Lecture - 16
AC Josephson effect, Superconducting Quantum Interference devices (SQUID)

We have been talking about, Josephson junctions. And the way it can be made is that, you take two oxidized niobium wires and then, put one against the other and press them. So, the basically the junction that is formed is a Josephson Junction, which is made of an oxide layer, which acts as a tunnel barrier, between the two superconducting materials, which are now we have as a superconductor in this case. So, we have seen that , how the current that flows in the superconducting or the Josephson Junction, through the junction actually oscillates, at a frequency which is large and the oscillation of this current is pretty much, reminiscent of the Young's double-slit experiment, in which there are as, there is interference between two light beams or coherent beams. And then, an interference pattern is produced. So, this current is actually the interference between, the two superconducting elements that are present there. So we have seen, towards the end that how the current and the work done, in the superconducting Junction, actually they are lagging in phase with respect to another.

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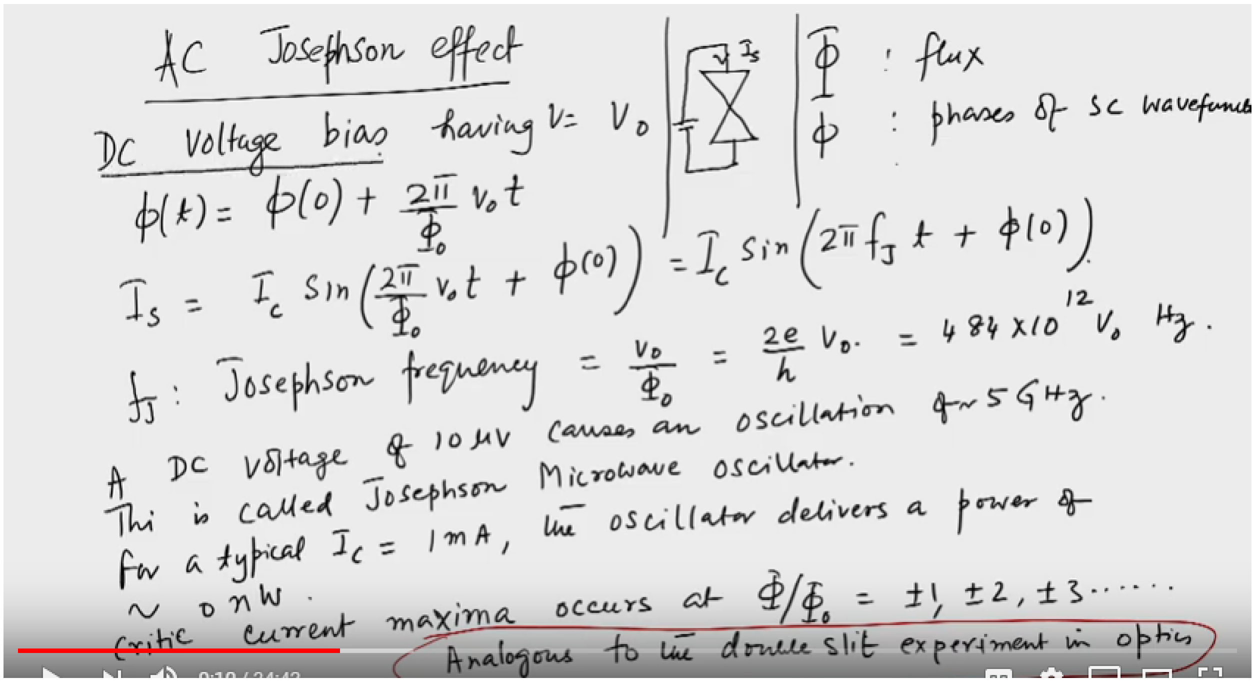

