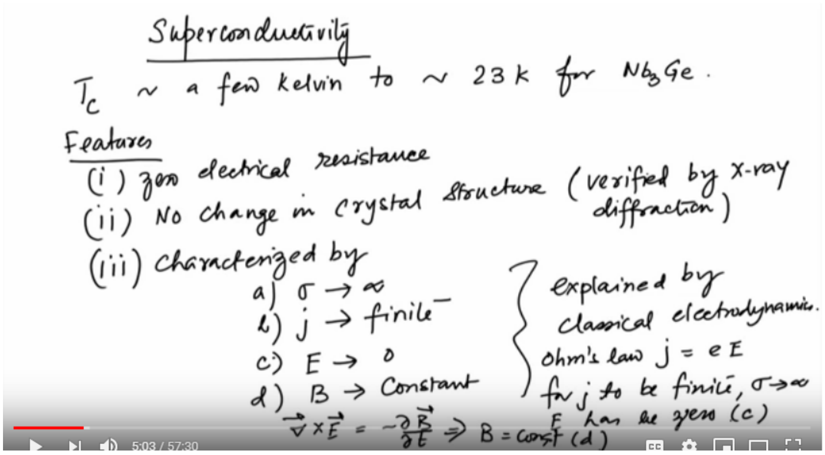
Lecture 2

A Brief Course on Superconductivity

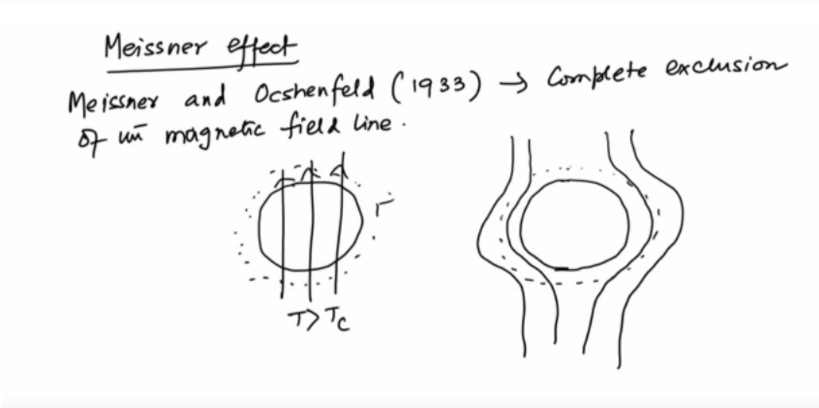
To continue with discussion on Superconductivity,

Refer slide time :(0:32)



I will repeat a few things, for your convenience. And so the sudden drop of resistitivity, below a certain temperature, is called as, ‘Superconducting Phenomena’, or this gives rise to Superconductivity. The temperature, below which the resistance vanishes, is called as a, ‘Critical Temperature’, which is a property of a particular material. And, so the TC, the Superconducting transition temperature is, usually of the order of few Kelvin, for the conventional Superconductors. Now this we had discussed yesterday, that there are classes of unconventional Superconductors, which are also known as, ‘High Temperature Superconductors’. And, there are not, as extensive knowledge about them, as they exist for the conventional ones. However, the TC’s really vary from, a few Kelvin, to about 23 Kelvin, for, this is for Nb3 Ge. And the features are; i - zero electrical resistance or resistivity, ii -No change in Crystal structure and this is verified by X-ray diffraction. Both, below TC and above TC. That is in a normal state and in the Superconducting state. And third, is that, it is the, state, the super conducting state is characterized by, (a) conductivity to be Finite, (b) The current density to be, still Finite, (c) Is that, the electric field goes to zero and (d) Is that, the magnetic field is constant. And this cannot be explained by, classical electro dynamics. Because, Ohm’s law says that j equal to Sigma e, for j to be finite, j is the current density, for j to be finite, Sigma has to go to, Sigma tends to infinity, then E has to be equal to zero, zero. So this the third, this is the (c) condition. And also that, curl of E equal to, minus Del B, Del t, that gives you, B to B constant, this is number d. So these are some of the features of the super conducting state.

