Lecture – 17
A Brief Course on Superconductivity
RF SQUID, DC SQUID, Application of Magnetoencephalography (MEG)

So, we have been discussing superconducting quantum interference devices, which are written as squids.

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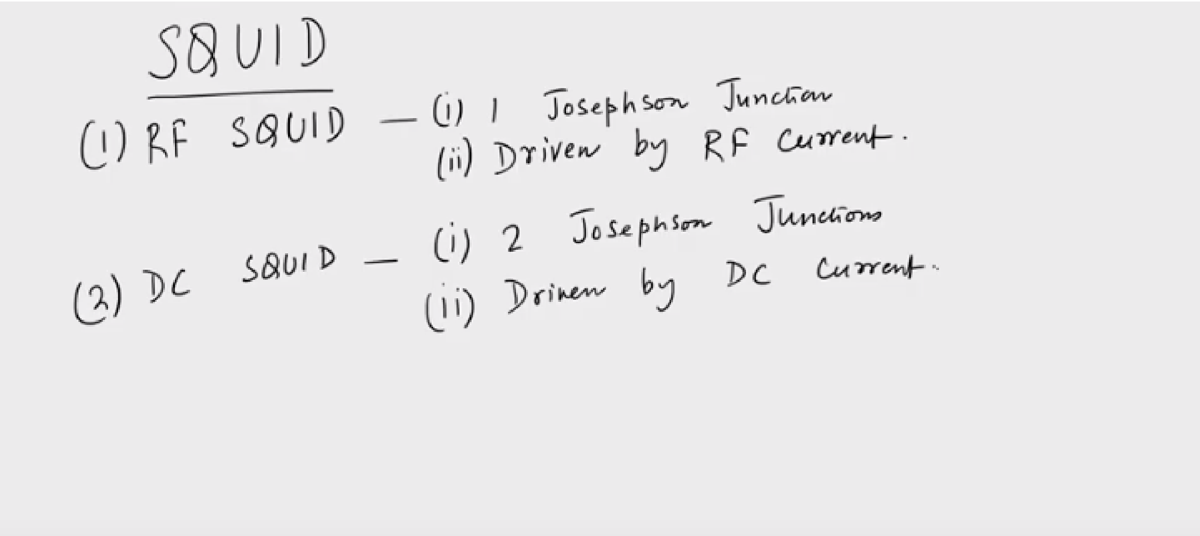

So, S for superconducting Q for quantum and I for interference and D for devices So, we have said that that there are two kinds of squids depending on the number of Josephson Junction and based on how they are driven or what kind of current that is input into the system? So, they're called as one is called as the RF squid radiofrequency squid all of you know that the radiofrequency is actually range over very large frequencies. these are like 10 to the power 4 to over 10 to the power 12 frequencies Hertz freak of frequencies. So, these are called as a radio frequencies So, one is called as the RF squid and this typically has one Josephs on Junction and driven by an RF current, and to of course what are known as the DC squids which are most commonly available in for commercial purposes as well as the discussions are more abundant in for the case of DC squids. I here as opposed to the RF squid there are two Josephson junctions and to the driven by a DC current.

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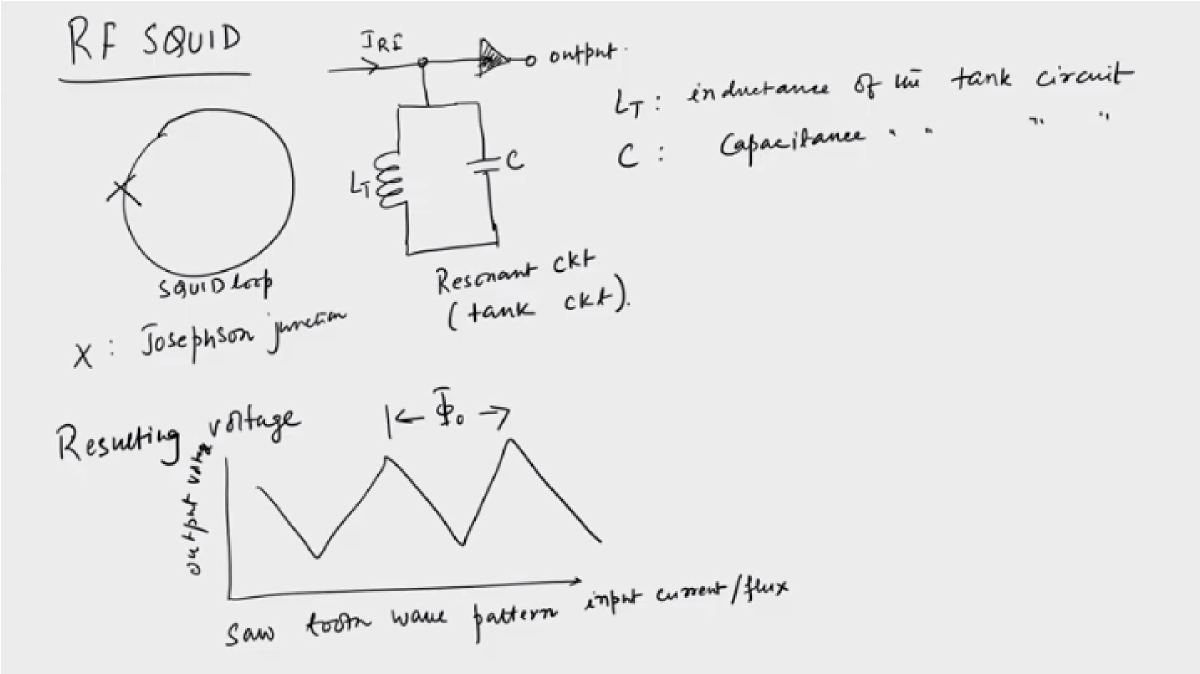

So, let us include every brief discussion on the RF squid though we will mostly consider DC squid for our purpose. So, all right So, RF squid. So, schematically it's shown as this is a superconducting loop, with a single Josephson junction. So, this represents the cross represents a Josephson Junction, and this is the squid loop, and this is inductively coupled to a tank circuit the world tank means something that can store energy just like water tank that we know that can store water. So, this is the tank circuit which is an LC circuit., So, this is the L. Let's call it a L_T t for the tank and there's a capacitance there, and this is driven by the an RF current. There's a RF current that flows into this circuit and so, there is a output which can be obtained from here. So, this is that output Okay?? So, L_T is the inductance of the tank circuit and c is the capacitance. So, basically, it's a LC circuit and so. Okay? And this called as a resonant, resonant circuit or it's called as a tank circuit, has we told. Okay? So, so, the tank circuit is driven by the RF current while the squid loop which is at the left and drawn here is inductively coupled to the inductance L_T of the tank circuit Okay? And the resulting RF voltage that we pick up from here from this point shows an oscillation with the input current or the flux in this saw tooth manner and so on. So, this could be either you call it the input current or it you can call it the input flux, and this is the output voltage. So, it has a sawtooth wave pattern, with a period which is given by the flux quantum. So, Phi_0. So, please distinguish between the capital Phi that denotes the flux quantum or Phi_0. Phi denotes a flux in general a flux and a small Phi that is related to the phase of the condensate wave function for the superconductor. So, we see a sawtooth wave pattern. So, any change of this flux, that is induced into the squid loop would show up in the oscillation pattern and we would know that there is a change in the magnetic field or the magnetic flux. In fact, it's misnomer. Because, there's no quantum interference takes place But itis the first squid to be made probably around but by Zimmerman and co-workers probably around 60s. Not too sure about the time.

