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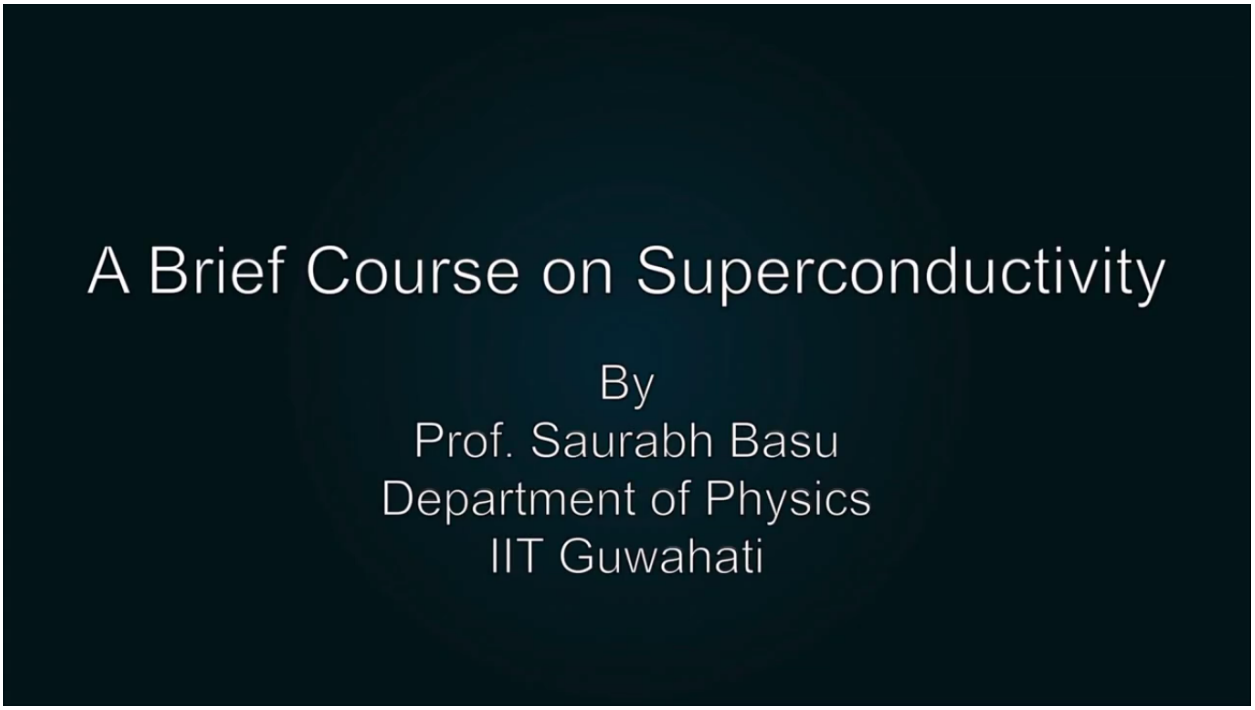
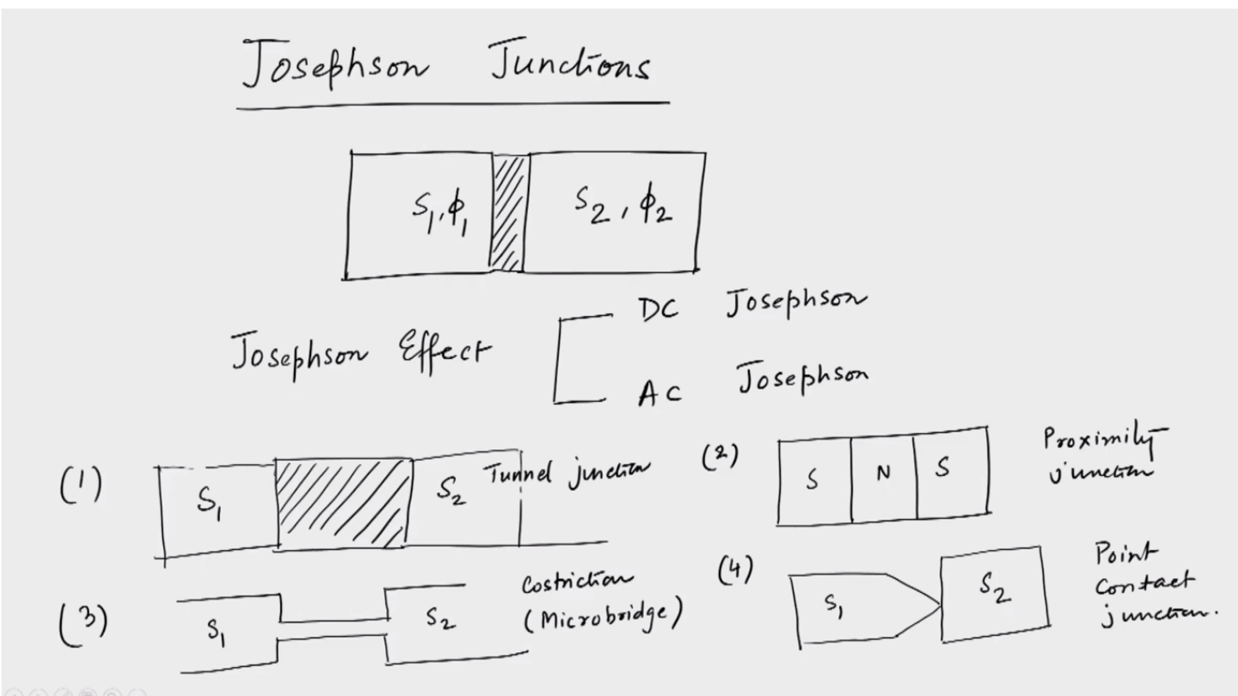




So today we are going to look at Josephson junctions which is at the heart of any circuits made out of superconducting materials so the superconducting circuits. So this is a very nice discussion on the superconducting properties or the quantum coherence properties of superconductors. So what happens when two superconductors are placed in close proximity to each other which could be separated by an insulating junction or a tunnel junction as it said. So we are going to discuss what are called as the Josephson junction.

