinuity can be computed. So, before we go on to computing the discontinuity of specific heat from the BCS Theory from the results of the BCS theory let us look at the thermodynamics.

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Discontinuity of specific heat: Thermodynamics

Gibbs' free energy per unit volume,  
 $G = U - TS - HM$  (Neglected PV term) (1)

$$dU = TdS + HdM$$
 (2)
$$dG = -SdT - MdH$$
 (3)  $dG = dU - Tds - SdT - HdM - MdH$ 

$M = -H$  for a perfect diamagnet or a superconductor

$$dG = -SdT + HdH$$
 (4)
$$G_s(H) = G_s(0) + \frac{H^2}{2}$$

$$G_n = G_s(0) + \frac{H_c^2}{2}$$

$G_s$ : Gibbs' free energy for superconductor  
 $G_n$ : Gibbs' free energy for normal metal.

So, we'll still call it a discontinuity of specific heat and so, this is thermodynamic considerations. Okay? So, we write down the Gibbs free energy per unit volume as G equal to u minus TS minus H M where we have neglected the PV term. So, that's the Gibbs free energy per unit volume and if you want call this as equation one now this is so, from second law of thermodynamics we know that D u equal to TDS plus h DM again we have not taken into the PV term and that is the expression for the

change in internal energy. So, if you put this if you take a differential of one and put two into one then what happens is that your DG becomes equal to minus SDT minus MD H. So, just to show you the skipped step we have a DG which is equal to D u minus TDS minus SDT as DT minus H DM minus MD H and since Tu equal to TDS and HDM so, they cancel giving rise to minus I'm sorry I had to cancel the wrong one s DT and you have h DM so, that cancels and, and gives me a minus SDT minus MD H so, that is equal to equation three let me just so, this is you can keep it saying that this is equal to a question three now what happens is that m equal to minus H four perfect diamagnets which is a superconductor or a superconductor. So, that gives me DG equal to minus SDT and plus HD H and then if I integrate this, I'll get a G, now I write an S for the super conducting so, this G s it's equal to G s 0, g s, this is at H equal to 0 and plus h square by, 2. Okay? So, this is, say equation 4 and let me use a different colour and to say that this is Gibbs free energy for superconductor. Okay? Then of course we want to compute the Gibbs free energy for the normal state and, basically along a critical curve where the superconducting and the normal states are same so, that is equal to G n which is equal to G s 0 plus HC square by two so, G n so, this is G s and G n is Gibbs free energy .Okay? Now this could have been GN 0 but GN 0 and G H 0 are same because, at s equal to 0 then they are they are same and now from Equation 3 one

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From Eq. (3)  $\left(\frac{\partial G}{\partial T}\right)_H = -S$  (6)

At equilibrium,  $S_n - S_s = -H_c \frac{dH_c}{dT}$  (7)

$S_n$ : Entropy of a SC in zero field

$\frac{dH_c}{dT}$  is always negative.

$\Delta C = C_s - C_n = T \frac{d}{dT} (S_s - S_n)$

$= T H_c \frac{d^2 H_c}{dT^2} + T \left(\frac{dH_c}{dT}\right)^2$

Taking a derivative of Eq. 7

At  $T = T_c$ ,  $H_c = 0$ .

$\Delta C = T_c \left(\frac{dH_c}{dT}\right)^2$

$H_c(T) = H_c(0) \left(1 - \frac{T}{T_c}\right)^{3/2}$  (8)

At the transition  $T = T_c$   
 $S_n = S_s$ ,  $\Delta S = 0$ .

And now from Equation 3 one can get, a del G, del T at a fixed H is nothing but equal to minus s ,so at equilibrium, SN minus SS ,which is nothing but minus HC and a d HC, DT. Okay? So, what I did was that I took a derivative of this equation number, let's call it equation number 5 and, and have calculated a 4 and 5 and have calculated this quantity .so, SN minus SS, which is equal to minus HC, HC is the critical magnetic field ,at which superconductivity is destroyed and DHC DT of course, this HC s is a function of T temperature and so there is a slope of this HC as a function of T comes and, and of course your s,s is, is the entropy of a superconductor, I'll just write it in short for