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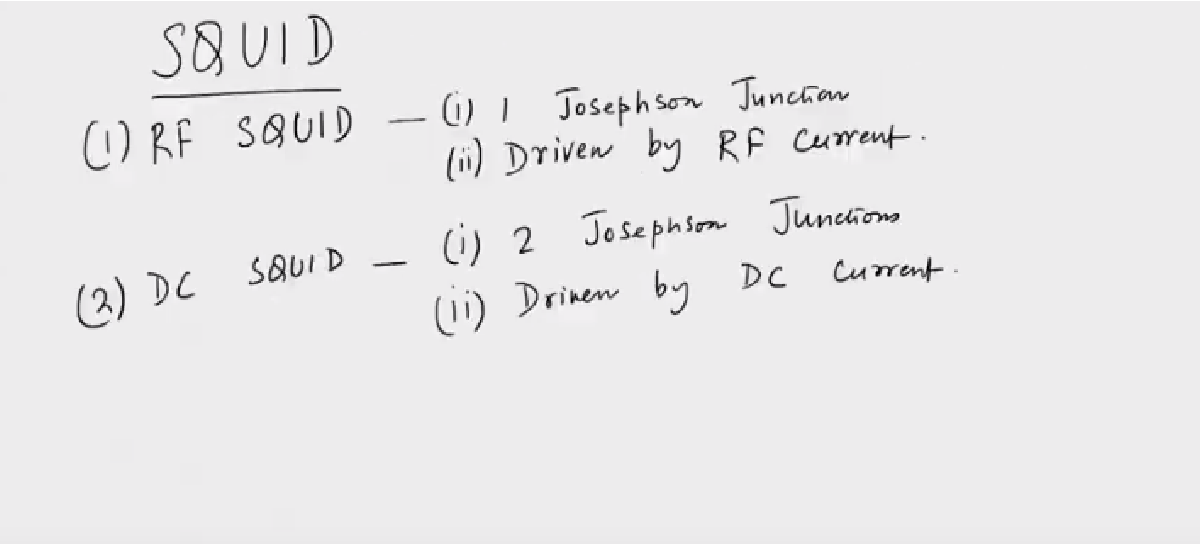

Let's come to that DC squid which is the most important part of the squid study of squids,
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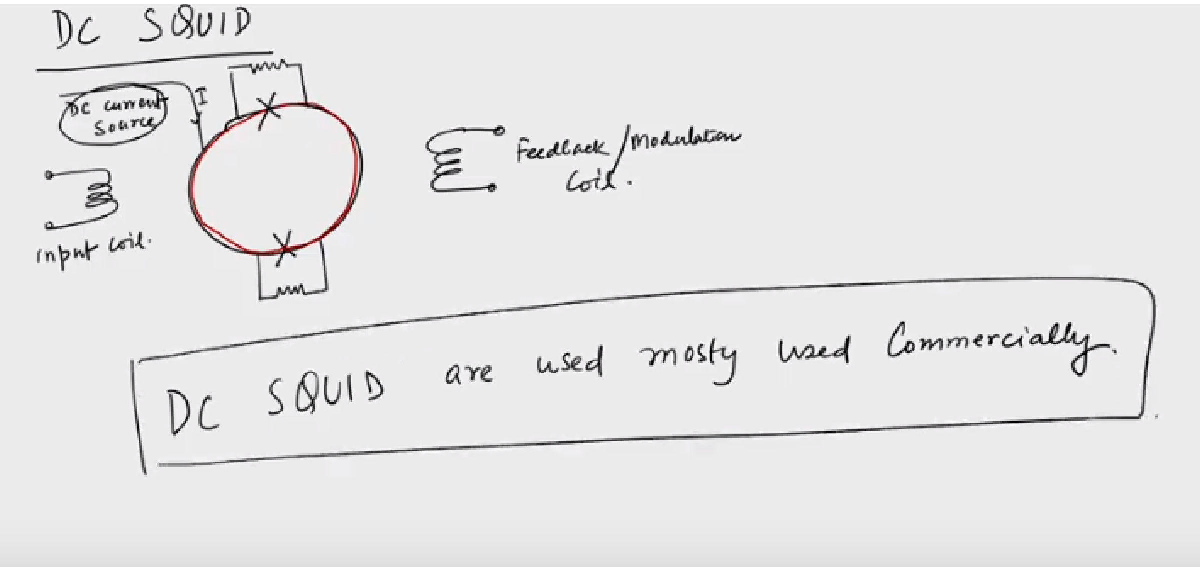

and as I said that they are most commonly available in for commercial purpose. So, again there are two Josephson junction as told they are connected in parallel, and these Josephson junctions are shunted with a resistance and this shunting is for the purpose of avoiding hysteresis effects or it's also to the mismatch of the Josephson junctions, by with a great degree of accuracy. It is also, possible not to use the shunt resistance, and I have a sort of mismatch junctions in DC squids. But that gives a much lower efficiency, than the shunted junctions. There's an inductive coil that is coupled here which has already been told. So, this is the input coil .and So, there is a DC current source which feeds current to this and, and the, the current that is fed or the bias current let's call it which is twice of IC where I Cis the critical current that we have discussed So, there is a DC current source Okay? So, there's a current that flows in and there is an output coil which is the feedback coil or the modulation coil. So, just to remind you that these arms let me draw it in a different colour. So, that you So, these arms are basically the, the superconducting wires, or superconducting specimen, this one, and this one, they are represented by the condensate wave function let's say given by size 0 exponential I Phi_1 and the other one is represented by size 0 exponential I Phi_2 and where Phi_1 and Phi_2 are the phases of the wave function, and ideally no current should pass through this junction classically. But however, we know that the current cannot be discontinuous. So, current actually flows through the junction and not only that the current actually fluctuates, with a very large frequency we have seen that for a micro volt of biasing bias voltage there is a large fluctuations of the order of 10 to the power 12 Hertz or so, that kind of a fluctuating current that flows through the Josephson Junction. So, neither of these circuits the input coil or the feedback or the modulation coil they are actually wrapped around the squid. But they are inductively coupled to this quid. So, that is the main idea. Now, if there is a change in the magnetic field that passes through the this loop of the squid. Then, there'll be a change in the output voltage detected, and this modulation, So, this actually the output voltage as we just have just seen for the case of RF squids that these voltages output voltages are periodic functions of the, the flux and there'll be and and the flux the periodicity is in it in units of Phi_0 which is a flux quantum, that we are very well aware of, and if there is a change in the magnetic field through the loop of the squid, the modulation profile changes and, hence we understand that and that there'll be there's a change in magnetic field ,and there are various examples or applications of this. We will not get into those details. But once again just to come to this shunt resistances. There are two junctions which need to be matched within a few percent and these shunt resistors actually help in achieving, that purpose and it's also, biased with a current which is twice of IC where I_C is the critical current and So, these are sort of so. As there is a So, there is a change in phase So, if there's a change in magnetic field or a magnetic flux, that flows through this, that flows through this loop. Then this will be this will induce a phase change in the condensate wave function, and that enhances the current through or one of the arms, of the squid. So, there is one arm on the left, and the other arm on the right. So, one will get a current slightly larger which is I_C plus IB and the other will get a slightly lower current I_C minus IB. So, as the this output as the flux external flux, increases or decreases through the loop, the voltage will change in a periodic manner as we have said, and the, the period being the Phi_0 which is the flux quantum. So, that is the main idea of this DC squid and the most DC squids are used mostly used commercially Okay? So, let us do the DC squids with a little more elaboration, and see that the expressions that we get for this.