So, let us talk briefly about the, AC Josephson effect. So, we have a DC voltage bias. And having voltage, having V 0. So, V equal to V 0, so that's the bias. So, we are talking about a phase, which varies as a function of time, which is some initial phase and plus the 2pi over Phi 0, which is denotes a flux quantum and a V naught T, we have seen that. So, our I S which is the, the current, through the junction that has a relation which is IC sine of 2pi over Phi 0 and V naught T Plus this Phi 0 that initial phase that is assumed. And this gives rise to, this can also be written as IC sine of 2pi f J T plus Phi of 0 and so, the Josephson frequency so this, F J is called as a, 'Josephson Frequency'. And this is equal to V naught over Phi naught and this is, if we write down. So, F Phi naught is a flux quantum, so please, distinguish that the capital Phi that we write is flux, whereas the small Phi that we write without the horizontal lines on top and bottom are the phases of the superconducting order parameter. Or the wave function: condensed set wave function. So, please distinguish this and so its V 0 by Phi 0, which is nothing but equal to 2 e over H V 0 and that, has a value, say 484 into 10 to the power 12 V zero Hertz. Okay? So, this is in the gigahertz range, so with a DC voltage of 10 microvolt causes an oscillation off about, approximately 5 gigahertz. So, this is called or rather it has a name that's called, 'Josephson Microwave Oscillator'. Because, this gigahertz frequencies actually denote the microwave frequencies and for a typical, IC we have defined IC on the previous occasion, in which I see is basically the resistance less current or the current in the Josephson Junction or rather the Josephson current, before it starts developing a voltage. So, for a typical IC of equal to 1 milli amps, the oscillator delivers a power of, of the order of 10 nano watt, so that's a small power, but one what one is interested in is a very large oscillation in the current that

flows through, the Josephson junction which classically being forbidden, because that's a tunnel barrier, so there's no current that's an insulating region, where no current should pass through.
So, as I said that these rapid oscillations are due to the quantum interference between the two junctions and the period is, given by the field, required to create one flux quantum Phi 0, which is 2 into 10 to the power minus 15 Weber meter square. And so, the critical current Maxima, happens at or occurs at Phi by Phi 0 equal to 1, 2, 3, etcetera and so on. So, this critical current maxima, occurs at Phi over Phi zero equal to plus minus 1, plus minus, 2 plus minus 3 and so on. Okay? So at, at these integer values, which can take both plus and minus values and these are analogous to the as I said, to the double-slit experiment in optics, all right. So, let us draw a small sort of diagram for this, which is, so this is a schematic of the junction. Okay? So, this schematic of this thing would be like, a DC voltage as we have said, connected here and this is connected to the Josephson junction and there is this current that is coming in, which as ice as we have written it as, IS. So that's the circuit for this, let's go to the DC with our AC voltage bias, so this is for a DC voltage bias, let's underline this and let's go for AC voltage bias.

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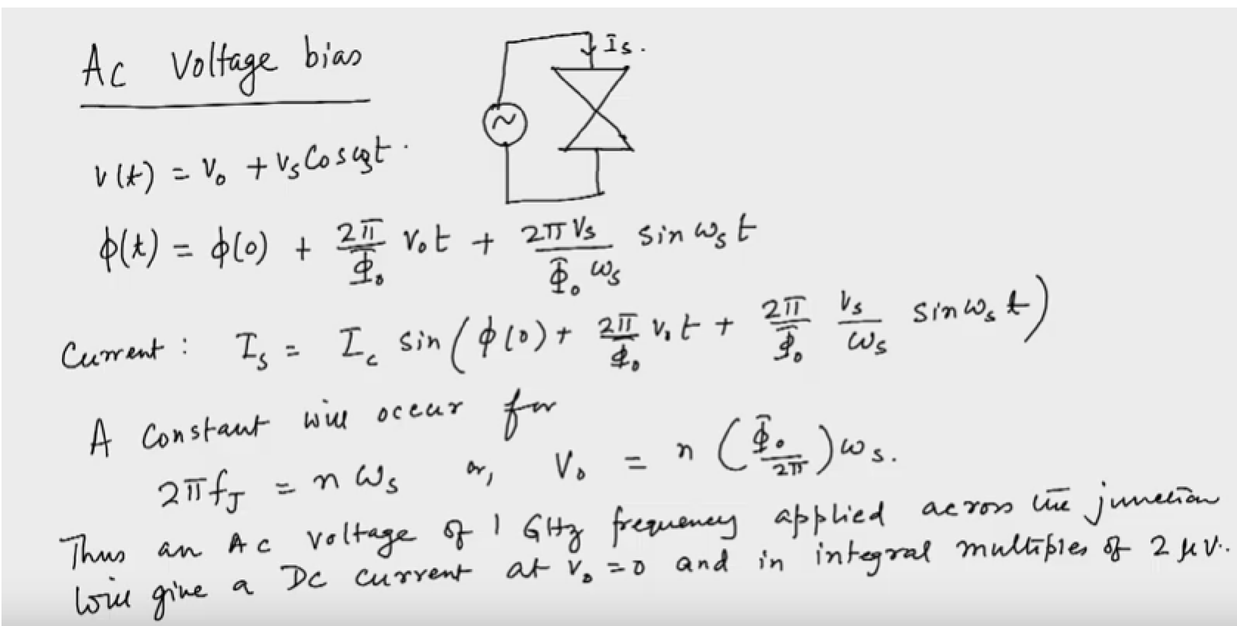

