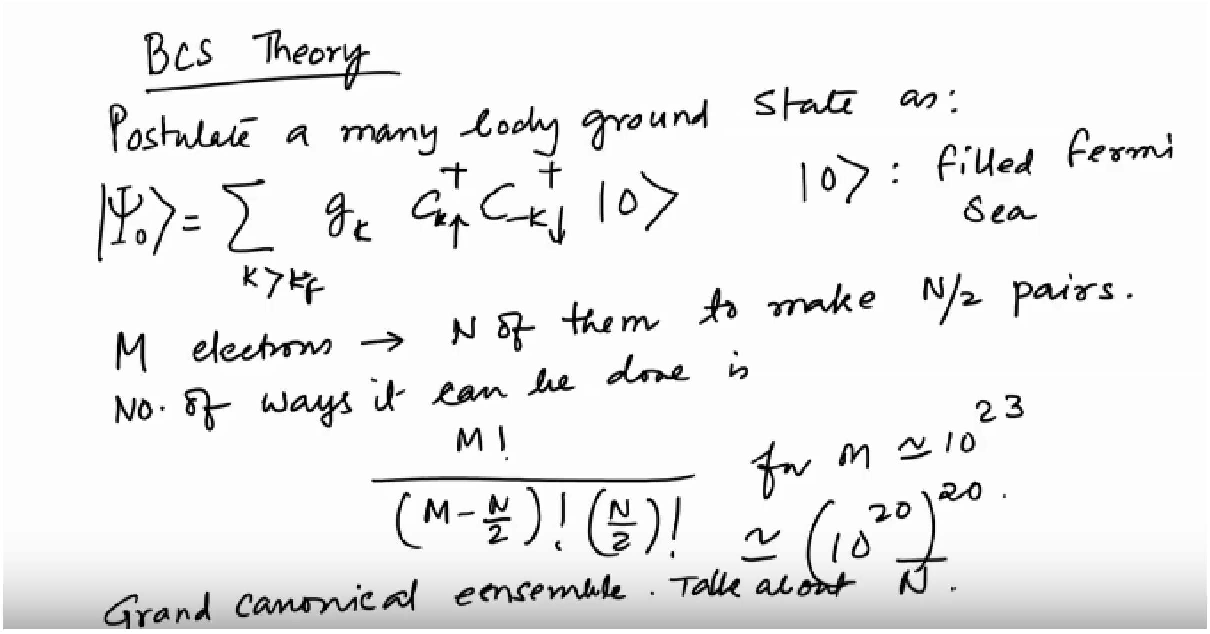
Lecture - 4

BCS ground state, variational calculation, expression for Tc

So having laid the foundation of BCS theory, that is how, Cooper established that there could be a electron, electron attractive interaction, mediated by phonons, let us go over to start,

Refer Slide Time :(0:47)



BCS theory and before, we start at least tell that, it involves some mathematics, that is algebra, which you should do? Because, every step cannot be shown in the in the class so, go through the steps, yourself, before you convinced, that the results, that we are coating, are correct. So, postulate many-body ground state, as this is a sum over, so this is like sigh zero and this is all K, greater than KF and there is a GK k up dagger C minus K down dagger, acting on the zero, here zero is not vacuum but, it's the filled, filled Fermi sea, GK is some amplitude of the wave function and ck up dagger C minus K down daggers at 2 is a pair, is a pair created with ,momentum K and minus K ,with an up and down spins and this is a many-body wave function ,a superconducting many-body wave function and then we have ,say m electrons and we want to choose, n of them to make n by 2 pairs and the number of ways it can be done, is M factorial, divided by, M minus n by 2 factorial and n by 2 factorial, for m equal to 10 to the power 23, this combination is equal to 10 to the power 20, to the power 20 .so, in principle we have to solve for these many GK's, in order to be able to write down ,a proper many-body state and which is an impossible task, what can be done is that? One can treat the problem statistically and in order to do that, one can also take a grand canonical ensemble, such that we don't keep the particle number of fixed and instead talk about an average number of particles. So, we shall talk in grand canonical ensemble. And talk about average n, we call it as n bar, instead of the number of particles.

Refer Slide Time :(4:35)

Bcs ground state:

$$|\Psi_G\rangle = \prod_{\vec{k}_1, \vec{k}_2 \cdots \vec{k}_M} \left(u_{\vec{k}} + v_{\vec{k}}\, c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} \right) |\phi_0\rangle$$

$$(\vec{k}\uparrow, -\vec{k}\downarrow) \longrightarrow |v_k|^2$$

$$(\quad " \quad) \text{ unoccupied} \longrightarrow |u_k|^2$$

$$|u_k|^2 + |v_k|^2 = 1$$

$\vec{k}_1, \vec{k}_2$

$u_{k_1} u_{k_2} \rightarrow$ No pairs

$u_{k_1} v_{k_2} \rightarrow$ 1 pair $(k_2, -k_2)$

$v_{k_1} v_{k_2} \rightarrow$ 2 pairs $(k_1, -k_1)(k_2, -k_2)$

So, Bcs ground state, BCS rolled down the ground state as, sigh G, it's equal to a product of all these K 1 ,K 2 and K n, or say m and this is UK + VK, ck up dagger, C minus K down dagger and acting on a Phi 0. So, this is the filled pharmacy, as we have been talking about. So, the probability of a pair to exist, so a pair formed off K up and minus K down, to exist is given by V K square and that this is unoccupied, is given by U K square, that's .Right? it has subscript and of course, the normalization says that the U K square plus, V K square, should be equal to 1, which means that the probability, that K up and minus K down, would be occupied, ur occupied or unoccupied and the total probability is equal to one. state two states K 1 and K 2 ,so the amplitude with UK 1 ,UK 2, represent no pairs, in these two states UK 1, VK 2 1 pair K 2 and minus K 2 and VK 1 VK to clearly distinguish or use and Vees there are two pairs for K 1 minus K 1 and K 2 minus K 2 both are occupied so, these are the notations so, a pair would be unoccupied with a probability UK mod square and a pair would be occupied would be given by V K square mod V K square