So two regions of superconductors they are placed in close proximity as I said and the quantum-mechanical phase of one of them is say Phi 1 and the right one is called as Phi 2 so we have two superconducting material. Let's call them as S1 and S2. This has a condensate wave function characterized by a phase Phi1 and this has a condensate phase Phi2 the phase of the condensate wave function and they are placed in close proximity and this area the one that is in between S1 and S2 is occupied by tunnel junction or an insulating barrier which we'll see.

And so in a normal material this would not have mattered because there is no phase relationship between the two materials that you place in the close proximity. However, for a superconductor or rather for two superconductors when you place them in this kind of a scenario then a current flows through the system even through the tunnel junction that lies in between will draw the tunnel junction with a shaded or hatched line. So current actually flows through this and this called as a Josephson current and this Junction is called as the Josephson Junction and as I said that just the way the transistors are important to semiconductor electronics, these Josephson junctions lie at the heart of these Junction, these superconductor based electronics. So basically strictly speaking the resistance-less currents that we have said the current that flows which is the Josephson current its resistance-less and its manifestation of the DC Josephson effect while there is another situation in which the current actually oscillates with very high frequency and that's known as the AC Josephson effect.

So this Josephson junctions they are or rather these -- let's talk about the effects now which is basically the flowing of the resistance-less current. So there's a DC Josephson effect and there's AC Josephson effect. And we are going to see what they are in just a while from now. All right so any weak coupling between two regions of superconductors which could be a tiny constriction which is also known as a micro bridge or there are even point contacts between two superconducting specimens they also can constitute the Josephson Junction.

So let us draw some pictures of these junctions. So let us -- so one is you have and I am course drawing it much larger than scale so these insulating or which is called as a tunnel Junction so this is called as tunnel Junction and these are superconducting just the way we have drawn it is just that we have enlarged the view or there could be – so these are the superconducting.

So let's just and then one can have SNS where N is a normal metal Junction so these are called as the proximity junctions. Third there could be a micro constriction so this is S1 and S2 and this micro constriction could actually be made of the same material as S1 or S2 and all three could be same but just that fact there's a constriction there the number of modes carrying current will definitely be affected by this and this is called as the constriction or what are called as the micro bridge and these kind of materials are quite important in the sense that in the superconducting technology because they are made by a variety of techniques, electron beam lithography being one of them and one can actually make such junctions and study the properties of material that is the transport, the charge transport or rather the current in these junctions or other properties such as from the sensing applications or them as switching devices, etcetera. that also can be done and forth so this point contact. So this is the superconductor and there's another superconductor and one is connected to the other by just a point contact. So these are point contact junction. So these are micro bridge Junction and so on. So these are some of the examples of Josephson junctions which –

Josephson junctions are characterized by small critical currents, I_c.
I_c is characterized when the system starts developing Resistance / voltage.

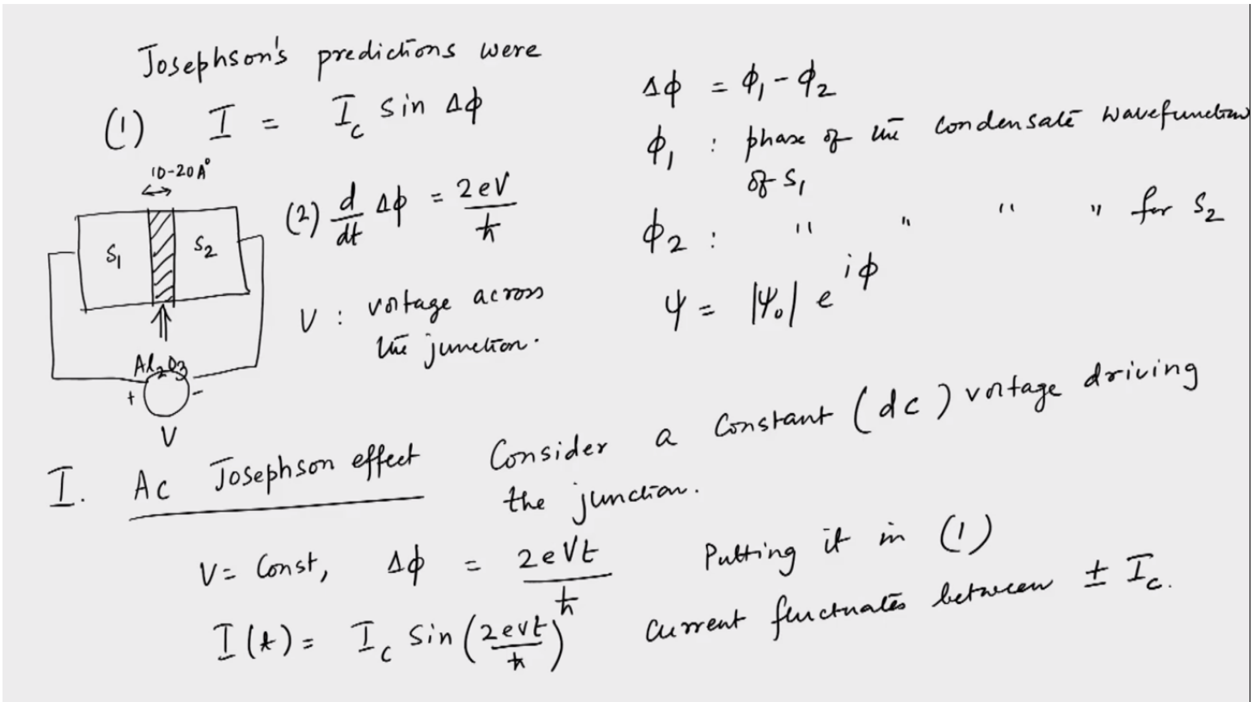
Applications

(1) Sensors,
(2) Variable inductors
(3) Switching devices
(4) oscillators
(5) Amplifier etc.
(6) SQUID (Superconducting Quantum Interference Devices)