So, the AC voltage bias. And it's the same diagram, accepting the fact that now we have a AC source that is biasing the junction. And so, this is like, this and just, just write it like this and so, there is a AC current that is there. So now, we have a V of T equal to some v-0 which is a constant part and V s cosine Omega T and so the Phi T that's the phase, as a function of time is equal to a Phi 0, which is a constant value that was present at T equal to 0 plus a 2 pi over Phi 0 that's the flux quantum and V naught T, which we have already seen and plus, an additional term, which is due to the, the voltage, AC voltage. And this is like a sine Omega s T; let's write it as Omega s T and so on. And so the current will be, its I s equal to IC sine of Phi 0 plus a 2 pi by Phi naught V naught T plus a 2 pi by Phi naught and V naught by sorry, this is V s, let's make it s, V s by Omega s and a sine Omega s T, let's see whether, this is V s. And so that's the, that's the current that flows through the junction and a constant current will occur for that is the time dependency will go away, will occur, now this will occur periodically not once, but as the time progresses it will occur, a number of times depending on the condition. So, this is the condition that 2 pi F J which is the oscillator frequency is n Omega s or V 0 equal to n Phi 0 by 2 pi into Omega s. So, basically it says that, AC voltage of 1 gigahertz frequency, applied across the junction, will give a DC current at v not equal to 0, slightly misnomer I mean, DC means direct current. But, we keep saying DC current and DC voltage and just as, a matter of you know convenience. So that at V naught equal to zero and also in integrals of, integral multiples of, of say two micro volt and so on. So, there's an interesting correspondence between, the Josephson junction and the LC circuit and this correspondence will be also, exploited in a device called as a, 'RF Squid'. And let's just give that,

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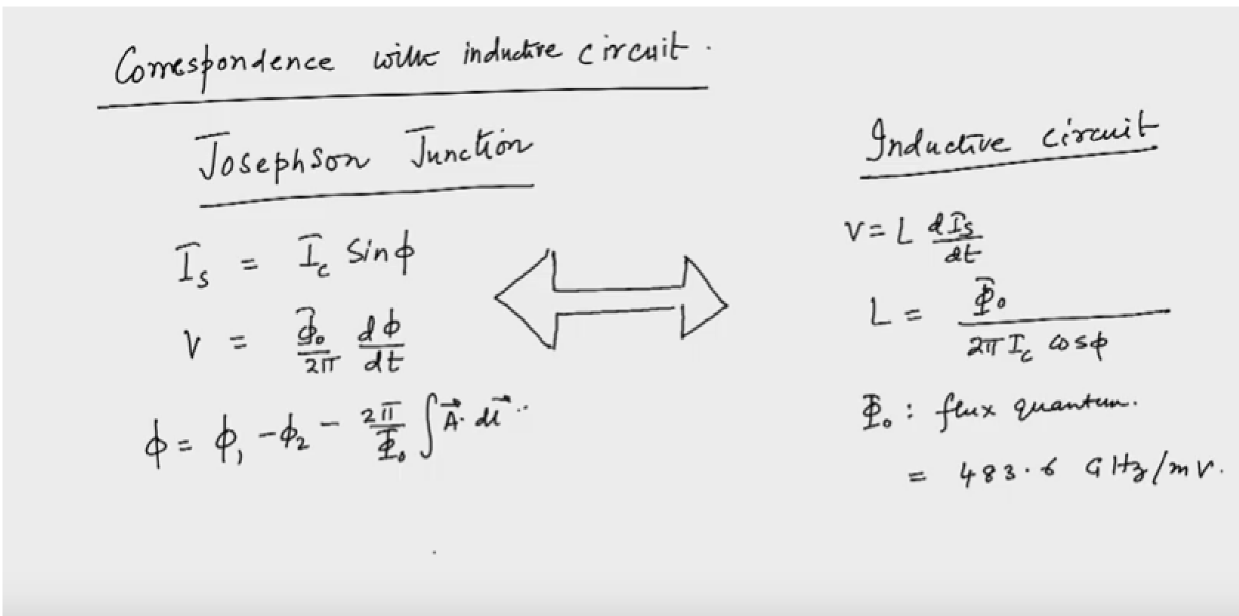

