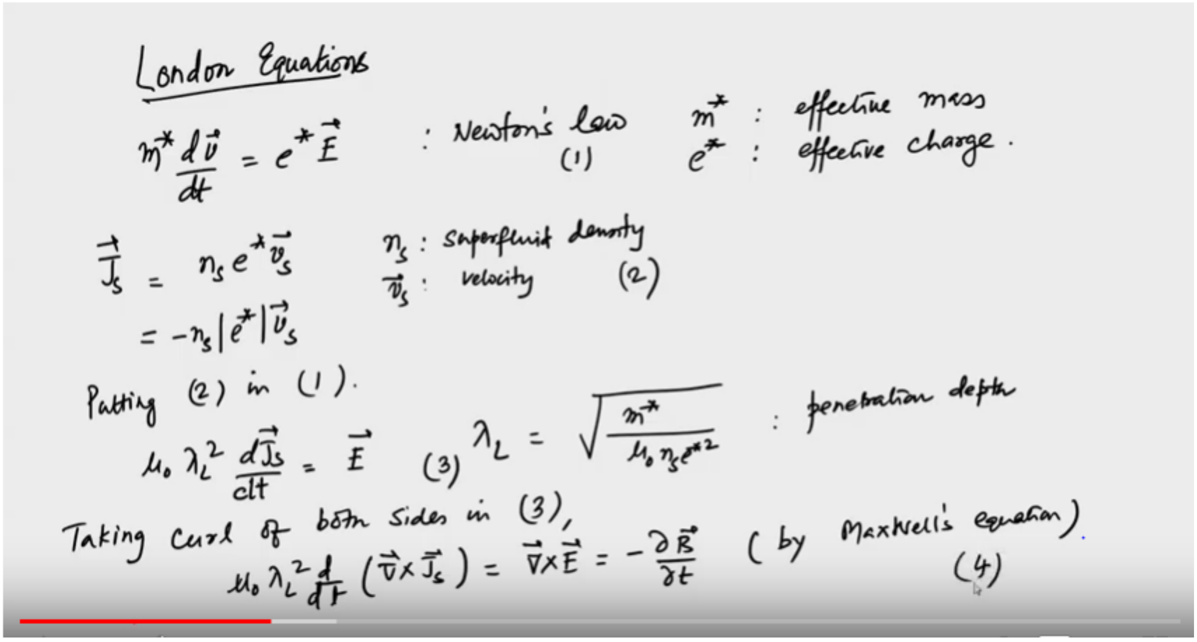
Lecture 06

London Equations, Flux quantization

\

So, good morning all of you. Let us look at the flux quantization, which is a very important concept, in the context of, superconductivity today. And in order to understand flux quantization, we will have to go back to the, London equations. The equations that we have learnt a little bit in brief, on this effect of electromagnetic fields, in superconductors. So, let's start with London equations and then we'll go over to the flux quantization.

Refer slide time :(01:04)