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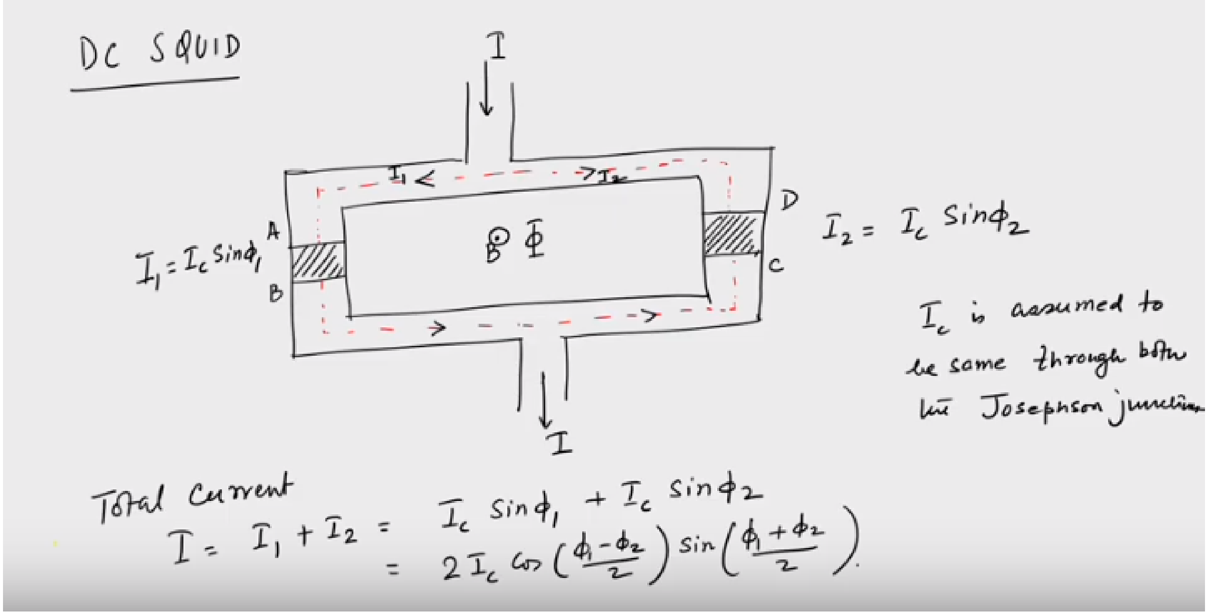

It's more useful for us to redraw this DC squid not in the form that we have drawn and the circular form. But it's it's just convention or rather is for my own convenience, that I'm drawing this squid as. So, there is a current that is coming in and so, there is this Okay? So, this is my alright So, these are the Josephson junctions that tunnel junctions Okay? which are by this dashed portion that is there, let me name them as A B on either side of the junction, and there's a C and D the input current comes I, which I, we have discussed that it's twice of I_C and the current actually goes out like this, and we are going to draw a coil for that let me take a different color maybe a red color and show a contour like this, hope you can see the, the red coloured contour that we have drawn all across the Josephson junctions. So, these places are the superconductors let's call it a so, let's go back so these are the portions let me show a highlighter so, these are the, the superconductors on both sides of the junctions and these are so, that that dotted line or the contour is drawn well inside the superconductor and so, there's a magnetic field that passes through this loop so, just distinguish between what I, have drawn in the last slide and this one there was a circular shaped loop that was drawn, and here here we have drawn a square shaped loop otherwise things remain the same so, there's flux threading the loop it's equal to Phi there's a magnetic field which is which is piercing through it we are seeing the the dot of the arrow which means it's coming towards us so there is a current that is flowing this is equal to I_1 we call it a I_C let's just write it on the left I_C sine of Phi_1 and this is a I_2 it's equal to I_2 equal to I_C sine of Phi_2.

So, these are the currents that flow through the Josephson junctions which are the tunnel barriers, and for convenience we have taken the I_C to be same I_C is assumed to be same through both the Josephson junctions Okay? and so, there's a so so, the current enters and it the current I_1 goes through the left portion and there's a current I_2 that goes through the right arm and that's pretty much it that's the setup that we are going to consider for our calculation, and this is just an approximation that the I_C or the critical current is considered to be same for both the Junction's it may not be they could be different, and and these so, these are the red line or the contour lies entirely within the Josephson junction and at a distance which is greater than the penetration depth of the sample.

So, our current the total current is, is I equal to I_1 plus I_2 which is equal to I_C sine Phi_1 plus I_C sine Phi_2 so, this is equal to a 2 I_C cosine of Phi_1 minus Phi_2 by 2 and sine of Phi_1 plus Phi_2 by2 Okay? Now this, is just applying the trigonometry I, have trigonometric identity.

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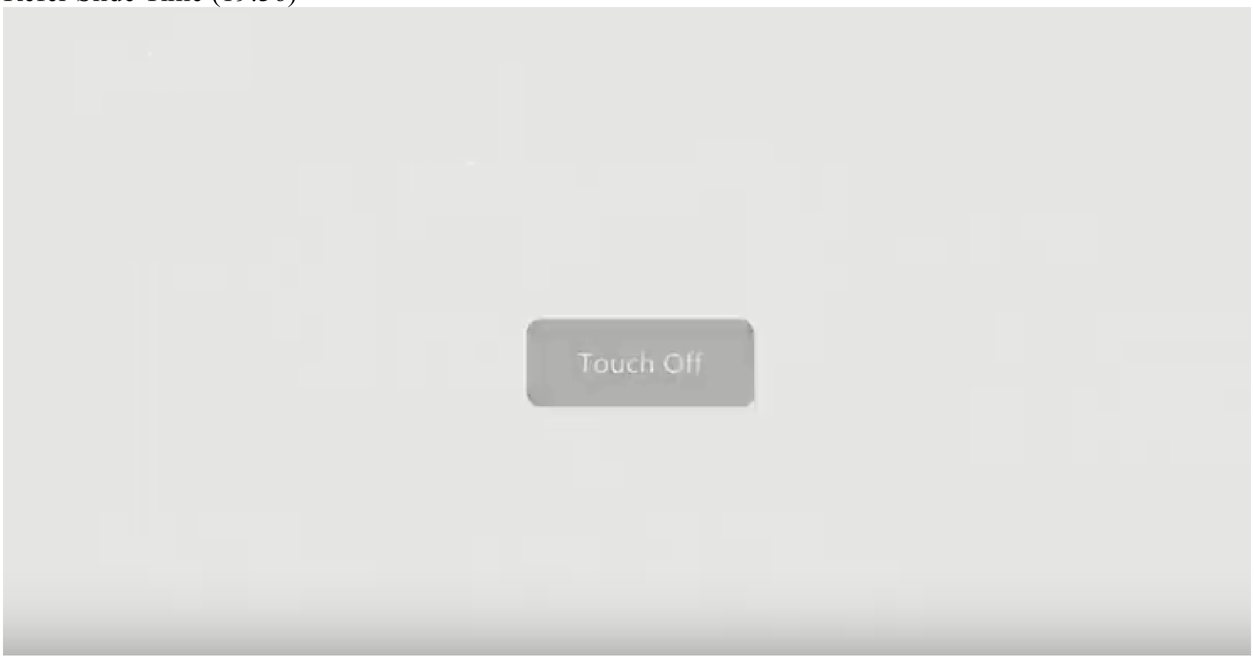

So, these are the that's the total current.