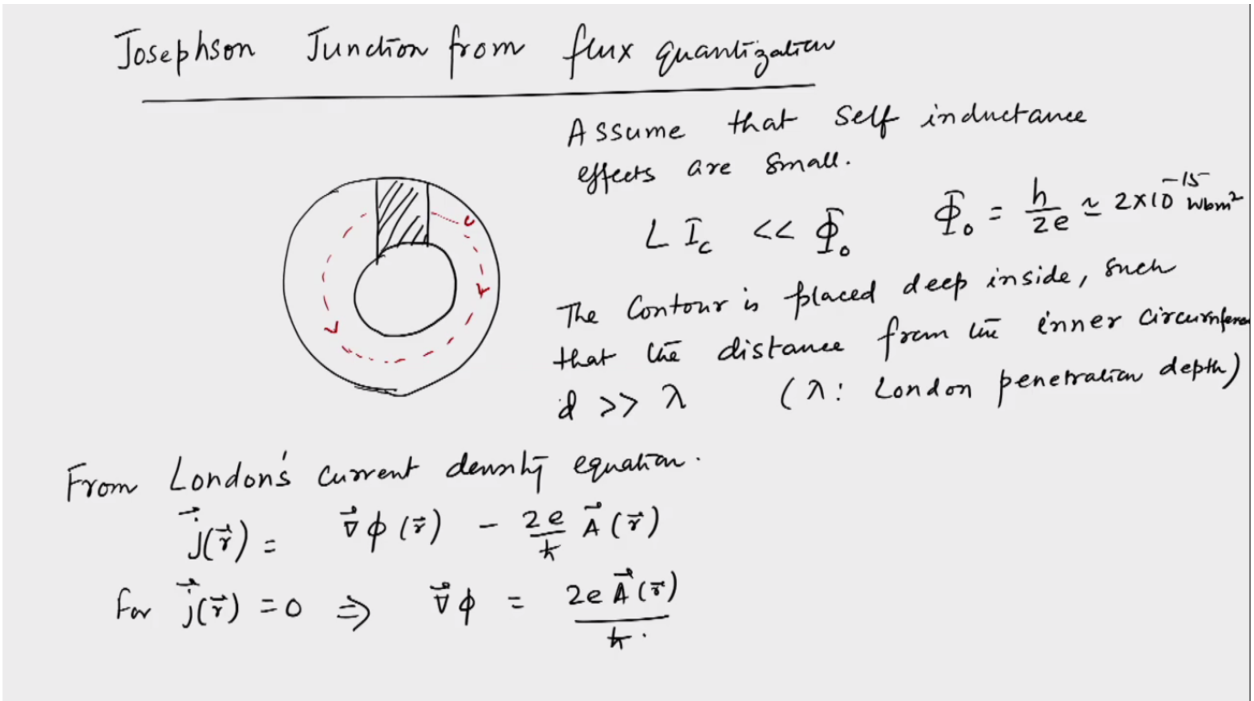
So mainly we are going to talk about the one that is shown in a serial number one. However, the discussion is not restricted to one. It could actually be extended to all these other three junctions that we have shown here.

One important thing about this Josephson Junction is that it has usually much lower critical current that is there than other just the superconductors. So critical current is a current beyond which resistance starts appearing or the voltage starts developing into the sample. So we will show a typical current voltage characteristic curve of Josephson junction and you will see that this is that point – so there is a we call it IC or the critical current and so basically the Josephson junctions small critical currents IC. IC is characterized by when the system starts developing resistance. So that's the limit of IC or rather IC is defined as the current that flows through the junction in a resistance-less condition. So developing a resistance is same as saying that it is developing voltage. So we'll say resistance / voltage. So these have much smaller IC as compared to the bulk superconductors. And as we have said that they're very important to the superconducting electronics and they can be used in a variety of situations. So applications. It could be sensors, variable inductors, switching devices, oscillators, amplifiers, and so on. So there are very large number of applications of these Josephson junctions. And as I said that they are actually central to all the superconducting electronics that one thinks about and the most important application is in the form of SQUIDs which are called as the superconducting quantum interference devices. So let's just call it a SQUID. Superconducting Quantum so well Quantum Interference Devices and we are going to take this up for discussion next after we are done with the Josephson junctions.

So let me just point out that so this is S then QU and then I and then D. So that makes it SQUID. So let us see that what are the predictions of these Josephson effect or rather what are the key equations and key sort of predictions and concepts that are involved in this Josephson effect.