superconductor as SC in zero, field. Okay? Now it's very important to understand that let's call this, give this, some name, what was in number earlier so this was 5 and let's give it as 6 and then, give it as 7.  $dH_C/dT$  is always negative. Because,  $H_C$  goes down as  $T$  is increased, the critical magnetic field, the value of the critical magnetic field, goes down, as the temperature is increased. So,  $dH_C/dT$  is always negative, which means the Right-hand side of equation number seven is always positive, which means that the entropy for the normal state is always greater than the entropy for the superconducting state. And that makes sense because the, the superconducting state is an ordered one whereas a normal state is a disorder and disorder always has larger entropy. Now, to calculate the jump in specific heat, which is defined as  $C_S - C_N$ , where  $s$  and  $n$  of course stands for the superconducting and the normal regimes, which is equal to  $T$  and there is a  $d/dT$  of  $S_S - S_N$  and this is nothing but equal to  $\int_0^{H_C} dH_C/dT$ . Now, I will take a  $d/dT$  of this equation seven. So, taking a derivative of seven, so this is equal to  $2 d^2 H_C/dT^2 + T (dH_C/dT)^2$ . Now, it's important to understand that there of course two terms there but, the first term is equal to zero because at  $T = T_C$ ,  $H_C = 0$  so, the first term vanishes but, the second term continues to be there and which is equal to  $dH_C/dT$  there is one square so, there's one thing that you should not forget is that even if at  $T = T_C$ ,  $H_C = 0$  but,  $dH_C/dT$ , exists which means the slope, of the critical magnetic field with temperature that continues to exist which gives rise to a discontinuity there so, that tells that so, this is the formula for the specific heat discontinuity so, if you know  $H_C$  as a function of  $T$ , that how it varies in fact incidentally the for most of the superconductors this  $H_C$  as a function of  $T$ , that varies as just the way we have seen so, it's  $H_C = H_C(0) (1 - T/T_C)^{1/2}$ , if you use this and calculate you will get pretty much very neat value for the specific heat now, these are empirical formulas that are not believed very well or beyond doubt but, however this for most of the conventional superconductors they seem to be following this behaviour with respect to temperature. So, from so, let's call this as equation 8, so from 7 it can be seen that at  $T = T_C$ , since  $H_C = 0$ , so  $S_N - S_S$  is equal to 0, so at the transition that is  $T = T_C$ ,  $s - n$  is equal to  $S_S$  which means that there is no latent heat so, the latent heat of transition is equal to zero so, which means  $\Delta s$  is equal to zero so, this all more I mean without any doubt establishes that the transition is a second-order however, of course away from  $T_C$  if there is a transition that could be first-order so, we have obtained form for the specific heat discontinuity and see that what the value is so, you can of course calculate from here a  $dH_C/dT$ ,  $DTA$ , take a square and multiplied by temperature and you'll get that so, this is at of course this thing and these are to be computed at  $T = T_C$  but, however we want to get it from the BCS theory let's see how to get that

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Specific heat discontinuity from BCS Theory.

$$E_k = \sqrt{\xi_k^2 + \Delta(T)} \quad f(E_k) = \frac{1}{e^{\beta E_k} + 1} = f_k.$$

$$S = -k_B \sum_i p_i \ln p_i \quad p_i: \text{probability of a microstate being occupied.}$$

$$S_{es} = -2k_B \sum_k \left[ (1-f_k) \ln(1-f_k) + f_k \ln f_k \right]$$

$$C_{es} = T \frac{dS_{es}}{dT} = -\beta \frac{dS_{es}}{d\beta} \quad (1)$$

$$C_{es} = 2\beta k_B \sum_k \frac{\partial f_k}{\partial \beta} \ln \frac{f_k}{1-f_k} \quad (2)$$

$$= -2\beta^2 k_B \sum_k E_k \frac{\partial f_k}{\partial \beta} \quad (3)$$

$$C_{es} = -2\beta k_B \sum_k \frac{\partial f_k}{\partial (\beta E_k)} \left( \frac{E_k}{\beta} + \frac{\beta \Delta E_k}{d\beta} \right) \quad (4)$$

$$C_{es} = -2\beta k_B \sum_k \left( \frac{\partial f_k}{\partial E_k} \right) \left( E_k^2 + \frac{1}{2} \beta \frac{d\Delta^2}{d\beta} \right) \quad (5)$$