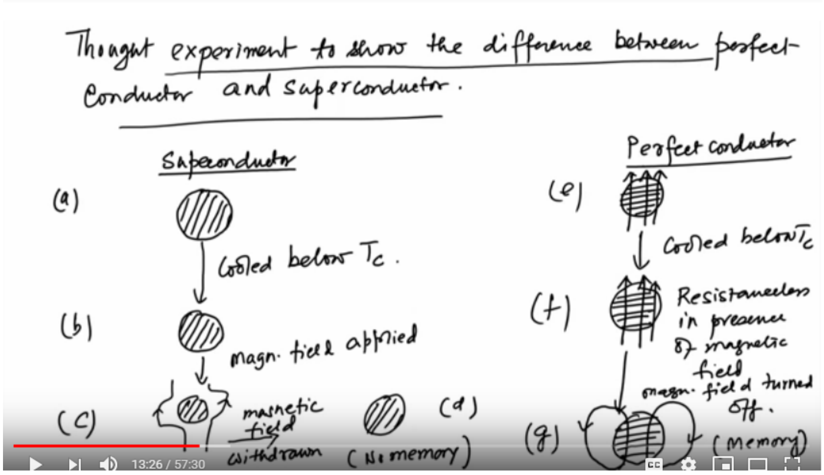
Refer slide time :(5:11)



Let us now look at, Meissner Effect. So the expulsion of the magnetic field, from the bulk of the superconducting material is called as, 'Meissner Effect'. So this complete and sudden vanishing of the, of the field, rather as the as the system goes in to a superconducting state, is really something that is, that distinguishes it from a, from an ideal or a perfect metal. Let us see that. So Meissner and it is usually known as Meissner Effect, but it's also, say it as, Meissner and Ocshenfeld. In 1933, they discovered that there is a complete exclusion of the field lines, magnetic field lines. So the way the experiment can be done is that; one can take a superconducting sample and then put a lot of iron filings, around that sample and then apply a magnetic field and along with that, cool the temperature of the specimen, so that it enters into a superconducting state. As it enters, as it is in a superconducting state, then the magnetic fields will be pushed, outside the sample and then iron filings would get lined up, in a regular fashion, outside the sample. So let us think that, this is what the sample is like. So the flux lines will go through this, if it is, T is greater than TC. And as T becomes, so the iron filings are all scattered in, around this superconducting sample and now, as T goes below TC, then these flux lines, the magnetic field lines are pushed out like this. And because of that, the iron filings will nicely align around the superconducting sample. So this is the tabletop experiment for seeing Meissner Effect. And it seems for superconductor, such as lead and tin and so on. So, now in certain materials, its only above a certain critical magnetic field, which is of the order of a fewer states, there's no expulsion of the magnetic flux. So, whence the superconducting superconductivity disappears, the material reverse into the, normal resistive state and the magnetic field, fully penetrates through it. And, so there are materials in which magnetic field can penetrate up to a certain extent. Beyond that, if you increase, still the value of the, the strength of the magnetic field, then it gets pushed out. And this is called as a, 'Type2

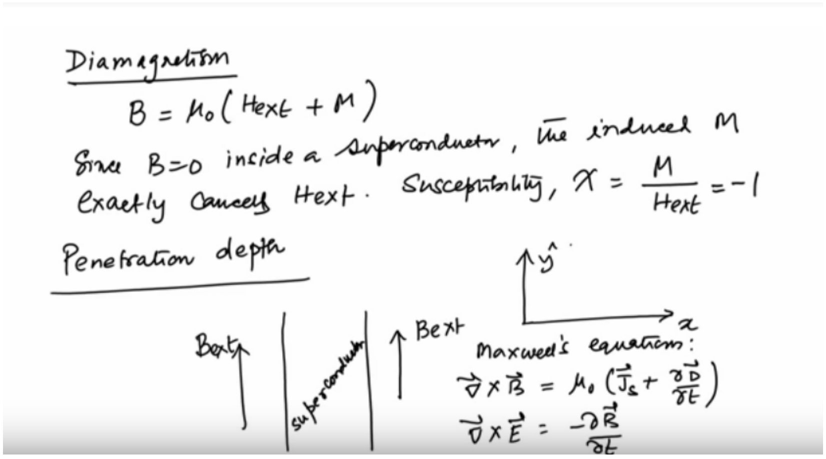
Superconductors.' Which allows, some, magnetic, for some range of magnetic field, for the magnetic field to penetrate inside the sample.