So Josephson predictions were, one, your current is actually is equal to sin Delta Phi where Delta Phi is equal to the phase. So this is equal to the change in phase so we'll write it as Phi1 minus Phi2 so we have a situation like this. So this is the tunnel Junction or the insulating barrier that lies in between and this is typically of the order of 10 to 20 angstrom and this is a superconductor 1 and the superconductor 2. And so this is usually alumina or the aluminium oxide which is well known insulator and the importance of this is that this is exactly made or there it is possible to make them in the form of a thin layer which is insulating. So it's usually alumina is used and so this is Phi1 minus Phi2 where Phi1 is equal to phase of the condensate wave function of S1 and similarly Phi2 is the same for S2. So anyway so this is your – so just to put things in perspective that we have the condensate wave function which is represented by an amplitude part and there is a phase part. So this is in general a complex quantity and the condensate wave function has this form and we are precisely talking about this Phi. So at two places in a superconductor the phase of the wave function is constant and when you put them in close proximity which is separated by 10 to 20 angstrom of an insulating barrier which is known as a tunnel Junction then the current that flows into through this Junction is actually a function of these, the change in the phases of these two condensate wave functions so there is a relationship between the current and the change in phase or rather difference in phase of these condensate wave functions for the superconductor 1 and superconductor 2. All right so this is equation number 1 and this is equation number 2 which talks about the derivative, time derivative of the phase. This is equal to a 2ev over h cross where V is the voltage that drives the junction and it could be a DC voltage or it could be an AC voltage. So we simply leave it at that so there is a plus and a minus. So there is a voltage that drives the junction and the voltage is actually related to the derivative, time derivative of the phase difference between the superconducting wave function or the condensate, the phases of the condensate. So V is voltage. If we have a voltage constant so situation one we will call it, so we will talk about the AC Josephson effect first.



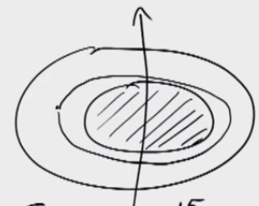
Consider a constant or a DC voltage driving the junction. So which means that V equal to constant and so if V is equal to constant then your Delta Phi is equal to – so V equal to constant so that means your Delta Phi is 2evt by h cross so that can be put back into 1. So putting it in 1 we have a current which depends upon time and it is IC sine of 2eVt over h cross. So you see the current is fluctuating between a plus IC and minus IC with the frequency that is given by this 2evt by 2ev by h cross. So that's the frequency and that could be pretty large depending upon the value of V that you are driving the junction with. So this is called as DC or rather this called as AC Josephson effect where the current fluctuates between plus IC and minus IC whereas I told that IC is the critical current which is defined for a given Junction.

II. DC Josephson effect,

$I(t) = I = \text{Const} \; (< I_c).$

$\Delta\phi = \text{Constant} = \sin^{-1}(I/I_c)$

Also from (2) $V = 0$ $\left(\text{as } \frac{d\Delta\phi}{dt} = 0\right).$



$\Phi_0 \simeq 2 \times 10^{-15}$ Wb-m^2

Let's now talk about the DC Josephson effect. Okay. So what happens is that it's not always possible to keep the voltage across the junction constant a better option is actually to keep the current constant so if we have a current that is constant. So It is equal to some I which is equal to constant and that current should actually be less than IC as we know that this current is less than the critical current and in that case for – so your Delta Phi is equal to constant as we see from equation 2. So if we have the current to be constant then Delta Phi has to be constant. If Delta Phi has to be constant then the second equation gives that V has to be equal to 0 which means that the bias voltage is equal to 0. So if that is the case so this is equal to a sine inverse I by IC so which means that the phases of the condensate wave functions denoting the two super conductors they remain constant at all times and the difference between the phases is given by the sine inverse of this I over IC where of course your IC is greater than I and also as we understand that from 2, so V is equal to 0 because of the fact that D Delta Phi Delta t equal to 0 so this is known as DC Josephson effect.

Now this Josephson effect can be very nicely understood from the flux quantization. So just to remind you that a what is flux quantization or why does it come. In a superconductor if we put it in a magnetic field then the information about the magnetic field actually enters through the phase the condensate wave function. So in addition to an intrinsic phase it picks up another phase which goes as a.dl where a is vector potential is a line integral of the vector potential which using a Stokes law can be written as b.ds where b is the magnetic field, external magnetic field and b.ds is nothing but the flux. So the phase of the wave functions get modified by the flux that is threading a superconductor. So we are talking about a multiply connected superconductor so that means that there is a superconductor with a single hole that is there. So if there is a magnetic field that is threading so the flux that goes through this is equal to b.a a where a is the area of this hole here. Now since the wave function has to be single valued if we make a turn of 2pi so which means that the phase here should be same as if you make a turn off by an angle 2pi you will still get the same phase because of this the flux that this superconducting specimen or this multiply connected superconductor that can thread can no longer be anything. It could just be multiples of a quantum of flux which has a value which we have said a number of times which is very close to 2 into 10 to the power minus 15 Weber meter square. There is a small factor I mean there's a small number slightly greater than 2 but we're just taking it as 2.

Now you understand that this is the whole idea of interest in this Josephson junctions and the squares that we will see will share it more elaborately but understanding that such small magnetic fields can be actually detected and in fact the magnetic field associated with human heart is of the order of 10 to the power minus 10 Tesla whereas the magnetic field associated with a brain is 10 to the power minus 12 to 13 or even lesser than that Tesla which is of the order of Pico Tesla and even smaller than that. Now what happens is that if there's increased brain activities such as a stroke or things such as that magneto and [00:25:15] is which is in short is called as meg which uses the principles of SQUIDs can actually detect such small changes in the magnetic field and no other instrument can actually detect such small magnetic field. We'll put things in perspective later where we show that where we are trying to determine magnetic fields which are much-much smaller than the Earth's magnetic field. So we can understand this Josephson effect from the perspective of this flux quantization.

