

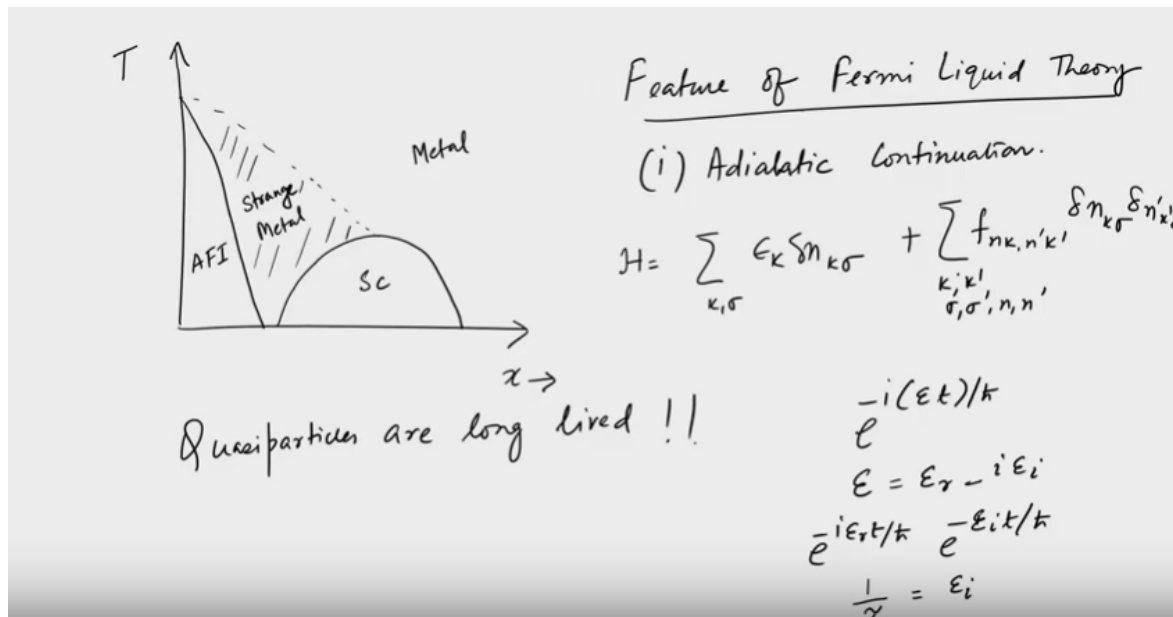
Lecture - 14

Quasiparticle lifetime, breakdown of Fermi Liquid

Theory in cuprate superconductors

So we have been trying to explain the anomalous, normal properties of this high-temperature superconductor. So, basically once again,

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just to remind you, the phase diagram, which is most important to us. So, this is the temperature axis and this is the doping axis, which causes, superconductivity, so there is a antiferromagnetic, antiferromagnetic insulating region and then there is a, superconducting region, let's call it this is, 'Anti-Ferromagnetic Insulator' we'll call it at, 'A F I' and there is a superconducting dome and if you draw an imaginary line, which goes comes very, near to the optimal doping point, where the TC is high, this region, of the phase diagram, should have been a metal. But, it is a very, strange metal it doesn't follow any of the characteristics, which have been given out or which have been proposed, by the Fermi liquid theory, to describe metals. And this is, almost a metal it's a nearly a very good metal, so we need don't, need to worry about. So, this is the region that we are, interested in maybe, we can just, draw a bit of shading here, so that's the region: that we are interested in. And our interest is, as I told you that it's accentuated or enhanced, by the fact that, none of the form e liquid Theory characteristics are being obeyed, by this metal. And to begin with, we have said a features of the Fermi liquid theory and those features, rest upon a very, important concept, contour other concept, called as the, 'Adiabatic Continuation'. So, there is a one-to-one correspondence, between the non-interacting problem and the weekly interacting problem and so, there are a relationship or rather there is a correspondence, between the non-interacting particles and what we call as the quasi particles? I have defined, what the quasi particles are, so when there are particle hole excitations, in the system, so there are, formation of these quasi particles and these are called as a, 'Landau Quasi Particles'. So, they are characterized by addressed mass, as opposed to a bare mass, of the electrons. So, this so, Landau's theory rests, on two ideas, one is that, there are, these quasi particles that are formed and due to the motion of the quasi particle, the Fermi the, fill Fermi sea: that recoils back, giving rise to because, of the momentum conservation and giving rise to, a an renormalized mass, for the quasi particles and in addition to that, there is another thing which we don't discuss at length: that is the energy of this quasi particles, actually depends, on the distribution, of other quasi particles in the system. And this landau, who had taken, into account by a, function which is called as a, 'F Function'. So, which is gives rise to a Hamiltonian, this is the Landau's Hamiltonian according to the Fermi