Refer Slide Time :(7: 18)

Average no. of particles $\bar{N} = \langle \sum_{k,\sigma} \hat{n}_{k\sigma} \rangle$

$$\bar{N} = \langle \tilde{\Psi}_G | \sum_k c^\dagger_{k\uparrow} c_{k\uparrow} + c^\dagger_{k\downarrow} c_{k\downarrow} | \tilde{\Psi}_G \rangle$$

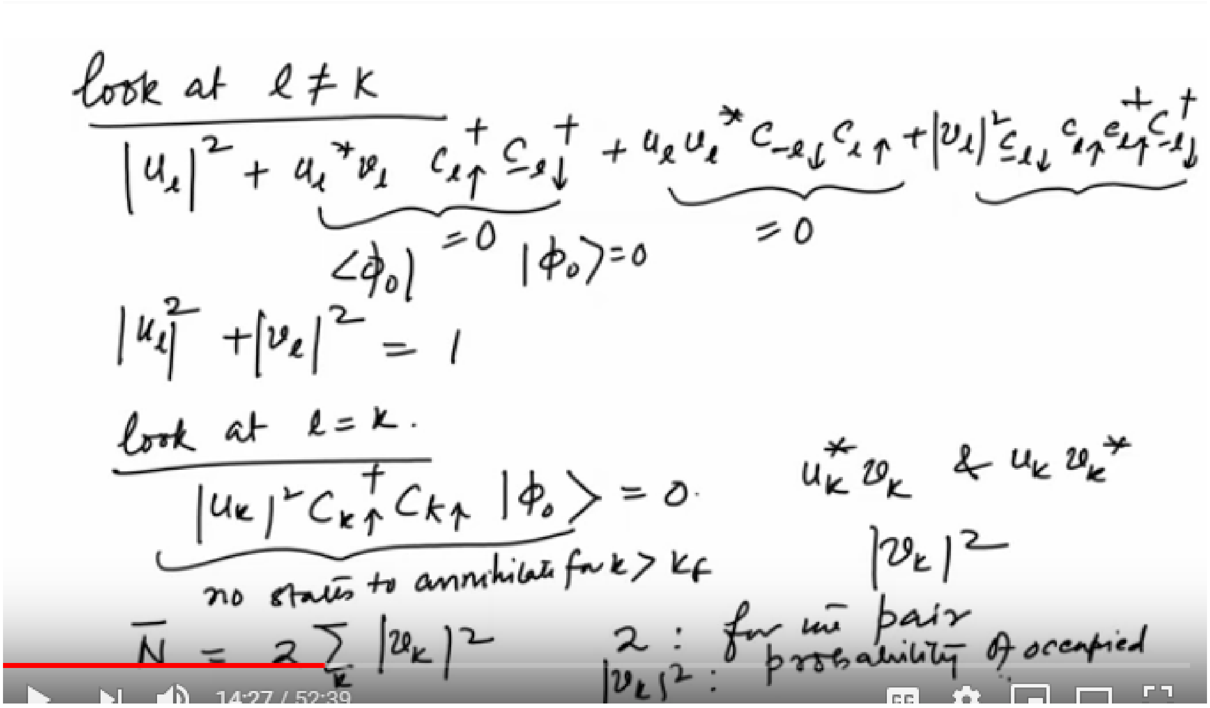
$$= 2 \langle \tilde{\Psi}_G | \sum_k c^\dagger_{k\uparrow} c_{k\uparrow} | \tilde{\Psi}_G \rangle$$

$$= 2 \sum_k \langle \phi_0 | (u_k^* + v_k^* c_{-k\downarrow} c_{k\uparrow}) \underbrace{c^\dagger_{k\uparrow} c_{k\uparrow}} (u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}) \prod_{k \neq l} (u_l^* + v_l^* c_{-l\downarrow} c_{l\uparrow}) (u_l + v_l c^\dagger_{l\uparrow} c^\dagger_{-l\downarrow}) | \phi_0 \rangle$$