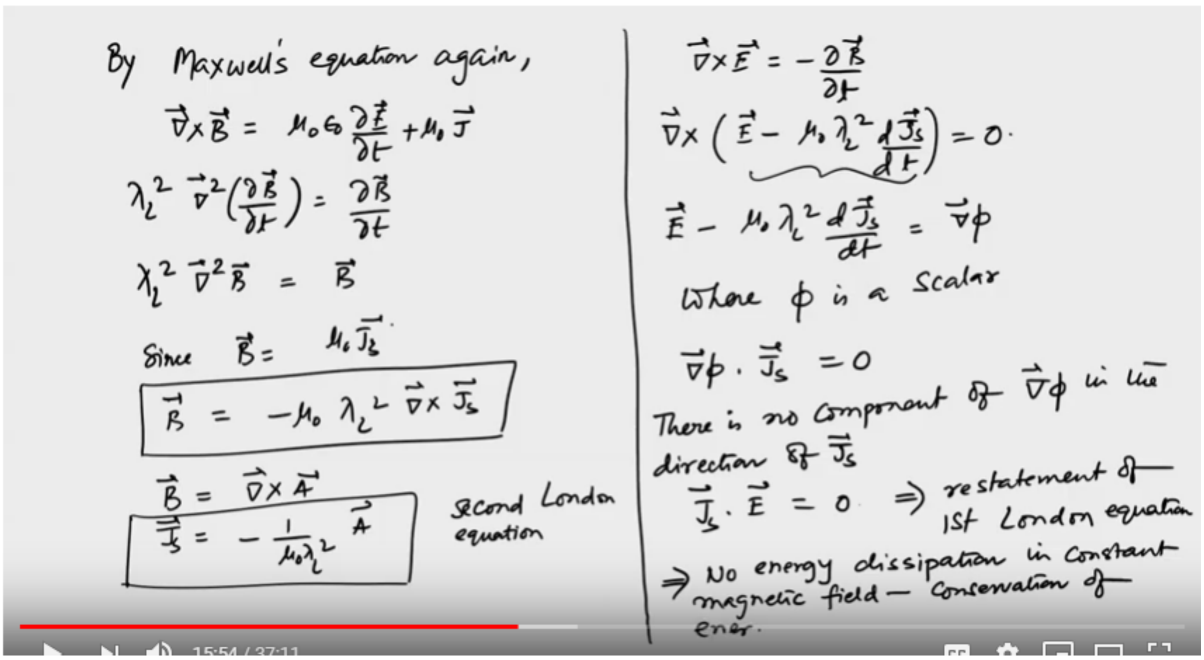


So, to start with London equations. So, what's the condition for the transport of charges, even if they're interacting amongst each other? So, if the charges are interacting amongst each other, they should create resistance. So, they should give rise to resistance. So, how is that that we are encountering a situation in which the current is flowing without any resistance, which means that the electrons would stop interacting with each other? We of course know partly the answer to this question. But here, what London and brothers London brothers actually they have suggested, is that, whenever there is a slowing down or there is a resistance that arises, due to the interaction of electrons, among the electrons, then there is a external electric field, that would overcome, the, this effect of the interaction and would eventually give rise to a resistance less motion. So, in order to, see that, let us write down Newton's laws for a charged particle, in an electromagnetic or rather in an electric field here. So, we write it with, M star, which is the effective mass of the particles that are contributing to the motion, which we know that, there are Cooper pairs or we are going to see more elaborately, the formation of Cooper pairs and so on. And then there is so, this is the Newton's laws which says that so, this is nothing but the Newton's laws for a charged particle of charge East our of course you can relate that charge Eastern is nothing but equal to twice of the electronic charge and this electronic charges.

It's as you know that it's a, its two EE or the Cooper pair but that will come to later let us simply look at this equation and try to understand that what it what it does? Or what it gives? So, m star is the effective mass and e star is of course the effective charge. Okay? And now the current density or the super current density is written as j s is equal to n s e star and V s this is all familiar to you j s is the current density n s is the super electron density, or the super fluid density as it's called basically it's a density of super electrons and V s is the velocity of these super electrons and of course Easter is just the electronic charge .so, that is the, the current density equation that one would get and we can write it using a mod star here of V s and the minus sign is just to make sure that the current density is actually in a direction which is opposite to the direction of the, the flow of the super electrons. Okay?

So, if you call this as equation one and call this as equation 2 so, if you put two into one, one would get the first London equation which is of the form that mu zero lambda L square a DJ s DT is equal to e where this lambda L is equal to m star divided by mu zero NS e star square and it's called as the, ‘Penetration Depth’. And we know that physically what it means is that the depth or the distance are two till in which the magnetic field can penetrate into the sample or more precisely this is the distance over which the magnetic field actually penetrates and falls into a value of 1 over e of the value that it has at the surface. So, it’s called as a penetration depth and it's one of the most important length scales of a superconductor. And m there's another length scale that we have looked at which is called as a, ‘Coherence Length’. And so, this is the equation that we get and let's call this as equation maybe three and then of course if I take the curl of both sides of equation three so, taking curl of both sides in 3 we have Muse mu zero lambda L square and a Del, Del T of curl of j s it's equal to curl of e which is nothing but equal to minus of del by Del T by Maxwell's equation. And now this requires minute of discussion here that you see there was a DJ DT or we can simply write it as DDT of that instead of writing. So, this is a DDT now you see that we need to take a Curl, Curl is a space derivative and because, there's a time derivative we can interchange the space and time derivative so we have interchanged that and I have written the curl inside and which is taken directly on j s and the DDT has come out. So, this call it as equation four