So, specific heat discontinuity from BCS theory. Okay? So, once again just to remind ourselves that the quasiparticle energies are given by  $\xi_k^2 + \Delta^2$  and so, this is there and, and of course we have the quasi particle distribution function is given by the Fermi distribution function, which tells that this is like exponential  $\beta E_k$  plus one, will call this as just  $f_k$  now, to remind ourselves the entropy expression in, in terms of these probabilities or, occupation or, the distribution functions is given by so, entropy is equal to minus  $k_B$  and this is equal to  $\sum p_i \ln p_i$  this formula you will see in your first course of thermodynamics where  $p_i$  is, the  $p_i$  is the probability, of microstate being occupied. Okay? So, in this particular case so, we are talking about the electronic contribution so, just to make sure that we write it  $e$ , and corresponding to the superconducting State we'll call it  $s, es$  which is equal to a minus  $2, k_B$  there's a sum over  $k$  and there is a  $1 - f_k$ , log of  $1 - f_k$  and plus,  $f_k$  log of  $f_k$  so, that's the form for the entropy where  $f_k$ 's are the distribution functions and we have talked about these  $1 - f_k$ 's, as the probability that these, the  $k$  States are not, occupied and  $f_k$ 's are the probabilities that this case states are occupied so, we have summed over them and so, that is the expression and so now, we can define the specific heat  $C_{es}$  which is equal to  $T \frac{dS_{es}}{dT}$  now, this is can be written as minus  $\beta \frac{dS_{es}}{d\beta}$  and the reason being that will just give you a derivation of each one of these steps let me first write down the steps and then that'll be easier for, for you to get a feel so we will call this equation as equation 1 and we'll give a derivation for this and then there is a  $C_{es}$  can be written as,  $2\beta k_B$  from here, from Equation 1 it can be these are all the same equations we are just simplifying a step or one step after another but, since we have to give a derivation for each one of them so, we are naming them as different equations and so, this is equal to a  $\beta k_B \sum_k \frac{\partial f_k}{\partial \beta} \ln \frac{f_k}{1-f_k}$ , call it as equation 2 and this is equal to nothing but  $-2\beta^2 k_B \sum_k E_k \frac{\partial f_k}{\partial \beta}$  and so, called this as equation 3 let's write it here so,  $C_{es}$  is equal to so, that's minus  $2\beta^2 k_B \sum_k E_k \frac{\partial f_k}{\partial \beta}$ . Okay? And this there's a well I mean  $\frac{\partial f_k}{\partial \beta} = \frac{\partial f_k}{\partial E_k} \frac{\partial E_k}{\partial \beta}$  and this is equal to a  $\frac{\partial f_k}{\partial E_k} \left( \frac{E_k}{\beta} + \frac{\beta \Delta E_k}{d\beta} \right)$  clearly so, there is a  $\beta \frac{dE_k}{d\beta}$  and call this as equation 4 and then finally we'll, have an another equation which is  $C_{es}$ , which is equal to a minus  $2\beta k_B \sum_k \left( \frac{\partial f_k}{\partial E_k} \right) \left( E_k^2 + \frac{1}{2} \beta \frac{d\Delta^2}{d\beta} \right)$  and so, on so, this is equation 5 so, finally we have got this equation 4 which is the expression for the specific heat due to electrons for the superconducting state of course these are, very

involved expressions and we need to derive them and it will not take much effort to derive them we will simply have to you know? Do it methodically

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$$\begin{aligned}
 C_{es} &= T \frac{dS_{es}}{dT} = T \frac{dS_{es}}{d\beta} \frac{d\beta}{dT} \quad \beta = \frac{1}{k_B T} \\
 &= -\beta \frac{dS_{es}}{d\beta} \quad \text{--- (1)} \\
 S_{es} &= -2k_B \sum_k \left[ (1-f_k) \ln(1-f_k) + f_k \ln f_k \right] \\
 \frac{dS_{es}}{d\beta} &= -2k_B \beta \sum_k \left[ -\ln(1-f_k) \frac{df_k}{d\beta} + \left( \frac{1-f_k}{1-f_k} \right) \left( -\frac{df_k}{d\beta} \right) + \frac{df_k}{d\beta} \ln f_k + \frac{f_k}{f_k} \frac{df_k}{d\beta} \right] \\
 &= -2k_B \beta \sum_k \frac{df_k}{d\beta} \left[ -\ln(1-f_k) - 1 + \ln f_k + 1 \right] \\
 &= 2\beta k_B \sum_k \left( \frac{df_k}{d\beta} \right) \ln \frac{f_k}{1-f_k} \quad \text{--- (2)}
 \end{aligned}$$