liquid theory, so there is an, ϵ_K , which is with respect to the, the Fermi energy and then there's an $N K \Sigma$, there is a K and Σ and then there is a factor, which depends upon if it's a $f N K$, n prime, K Prime and $\Delta N K \Sigma$. So, this is, we can write it with a, small fluctuation in the density, so it's a, $\Delta n k \Sigma$ and Δn prime, K prime, Σ , so there's a Σ prime as well and this is a sum over of course your, K prime, K , K prime, Σ , Σ Prime and n , n prime etc. So that's, the F function by which takes into account and the energy of these of these Quasiparticle, because of the distribution of all other Quasiparticle. So, the by in large the summary of this lecture is that, the whole, idea of Fermi liquid theories, rests upon the formation of the and, of the quasi particles and this quasi particles are long-lived. So, let's just write this point, just to remind you: that we have, said that this quasi particle wave function, must be, the at least the time evolution of that is, represented by this $\epsilon I T$ by, H cross, where because of the interaction term this energy picks up a real as well as an imaginary part, ϵR and ϵI , both are real, but, this is the imaginary part, because of this I factor or we can write it with a minus sign, which is more, meaningful in this context and so, this becomes equal to minus $\epsilon R T$ by, H cross, which is just like the, the particle that we have, the non-interacting particles accepting that, the now the energy has changed, which is given by the real part and this is the minus $\epsilon I T$ by, H cross and that's so, you're the quasi particle lifetime that is, how long these quasi particles will live, is actually, given by these ϵ eyes. Okay? So, this scattering rate or decay rate or the lifetime associated with the quasi particles, are the imaginary part of the energy of these quasi particles. So now, Landau's prediction makes an important statement, about this quasi particle lifetime and

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According to Landau's theory: $\tau_e^{-1} \sim T^2$ (T : absolute Temp.)

Resistivity ρ also goes as T^2

In plane resistivity, $\rho_{ab} \sim T^2$

But experiments in High- T_c Superconductor

$\rho_{ab} \sim T$

\rightarrow FLT is breaking down.

Scattering rate $\tau_e^{-1} = \frac{2\pi}{\hbar} \sum_{\text{final}} \left| \langle i | H_{\text{int}} | f \rangle \right|^2 \delta(E - E_f)$

\downarrow final \downarrow initial state \downarrow final
 $\sim E^2$
 $\sim T^2$