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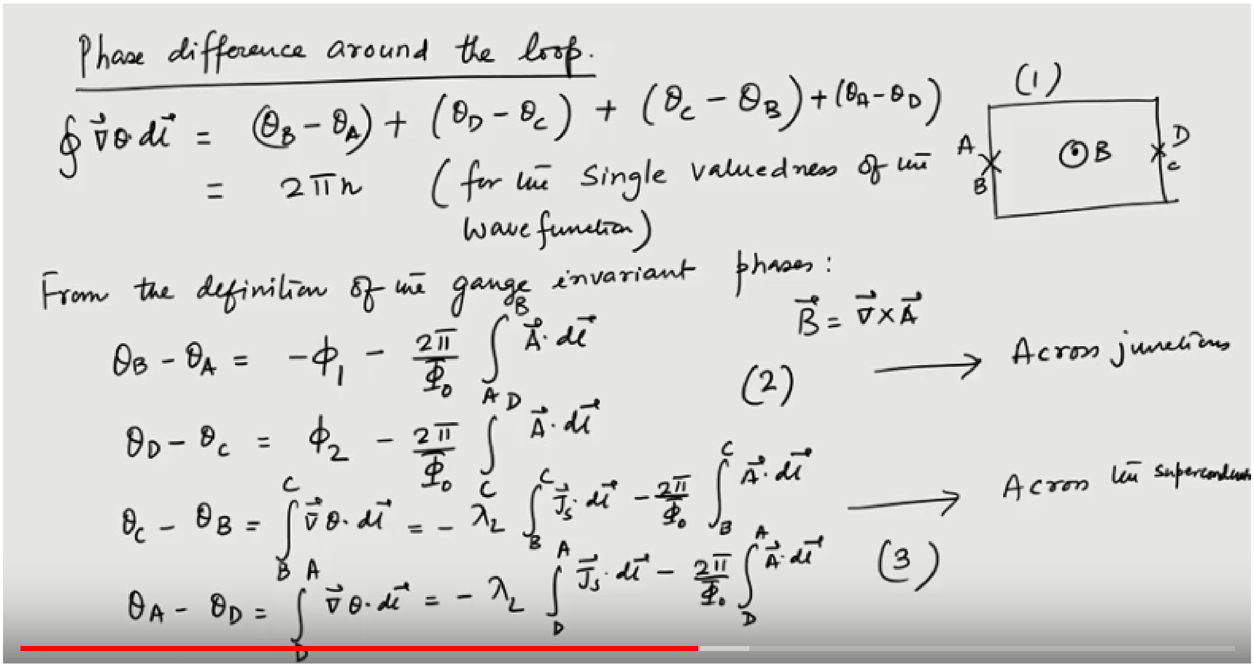

Now, the important thing is that the phase difference around the loop is gradient of theta dotted with dl which is equal to a theta$_B$ minus theta$_A$

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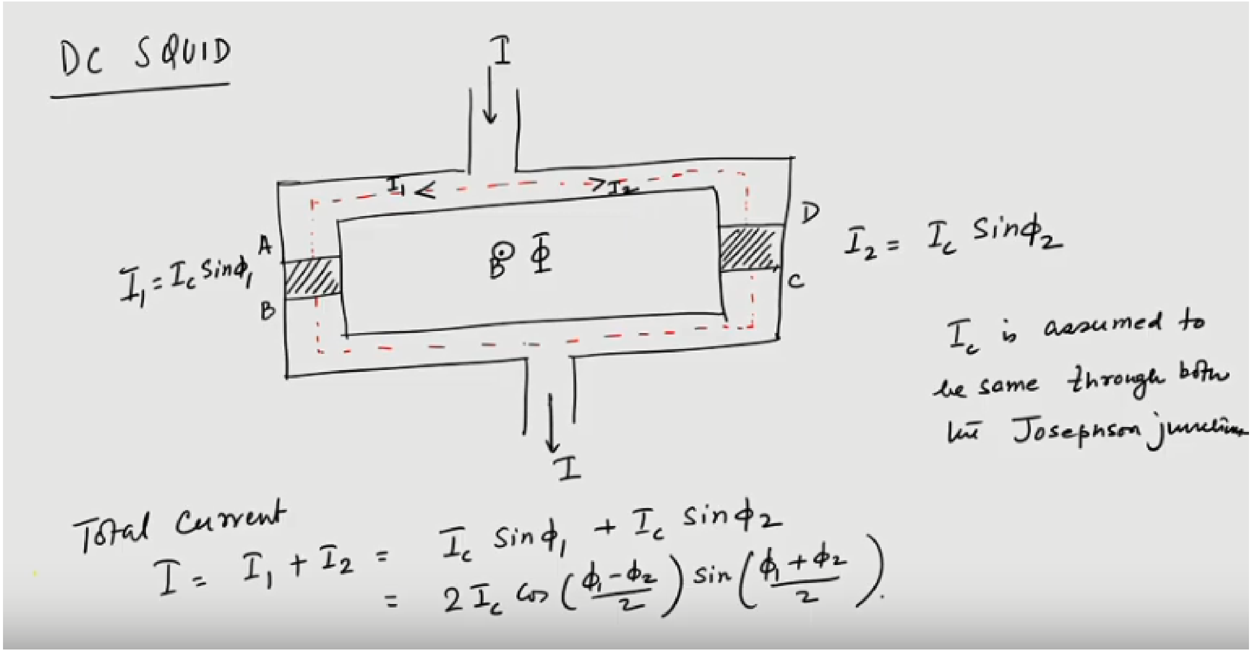

so let's just go back for a second to the diagram so, theta$_B$ and theta$_A$ so, it's just a convention that is assumed that so, the current is flowing like this so, the theta$_B$ is at a larger potential it could be at a larger potential than or the phase and then that of A, or it could be the reverse but we have simply taken it to be the total the total change in phase dotted with the line integral, and over the entire contour is taken to be like this so theta$_B$ plus theta$_A$ plus a theta$_D$ minus theta$_C$ plus a theta$_C$ minus theta$_B$ and so, we are we are here so we have so, these are the junctions so there's a B, and is a K there's a parallel junction which is a C and D so these are so these two are across the junction B minus A and D minus C and, this is C minus B is within the superconductor and there is also a theta$_A$ minus theta$_D$ so, that is the total change in phase and this has to be equal to a 2 pi n and which is for the single valuedness of the wave function.