$$\langle A\phi | \psi \rangle = \langle \phi | A^\dagger | \psi \rangle$$

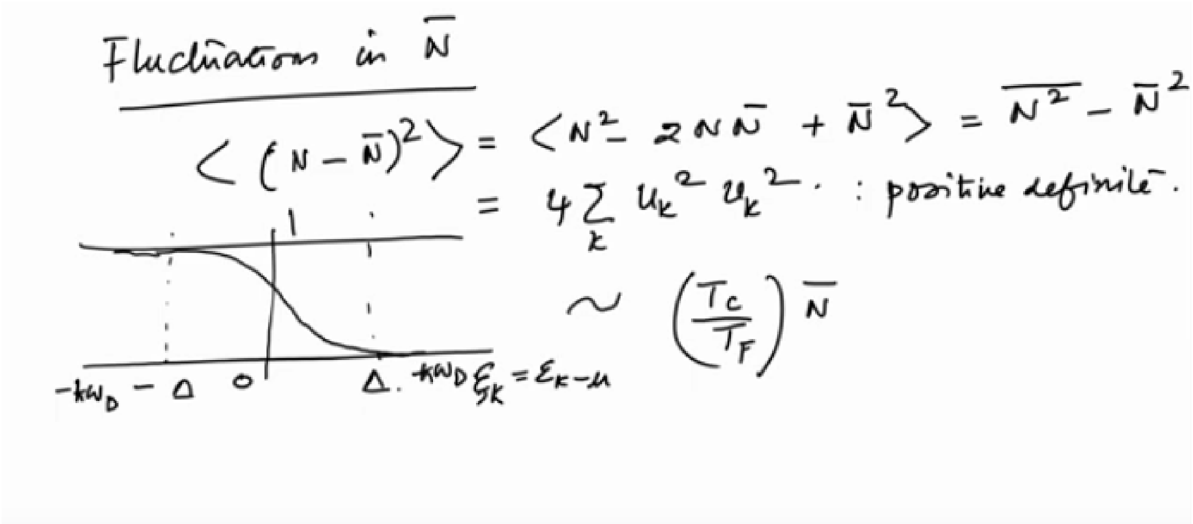
so, average number of particles n/bar, this is equal to so, the sum over k Sigma, and NK Sigma which is an operator and this is equal to we can write this as a sigh G, sum over k, ck up dagger, ck up that's the number operator for up, spin and now we'll have also sum over spins so, ck down dagger, ck down and this expectation has to be taken between the ground state that we have written here, that's the ground state postulate of the VCS ground state so, we have to take this thing here and this can be written as because there is no preference over one spin on another, so we can simply write this as twice, of size g sum over k, ck up dagger, ck up and aside g, this I am writing it once but then later on I'll skip so this will be a 5-0 UK star, plus V K star, C minus K, down CK up, that's, that's the size G, here the brass IG and now I have a ck up dagger, CK up and now I have a UK plus, a VK, ck up dagger C minus K down dagger and now, I'll have terms which are so, this is the same case as it's there in the operator NK Sigma and now, I will also have all the other terms in which K is not equal to or rather key, is not equal to L, all the other indices which are not same so you L star, plus a VL star, C minus L down, C L up and now I'll have a UL plus, a VLC L up dagger, C minus L, down dagger and there is a 5- 0 here, Okay? So, this corresponds to K not equal to L and the top one is for k equal to L so in principle UK and, VK are generally complex quantities so, let us write that UK and, VK are generally complex that's why the stars are written separately and so what we have done here is that we have used, a Phi, Shi as a Phi a dagger side .Okay?

Refer Slide Time :(10: 58)



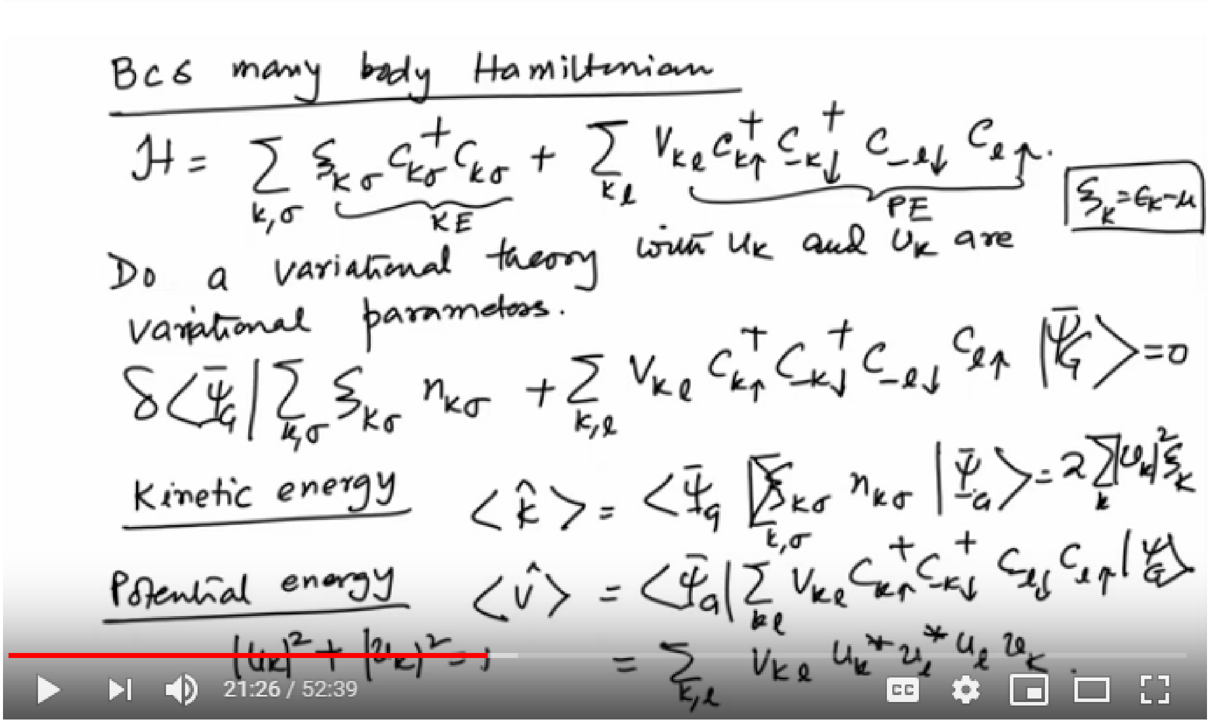
Now, look at the term, L not equal to K, that's a term, term that's written later. now, you can understand that, this will have a ul mod square, then U l star v L with a CL up dagger C minus l down dagger plus ul V L star and C minus L down C L up and plus a VL mod square C minus L down C L up C L up dagger C minus l down dagger. Now, you can understand that, so this is coming from the product of this term and so there are four terms, which are here. Now, you can see that this term gives you zero. Because, it will create a pair and so we'll change the occupancy of pairs in the ground state. So, it will have zero expectation value, so this term is not equal to zero. But, then when you take it between this, then that is equal to zero. This is, what I mean, and similarly this term will also idle equal to zero, when taken between the, the field for me see. Now, this simply adds a normalization, that it creates a pair and then it anneal it's a pair. So, ultimately what happens is that? So, L naught equal to K, gives you u L square, plus a V L square, equal to one. and now, let's look at the, L equal to K .this if you look at it carefully, you'll have a term which is a UK mod square ,CK up dagger c k up, acting on Phi 0, would give me 0. and u k, VK because, this ,this tells you that there are no States to ungulate, for K greater than KF and UK,VK the cross-terms ,both UK star VK and u k,v k star will give 0 for the same reason as its told above ,so the only term that contributes is a term with V K square. So, this n average, which is equal to twice of K, V K squared. So, that's a result, so 2 comes, because of the pair and VK square is the probability of occupied States.