correspondence with LC circuit. Okay? So, a Josephson Junction or rather an inductive circuit, what is called as? So, Josephson Junction so, this is really not an LC circuit. But, rather an inductive circuit. So, there's a Josephson Junction, inductive circuit and so here, I s goes as, I C sin Phi and V is equal to Phi 0 over 2 pi D Phi DT or this is this converted. So this is a D Phi DT so that's the current and the voltage and now, we can have a correspondence along with, so this is equal to V equal to L dIs DT in an inductive circuit and also, the L is given by the Phi 0 divided by 2 pi I C cosine Phi, Phi 0 is of course the flux quantum, which has also it can be written, just in addition to what we have said, it's 2 into 10 to the power minus 15 waiver meter square, it can also be written as 483 point 6 gigahertz per milli volt and here the Phi is nothing but, equal to Phi 1 minus Phi 2 or Phi minus Phi 1 and minus, minus 2 PI over Phi 0 A dot DL. So that's the correspondence between the two, with this knowledge of the Josephson junctions, we are going to get into the superconducting circuits and one of the things that we discuss, there is a device called a, 'Squid'. Which is known as, the superconducting quantum interference device. Okay? So let us, get into the discussion of squids.

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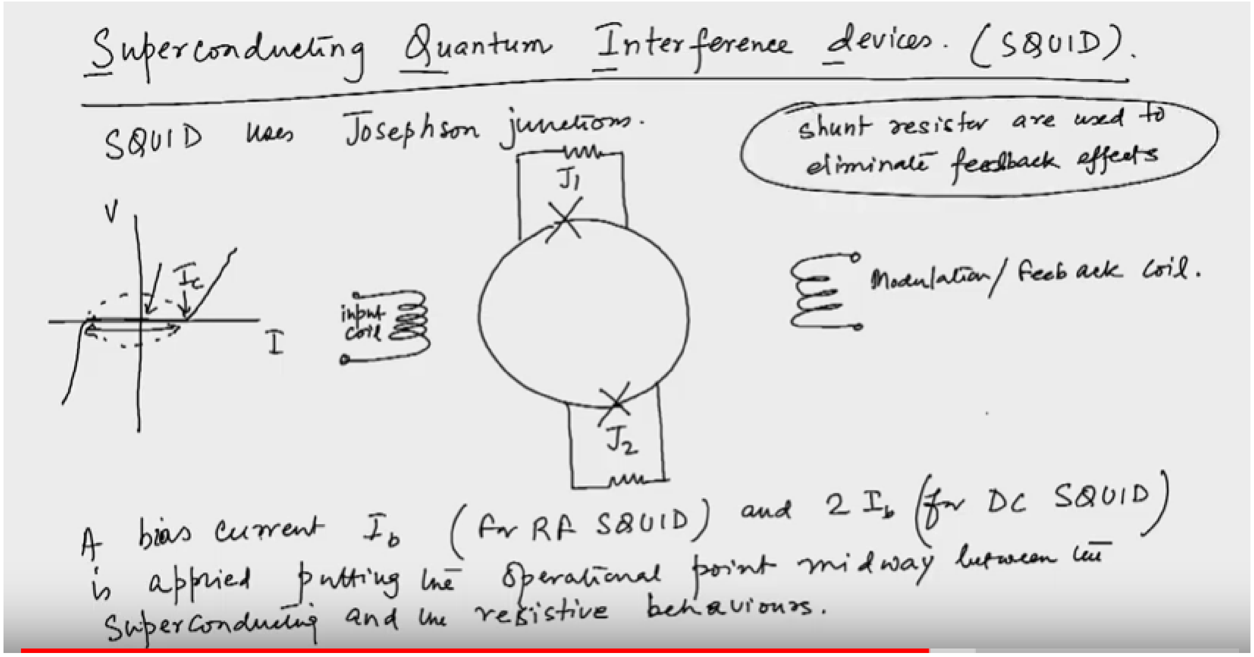