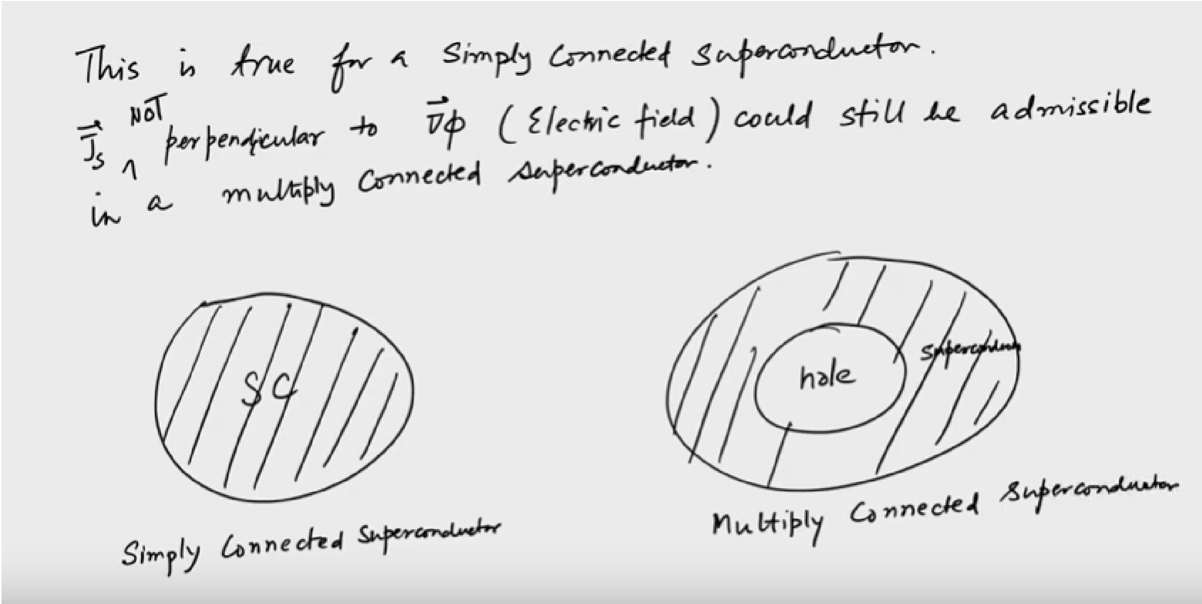
Refer slide time :(08:04)



Let us look at what happens to this the right-hand side of the equation by using Maxwell's equation again, we can write down the curl of B this is equal to a mu 0 epsilon 0 del e del T and a plus mu 0 J and if we use this then we can write down this lambda L square del square of del b del T this is equal to a del V del T and this equation since it's true, for the rate of change of magnetic field with time it should also be true for the magnetic field itself and we can write this down. So, this is a special case