Refer Slide Time :(14:30)



Now, second thing, is fluctuations in n bar. So, that's given by n minus n bar square, which is equal to n bar square n square minus or minus twice of n, n bar plus, n bar square, which is equal to a n square bar, minus n bar square .so, this if you again, repeat the same calculation and use the same logic to cancel out terms, this is given by 4 K, UK square ,V K Square and this is positive definite, in fact VK as a function of K, goes from 1 to 0 ,whereas UK as a function of K, goes from 0 to 1. So, in a all happening in an energy range, which is given by K T C. So, if you write down, the variation of this, so this is my, this is my epsilon K or Shi K ,which is equal to epsilon K minus mu and the VK drops from 0 to 1 and this is my superconducting gap Delta, there is minus Delta 2, plus Delta and this is my, so this thing is my minus H cross Omega D ,to this as H cross Omega D. So, in this range, V K becomes from 1 to 0 and so, the sum above goes as, T over or TC over TF, whole to the power n bar. So, this is the practically, if you want to estimate, the fluctuation in n, so this goes as that. So let, us write down a many a Bcs many-body ground state

Refer Slide Time :(17:07)



Oh, rather let's write down, a Hamiltonian and the Hamiltonian is a single particle term ,epsilon case, Saiki a sigma, c k Sigma, dagger CK Sigma plus, k l, VK l ,ck up dagger, c minus K down dagger ,C minus l down, C L up, that's the Hamiltonian and of course we know that, Shi k equal to epsilon K minus mu, we have already argued, that BCS theory cannot be obtained by doing a perturbation theory, of any order. So, we'll do a variational calculation instead. So and the variational calculation, with these small uk, vk. which are the occupancies, will be used as a variational parameter .so, what we have to do is that, we have to take this variation and it's a K Sigma, I k Sigma, n KS Sigma plus, K L, VK l, ck up dagger C minus K down dagger C minus l down C L up, that's the so we have to take a variation of this and put this equal to zero. So, let's look at the kinetic energy term or the single particle. The first term, here. So, this is the kinetic energy and this is the potential energy and this is given by, let us call this as kinetic energy operator ,which is Shi G ,sum over k Sigma, Sigma n KS Sigma, this you should work out and get this thing as, almost we have gotten this, when we did the average number, this comes out to be two, sum over K V K Square and Shi K and similarly for the potential energy, we have V this is equal to a Shi G and then there is a sum over k l, VK l ,ck up, dagger C minus K down dagger C minus l down C L up and this and this comes out to be K L, VK l, UK star ,V L star, u l ,VK. So, these are the kinetic energy and the potential energy these are the expectation values of those, with of course, the constraint, as u K square plus, V K square equal to 1. Now, because of this constraint,

Refer Slide Time :(21:30)

$$(u_k, v_k) = (\sin\theta_k, \cos\theta_k)$$

$$\frac{\partial}{\partial\theta_k}\left[\sum_{k'} \xi_{k'} (1+\cos 2\theta_{k'}) + \frac{1}{4}\sum_{k',l} V_{k'l} \sin 2\theta_{k'} \sin 2\theta_l\right] = 0$$

$$-2\xi_k \sin 2\theta_k + \sum_l V_{kl} \cos 2\theta_k \sin 2\theta_l = 0$$

$$\text{Define,} \quad \Delta_k = -\frac{1}{2}\sum_l \Delta_{kl} \sin 2\theta_l$$

$$2\xi_k \sin 2\theta_k = \sum_l V_{kl} \cos 2\theta_k \sin 2\theta_l = -2\Delta_k \cos 2\theta_k.$$

$$\tan 2\theta_k = -\frac{\Delta_k}{\xi_k} \Rightarrow \frac{\sin 2\theta_k}{\cos 2\theta_k} = -\frac{\Delta_k}{\xi_k}.$$

One can actually take a pair and reduce so, a pair of variables like this and reduce the number of variables from two to, one see it's always very difficult to do a variational calculation with two variational parameters, then you have to look for the minimum in a space, in a two-dimensional space it's much easier to look for a minimum in a wonderment on a, one-dimensional line and that is now given by, this single variable theta K. So, we can take UK equal to sine theta K, and VK equal to cos theta K, you can take the other combination that is UK equal to cos theta K and, VK equal to sine theta K, but it seems that this combination works better so, now what we do is that we take a variation with respect to theta K, of this now I'll use a dummy, variable K prime, 1 plus cosine 2 theta K, once again this algebra you should do because we have come from a BK Square, which we have written as 1 plus cosine 2 theta K and, there is a K prime L so, there is a VK prime L sine – 2 theta K, prime sine 2 theta L and put this equal to 0, in order to do a variational calculation and see, that what is the exterimum value of theta K, which minimizes the energy so minus 2 Z k, sine 2 theta K, after you do this derivative so, it's L and there is a V,K, L cos 2 theta K, sine 2 theta L equal to 0, if we define Delta k equal to minus ½, of sum over L V,K,L sine 2 theta L, they using this and putting it, into this equation one gets a nice equation such as 2 Psi K, sine 2 theta K, equal to sum over L V,K, L cos 2 theta K, sine 2 theta L, equal to minus 2 Delta K, cos 2 theta k, plus 2 Z ,2 Z K sine 2 theta K is equal to minus 2 Delta K, cos 2 theta K now, this equation gives us tan 2 theta K, if I divide or, rather bring this below its equal to a minus Delta K by, ZK so, this can also be written as sine 2 theta K, cos 2 theta K, which is equal to minus Delta K by ZK,

Refer Slide Time :(25: 10)