So, now let us write down so let's write this as equation 1 and, we can also write that from the definition of the gauge invariant phases theta$_B$ minus theta$_A$ is equal to a minus Phi$_1$ that's the phase of the condensate we have just taken it for a convenience rather as a matter of convention to be like this, and there's a 2 pi by Phi$_0$ and A2B and A dot BL which is the vector potential A is the vector potential, such that a B equal to curl A. So, the magnetic field and the vector potentials are related by this equation. So, that's across the junction and also across the other Junction is theta$_D$ minus theta$_C$ is equal to a Phi$_2$ and minus 2 PI over Phi$_0$ remember this small Phi on the first term on the right Phi$_1$ and Phi$_2$ are the phases of the condensate wave function while the Phi$_0$ that's written here, that denotes the flux quantum and so this is again across C to D A dot dl so, there's a magnetic field that is threading the loop so, that is because there is a magnum vector potential that is there so, these are for so, this let's write it for 2 and these are across junctions, and what about the super conductors so, the super conductors will have theta$_C$ minus theta$_B$ which is equal to a B to definition is B to C gradient of theta dot it dotted with dl it's equal to a minus lambda$_L$ B2C J_S dot dl minus 2 PI over Phi$_0$ B 2 C A dotted with dl and so, this is one of the things I, remember that this is coming from the London equation lambda L is the penetration depth London penetration depth, and J is dot DL and then 2 pi by Phi$_0$ this is for the superconductor Okay? For the junction, of course this term is not there and then the other thing is theta$_A$ minus theta$_D$ which is equal to the A2 it's D to A so, D to A gradient of theta dotted with dl equal to minus lambda L D to A J_S dot dl and minus 2 pi over Phi$_0$ D 2 A A dot dl and this is across the superconductor. So, we have so let's call this equation as three.

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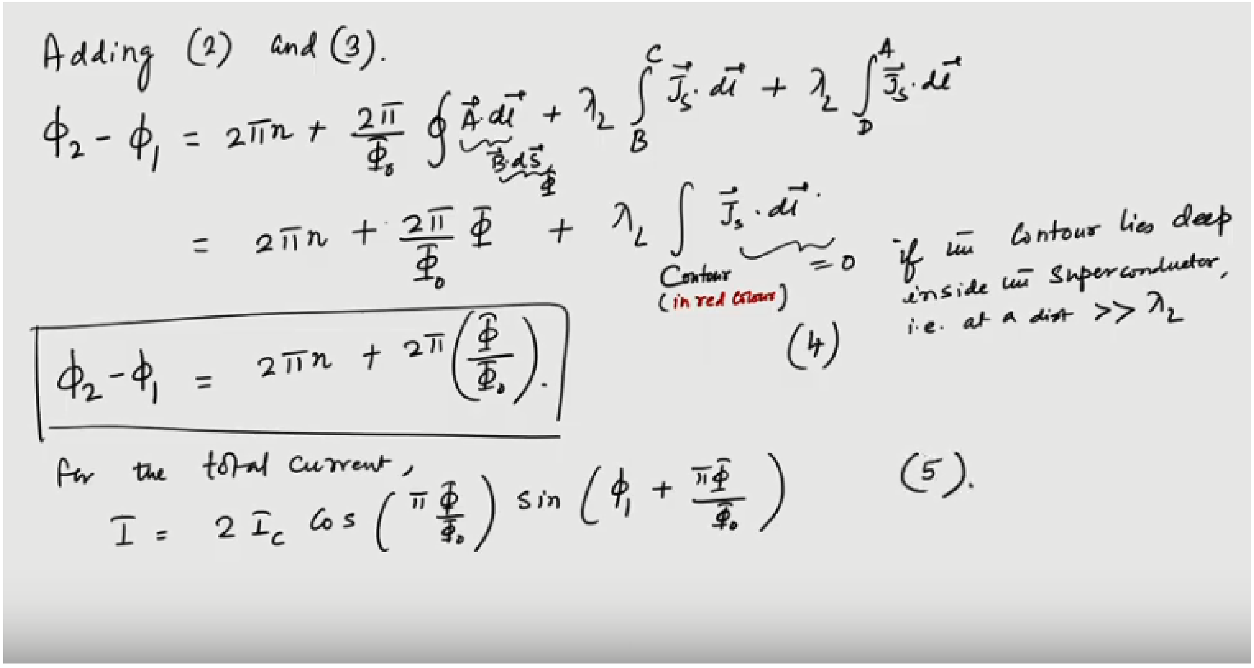

So, if we add this two and three in order to get one then we'll we'll be able to write this as so adding two and three we can write it as Phi$_2$ minus Phi$_1$ it's equal to a 2 pi n plus r 2 PI over Phi$_0$ closed integral of A dot dl plus a lambda$_L$ B to C at J_S dot dl plus a lambda$_L$ D to A J_S dot dl so, this is of course by assuming that these Josephson junctions are finite assume infinitesimal width, and so, these are that that's the change in the or rather the difference in phases of the condensate for the two superconducting content sets, and so this is equal to 2pi n plus 2Pi over Phi$_0$ now this quantity is nothing but B dot ds by using 'Stokes law' because it's curl A dot ds and so, this is a curl A dot ds and curl A is nothing but B, so it's B dot ds which is nothing but the flux. So, this is that flux Phi and plus a lambda$_L$ over the contour so, don't come so I, will let me just write contour in red color so, this is J_S dot dl now, this will be zero if the contour lies deep inside the superconductor, that is at a distance at a distance D from the from the inside of the loop at a distance much, much greater than the lambda$_L$ or the London penetration depth.

So, we are landing up with the expression that this change in the condensate phases phases of the condensate is nothing but a 2 pi n and 2 pi and now there is a ratio of Phi by Phi$_0$ Okay? So, this is the equation that we need, and we also you know is important to us which is we know from the Josephson junction, that there is a relation of this kind now for the total current the total current I, is given by twice of I$_C$ and cosine of Phi Phi over Phi$_0$ and sine of Phi$_1$ plus a Pi Phi by Phi$_0$ so, see that I as a function of Phi$_1$ has got an amplitude which of course modulates, and there is a sine part that goes with it and,