So, it's this S Q I and D so that is a SQUID. Okay? So, this quiz I use, Josephson junctions. And the purpose of this quiz is to actually measure very small variation of the magnetic field. So, if in a particular circuit or in a particular physical phenomena, if there is a very small change in magnetic field or the flux associated with it, there is no other device that will measure it, down to 10 to the power minus fifteen vapor meter square, than that of, of a squid. And in fact it's interestingly, interesting to know that, the magnetic field associated with human heart is about 10 to the power minus 10 Tesla, whereas the magnetic field associated with the brain is about, 10 to the power minus 13 to, 10 to the power minus 14 Tesla. So, if there is an increased activity or there is a magnetic field, increase in the magnetic field due to activity in the brain, such as stroke or something, these squids are the devices that can, detect changes in magnetic changes in this magnetic field or the flux associated with it, there is a device called as a, Micro a Magneto an Magnetoencephalography M eg, which measures these changes. So, basically each neuron is taken as a magnetic dipole, which gives a you know a magnetic field which is of the order of 10 to the power of minus 13 tests, let's say for example. And these there's the small change, such small changes in the magnetic field, in the brain can also be detected by a square. So, squids are very, very useful devices, which are as I said, mainly used for understanding or recording changes his magnetic field of that small order, usually for other measuring devices a 10 to the power minus 14 or 10 to the power minus 13 Tesla would come as, within a noise so or the error bar of that or the I mean, within the accuracy of the measuring device, so one cannot think of measuring such a small magnetic field, but squids can do it. Okay? So ,what it does is that, we will just draw squid generic squid, so basically there is a, so there is a junction here, there is a junction here, there could be one Junction as well, will see that and so this is one of the Josephson Junction let us call this as, ‘J1’. And let's call this as, ‘J2’. And there could be shunt resistances, but however that is not necessary, but usually in commercial DC squids there are, these shunt resistances being there and this shunt resistance is what the purpose of them, to have it is to avoid, effects such as hysteresis effects, so which we have seen that, the Josephson junctions have current voltage characteristic as this, so there is a so this rises like this and this fall's like this, so this is that Josephson current, so this is that Josephson current and this is called as the, ‘IC’.

We have discussed this and as, the voltage starts developing one actually defines a critical current. Now, if you do not use this shunt resistance is there are two things that can happen, one is that the efficiency of the squid's will go down and secondly as you trace back the curve, you'll have a hysteresis that is, that will probably come in here, so one will have an area that is formed here and there won't be a straight line grazing the current axis completely, so that is the reason these shunt resistances are used. So, as we have or rather we are going to show that, that these coils, the input coils and the output coils are never wound around the Josephson junction. But, they're inductively coupled, so there is an input coil here and so, this is a input coil and there is a output coil, which is also known as the modulation or feedback coil or the modulation coil can be different ,than the feedback coil but there, they are placed here, modulation feedback coil and the voltage that is detected in the modulation coil, of the feedback coil is an oscillatory function of the flux that is threaded, through this loop of the squid or these Josephson junctions and there is a change in the magnetic field, there is a change in the threading of the flux and that shows up, in the voltage characteristic curve, voltage as a function of Phi. Now, this voltage as a function of Phi has got a periodic modulation with a period as, Phi naught which is the flux quantum. And so, one can actually put a bias current, colita IB which is for RF squid, will define what is our F squared and 2 Ib for DC squid is applied, so putting the operational point, a midway between the superconducting and the resistive behaviors. What I mean is that? It's put roughly here, so which is between the or, or around that point, so that it's between the resistive the superconducting and the resistive behavior. And as I said here, let's write that the shunt resistance is resistors are used to eliminate feedback effects. Okay?