Use the definitions,

$$2u_k v_k = \sin 2\theta_k = \frac{\Delta_k}{\sqrt{\xi_k^2 + \Delta_k^2}}$$

$$v_k^2 - u_k^2 = \cos 2\theta_k = -\frac{\xi_k}{\sqrt{\xi_k^2 + \Delta_k^2}}$$

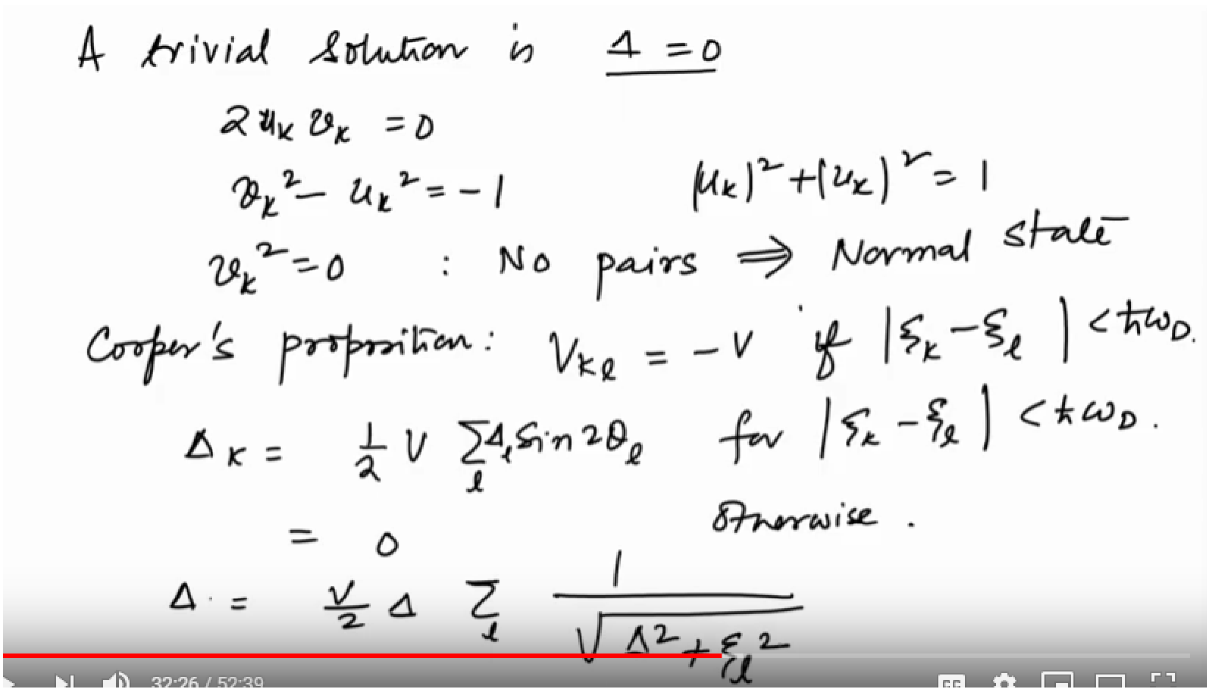
Δ_k assumes a form,

$$\Delta_k = -\frac{1}{2}\sum_l V_{kl} \sin 2\theta_k .$$

$$\boxed{\Delta_k = -\frac{1}{2}\sum_l V_{kl} \frac{\Delta_l}{\sqrt{\Delta_l^2 + \xi_l^2}}}$$

Now, if we use the definitions, that 2UK, Vk, which is equal to sine 2 theta K, this is equal to a delta K, divided by psi K square, plus a Delta K square, so this is my sine 2 theta K, definition of sine two theta K which is also equal to twice of U K, VK and also V K square, minus u K square, which is equal to cosine two theta K, which is equal to psi K divided by, root over psi K square plus Delta K square, there is a little bit of understanding that needs to be done here, is that this particular choice, we could have taken the other choice also that is a vice versa but, this choice fits all the definition and we have taken the cost 2 theta K, to be negative, because if Z K is large which means that epsilon K is much, much greater than the chemical potential mu, then VK should go off to zero, which is apparent from this diagram, so that's why the cosine 2 theta K is taken with a negative sign, alternately we could have taken the sine 2 theta K but, that would not have satisfied the conditions, or the boundary conditions, that we have so, hence the quantity Delta K assumes, a form that Delta k equal to minus half, of sum over L, VKL, sine 2 theta K and hence this is equal to minus half, sum over L, VKL, Delta K divided by putting the value of sine two theta K here, is Delta L square **plus I** else so that's the definition of Delta K so, a trivial solution so, we have to solve for, these in order to solve for Delta K a **priori let** us say that the Delta K, is really the energy gap, or the superconducting energy gap and we have to solve for it in order to find that what is the or how, does the gap vary, with different parameters especially say V and ZL or how does it enter into the expression of the gap, in order to see that we can also look at this expression and see that the trivial solution is Delta equal to rather this is Delta equal to 0 is a trivial solution, now you see Delta is there on both the sides of the equation here, it is just a standalone Delta K and here it's sum over L Delta L, and K is not equal to L, so for a given momentum value L, Delta L is a value and Delta K has to be computed by summing over all those Delta L but, since we are solving for Delta L we don't know what it is, so we cannot so the unknown, quantity appears on both the sides

Refer Slide Time :(29: 11)