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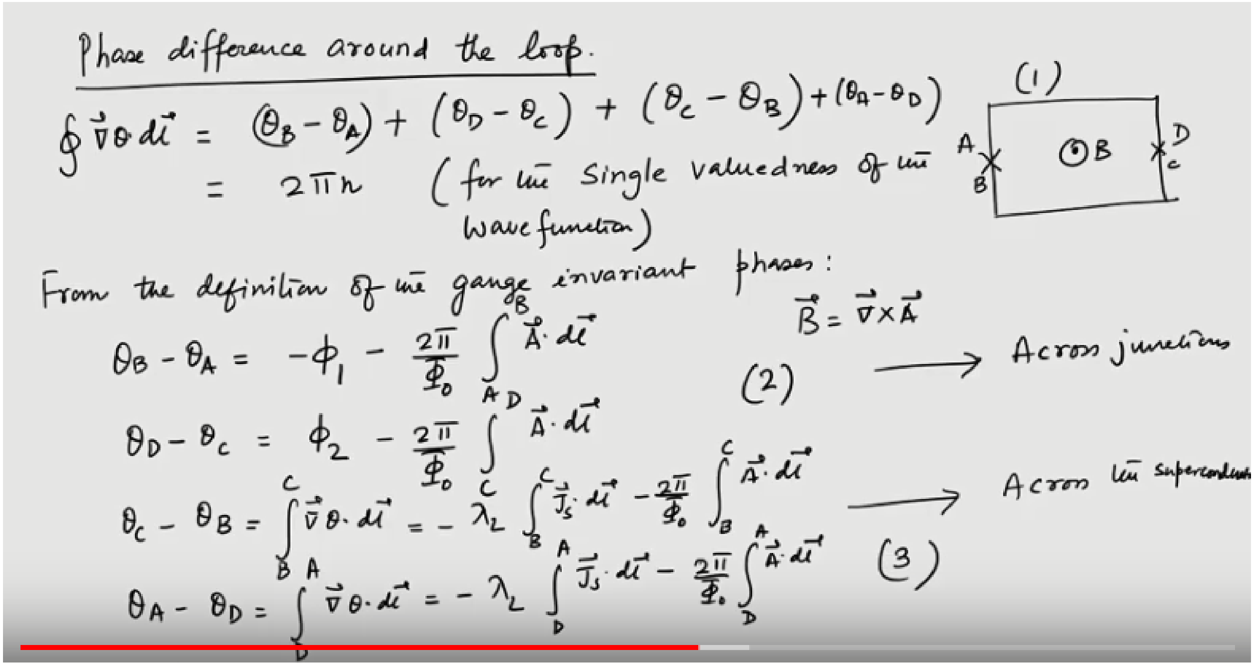

let's call this equation as maybe we have called it as 3
so, we can call this as a 4 and maybe this as say 5.
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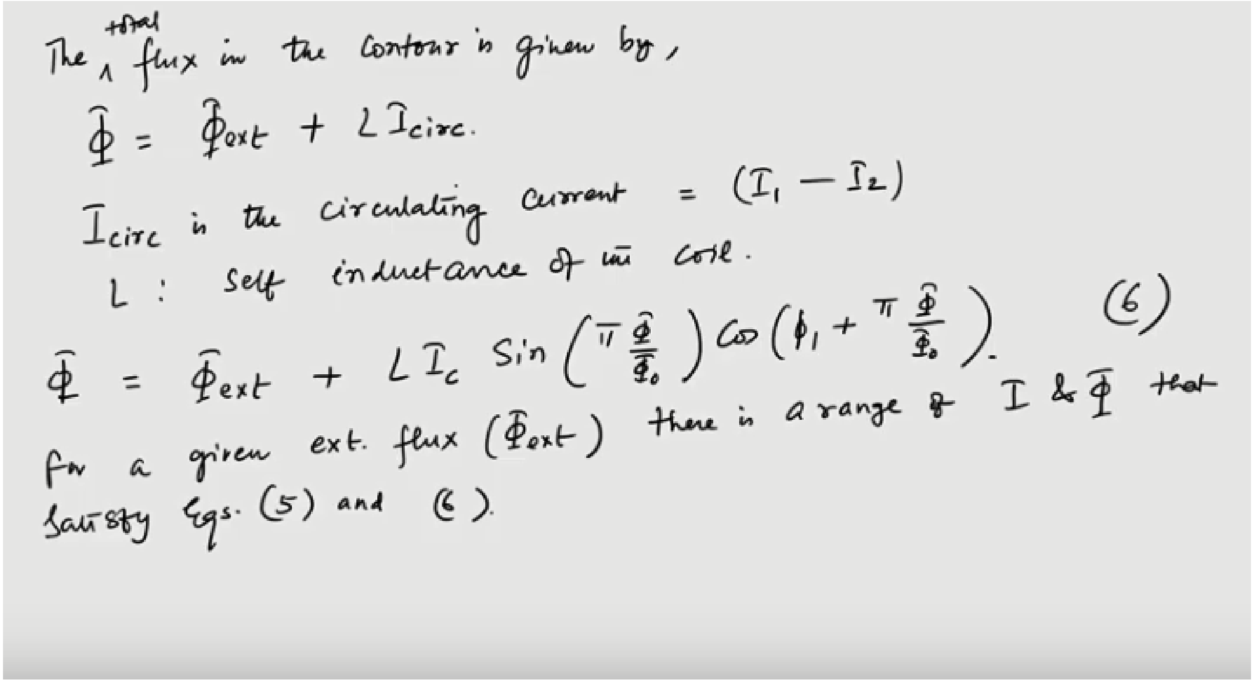

And, then the flux in the contour so, that's the circulating current now, the circulating current so that's the relationship between so assuming that there is a self-inductance of this squid circuit, or the squid loop which is given by L, so that will be the five the total Phi or the flux would be the external flux due to the external magnetic field and the L_i where L is the self-inductance of the loop and I circulate or I_{circ} is the circulating current so I_{circ} is the circulating current equal to I_1 minus I_2 Okay? That's the current that is circulating because they're in opposite direction I_1 and I_2 and elf L is the self-inductance of the coil, or the loop Okay? So, the total flux so this is the total flux so, Phi can be written as Phi equal to Phi external and the plus LI_c sine of Pi Phi over Phi zero and cosine of Phi_1 plus PI Phi over Phi_0 and so on.

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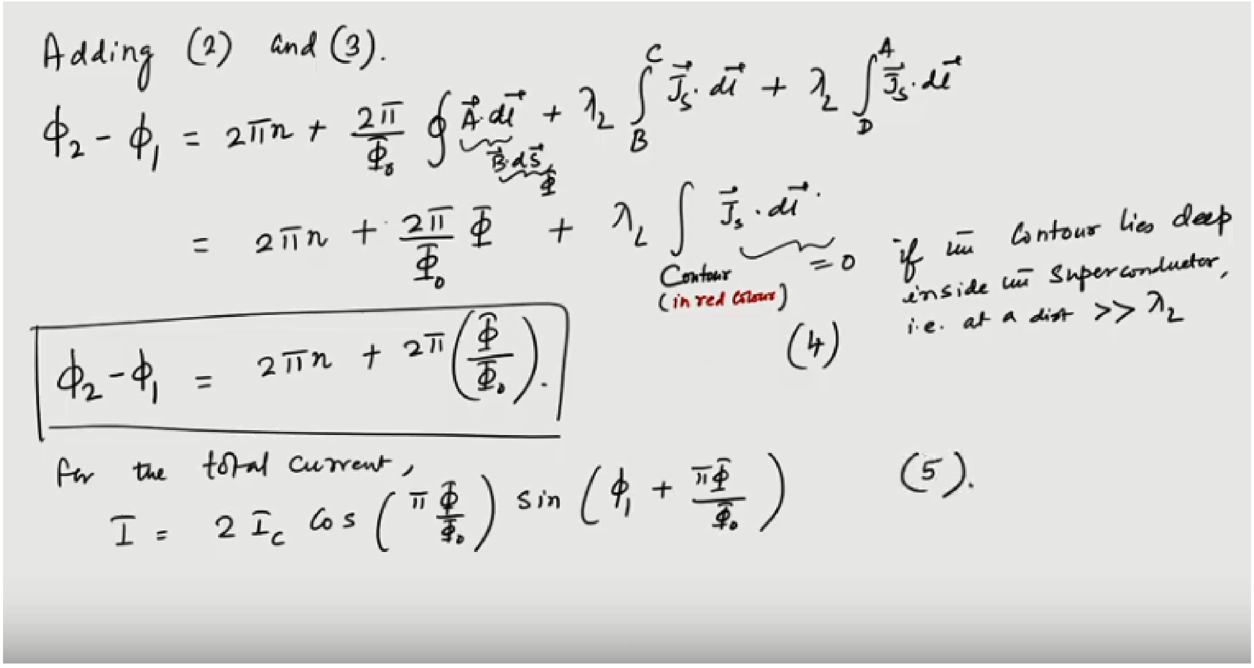

For given so, let's look at this equation and equation 5 and 6 so this is 6 so, we have for a given external flux which is Phi_{ext} which is the first term on the right there so, there's a range of of I and Phi that satisfy equations 5 and 6 and of course? one needs to know I, see which is the resistance less current or the critical current that flows into the squid loop.