so, let me derive equation number one so, CES is equal to RT DS, es DT which is equal to T, or D s e s beta D beta, DT I can write that as a chain rule which all of you know? Where so, we are writing it as full differentials and since beta equal to a 1 over KT D beta DT is nothing but minus 1 over KT square. Okay? So, that gives me this is equal to and then 1 T will cancel and we'll be left with a minus 1 over KT which can be written as minus beta D s, es, D beta so, that's equation number 1 which is what we have seen. Okay? Now, for equation number 2 you remember that we have s, es is equal to minus 2 K and there's a sum over K, 1 minus FK these derivations are important and for you to learn them is also important that's why I am doing it each step 1 minus FK and plus FK, log of F K so, D s, es D beta is minus 2 K P beta, sum over K minus log of 1 minus FK and so, this is equal to and then D FK, d beta and plus 1 minus F K, divided by 1 minus FK and, then a minus D FK, d beta please do them carefully and you know? Not making mistakes because, this if you make a mistake then and a plus RB FK D beta which Ln, FK and plus FK / FK d FK d beta. Okay? So, that sort of you know it's going to cancel things - 2 KB beta sum over K and a d FK, D beta and log of 1 minus FK minus 1 plus log of FK - plus 1 so, this one will cancel and we can write this as 2 beta, K be sum over k, d FK D beta and these two logs can become combined writing it as FK divided by 1 minus FK and if you see that this is equation number 2 this is equation number 2. Okay? So, that's, that's the same, equation that we have derived Okay? So, this is equation 1 and, this is equation 2, let us look at the other things as well

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$$\begin{aligned}
C_{es} &= T \frac{dS_{es}}{dT} = T \frac{dS_{es}}{d\beta} \frac{d\beta}{dT} \quad \beta = \frac{1}{k_B T} \\
&= -\beta \frac{dS_{es}}{d\beta} \quad \text{--- (1)} \\
S_{es} &= -2k_B \sum_k \left[ (1-f_k) \ln(1-f_k) + f_k \ln f_k \right] \\
\frac{dS_{es}}{d\beta} &= -2k_B \sum_k \left[ -\ln(1-f_k) \frac{df_k}{d\beta} + \left( \frac{1-f_k}{1-f_k} \right) \left( -\frac{df_k}{d\beta} \right) + \frac{df_k}{d\beta} \ln f_k + \frac{f_k}{f_k} \frac{df_k}{d\beta} \right] \\
&= -2k_B \sum_k \frac{df_k}{d\beta} \left[ -\ln(1-f_k) - 1 + \ln f_k + 1 \right] \\
&= 2\beta k_B \sum_k \left( \frac{df_k}{d\beta} \right) \ln \frac{f_k}{1-f_k} \quad \text{--- (2)}
\end{aligned}$$

So, now you have  $f_k$ ,  $1 - f_k$  is equal to nothing but, exponential minus beta  $E_k$  log of exponential minus beta  $E_k$ , it's equal to  $X$   $1 - \beta E_k$  and so, this is of course equation number 3 and the equation number 4, is we need to derive it with then it's a day  $f_k$   $d\beta$  can be written as  $df_k$  and there is  $d$  of, beta  $E_k$  and then it's a  $df_k$ ,  $d\beta$  if you take beta  $e^{-\beta E_k}$  is equal to  $X$ , then  $d f_k$   $d\beta$  or  $\frac{df_k}{d\beta}$   $d\beta$  does not matter so, this is equal to  $df_k$ ,  $dX$  and,  $dX$ ,  $d\beta$  which is equal to  $df_k$ ,  $d$  of beta  $E_k$  and  $E_k$  we are trying to sort of do a derivation backward and so, this is equal to minus 2 beta square  $k$  and sum over  $k$  that's equal to so,  $E_k$  and  $\frac{df_k}{d\beta}$   $E_k$  so, this is an  $e_k$  plus a beta  $dE_k$ ,  $d\beta$  so, that's equation number 4 and so, this is equation number four and we know, that your  $df_k$ ,  $dX$  is equal to  $d$ ,  $f_k$ ,  $dY$  and a  $dy$   $dX$  so, if you take your  $x$  equal to beta  $E_k$  and your  $y$  equal to  $e^{-\beta E_k}$  then your  $df_k$   $d\beta$   $e^{-\beta E_k}$  equal to  $d f_k$   $d\beta$   $e^{-\beta E_k}$  and  $dE_k$ ,  $d\beta$   $E_k$  it's equal to  $1$  over, beta  $d f_k$   $dE_k$  so, so that is, is there and then of course your beta  $E_k$   $dE_k$   $d\beta$  this is  $e^{-\beta E_k}$  equal to equal to beta by 2  $e^{-\beta E_k}$   $\Delta^2$  by  $d\beta$  and your well this is  $d\beta$  and because, your  $E_k$  is equal to  $\Delta^2$  by  $\Psi_k^2$ . Okay? So, basically this is all so, this is equation number five and this is what we have derived if you if you look at it here this is that equation that we have derived now what's the meaning of this equation and what it really tells us is that the first term is due to the usual quasiparticle contribution to the specific heat of electrons. Okay? The second term is important it term would not have been there if there is no gap or the gap does not depend upon temperature this comes because, the gap has a dependence of temp on temperature. So, that's why the specific it now derives a contribution from two terms one is the usual from the quasiparticles the other from the temperature dependence of the specific the gap.