So, a trivial solution as we say, Delta equal to zero so, what does Delta equal to zero gives it gives that to UK, VK equal to zero because, you see that to UK, VK equal to Delta K so, if delta k equal to Zero, to k UK VK equal to Zero, from this equation so to UK VK equal to zero and that tells that we also have a VK square minus, UK square, equal to minus one but, then we have the normalization condition is that UK mod square, plus became on square, it's equal to one that it tells that VK square equal to zero, so at Delta equal to Zero, implies that VK square equal to zero so there are no pairs and hence, this should correspond to the normal state, so the trivial solution is important because it talks about the normal state but, at the same time it gives a meaning to Delta now, Delta can be used as an order parameter for the superconducting transition because, at normal state Delta is equal to zero and Delta is not equal to Zero, for the superconducting state so, now go to the original cooper's, proposition that VkL, equal to minus V if ZK minus, Psi L is less than H cross Omega D, thus Delta k equal to ½, V sum over L, sine 2 theta L, for Psi k minus, Psi L is less than H cross Omega D, equal to Zero, otherwise. So, this tells that as if Delta doesn't depend upon K, it simply depends upon exist a number so, that is even more convenient because Delta is now, can we a truly thought as a number which when is non zero, will give a superconducting state however when it is zero it will give rise to a normal state so, then we have Delta L, which has to be put here yeah, so this says that if it's, sum over all L, Delta loses the K dependence so, it's just a, just a number and has no K dependence so, we can write this as Delta equal to, a V by 2 Delta sum over L 1 divided by, root over Delta square plus, is IL square and we can cancel Delta from both sides

Refer Slide Time :(32: 31)

$$1 = \frac{V}{2}\sum_{l}\frac{1}{\sqrt{\Delta^2+\xi_l^2}}$$

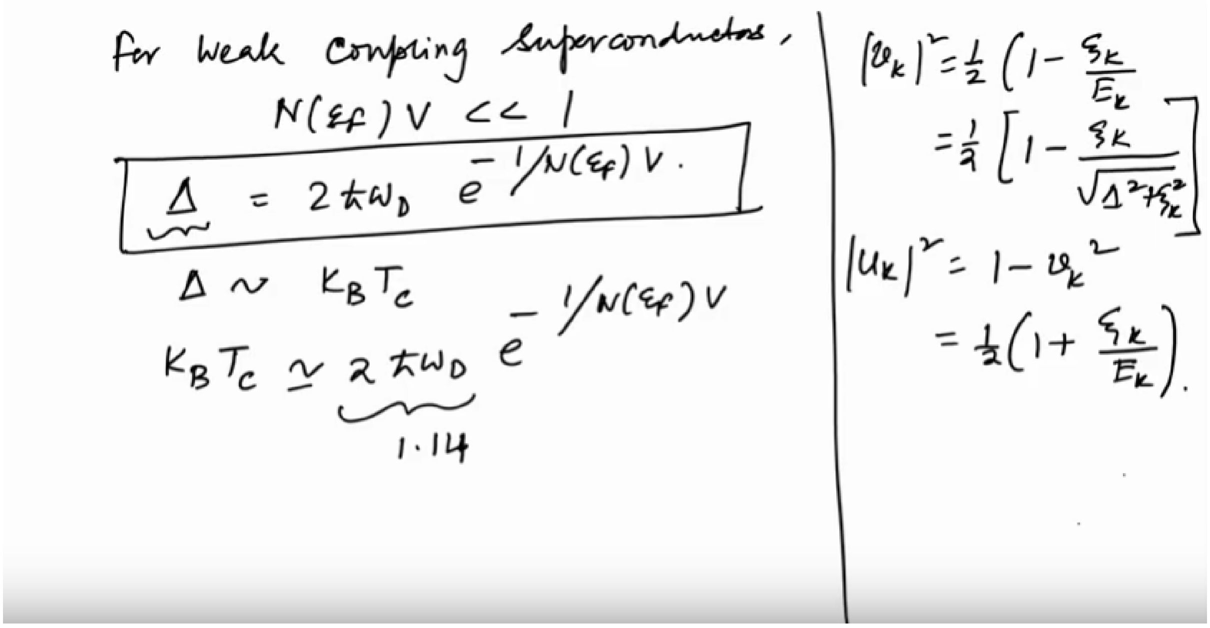
$$\frac{1}{V} = \int_0^{\hbar\omega_D}\frac{N(\xi)\,d\xi}{2\sqrt{\Delta^2+\xi^2}} = \frac{N(\varepsilon_F)}{2}\int_0^{\hbar\omega_D}\frac{d\xi}{\sqrt{\Delta^2+\xi^2}}.$$

$$\frac{2}{N(\varepsilon_F)V} = \int_0^{\hbar\omega_D}\frac{d\xi}{\sqrt{\Delta^2+\xi^2}} = 2\sinh^{-1}(\xi/\Delta)\Big|_0^{\hbar\omega_D}.$$