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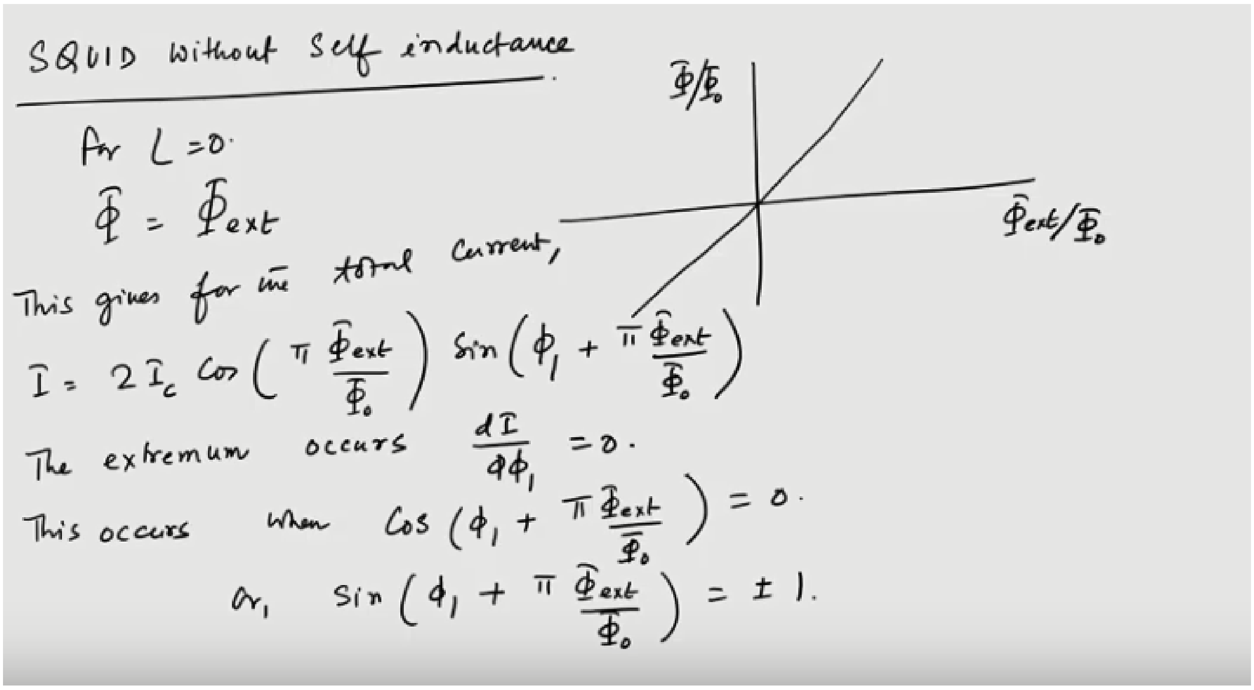

So, let's just talk about squid for a moment squid without self-inductance Okay? So for L, equal to 0 you have Phi equal to Phi external Okay? So, what happens is that your 5vs5 external so, Phi external normalized with Phi_0 and this is Phi over Phi_0 but that's like a straight line that passes through the origin all right? So, this is so this is Phi equal to Phi external so, which gives for the Total Current I equal to twice of I_C cosine of pi Phi external divided by Phi_0 and sine of Phi_1 plus Pi Phi external by Phi_0 remember that there is a relation between Phi_1 and Phi_2 so, 1 this is also a function of Phi_2 so, the extremum of this current or rather the you know the maximum so, let's just write extremum because we don't know before we take a second derivative whether it's a maximum or a minimum so, the extremum going to 0 is only saying that it's an extremum or the function going to zero says only about the extremum condition. So, the extremum occurs when the only variable is of course Phi on the right-hand side for dI d Phi_1 it's equal to 0 so, this occurs when cosine of cosine of Phi_1 plus, Phi Phi external by Phi_0 it's equal to 0 or or equivalently the sine is completely out of phase as we know it's Pi Phi external by Phi_0 it's equal to 1 or plus minus 1.

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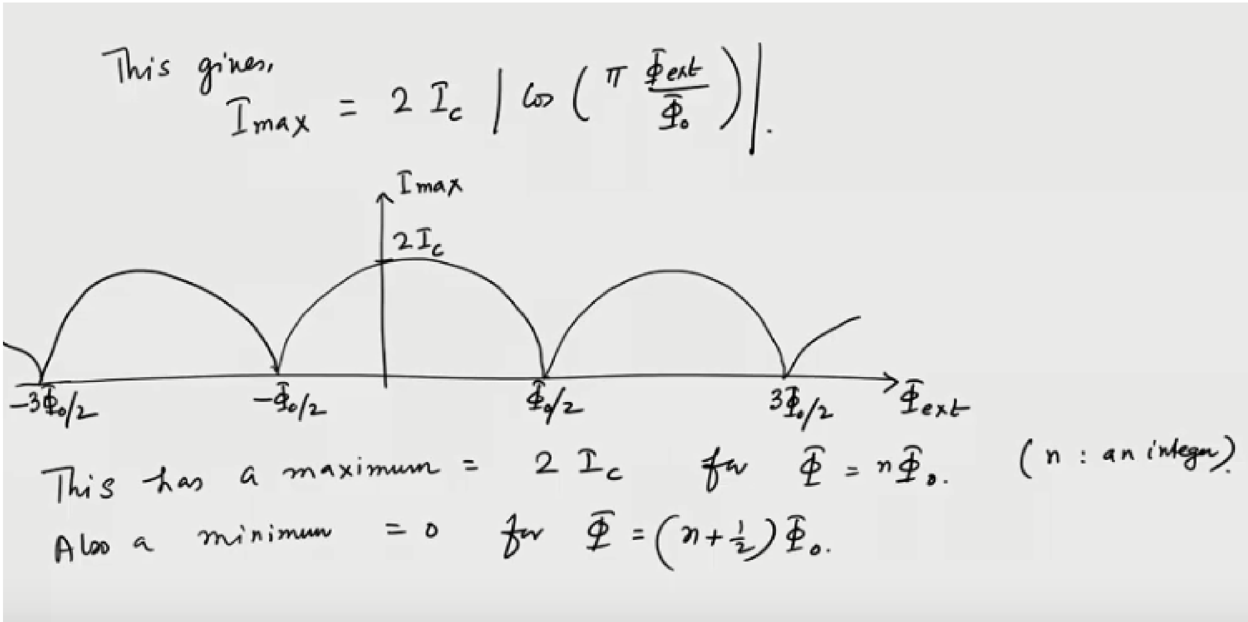