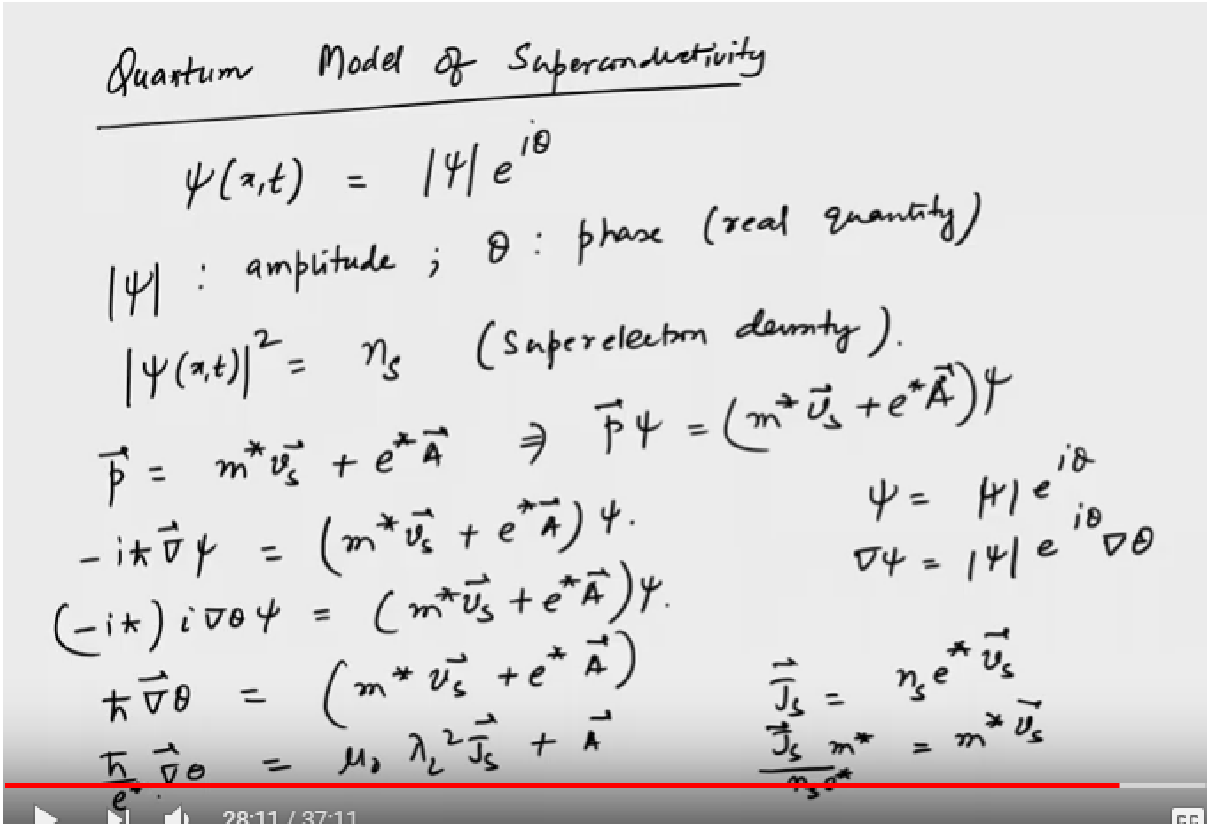
that we are considering that where the rate of change of magnetic field is actually taken to be same as the magnetic field. So, we have ignored or rather we have taken this the value of the magnetic field at the surface to be equal to 0. And in which case we can write down the same equation as for simply B and that is written as so, let's write it with this so, it's a this is equal to B and since B is equal to MU 0 J s. So, this B actually looks like it's equal to minus mu 0 lambda L square curl of J s. Okay? So, this is the equation this is one of the London's equations where the magnetic field is expressed in terms of the current density of the curl of the current density. So, if we introduced introduce the magnetic vector potential which is defined by be equal to curl a and then of course the j s can be written as minus 1 by mu zero lambda L square e because, of this of course that your B is equal to curly. So, that is the relation between the J s and E so, this is basically called as the second London equation, equation and just to rectify that we have not written this one was written as the first this one is written as a first London equation all right. So, these are the two learning equations that we get but what are they good? For what do they convey? And can we extract some more information out of them that's the question. So, of course we know that curl of E is equal to minus del be del T and the curl of so, e minus mu 0 lambda L square del J s del T or we can write it as DJ as DT as well.
So, this is equal to 0 so, which means that if a curl of vector is equal to 0 that vector should be written or rather we one should be able to write that vector in terms of a gradient of a scalar quantity. So, e - e - mu zero lambda L square we'll call it DJ s DT just to make matters simple let's change that as well here. And so, this is equal to nothing but a Phi where Phi is a scalar. So, once that is done one can easily understand that the grad phi dot j s it's equal to zero because, of the reason if i take a dot product with the current density the or the super current density that should give me equal to zero. So, that tells that there is no component of grad phi in the direction of J s. Okay? And so, which means that the J s dotted with the electric field should be equal to zero and which is basically a restatement of nothing but the first London equation. Okay? And this is basically it looks very reasonable for the reason that, that there's no the energy dissipation is zero, in a constant magnetic field, which should be correct, because, otherwise it would have violated conservation of energy. So, this in a way, it says that, no energy dissipation, in a constant magnetic field. That is because, the magnetic force is, is a low-range fortress, which goes as V cross B and the energy dissipation is called as a, power. And the power goes as, f dot V so, if V cross B is dotted with V, then that has to be zero, this is a common wisdom, and this is basically saying the same thing. So, and it says that the conservation of energy is valid or A very important thing at this moment this is strictly true for what is called as a simply connected superconductor. Okay?