So let's – so in order to do that we'll have to take a superconducting ring because we want a a hole which the magnetic field can thread and we'll also have a Josephson junction which means we have a insulating barrier. So we'll take a thick ring. I will tell you the reason for taking a thick ring. So this is the thick ring. So there's a hole in between and there is a superconducting material and we take this region as the tunnel Junction or the insulating barrier that we have. So we have a superconductor on the left of the tunnel junction and we also have a superconductor on the right of a tunnel junction which are of course connected. Now let us draw for our convenience that's going to be immediately clear that we will draw a contour which is weld inside the superconducting specimen. Let's call that contour as C and we can give a direction to this. It could be the contour I mean when we take the line integral about the contour we can follow this path and let's denote these points as A and B and so this is same as A and this is same as B inside the hole. And so we have this set up which is a Josephson junction and we are going to use the flux quantization and it's important to understand that the self inductance effects are small. So self inductance of the loop itself does not interfere into our considerations in a dominant way. So what we mean by this is the following so which means that LIC is much smaller than Phi naught, where not equal to h over 2e and the value you know it's 2 into or 10 to the power minus 15 Weber meter square. Okay.

so the self-inductance of the loop multiplied by the critical current that flows which is a resistance-less current that flows through the junction is is much smaller than the Phi naught which is the quantum of flux which has a value which we know. And the contour is placed deep inside such that the distance from the inner circumference. Let's call that d is much much greater than lambda which is the penetration depth, lambda penetration depth.

Now if all these things are correct then what happens is that the magnetic field is of course zero in the specimen and we can write down from London's current equation. So your jr this is equal to its a gradient of Phi where Phi is of course the phase minus 2e by h cross Ar this you can see your notes. So this called as a London equation. For jr equal to 0 which is a current density in the superconducting specimen equal to 0 this how the Meissner effect is actually derived. So this is equal to 0 that gives that this grad Phi is equal to 2eAr by h cross.

Doing an integral along the Contour C

$$\int \vec{\nabla}\phi \cdot d\vec{l} = \frac{2e}{\hbar}\int \vec{A}\cdot d\vec{l} = \frac{2e}{\hbar}\int \vec{B}\cdot d\vec{s} = \frac{2e}{\hbar}\Phi = \frac{2e}{h}\cdot 2\pi\,\Phi = 2\pi\frac{\Phi}{\Phi_0}.$$

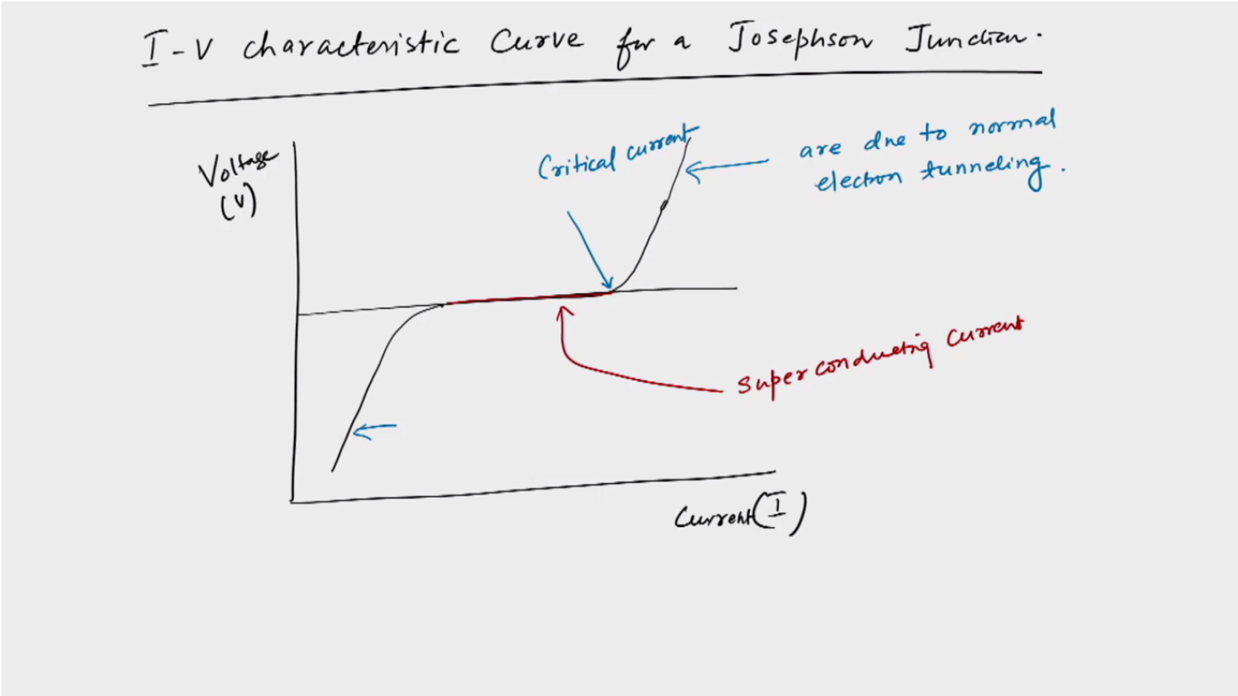
$$\boxed{\Delta\phi_{A\to B} = 2\pi\frac{\Phi}{\Phi_0}}$$

$$\frac{d}{dt}\Delta\phi = \frac{2e}{\hbar}v(t). \longrightarrow$$ Josephson's second equation