Refer slide time :(9:17)



Now let us do a thought experiment of, to show the difference between Perfect Conductor and Superconductor. This has been told yesterday or rather in the last discussion, through slides. Just wanted to make it little more clear, by drawing the diagram. So this is a typical superconductor and this is a Perfect Conductor. And so, see the state (a) is that, this is a, a, superconducting materials and this is cooled below TC and then it becomes superconducting and then magnetic field is applied, applied and then, there are, you know, the expulsion of the magnetic field, as we have said. And then magnetic field is withdrawn, field withdrawn, superconductor goes back to its original state (No memory). And what happens to a perfect conductor? So this is a perfect conductor, and so these are external, its in an external field, so its cooled below TC. The flux still penetrates, but it is resistance less, in presence of magnetic field. Now when field is withdrawn, it goes into a state, which is, so this is magnetic field, turned off, off and it has a memory. So because the perfect conductor does not have Meissner Effect, this is what happens. So if you compare, so this is (a), this is (b), this is (c) and this is (d) and now we have this, let's call it as, (e), this is (f) and this is (g). So if you compare (d) and (g), it is easy to see that a perfect conductor depends on its history and Meissner Effect does not happen in perfect conductor. And that's what distinguishes perfect conductors from superconductors. So the total exclusion of the magnetic field from the inside of the superconductor is a property, which is called as, 'Diamagnetism',

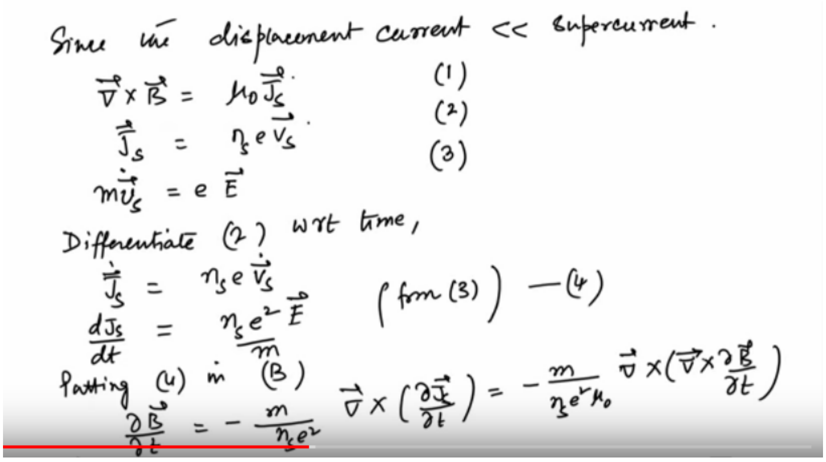
Refer slide time :(13:51)



And you can understand this as follows; So we have B equal to mu 0, H external plus M, H external is the, externally applied, magnetic induction, we have not written in vector form, but they are vector equations. So since B equal to zero, inside a superconductor, the induced M, M, exactly cancels, H external. So susceptibility, Chi equal to M by H external, becomes equal to, minus 1, and, the, as we have said earlier that, nothing is more diamagnetic than a superconductor. In fact in best of the, the, the diamagnetic metals have susceptibility which are extremely small of the order of 10 to the power of minus, to the 10 to the power minus 5 to the 10 to the power of minus 6.

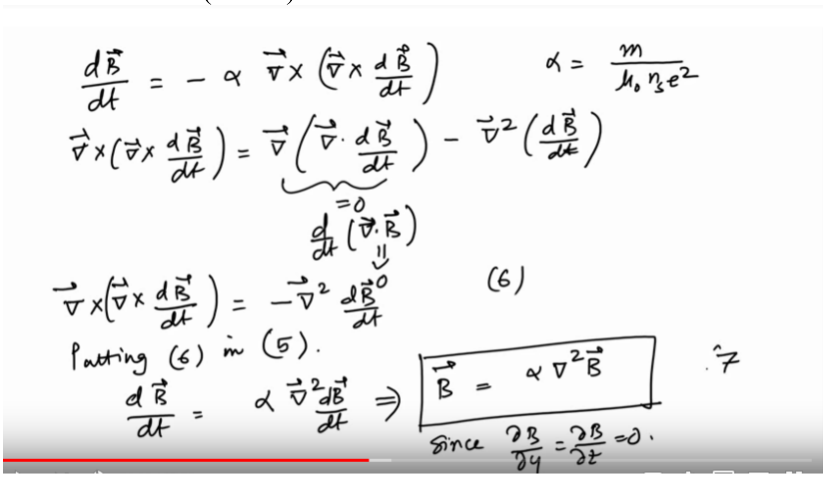
Now, let us look at the, electro dynamics on super conductors. And particularly we are going to talk about penetration depth. So let us look at Penetration Depth. And before we do any calculation, let us say that, we have claimed several times the magnetic field it totally expelled. But the fact is that that it enters only up to a certain distance, which is called as the, 'Penetration Depth'. Okay? Now consider the following geometry, to compute the distance through which it, it penetrates. So this is the superconductor and one has applied an external magnetic field, here. And let's call this direction Bx and this direction By. So this is the geometry of the sample. Now the Maxwell's equations can be written as, Curl of B equal to mu zero, Js plus del D and del t and curl of B, these are the, last two Maxwell's equation, minus del B, del t. Okay? So this is the equation, Maxwell's equation, we all are aware of. This is classical electro dynamics. We are simply doing classical electro dynamics and superconductors and trying to get some information on, how much magnetic field can penetrate inside a sample, before it gets completely exp, expelled. So there is a certain critical depth, up to which the magnetic field can penetrate. And this by no means contradicts the statement that we have made earlier, that in superconductor, the classical electro dynamic laws are not strictly valid. We are simply trying to get some information out on a quantity called as, 'Penetration Depth'. And to tell you a priori, the reason is that, that this is one of the important length skills of the problem. And this length skill, along with another one called as the, 'Coherence Length', determines the material to be, of Type 1 or of Type 2.