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(i) At  $T \ll T_c$   $C_{es} \sim T_c e^{-T_c/T}$

(ii) Near  $T_c$   $\Delta(T) = 0$ , we can replace  $E_k \rightarrow |\xi_k|$ .

First term  $C_{es}(T) = \gamma T = \frac{2\pi^2}{3} N(\xi_F) k_B^2 T$

(iii)  $\frac{d\Delta^2}{dT}$  is finite below  $T_c$ . (large)  
is zero above  $T_c$ .

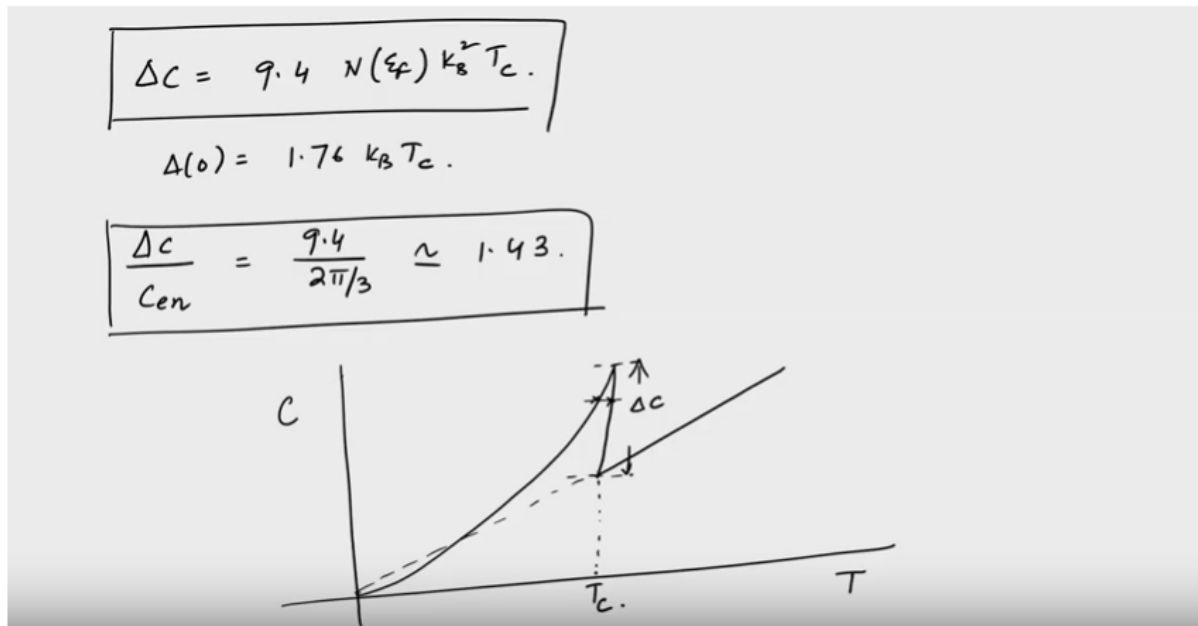
$$\Delta C = (C_{es} - C_{en})|_{T=T_c} = N(\xi_F) k_B \beta^2 \frac{d\Delta^2}{dT} \int_{-\infty}^{\infty} \left( -\frac{df}{d|\xi|} \right) d\xi$$

$$= N(\xi_F) \left( \frac{d\Delta^2}{dT} \right) |_{T=T_c} \quad \frac{df}{d|\xi|} = \frac{df}{d\xi}$$