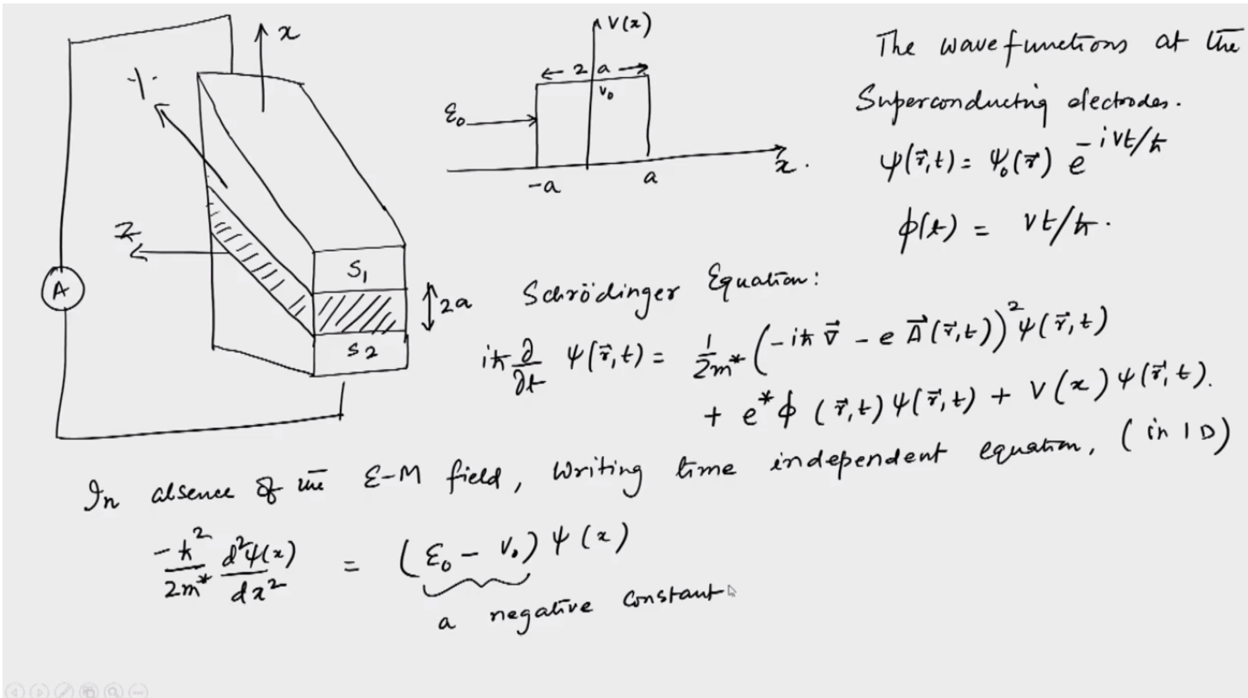
$v(t)$ is the emf developed or the voltage drop across the junction (between A and B).

and if we do an integration doing an integral along the contour C we have a grad Phi.dl this is equal to 2e over h cross a.dl which is equal to 2e over h cross b.ds and this is equal to 2e over h cross Phi that's the flux that threads the ring, superconducting ring because of the external magnetic field b. So this is equal to 2e by h into 2pi. This h cross can be written as h over 2pi and then Phi and then 2e over h is nothing but 1 over Phi0 so this becomes 2 pi Phi over Phi0 where Phi0 is the flux quantum.

So that tells you that Delta Phi that is the value of the change in the phase of going from A to B along the contour C so this is equal to nothing but it's a multiple of Phi over Phi0. Now this is an important equation that tells that the superconducting phases will be different by not an arbitrary quantity but by a quantity that is a multiple of Phi naught. So Phi over Phi naught is actually a number and then of course that has to be multiplied by 2pi and so there's a relationship between the external flux and the change in the wave function between the points A and B. so we can write going from A to B. So if we take this equation and take a derivative of this equation then we have to take a derivative because Phi naught is a constant. We can take a derivative of Phi. So d Phi dt will be nothing but the induced emf and we can write it down again as 2e over h cross v as a function of t and this is nothing but Josephson second equation which we have written by 2 where a v of t is nothing but the emf developed or the voltage drop across the junction between A and B.



So what is the current versus voltage characteristic for a Josephson junction? Okay so if we plot V this way and I this way then of course at the middle so we have -- so this is your V, voltage in volts and this is your current I. So this is this superconducting current. So let's just shade it with a different color. So this is the Josephson current or the superconducting current. So these are – whereas this and as well this are due to normal electron tunneling and this point this is called as the critical current. So basically when the voltage starts developing or the junction starts developing resistance is what defines as the critical current. Okay.



So let us do a small calculation of a junction, a Josephson junction and see the current characteristics and so on.

So we are just rotating our picture by a little bit such that you get a full three-dimensional view. So these are – that's the insulating barrier which is – and there is this superconductor S1 and S2. So this is your – suppose this is your X-axis and this is your Z-axis and this is your Y-axis then we have there's a current that is there's an ammeter and this is been biased. So this has say a width of 2a and so on. So this is your the picture of a junction. What is important for us is that we have a junction like this from -a to plus +a such that the width is basically to 2a and there's an incident energy is equal to Epsilon and this has the potential barrier potential is equal to v0 and this is of course your V as a function of X and this is your X and so we are going to solve a single particle Schrodinger equation in order to see or rather calculate the current. It's important for you to understand that the current is continuous across the junction. So there is a current that is flowing and the current cannot just disappear coming at the border of the superconducting and normal. So the current will be continuous and for the current to be continuous we can -- so it's entering into a regime which is classically forbidden. So this it will undergo a decay and we will have to calculate that or rather take into a consideration by writing down the Schrodinger equation and taking into account the boundary condition, proper boundary condition at the edges. So the wave functions are the superconducting electrodes. It's equal to psi rt which is equal to psi naught r which is the amplitude and the phase which is we have discussed that it goes as the vt by h cross where theta of t or Phi of t rather we were writing it as Phi so Phi of t is equal to vt over h cross and so on. So it's a DC voltage that is applied. So we'll have a Josephson current there which is otherwise classically forbidden. So we can write down the one the single particle Schrodinger equation as – so this is equal to ih cross del del T of a psi rt that's a time-dependent Schrodinger equation. It's a 1 over 2 m* where m* is the effective mass minus ih cross del that's the mechanical momentum and then there's a canonical momentum which can be obtained in presence of a magnetic field like this and this psi, so this the first term is the minus – so it's basically the p square over 2m now p has become minus ih cross del minus ea and there is an electromagnetic field. So we have a e* Phi which is characterized by a scalar potential Phi and a vector potential a. So this is rt psi of rt and we will finally write for a barrier, so this is V of X psi of rt. Okay. And in this particular case of course VX V of X is equal to constant so in absence of the E-M field just switching it off now and writing and writing down the time independent equation one gets minus h square by 2m* so m* and e* are included because they are effective masses and effective charge of the particles. So this is and d2 psi I'm writing it in of course in now in one dimension. So this is equal to d2 psi dx2 this is equal to Epsilon 0 minus V0 and psi now this is only a function of X we are writing in one dimension. So we write 1d.