Refer slide time :(18:23)



So, what happens is that, since the displacement current. So, let me go through it again. Curl of B, is the, the curl of the magnetic field, this is mu zero is the permeability. J is the super current density. And, del D, del T, the, variation of the displacement current density or displacement current and this is that, e and B are the electric and the magnetic fields respectively. Now the displacement current variation, or displacement current, rather. This is the, which is the, time derivative of the displacement vector. The displacement current is, far smaller than the super current. So in this term you can neglect this one. So neglect this and write the Maxwell's equation as, curl of B equal to mu zero, Js and Js is nothing but, its equal to, nse, vs. Which is the, the super current density, is the, density of the super electrons, multiplied by the electronic charge and the velocity of these super electrons. And equation of motion, of these super electrons can be written as, in an electric field to be, Mv as dot, which is nothing but, Newton's Law, Mv as dot equal to, charge times the electric field, in an external electric field that we are talking about. So if we call this as (1), this as (2) and this as (3) and call these equations, maybe as, (A) and (B). So, what happens is that, if you put three or rather, a if we have, if we take, differentiate (2) with respect to time. So JS dot equal to, nse, Vs dot. I can replace Vs dot from this equation, equation (3). So ns, e square, e over m from (3). So that's the equation for Js dot. So this is equal to dJs, dt. This is how the super current varies with time. Now if I put (4)? Let's call this as (4), If I put (4) in, in (B)? Putting (4) in (B), I will have a del B, del t, which is equal to, minus m by nse square, curl of del Js, del t. And this is nothing but, now I'll use (4) to replace dj as dt, as this is equal to, minus m over, nse square, mu zero, curl of, curl of del B, del t. So that's called curl and curl of this.

Refer slide time :(22:55)



Hence, what we have is, that the, dB, dt. Now I can write is as a, complete differential, is equal to, minus alpha times curl of, curl of, dB, dt. So this is a equation for dB, dt. And, Alpha is given by, m divided by, mu naught, ns, e squared. That is, what this is defined as, Alpha. So now, this is written as, the right hand side is written as, curl of dB, dt, this is a gradient and divergence of dB, dt and minus, del square, dB, dt. Now this term is equal to zero, because one can swap the order of space and time derivative and can write this as, ddt of divergence of B, which one knows by, the second Maxwell's equation, that this is equal to zero. So this term, becomes equal to zero. And hence my curl of, curl of B, or dB dt rather, it is nothing but, equal to minus del square times, dB dt. So if you put this in to the equation number, let's call it as (5), putting (6) in (5). dB dt is equal to alpha and this, dB dt. So doing a time integral, this can be written as, B equal to Alpha, del square B. So this is the equation, that gives the space variation of B. And, now since the variation according to this diagram, the variation can be in the X direction. So let us convert this 3 dimensional equation here, by, to 1 dimension, because we can safely assume that, del B, del y and del B, del z, to be equal to zero and hence I, we can write it down as a, being equation 7,

Refer slide time :(26:20)

$$B_x = \alpha \frac{\partial^2}{\partial x^2} B_x .$$

$$B(x) = A e^{-x/\lambda_L} + B \exp^{x/\lambda_L} \qquad \alpha = \lambda_L^2$$

$$B(x) = \underbrace{B_{ext} e^{-x/\lambda_L}} + B_{ext} e^{x/\lambda_L} .$$

At $x = \lambda_L$.