$$\Delta = \frac{\hbar\omega}{\sinh\left(1/N(\varepsilon_F\right.}$$

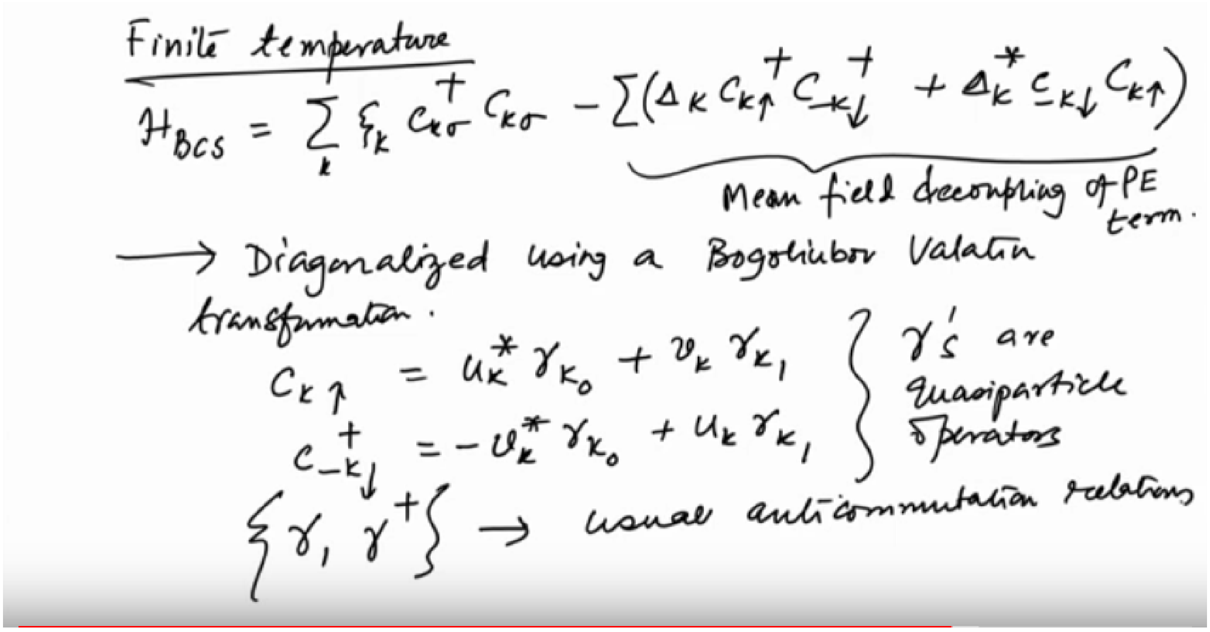
And this gives rise to an equation which is a V by 2, L and a root over Delta square plus so, there is no Delta L, so it is equal to Zi L square now, this is the equation for Delta, you may not see Delta on the left hand side to solve for but, there is Delta in the right Hand side and in the denominator and, in the square root of a denominator that tells that it's a highly no linear equation and you have to solve it either by, a root finding method or one of the root finding methods such as Newton raphson, or bisection method if you want to do it, using a computer that is numerically. We can also solve this problem analytically by, converting this sum, sum over L to an integral which is of this form remember the V is the strength of the attractive potential so, it's once when we convert the summation into an integral we need to bring the density of states or we can write down the density of states like this and do this, integral such as so there's a 2 and Delta square plus, y square **besides** the variable now, the integral will be from Zero to H cross Omega D, this is what we have said earlier and Cooper had explained that how, the pairs have to be formed within an energy shell of H cross Omega D from the Fermi surface, so its measured from the Fermi surface, and now n of Zi the detailed feature of n of Z is not required because we know that this whole phenomena is occurring at the Fermi level so, we can write this as n of epsilon F, by 2 Zero to H cross Omega D, design and a root over Delta square plus size square and that tells us that, this is equal to 2 by n V and this is Zero to H cross Omega D, DS I root over Delta square plus Z square and this is equal to two sine hyperbolic inverse, Z by Delta and from zero to H cross Omega D, if we put these values and rearrange then we'll get Delta equal to H cross Omega divided by sine hyperbolic, 1 divided by n epsilon FV, now since.

Refer Slide Time :(35: 50)



We are talking about a for week coupling super conductors n epsilon FV is much smaller than one, thus Delta assumes a form which is to H cross Omega D, exponential minus 1 by n EF of V so, we are getting a similar expression for the energy gap as we have done by solving a two particle Schrodinger equation this tells that this much of energy has to be supplied in order to break a pair and go from superconductor to a normal state and in BCS, theory this energy gap is, is a scale which is given by, the temperature scale so, Delta is of the order of KT C and so, a KT C becomes equal to one point, one four it is actually two here as we have written there and very accurate calculation shows that this is equal to one point one four and so, the TC expression is obtained from here we'll just do it in a minute so, this gives the how, the energy gap depends upon the phonon energy spectrum and how the density of states at the Fermi level come into the picture and the strength of the attractive interaction is also there which is V and we also have an epsilon F, multiplied by V, is much smaller than one which is relevant for a weak coupling superconductor so, let us give you all these occupation probabilities, or what are also called as coherence factors, that we have a VK square, equal to half of 1 minus psi by Ek, which is equal to half of 1 minus Zi K, divided by root over Delta square plus Psi k square so, that's VK and you K square, is simply equal to 1 minus VK square, which is equal to 1/2 1 plus Zi K Y, so these are important because these decide the behaviour of the gap the K dependence of the gap.

Refer Slide Time :(39: 04)



Now, let's go to the finite temperature so, we have seen that list so, let's write down the BCS Hamiltonian once more ,which is a mean field BCS Hamiltonian so, this is equal to Zi k, c k Sigma dagger, c k Sigma and in the mean field picture we have a minus delta k, ck up dagger, C minus K down dagger, plus a delta K star, C minus K down, CK up, this would be obtained if you take the mean field decoupling, of the P term of the potential energy term these kind of Decouplings we have done earlier and this equation or rather this Hamiltonian can be diagonalized, using a bogoliubov of valaten, transformation where the c-operators, are transformed into quasiparticle operators of this form ck up, equal to UK star gamma k zero, plus a VK gamma K one, and a C minus K down dagger equal to minus BK, gamma K is zero, plus UK, gamma K 1 and so gammas are quasiparticle operators so, they have so gamma, gamma dagger the have usual, anti commutation relations as the CK, CK daggers.
Refer Slide Time :(41: 54)

A generic form for the gap.

$$\Delta_k = -\sum_l V_{kl} \langle c_{-l\downarrow} c_{l\uparrow} \rangle$$

$$= -\sum_l V_{kl}\, u_l^* v_l \langle 1 - \underbrace{\gamma_{l_0}^{\dagger} \gamma_{l_0} - \gamma_{l_1}^{\dagger} \gamma_{l_1}}_{1-f(E_l)-f(E_l)} \rangle$$

$$\langle 1-\gamma_{l_0}^{\dagger}\gamma_{l_0} - \gamma_{l_1}^{\dagger}\gamma_{l_1}\rangle = 1-2f(E_l).$$

$$\Delta_k = -\sum V_{kl}\, u_l^* v_l \underbrace{\left(1-2f(E_l)\right)}_{\tanh(\beta E_l/2)}.$$

with $V_{kk'} = -V$

Now, a generic form of the gap now, we call Delta as the gap because we have established that Delta is nonzero for the superconducting state and, zero for the normal state so, this is equal to a minus, you can call it L and, a VkL and, a c minus L, noun C L up, so a little bit of algebra in terms, of this gamma operators, will yield VK L, u L, or UK star, VK and 1 minus gamma, L Zero dagger gamma, L Zero, minus gamma L 1, dagger gamma L 1 and this at finite temperature is given by, each one of them will be given by a Fermi distribution function which is or this is e L and, thus 1 minus gamma L Zero dagger gamma L Zero, minus gamma, L 1 dagger gamma L 1, it's equal to 1 minus twice, of L so, to say so Delta K putting it back into this equation the gap equation, it's equal to minus VK L, u L star u L so, this should be L at you ,uL star v L, u L star VL and a 1 minus 2, f e L and this is nothing but tan hyperbolic beta eL by 2 so with vkk, prime equal to minus V, which is Cooper's assumption.