Refer Slide Time :(16: 10)



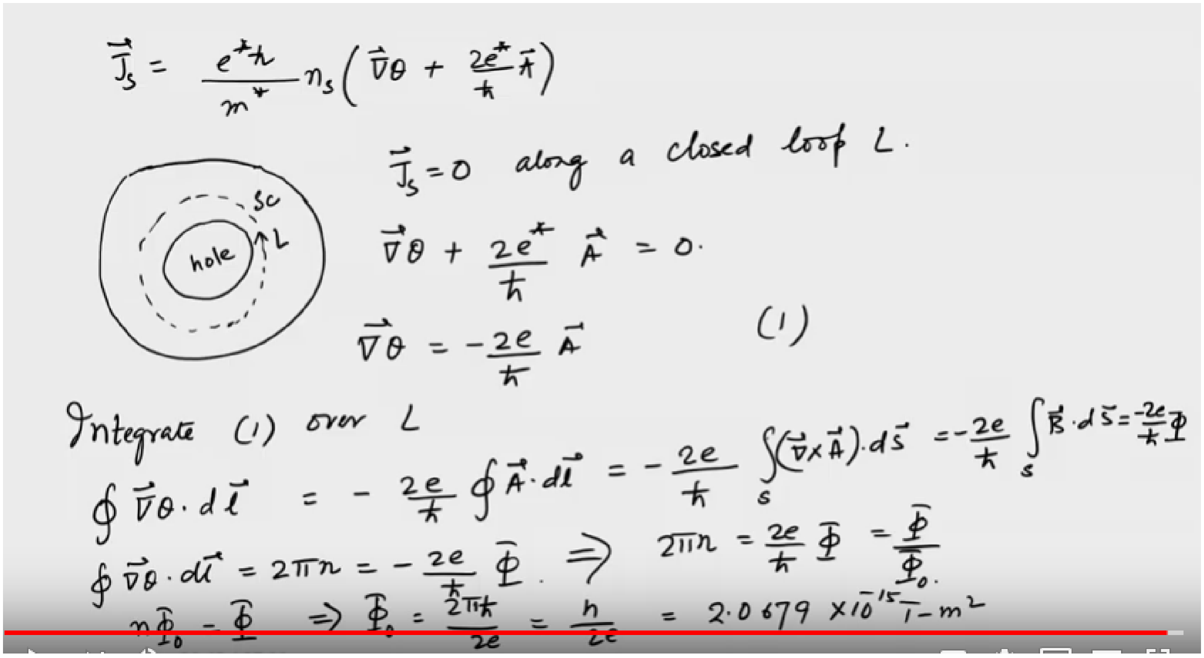
so, let me write that in big and bold, if we have a multiple connected superconductor I will just tell you what multiply connected superconductor means, in a multiplicative superconductor there could be a component of e that is if we do, it with the super current density it may not still give it equal to zero which means that, the grad Phi, perpendicular to J s, may still be admissible for a multiplicand acted superconductor so, let me write that once again so, J s perpendicular to grad Phi, which is nothing but the electric field, or say that not perpendicular to grad Phi, could still be admissible in a multiplicand acted superconductor so, this is one of the important statements that we make here, and in order to justify that and get a connection with flux quantization, we'll define what a multipliconnected, superconductor is so, this if this superconducting specimen so, this is a superconductor everywhere . Okay? So, it's there is no hole there is nothing so this is called as a simply connected superconductor and there is a hole there so, this region where I write hole is not superconducting so, I put a hatched area to denote that it's superconducting so, this area is superconducting and the whole area is not so, this is anything other than superconducting so, this is called as a multipliconnected, superconductor so, why did we introduce this in the first place the reason that we introduced it, is that we wanted to show flux quantization, that if there is a there's a hole in a superconducting region and the superconductor is called as a multiplicand acted superconductor put it in a magnetic field, the flux lines, will thread or penetrate through this hole and is cannot, take any value it takes only values that are perpendicular sorry, that are quantized, that are quantized in some known quantum which is called as a flux quantum will tell you the value of that and so, there could be one flux quantum, or there could be two flux quantum, or there could be three flux quantum but, nothing in between this there's no half, or one-and-a-half flux quantum and so on, so that's one the other thing is that what in line with the preceding, discussion we Have said that the grad Phi, dot J s is equal to zero, or rather the e dot J s is equal to zero, so basically there is no energy dissipation and E and J s, are always perpendicular to each other and that's why the dot product vanishes because you know? The dot product comes with of two vectors that come with a cos theta, theta is the angle between, the two vectors and if theta is equal to 90, degree then of course the angle goes to so the COS, of that angle that becomes zero and so the whole quantity vanishes so here, we want to show that also, in a multiplicative superconductor it the e and J's, they don't have to be necessarily perpendicular now, in order to show both of these let us look at the basically we'll just we have looked at enough, quantum model of super conductivity and going to see enough of that in the future but, however just to formally once again write it.