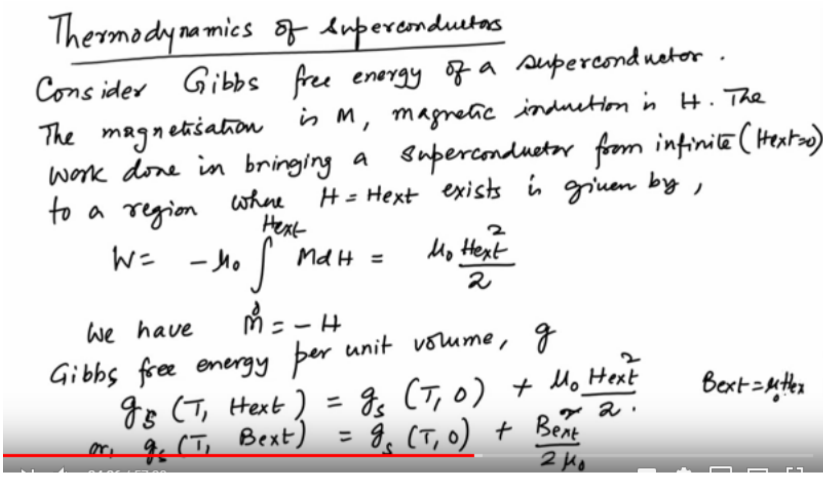
$$B(x) = \frac{B_{ext}}{e}$$

λ_L is called as the penetration depth.

$\lambda_L \sim 500\ A^{\circ}$ in ordinary Superconductors.

We can write it down as, Bx equal to alpha and d2, dx2, Bx. This is really a, a partial differential, if you like, it’s like, del, del and so on. And this has a simple solution, which is equal to, B external, well this is like, let’s not write, B external now. We can write it as, the solution of this, can be written as A exponential, minus x over lambda L, plus B exponential, x over lambda L, where lambda L is nothing but, so alpha is, lambda L square and one can easily see that, that these, because as x goes to zero, so this superconductor is, say from zero to some, d, that’s the width of the superconductor. In that case, at, x equal to zero, B is equal to, B external and so on. So this can be written as, B of x equal to B external, x minus, lambda L plus B exponential, lambda L, and of course, this is blow up as, x becomes large. So we can simply, so this is again, so B external. So we can only talk about this term. And this term says that the B, which is inside, which exists inside the super conducting sample. Falls of as falls off as exponential minus x over lambda L. So at X equal to lambda L, be X, equal to B external, divided by E. So which becomes equal to e minus one. So this is called as, lambda L is called as the,’ Penetration Depth.’ Lambda L is of the order of, about 500 Angstrom, in ordinary or conventional Superconductors. All right. So we'll come back to this discussion when we talk about the other energy scale, for the superconductors, namely, the coherence length and distinguish between the type 1 and 2 superconductors.

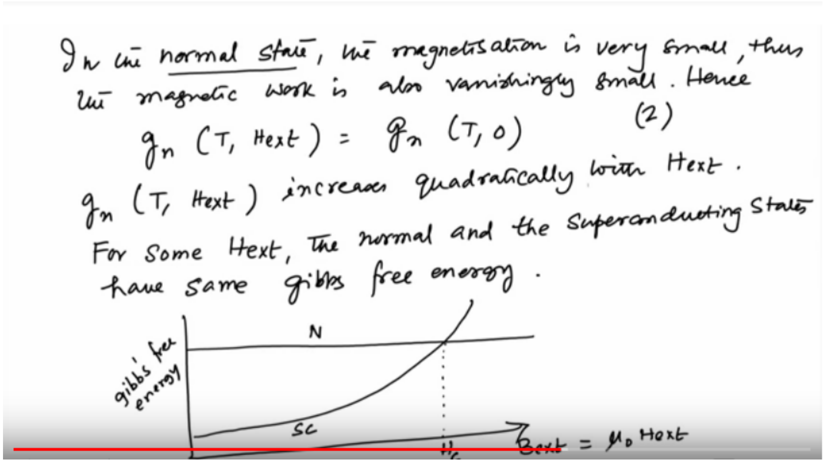
Refer slide time :(29:35)



Meanwhile, let us do the thermodynamics. So far we have been doing electrodynamics, now let us do thermodynamics of. So thermodynamic route is usually the simplest route, to study phase transitions. And so let's consider, Gibbs free energy, of a superconductor So if the magnetization is M and the magnetic induction is H, then the work done in bringing the superconductor, into a region, where the magnetic induction H external, from where the magnetic induction is, H external, that exists, from a region, which is far away where the H external is equal to zero, is given by the work done expression, which I am going to write. Let me write this, because these are important points. So the magnetization is M, M and magnetic induction is H, the work done in bringing a superconductor, from infinity, where H external, equal to zero, to a region, where h equal to h external, exists, is given by, W equal to minus mu 0, 0 to H external, MD H, which is equal to mu 0, H external Square, over 2. So that's the work done, in order to bring it, from infinity to a place where H external, external magnetic field or magnetic induction exists. So, we have m equal to minus H, for a superconductor. So let's write down the Gibbs free energy.