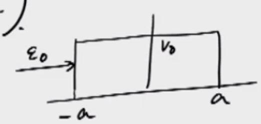
Alright. So now this you see a negative constant because Epsilon 0 and V0 the Epsilon 0 is the energy of the incident particle. So this is a negative constant. So you know what happens when you have a negative thing that appears here. You have a decaying solution.

$$\psi(x) = C_1 \cosh\left(\frac{x}{\xi}\right) + C_2 \sinh\left(\frac{x}{\xi}\right) \; ; \quad \xi = \sqrt{\frac{\hbar^2}{2m^*(V_0 - \varepsilon_0)}} .$$

Solution

Current

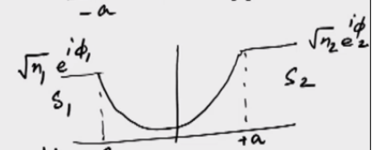
$$J_s = \frac{2e^*}{m^*} \, \mathrm{Re} \left\{ \psi^* \frac{\hbar}{i} \nabla \psi \right\} = \frac{e^* \hbar}{m^* \xi} \, \mathrm{Im} \left(C_1^* C_2 \right)$$



Applying Boundary Conditions.

$$\psi(-a) = \sqrt{n_1} \, e^{i\phi_1}$$

$$\psi(+a) = \sqrt{n_2} \, e^{i\phi_2}$$



$$C_1 = \frac{\sqrt{n_1} \, e^{i\phi_1} + \sqrt{n_2} \, e^{i\phi_2}}{2\cosh(a/\xi)} \; ; \quad C_2 = \frac{\sqrt{n_1} \, e^{i\phi_1} - \sqrt{n_2} \, e^{i\phi_2}}{2\sinh(a/\xi)}$$

$$J_s = J_c \sin(\phi_1 - \phi_2) = J_c \sin \Delta\phi . \qquad J_c = \frac{e \hbar \sqrt{n_1 n_2}}{m \xi \sinh(2a/\xi)}$$

So we have a solution which is either you can write it in terms of exponential but in a constrained geometry we usually write it in terms of cosine hyperbolic and sine hyperbolic. So this is C1 cosine hyperbolic x over xi plus a C2 sine hyperbolic x over xi. Just to remind you that if you have a second order differential equation if the C1 cosine hyperbolic x over xi I will write what xi is, is a solution and C2 sine hyperbolic or rather just sine hyperbolic x over xi is a solution then a superposition of that with constants arbitrary constants such that constrained to the fact that C1 mod square plus C2 mod square is equal to 1 is also a solution where xi is nothing but the momentum vector which is written as h square by 2m* V naught minus e naught. All right so then of course the current is given by of course it has a longer expression that you are familiar with. It's h cross over 2mi psi star di* idx minus psi di* dx and so on that can be written in a compact form as 2e* over m* and a real part of psi* h cross by I and write a grad psi but it means this idx so this if you compute it it becomes equal to m* psi imaginary part of a C1*C2 so that tells you that at the boundaries applying boundary conditions. So psi of minus a it's basically equal to so what happens is that you have in this particular geometry which we have drawn in the last slide. So we have a minus a to plus a and a barrier is v0. There is this epsilon 0. So here what happens is that you have the wave function that decays and then of course it grows in order to connect smoothly to the other region. So this is a minus a and this is a plus a. So this is – so that tells you that this is equal to a root over n1 where n1 is the superfluid density of superconductor 1. So this is a S1 and S2 so which is characterized by a root over n1 exponential I Phi 1 and this is root over n2 and exponential I Phi 2 and so on. So this is equal to exponential I Phi 1 and psi at plus a it's equal to root over n2 exponential Phi 2. So we are – in a general sense we are talking about two different superfluid densities where these n1 and n2 are actually mod size 0 square which the amplitude square.

So if you apply these boundary conditions to these solutions that are written here so these are the solutions of the Schrodinger equation. So if you put them there then you can evaluate the coefficients which goes as n1 exponential I Phi 1 plus n2 exponential I Phi 2 divided by 2 cosine hyperbolic a oversee xi and for c2 1 as n2 exponential I Phi 2 sorry n1. So it's n1 exponential I Phi 1 minus – did I write – it's n2 exponential I Phi 2 divided by 2 sine hyperbolic a over xi. So these are we have now calculated this so your JS which now can be put there so JS becomes nothing but a JC and sine of Phi 1 minus Phi 2, so this is equal to a JC sine Delta Phi and where JC is equal to eh cross root over n1 n2 divided by mxi sine hyperbolic 2a over xi. So we have achieved quite a bit. You see that we have written down predictions of the Josephson current that the current would be oscillating in this particular fashion or we wrote it of course for the current and we derived it for the current density but of course if we multiply it by the area it remains the same. So there is a sinusoidal variation and the argument of the sine function that involves the change or the difference in phase between the two superconductors.

So now if we want to go back to our electromagnetic field so then it becomes – so your Delta Phi.