Refer Slide Time :(21: 52)



so, we'll say quantum model, of super conductivity and very interestingly this has relevance with the fact that the quantum mechanics is usually seen in microscopic and, sub microscopic scales, or rather very small length scales and inside, an atom and maybe inside, a nucleus and so, on however super conductivity is one such example in which quantum phenomena, or quantum mechanics, manifests itself in macroscopic scales. Okay? So, we can have a big superconductor which could be you know? Extending over many, many sort of coherence length or penetration depth and still the quantum formalism, or the formulation of quantum formulation, of superconductivity still holds and we can write down a wave function, corresponding to that quantum regime and let's call that wave function as si, of X T, where it is written as an amplitude and a phase where theta is called as a phase so, theta si, is the amplitude and theta is actually called as a phase and it's a real quantity. Okay? But, what is the physical significance of that the mod size square, is the super current density, or the super electron density as we know so, in a homogeneous superconductor NS, is does not depend upon the space variable and in an inhomogeneous superconductor it can it, can depend upon the space variable or space coordinates so, let's write down the canonical momentum, for a particle of charge East star and, mass M star so, that is given by P equal, to M star vs. plus a e star A, so this is the in an electromagnetic field, where a is the vector potential of course we can choose a gauge in which Phi equal to zero and you know? Also, the divergence of a equal to zero so, this is can be written as a minus I h cross, del psi, by writing P as so, this can be you know? In a way this can be operated on psi, and this m star Vs, plus e star a si, and so, on and then you write P is equal to minus IH cross del, so this becomes equal to minus IH cross del, C which is equal to m star Vs, plus a East are a si, and so on and now, then of course your psi, is equal to as written above its equal to psi, to the power I theta, so del psi, is nothing but, equal to psi, e to the power I theta, del theta and, and at this moment we are assuming, that we are talking about a homogeneous superconductor and in which psi, is this more amplitude is independent of position and it's only the phase that depends on the special coordinates so, if you put it here, then it becomes equal to a minus IH cross and there's AI Delta Theta and psi, this is of course equal to a M star, Vs, plus e star a psi, if you do a simplification then that becomes equal to because, your minus I, into I will become equal to 1 H Delta Theta so, this is equal to M star, V S Plus, e star, A and this is psi, well I mean I've taken off psi, from both the sides and this can still be written as H cross, by e star, Delta Theta this is equal to a mu 0 lambda L square, J s + e, that's because the J s is equal to NS, e star, V s that's the equation for or the defining equation for the current density so, the J s / NS, e star into M star, is equal to M star, V s and then I can introduce this lambda L, which was defined earlier by, this equation by this lambda L square is equal to M star divided by mu 0 NS, s square or e star square and that's is going to give me this and so, this is basically so, we can write down the so, this is an equation that is coming from the London equations and that's why all this exercise of explaining this London equation is being done, let's see what it takes us from there

Refer Slide Time :(28: 25)