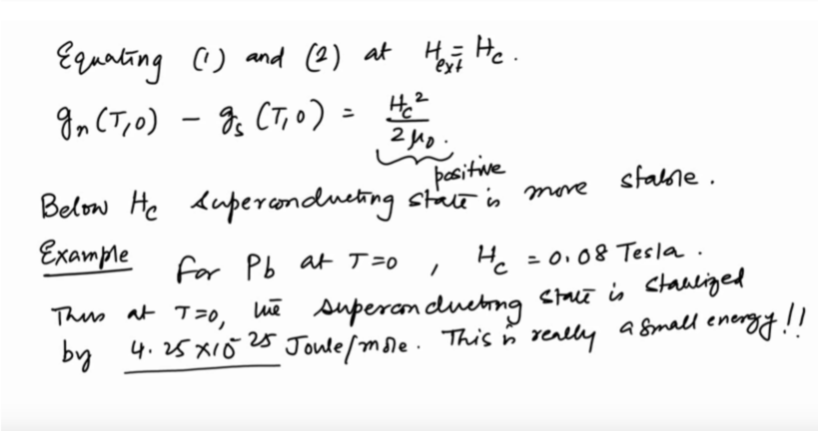
So Gibbs free energy, per unit volume, volume is given by, G. Let's write it with a G, small g so g s, s stands for super conductors, in T and at a temperature T, and a magnetic induction, external, which is equal to gs t0 and then Plus this extra energy that, it acquires, because of, because of the H external, which is what we have, shown here. So or Gs, the same statement, if you want to put it in terms of the magnetic field. It’s just, there'll be a, this B external Square, divided by 2 mu 0, because of the Relationship between, H, so your B External, equal to mu H, mu 0, H external.

Refer slide time :(34:40)



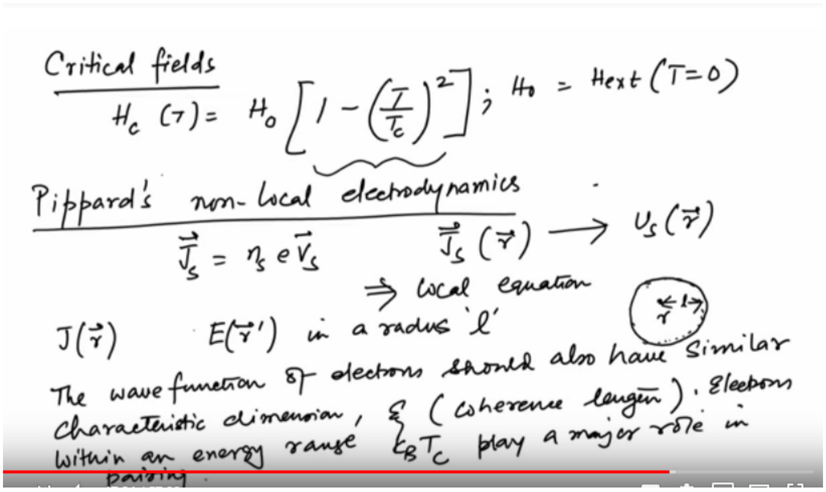
So, in the normal state, the magnetization is very small. Thus, the magnetic work done, is also vanishingly small. Hence, Gn, th external, equal to, gn t0. So in the superconducting state, gs Tand at a given magnetic field or magnetic induction, there is a, extra term, B external square by 2, mu 0 or mu 0 H external square by 2. However in the normal state, the magnetization is known to be, very small. And, we do not have this, magnetic energy contribution, as it's seen here. So now this tells that, equation 1 tells you that, there is a B dependence, for GS. Whereas there is no B dependence, for or B or H dependence for, gn. So Gn, TH external it increases quadratically, with H external. So it increases quadratically with H external. So for some H external, equivalently be external, the normal and the superconducting states, have same energy, have same Gibbs free energy. So what it says is that, if you plot, the Gibbs free energy, so this is, Gibbs free Energy, as a function of either be external or H external, doesn't matter, they just get scaled by this. And so for the superconducting state it's like this and for the normal state it's like this. So this crossing point is called as, HC or BC, depending on which language you want to use. So below HC this is the, this is that of, superconductor SC and this is normal. So the SC state, the superconducting state, has lower energy, for all values of external field or induction, below HC or BC. And as the external field, crosses this HC, then the superconducting state, has higher energy and the normal state has lower energy, as you can see it here. So then, beyond that the normal state stabilizes. So what one can look at, is that, we can call this as equation one or rather and this has equation 2 and if we equate 1 and 2, at 8 equal to HC,