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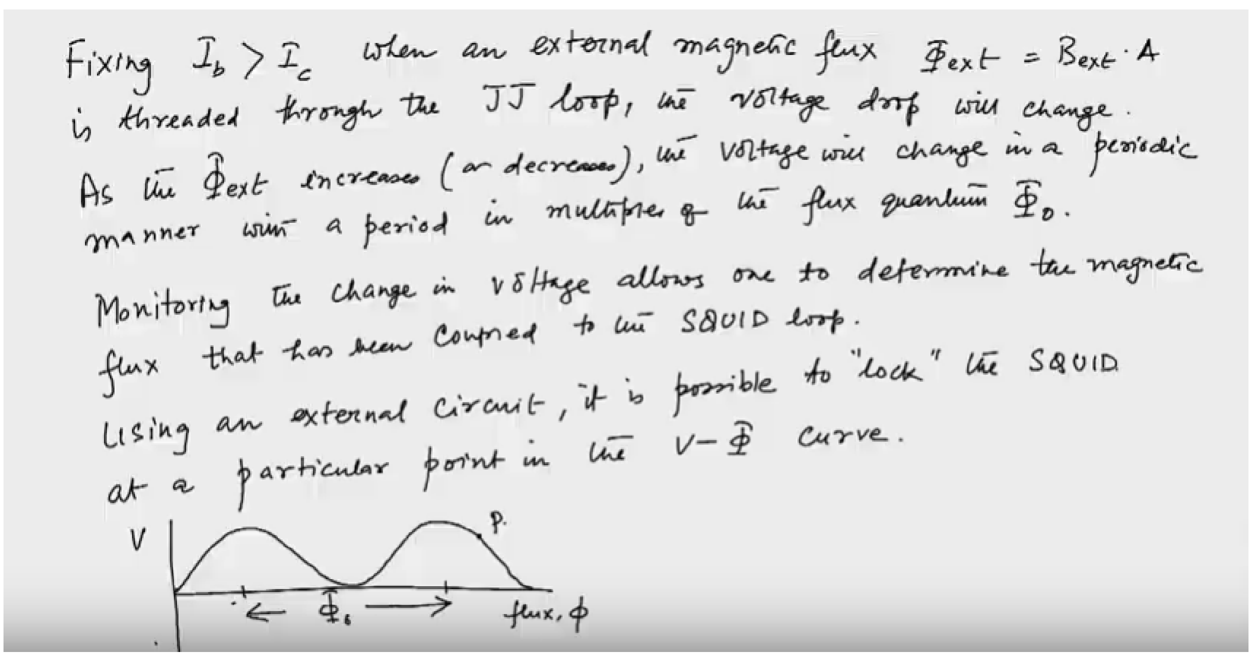

So, fixing Ib greater than Ic that is the critical current, when an external magnetic flux, phi ext equal to b ext dot a, where a is the area of the loop of the squid is threaded, through the so, JJ means Josephson junction, JJ loop, the voltage drop will change. So, voltage drop in the feedback coil will change, as the or other Phi ext increases or decreases, the voltage will change in a periodic manner, with a period in multiples of the flux quantum Phi zero. Okay? So, basically monitoring there's the principle of the squid, the change in voltage, allows one to determine the magnetic flux that has been coupled, to the squid loop, which means in the feedback coil. Now, using an external circuit, it is possible to lock the squid, at a particular point in the v phi 0 curve. So, our V Phi curve rather, V Phi curve. So what, what it look like is the following that you have a V Phi, so V and flux Phi and so it oscillates like this and so on. And this distance from the, the period from one Maxima to the other Maxima is Phi 0 or multiples of Phi 0. Okay? So, we can actually lock this thing in somewhere for the operating reasons, we can lock it say somewhere here and it's usually locked, where the DV, D Phi is maximum that's the slope of this V Phi graph is, maximum for the maximum sensitivity or performance of this squid. So, the squids have a key factor in the development and commercialization of ultra sensitive electric and magnetic you know, measurement systems in many cases squids offer the possibility of measuring, very small magnetic fields, changes in the magnetic fields, where no other methodology, measuring methodology is available. So, it has a very large variety of sensing applications and not only that, there's other applications as well, so let me write that down.