$$\Delta(T) = 1.74 \Delta(0) \left( 1 - \frac{T}{T_c} \right)^{1/2}$$

So, as so, basically just we are almost completion towards you know, nearing completion for the calculation of for the specific heat discontinuity. So, at  $T$  much smaller than  $T_c$  your  $C_{es}$  is exponentially small it goes as  $T_c$  exponential minus  $T_c$  over  $T$  where  $t_c$  is much smaller than I mean  $T$  is much smaller than  $T_c$ . Okay? So, near  $T_c$   $\Delta(T)$  goes to 0 so, we can replace  $E_k$  by  $|\xi_k|$ . Okay? So, the first term first term in the expression of the specific heat which we told about the quasiparticle the first term that boils down to  $C_{es}$  the second term of course is equal to 0 the so, so this is equal to it's not so, the second term of course has a value and this is equal to a  $2\pi^2$  square by 3  $n$   $E_F$  and  $k$  squared  $T$ . Okay? So, that's a linear contribution to the specific heat which we are a very well exposed to in the context of the metallic specific heat. Now the second term that is the  $D \Delta^2$   $\frac{d\Delta^2}{dT}$  which is this term the second term in equation 5 is  $d \Delta^2 \frac{d\Delta^2}{dT}$  is actually finite is finite below  $T_c$  and in fact it's basically it's large not only finite it's large because, the variation of  $\Delta$  with respect to temperature is very sharp there but this is equal to 0 above  $T_c$  when  $\Delta$  completely vanishes altogether. So, so now this gives rise a discontinuity so this term gives rise to a discontinuity because, this is finite just below  $T_c$  and its zero just above  $T_c$  so,  $\Delta C$  which is equal to  $C_{es} - C_{en}$  which is at  $T$  equal to  $T_c$  its equal to  $n$   $\epsilon_F$   $k_B \beta^2$   $\Delta^2 \frac{d\Delta^2}{dT}$   $\int_{-\infty}^{\infty} \left( -\frac{df}{d|\xi|} \right) d\xi$  this I has to be in mod now of course this is equal to  $n$   $\epsilon_F$  and minus  $D \Delta^2 \frac{d\Delta^2}{dT}$  at  $T$  equal to  $T_c$ . Now basically what we're we have used that  $\frac{df}{d|\xi|}$ ,  $\frac{df}{d|\xi|}$  is same as  $\frac{df}{d\xi}$  which is an even function so now if you use this formula that we have said that  $\Delta(T)$  is equal to one point seven for  $\Delta(0)$  one minus  $T$  by  $T_c$  whole to the power half and calculate this  $D \Delta^2 \frac{d\Delta^2}{dT}$  evaluated at  $T$  equal to  $T_c$  then these

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Delta C can be computed which comes out as nine point four n epsilon F and KT square PC so, that's the value of the discontinuity So, this is the magnitude of the discontinuity where of course your Delta zero as told earlier during the discussion is equal to KT C. So, the normalized discontinuity if you are interested in that that is the discontinuity divided by the value of the metallic specific heat which is a well known quantity so, this is equal to that - PHI square over three value which is what we have just seen here this 2 phi square by 3 and this becomes equal to 9 point 4 divided by 2 pi square by 3 and this is equal to 1 point 4 3. So, this is the value or rather the important result that is important in the context of experiments that is measurable that this is the jump in the specific heat and what have is that if you plot the specific heat and as a function of T it as I told earlier that it grows like this it drops like this there's a vertical drop I'm so sorry this has to be shown and then of course it goes like this. Okay? So, this is this is at TC. Okay? And there is a this discontinuity here is the Delta C and if you want this discontinuity to be you know to be divided by the Cen which has a value where is the normal specific heat goes like this the dashed line and then inter. So, the normal state specific heat goes like this so, basically this you know is a hallmark signature of second order phase transition so, basically you see that if there is that these two values just below TC and maybe just above TC you. Right? I mean give the you increase the temperature but the specific heat acquires the same value and this is the fact that the, the latent heat of course is not there and it's a feature of a second order phase transition. So, we have comprehensively given an account of what would be the experimental verification of superconductivity that is how super conductivity would be realized and of course there are many other things like a field cooled and is it zero field cool protocol that we have given earlier but importantly the Meissen effect which is basically the diamagnetism and the specificity discontinuity, along with a sharp drop of, the resistivity below TC, is our, what characterized superconductivity.