Refer slide time :(39:34)



H external equal to HC, then G n t0 minus G s t0, equal to HC square, over 2 mu 0. So this is positive and because this is positive and because this is positive, the superconducting state is more stable, than the normal state, below h equal to HC. So let's give an example, for lead, at T equal to 0, HC equal to 0.08 Tesla.
Thus, at t equal to 0, the superconducting state is stabilized, by 4.25 into 10, to the power minus 25, Joule per mole and this is really a small energy. So it's quite amazing that, such a small energy actually stabilizes a superconducting state. But that is true, of the order of 10 to the power minus 25 Joule per mole.

Refer slide time :(41:59)



Let's now talk about the critical fields, which is what we are just introduced. And the variation of the critical fields, with temperature, is given by, we give this without proof, is HC of T it's equal to H 0, will tell you what H 0 is. It's 1 T, 1 minus T by T C whole square, where H 0, equal to H external, at T equal to 0. So this is the externally applied magnetic induction, at t equal to zero, so that's H 0. So it is clear, that, at T equal to TC, this term vanishes, so HC of T, goes to 0. So there are many things, that are, important, in this particular context. And we have talked about electrodynamics and we have talked about thermodynamics. And we found that some important information is encoded in both of them. Now let us talk about, another interesting comment, from Pippard, which is encoded in Pippard’s local electro dynamics or rather non-local electrodynamics and let's see what it says. If you look at this equation, J is equal to ns eVs, this is what we have written, this looks like that, J s at r is related to, V s at r, so it's a local equation. So J s at r, is simply determined by, the V s of ah, at that point r. What Pippard said that, the current density at a point r, depends on, so J's at r, it depends on e, the electric field, at r prime, so, which is centred around r, in a radius, say, L. Okay? So at a given point, r, the electric field, the, the current density at this point, will depend upon electric field, in a region, which is, spread all over r prime, where r prime is centred around r, with a radius, which is L. So every point inside this circle is r Prime. And all of that, those r primes, will contribute, to the current density, at r.

So this is the non locality, which Pippard thought, is more relevant and realistic and this can be, you know, I thought of to be, a spread over, a region, of radius L. And so this L is actually, related to the characteristic dimension, of the electron wave function. And so this l, is related or rather, let us write it, the wave function of electrons, electrons, should also, have similar characteristic dimension that is extent, which is called as, let's call that, that Zi, which is called as a coherence length. So electrons within an energy range, KTC, where TC is the transition Temperature, play a major role, in, pairing, so this is pairing of Electrons.

Refer slide time :(47:37)

The moments of these electrons have an uncertainty

$\sim \Delta p \sim \frac{\Delta E}{v_F} \sim \frac{k_B T_c}{v_F}$.

Thus the position uncertainty $\Delta x \sim \frac{\hbar}{\Delta p} \sim \frac{\hbar v_F}{k_B T_c}$.

$\xi = a \frac{\hbar v_F}{k_B T_c}$ $\quad a \sim 1$ $= 0.8$ in BCS theory.

$\Downarrow$

Coherence length. Pippard suggested that:

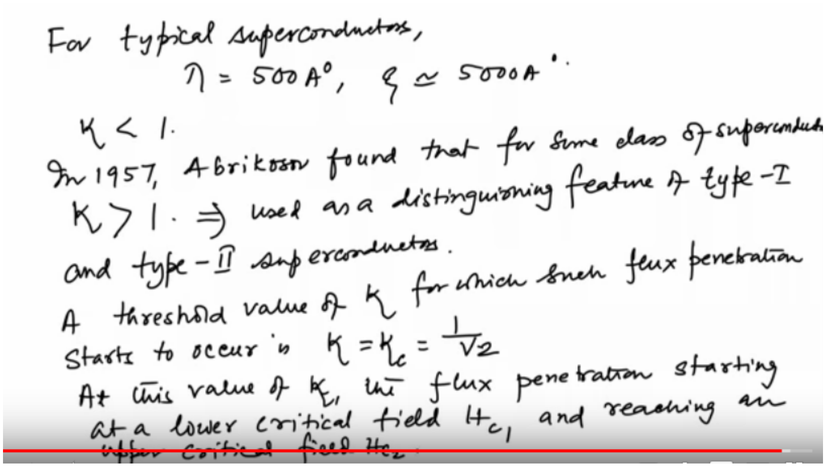
$$J_s(\vec{r}) = (\text{Const}) \int \frac{\vec{R}\,[\vec{R} \cdot \vec{A}(\vec{r}\,')]}{R^4} e^{-R/\xi} \, d\vec{r}\,'$$

Introduce a dimensionless parameter.