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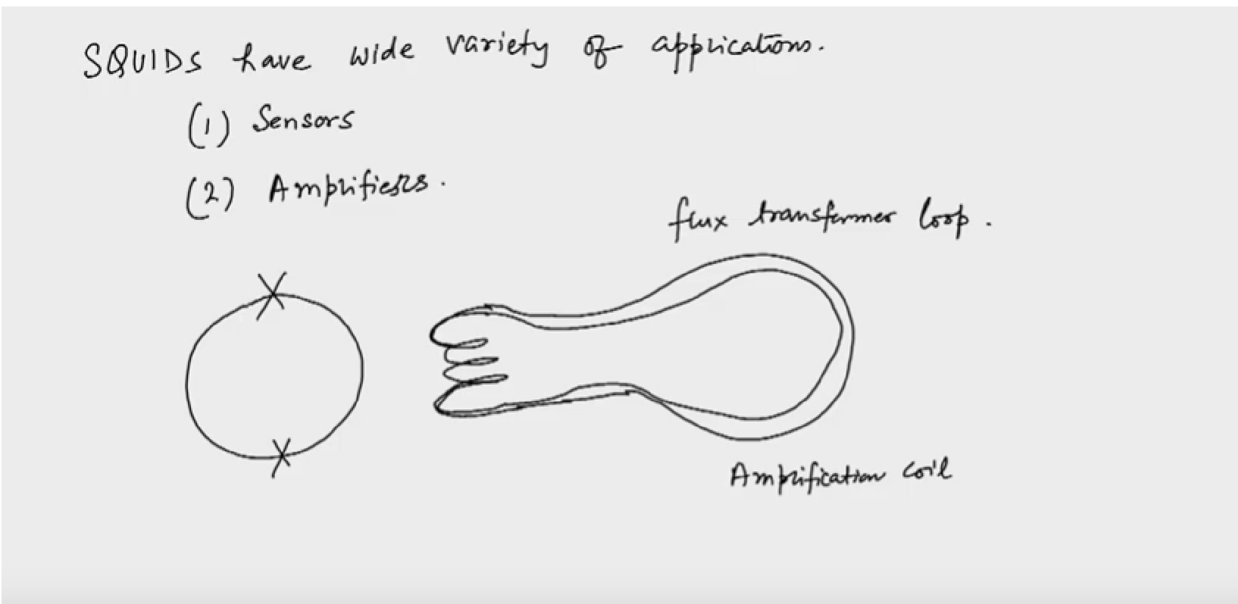

Squids have wide variety of applications. One is this is a sensing application or use them as sensors. And then also use them as amplifiers, amplifiers. So, basically what can happen is that, so usually as we have shown a squid, squid has a so this is a basically a two Junction squid, which is known at DC squid. But, in any case, so this there is a certain amount of loop or associated with DC with a squid and this if you try to increase the loop area, of course it will increase the, the performance or the sensitivity, but at the cost of also increasing the self inductance of the coil. And this increase in self inductance, actually overcompensate for the gain in sensitivity. So, the performance of this or rather, this can be actually coupled, inductively coupled with a, a large you know, sort of and so on. So, this is that amplification coil, which can be threaded or rather he inductively coupled and which can have a very large area. So, this will so this is as a is a flux, transformer loop, so instead of increasing the area of the loop of a squid, one can inductively coupled with a large loop, which will give the amplification. So, there are these applications and in the next, we will talk about the, the RF squid and DC squids in some details and do some calculations or calculate the current characteristics current and voltage characteristics of the of the DC squid. DC squids are mostly used for commercial purposes, RF squid it uses one Josephson junction and in fact it's a misnomer because no, quantum interference takes place in RF squid. However we'll describe that and as, also described the DC squid. And it's applications.