So, I can write down the J s, it's equal to e star, H divided by M star, ns and there is a del theta and plus a to e star by H, by H cross and A sorry, the A, has to be in the numerator A and so on I believe this is H cross one can check, that so this is the expression for this the current density, or the super current density just to remind you that let me also write down the lambda L square, is equal to M star mu 0, NS e star square . Okay? So, that's the that's the equation that one gets and so, J's becomes equal to this and we're of course all these things are very familiar to you, theta is the phase of the wave function so, we are getting a gradient of the phase and let us see that how this gives rise to the quantization let us take a multiplicative superconductor like this. Okay? Which has got of course this closes smoothly do not so there is a hole there. Okay? So, this region is superconducting and this region is a hole, let me take a path, I draw it by a dashed line a path. Okay? And this path let me, call this as L and of course because, it's in the superconducting it falls into the superconducting specimen the J s, should be equal to zero here, because otherwise of course so, this J s should be equal to zero, along this line or inside the superconducting sample and that tells that so, along a closed loop l so, in since we have taken a closed loop, the net superconducting current density along the closed loop, would be equal to 0 which means the coefficient cannot be equal to zero so, the term that is there on the. Right? Inside the bracket should be equal to zero, which says that delta theta plus a two, e star, over h cross a has to be equal to zero, which says that delta theta has to be a minus, 2 e by H cross A and that is equal to that so, this is the variation of the phase and its relationship to the magnetic vector potential so, if you integrate, let's call this as equation 1 here so, if you integrate 1, over L then one can get so, it's a closed integral so it's a gradient of theta dotted with DL, so we take a small element of length along that capital L, or that contour the dashed contour is equal to a DL, which is nothing but equal to minus 2 e by, H cross and this is equal to a dot DL so, I can use Stokes theorem, which says that the line integral, or the closed line integral, of any vector can be written as the curl of the, the surface integral of the curl of the vector which means this is equal to minus 2 e, by H cross and this is equal to a curl, of A, d s where this s is basically, the surface area of the I mean where the contour is found by, L and s is the corresponding surface area now since you know? That a curl of a equal to B so, this is equal to minus 2 e by H cross and this is a B dot, d s and B dot d s, is nothing but the magnetic flux so, this is equal to a minus 2 e, by H cross and a Phi, which is called as a magnetic flux, let's write it with a capital Phi so now, remember that our theta is actually the phase of the wave function now, we always demand that the wave function is a single valued quantity Okay? Wave function cannot be multiple valued unless, we make a full circle of the phase, otherwise it will lead to the ambiguity, in the probability, density if we do not enforce the single valued nests of psi, so this is then equal to if you enforce single valued nests of size so this is equal to 2pi, n it's equal to minus e by 2 H cross Phi, that is coming from the, the top line so, that tells that so this means that 2 pi, n equal to 2 e over H cross, Phi it's equal to Phi over Phi 0, where of course we have written that n Phi, 0 it's equal to Phi so, the flux through the hole that you are seeing, should be quantized in terms of a Phi 0, certain Phi 0, and it can only take values n equal to 1 and equal to, 2 n equal to, 3 and so, on n equal to 0 is a special case where we have a simply connected superconductor and this of course gives that the Phi 0, value is equal to 2 phi, H cross over 2 e, which is equal to H over 2 e and if you put, the value of H and E we comes out to be H is of course you're, the Planck's constant which is six point, six ten to the power minus 34 joule-seconds, is equal to one point six, ten to the power minus, 19 coulomb, all these things you will find in any of the data book that one comes across and this is equal to a two point, zero six seven nine into ten to the power minus 15, Tesla meter square so, this clearly shows that the flux that can pass through the hole, is quantized and the quantum of flux, which is an important quantity the quantum of flux is this value two point, zero six seven nine ten to the power minus 15 Tesla, meter square it has of course the magnetic field multiplied by the area that dimension and so, these are experimentally observed and this comprehensively shows, that electronic the, the e Star, that takes part in this particular calculation is nothing but equal to 2e. So, which says that there are two, electrons that are responsible and that are relevant for these you know? The carrying of the super current and which we know, are called as the, either you call it them as, ‘Super Electrons’, we know that they are called as the,’ Cooper pair’.