$\kappa =$

So the moment of these Electrons, have an uncertainty, of the order of Delta P, which is equal to Delta e, by H, this is V F, which is nothing but, K T C over V F, thus the position uncertainty, so we are just using a, Heisenberg's uncertainty relation, the position uncertainty, is Delta X, is equal to H cross, by Delta P, H cross V F by, K T C, thus Zi equal to a, h cross, V F by K T C, where a, is a number which is of the order of one and a equal to, point eight, in BCS theory. So this introduces and others, in length scale for the problem, which is known as the, 'Coherence length'. So a coherence length, is the second length scale in addition to the, the penetration depth that we have introduced earlier. So people suggested that, so, basically, that J s at r, should be written as an integral, rather than writing it as n s into e into V s. So this is equal to some constant, which is not so important. So this is our R dot, A, r prime, this is that r prime, that we were talking about, and R, and this is a R by, Zi, D R Prime. So this is how. the non locality. in the, the super current density comes. that the super current density. at a given position R. depends on the electric field. in this particular fashion. that there is a region. which is centred around r prime, that every point in that region, contributes to the electric field. and a characteristic length scale, comes out of it and we have introduced this earlier. Now we can talk about, dimensionless, introduce a dimensionless parameter, kappa, which is equal to lambda by, Zi, and these are ,

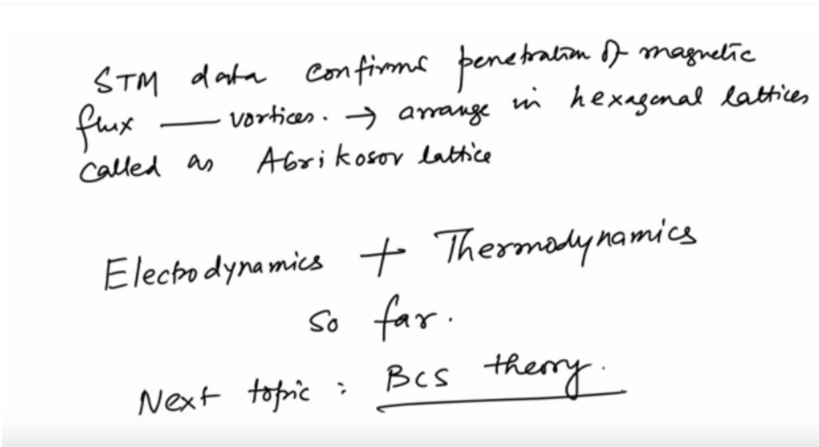
Refer slide time :(51:35)



So, for typical superconductors, lambda equal to 500 angstrom, as we have already said, Zi equal to about 5000 angstrom, so Kappa is typically less than 1. But however in 1957, Abrikosov Found, that for some class of Superconductors, this Kappa can be greater than 1 and, this is used, as a distinguishing feature, of type 1 and type 2 superconductors.

So to remind you, in type 2 Superconductors, the flux lines penetrate, the sample, till some, threshold value of the magnetic field and if we increase it, beyond a certain threshold, then super conductivity, disappears. So a threshold value of, value of Kappa, for which, such flux penetration occurs, or starts to occur, is Kappa. It's say, that value of Kappa is called, 'Kappa C', which is equal to 1 over, root 2, so I mean, this is in terms of that. So, okay. So this says that, at this value of, at this value of, Kappa, the, the flux penetration, starting at a lower critical field hc1, hc1 and reaching, reaching, an upper critical field. So the scanning tunnelling microscope Data,

Refer slide time :(55:25)



STM data, confirms, confirms penetration of, of magnetic flux, flux which are called as, 'Vortices'. And these vortices, actually line up, in the form of a hexagonal lattice, in irregular lattice, in the form of an hexagonal lattice. And they arrange, in hexagonal lattices and which are called as called as, 'Abrikosov Lattice'. So we have studied mainly, the electrodynamics and thermodynamics, of superconductors, so far and have gotten quite a few, information, useful information, about superconductors. And the next thing would be, doing BCS theory, which is a, Microscopic theory of Superconductors.