In presence of the EM field.

$$\Delta\phi = \phi_1 - \phi_2 - \frac{2\pi}{\Phi_0}\int_1^2 A(\vec{r},t)\cdot d\vec{l}.$$

Rate of change of phase,

$$\frac{d\Delta\phi}{dt} = \frac{\partial\phi_1}{\partial t} - \frac{\partial\phi_2}{\partial t} - \frac{2\pi}{\Phi_0}\frac{\partial}{\partial t}\int_1^2 \vec{A}(\vec{r},t)\cdot d\vec{l}.$$

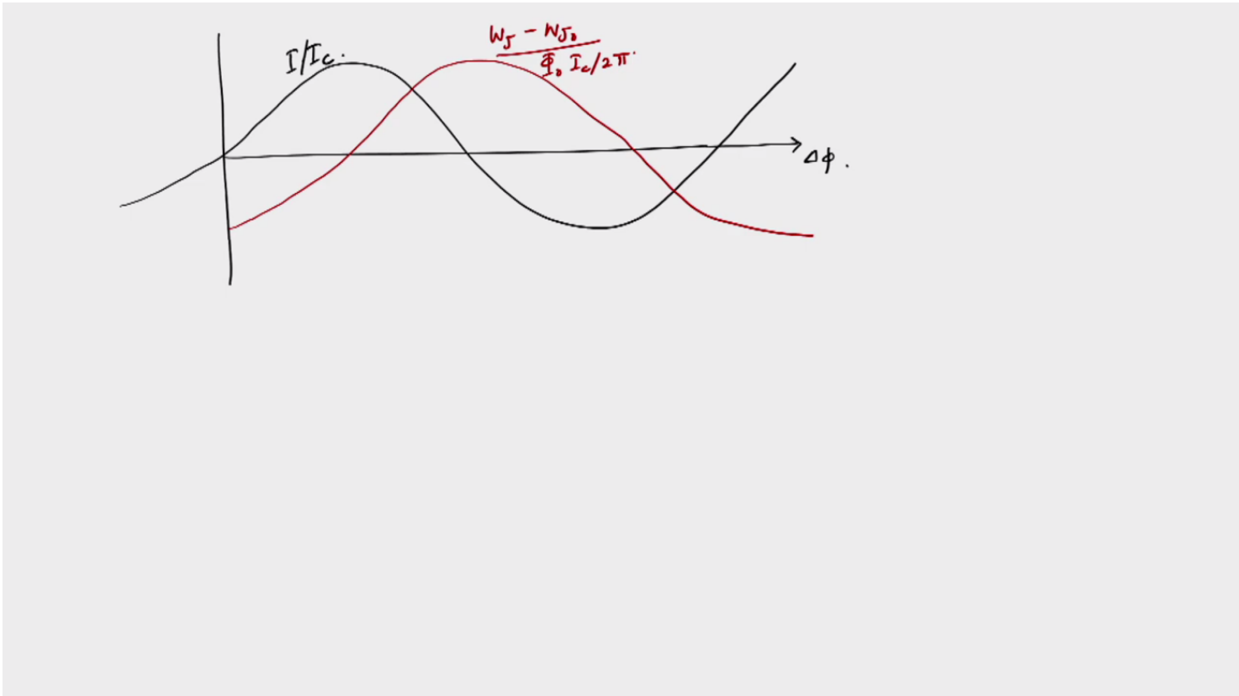
Work done is $W_J = \int_0^{t_0} I V\,dt = \int_0^{t_0} I_c \sin\Delta\phi'(t)\,\frac{\Phi_0}{2\pi}\frac{d\Delta\phi'}{dt}\,dt$

$$= \frac{\Phi_0 I_c}{2\pi}\int_0^{\Delta\phi}\sin\Delta\phi'\,d\Delta\phi' = \frac{\Phi_0 I_c}{2\pi}\left(1-\cos\right.$$

$I = I_c \sin\Delta\phi$

$V = \frac{\Phi_0}{2\pi}\frac{d\Delta\phi}{dt}.$

Now in addition to the Phi 1 minus Phi 2 it picks up the same phase that we talked about between 1 and 2. So these are these minus a and plus a and then this dl. So the rate of change of phase is a d Delta Phi Delta t this is equal to a del Phi 1 del t minus del Phi 2 del t and minus 2 pi over Phi 0 del del t of 1 to 2 Art.dl. So in general I mean this is just a convention but you will find different conventions at different places in literature. Josephson junction is represented by a symbol like this. It's like two inverted triangles that are connected here and so there is a V. So this basically has I equal to IC sine Delta Phi and a V equal to Phi 0 by 2 pi d Delta Phi dt. So these are the equation. So one has the work done by such a junction. So it's WJ there's a Josephson junction it's in a time 0 to t0 which can be chosen. So this is equal to 0 to t0 IC sine delta phi prime which is a function of t and a Phi 0 over 2 pi now it's a V which is a d Delta Phi prime dt and dt so on. It's just the Phi prime is just the dummy variable that we have taken here and if we integrate it, it is a simple integration to perform. It's equal to 2pi and so it's from 0 to some Delta Phi and the sine of Delta Phi prime d Delta Phi prime. So this is basically the integral that comes out and this is equal to I0, sorry Phi 0, Phi 0 IC over 2pi and 1 minus cosine Delta Phi. So that's the work done or that's the energy stored in the junction.



So it can be actually used to store the energy and a simple plot of the current and the work done can be shown as like this. So there is this the current fluctuates like or other oscillates like this. So this is current IS or I over IC and let's draw it with a different color. So this is your WJ minus WJ0. This is just a normalizing factor that is being used Phi 0 IC over 2 pi that variation is shown with Delta Phi. So this is a function of Delta Phi. And so there's a sinusoidal variation that one can see it here and so basically this is a in a brief description of the Josephson effect that one needs to know and the reason that we have introduced is that that we are going to do SQUIDs and some applications in very brief and in order to understand SQUIDs both the DC and the RF SQUIDs one needs to know and learn Josephson junction.