So, this gives of course the I_{max} which comes from the extremum condition is in fact the maximum here is 2 I_C and the cosine of pi Phi external by Phi_0 and so, a plot of this looks like this so, there is so this and so on Okay? These are of course of same height maybe it doesn't seem so and it's very symmetric about 0 the drawing is not all that good, but you should it's slightly harder to draw accurate on this panel. So, this is the I_{max} as a function of the Phi external, or the external flux that is threading the loop and that has a sinusoidal dependence so this is minus Phi_0 by 2 this is Phi_0 by to see that it has an exact period of so, this is like this so this is minus 3 Phi_0 by 2 and this is plus 3 Phi_0 by 2 and this of course goes like this so, this is the so this has a maximum equal to 2 I_c which is shown on the plot on the y-axis so I, is plotted the current is floated rather the maximum current is plotted as a function of the external flux that is threading the squid loop and so for for Phi equal to some n $Phi_{0,}$ and of course an integer so also we see a minimum equal to 0 for Phi equal to n plus half phi_0 Okay? So, these are the n is an integer of course including 0 so, these are the current variation with respect to the flux and as we have said a number of times that this variation of current is a periodic function of the flux, and there is a change in the external flux because of a change in magnetic field there'll be a change in this periodic evolution of the current, and also of course of the detected I mean the the voltage that is detected in this feedback loop, that also will change and will have a variation like this.

So, just to summarize that we, have done in this last part of this course that we have done Josephson junctions and and then how they are applied to the you know the theory of squids, and how the the current variation or the voltage variations are there and we have discussed briefly that there's RF squid which uses one Josephson Junction, and DC squids which are most commonly available they use two Josephson junctions in parallel, and driven by a DC current which which is twice of the the critical current and this is that current variation that is shown here.

So, just to summarize the entire course that we, have started with basic idea of super conductivity Meissner effect and London equations and so on, and then we have talked about BCS theory in details and and understood that how a pairing attraction, attraction, in the electronic channel takes place which causes pairing and hence that causes superconductivity and these are conventional superconductors so gap equations are derived, and simultaneously we have seen a phenomenological theory of superconductivity which has been attributed to the Ginsberg Landau theory which writes down a free energy functional minimizes it, and gets various information that are related for for this for these superconductors in fact they preceded the BCS theory because it's a phenomenological theory BCS theory is a microscopic theory of superconductivity.

And then we, have talked about you know flux quantization we had talked about the new unconventional superconductivity d-wave pairing and we, have also talked about the non-form a liquid features that are very apparent in from the experiments in the case of high TC superconductors, which are the cupids superconductors and finally, we have seen what are called as Josephson junctions and how Josephson junctions can be used to make squids and these squids formed interesting sort of addition to this discussion.

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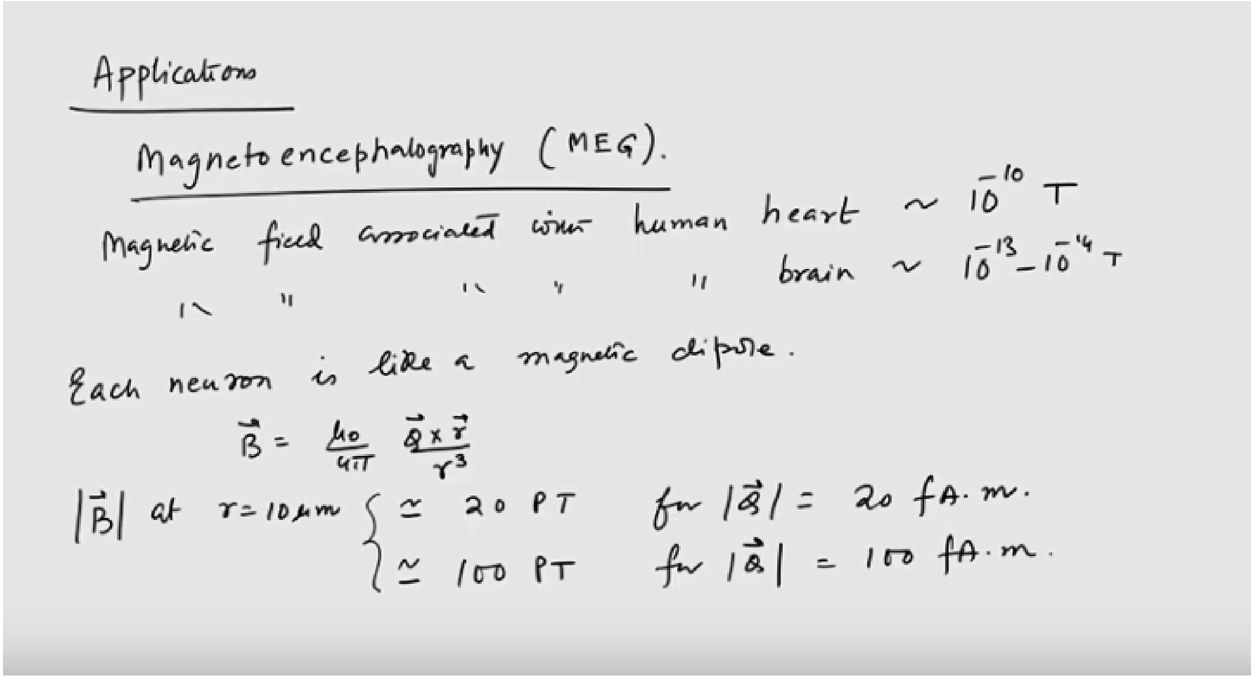

Before I, wind up just say one thing about the application of squids way so, basically there is a something called the magneto encephalography and in short it's called MEG so, basically it's done with respect to the brain and so, as I said that the magnetic field associated with human heart it's about 10 to the power minus 10 Tesla, while the same associated with human brain is about 10 to the power minus 13 to minus 14 Tesla and as you see that they're only squids can determine if, there is any change there so if you think that each neuron is like a magnetic dipole so, we can calculate using the bio Seibert law the magnetic field due to such a dipole which is q cross r by r cube so, this is a dipole model q is like the like the current or something and that is there so, this is B is about so B at r equal to about 10 micrometer is approximately equal to 20 Pico Tesla for, for Q equal to something like 20 for me ampere meter and this is when this the neurons become more active and it becomes like something around hundred for me ampere per meter this becomes something like 100 Pico Tesla for Q to be equal to 100 for me ampere per meter and so on, at the same distance this is roughly an order of magnitude. So, this change can be detected by this MEG technique and suppose there's a trauma that a patient is having at the brain and then this is the kind of change in magnetic field that would be recorded and, and the only instrument that can record it is a squid.

So, there are various other applications of squid which we are not getting into details so, these are the sensory applications and, and, all that so by and large this is where we should stop, and maybe you know sort of take a review of various things that are or rather I, mean it the, the study of super conductivity doesn't stop here, and there are lots of other directions in which studies can be can start, and we of course hope that with this introductory course you would be able to build upon various things related to superconductivity the field is still very open and lots of research is currently going on.