Refer Slide Time :(44: 43)

$$\frac{1}{V} = \frac{1}{2}\sum_{K} \frac{\tanh\left(\frac{\beta E_K}{2}\right)}{E_K}.$$

$$\frac{1}{N(\varepsilon_F)V} = \underbrace{\int_0^{\beta_c \hbar\omega_c/2} \frac{\tanh x}{x}\, dx}_{\ln\left(\frac{2e^{\gamma}}{\pi}\beta_c \hbar\omega_c\right)} \qquad x = \frac{\beta E_K}{2}$$

γ : Euler's constant $= 0.577$

$$\frac{2e^{\gamma}}{\pi} = 1.14$$

$$\boxed{k_B T_c = 1.14\, \hbar\omega_c\, e^{-1/N(\varepsilon_F)V}.}$$

I get this equation as 1 over V, equal to ½ sum over K, tan hyperbolic beta e K by 2, by E K again converting that sum into the integral and using the density of states to have a value that is at the Fermi level so, this is equal to a 1 in this and a Zero to beta C H cross, Omega C by 2, a tan hyperbolic X by, X, DX where X equal to beta e K by 2, this integral as a standard value which is given by log of 2 e to the power gamma, by PI beta C H cross, Omega C where beta C is equal to 1 over KT C and gamma is the Euler constant, which has a value 0.577 and hence this 2, e to the power gamma by PI, has a value 1 point 1/3, or ¼, around 1/4 and, that if you simplify this it comes out as KT C, equal to 1 point 1, four H cross Omega C, exponential minus 1 by n epsilon F V, this is exactly the formula for TC, that we have got so, this is a how PC varies, with the phonon frequency and the density of states at the Fermi level and V

Refer Slide Time :(46:42)

Thus, $\frac{\Delta(0)}{k_B T_c} = \frac{2}{1.14} \simeq 1.76$

$$\boxed{\frac{2\Delta}{k_B T_c} \simeq 3.52}$$ → Followed by all weak coupling phonon mediated Superconductors.

Deviates strongly for the high-Temperature Superconductors.

So, thus Delta zero so, that's a value of the gap at T equal to zero, it's equal to 2 divided by one point one four, which is equal to one point six, seven nearly and that tells you that this is a feature of BCS, super conductivity that twice, of Delta divided by K TC, it's equal to three point five two, this is followed by, all weak coupling, phonon mediated superconductors .Okay? so, this is the a feature, or rather a property of this that the twice, of the energy gap verse / TC, or k TC, should be a number which is 3.5 Two, it deviates strongly for the high temperature, superconductors so, let us summarize what we have seen so, for we have genetically talked about

Refer Slide Time :(48:30)

Summary

Properties of superconductors

origin of attractive interaction.

$T_c \sim f(V, N(\epsilon_F), \hbar\omega_c)$.

The properties, of superconductor and those properties include the Meissner effect that is the electromagnetic response, then the thermodynamic response and the conducting properties, or rather the resistivity how it suddenly drops to zero below certain critical temperature, or threshold temperature and so, on and then we have gone on to talk about what is the origin of attractive interaction and this tells explicitly, that there is a phonon part involved the role of phonon is very apparent because, of the isotope effect that we have seen because the Debye, frequency actually scales as the ionic mass so, the lattice is or rather the TC scales with the ionic mass so, the involvement of the lattices very clear and there we can actually for us narrow, energy range we can get an attractive interaction between the electrons so, the wave function of the electrons will have to actually select this energy range to form a bound pair and then we have written down a many-body ground state and have taken a Hamiltonian, which is a generic Hamiltonian, where an interaction is taking place between two particles for a particular form of this interaction term that is V, KK prime equal to a minus B for, the epsilon Zi K minus, Zi K prime to be falling in this energy interval H cross Omega D, we find that the gap or rather we have been able to write down the equation of a gap and also at finite temperatures have able to write down the temperature dependence or rather the, the TC how the, TC that threshold temperature the critical temperature depends on these quantities such as V that's a attractive interaction the density of states and the phonon frequency call it Omega D or Omega C. So, this is in a nutshell, what is super conductivity all about and so, as you either destroy super conductivity by, using temperature or thermal effects, or you destroy super conductivity by applying magnetic field the state goes on to a, a normal state, a metallic state which is apparently not the case for the high temperature superconductor and they have many other complications there are very little some consensus whether phonons are involved into this pairing mechanism, or there's something else that is involved in any case and also these are behaviour or rather

Refer Slide Time :(51:52)

Thus, $$\frac{\Delta(0)}{k_B T_c} = \frac{2}{1.14} \simeq 1.76 ;$$

$$\boxed{\frac{2\Delta}{k_B T_c} \simeq 3.52}$$ → Followed by all weak coupling phonon mediated superconductors.

Deviates strongly for the high-Temperature superconductors.

this to Delta by KT C, that we have found to be three point five two, is a value that is much higher maybe five between five and, six and which are not at all, called as the weak coupling superconductors and these high-temperature superconductors they belong to a class of non weak coupling rather strong coupling superconductors.