$$R = \rho \frac{L}{A} = \rho \frac{L}{L^2}$$

$$= \rho / L = \rho L^{-1}$$

$$\rho = R L$$

$$\rho = R L^{D-2} \quad D: \text{dimension}$$

In $D=2$ geometrical

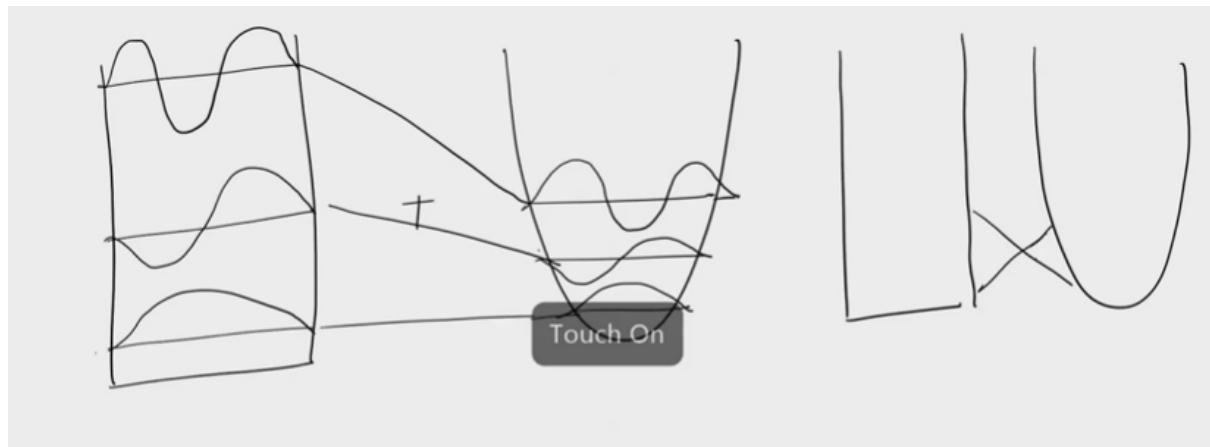
$$\rho = R$$

that is called as the, so, let's just write this in full, this scattering rate or the lifetime, it goes as T -square. Okay? And it's an important thing: that, this if one actually, can show that, this scattering rate goes as the, square in temperature, there is an absolute temperature, then of course the resistivity, also goes as, we are not going to prove this, but, these are results that we are quoting: that these Quasiparticle lifetime or the scattering rate or the decay rate, whichever name you want to give it to this, τ_e , e for the electron. So, are the quasi particles for that matter: that goes the, the temperature dependence of that is this and the row, which is the resistivity, now this resistivity, also goes as these quasi particle lifetime the temperature dependence is that of the quasi particle lifetime and that goes as

T square, as well. Now, what resistivity that we are talking about? We are here talking about the in-plane, resistivity, so this is like, what is called as the row a B in the experimental language? So, this is the resistivity of the copper oxide plane and that, goes as T square. So, just to give you a small story here: that row actually, which is, what you know that? Okay. The resistance, of any material, in three dimension that goes as row, into L over a, where L is the length of the wire and a is the area, of cross-section and row is the resistivity, which is a property of the material, so this if you, do a dimension, so l as a dimension of length and a has a dimension of L square. So, this is like row, R divided by L, so it's equal to, row L minus one, so row equal to R into L. So, there's a geometric factor that is there in three dimension, so if you really, try to understand this, goes as, L to the power, 2 minus D, where D is the dimension. Okay? So, because D equal to three or rather if we write it this way, then of course this goes as D minus 2, D minus 2, not 2 minus D is d minus 2. So, if D is equal to 3, then row goes as, R into L, so there is a geometrical factor, this is: that is there, however, in actual experiments, one measures are, the resistance, whereas in a theoretical sense, one wants to talk about row, which pertains to the property of the material. So, if we need to, do away with, the geometry or just the external parameters, we should actually talk about row, but however, in two dimension, it really doesn't matter because row is equal to R. So, there is a perfect agreement, between experimentalists and theorists and that's why? One doesn't really; care too much, so this is, same as the resistivity, so the in-plane resistivity goes as, T Square, according to Fermi liquid theory. But, experiments in the hi TC superconductor, shows row a B, is proportional to T. So, it's not a square off it's not quadratic in temperature, but, it goes linearly in temperature. So, this is a clear, indication that Fermi liquid theory, is breaking down and if Fermi liquid theory is breaking down, one needs to understand that why is it happening? So is it, related to the fundamental assumptions that we made, for the adiabatic continuation: that is in adiabatic continuation, if you understand that we have said: that there is an equivalent description, of the non-interacting system, with that of the interacting system and there is a one-to-one correspondence and not only that, there is a quantity: that can be used as a marker to describe both the systems that is a non-interacting system and the weakly interacting system and we have said that, that could be the node of the wave function, the node remains same and there's a one-to-one correspondence between, the two the non-interacting, plus the non-interacting, plus a weak interaction superpose these two systems. So, if that has to break down, one thing can happen is that, these the quasi particle description that we have, started off with: that is this one-to-one correspondence, between the non-interacting and the weakly interacting system that may be going down and that could be going down for two reasons, one is that, that the quasi particles are no longer long-lived, they could be actually, short-lived and decay fast, in a way the, the, the scattering rate, can be calculated by this is the tau e, which is, what we are talking about it's, like two pi H cross and then then sum over the final states and then initial and H, interaction and a final state and this and then there's a conservation of energy, here to sum over the final state. So, these are final, so this is the final state and this is the initial state. And n this quantity is by and large taken as momentum and energy dependent, which is true at low energies and close to the Fermi surface. So, this tells that, the so this basically, goes as if you want as epsilon square and it also goes as T square. So, this is the assumption and of the form a liquid theory and it says that, these are at low temperature or low energies and low temperature, these are very long-lived, quasi particles. But, suppose that does not happen, then we have to understand, why it does not happen? And one can understand it that, since we are close, to a phase transition, the fluctuations, in the order parameters become large and when the fluctuations, become large, it becomes difficult, to have these, matrix elements to be having small values and these matrix elements actually, can become large and they start depending on energies and, and so on and momentum and so on momentum, transfer between the initial and the final states and in which case, these interaction terms or the interaction matrix elements between the initial and final states no longer remain small and these tau e, no longer remains, in finite or rather they, sort of, so this is, in fact it's

like the, it's the inverse of that these are basically, we are talking about, the inverse of that, which is so, these becomes large, so this becomes small, the Quasiparticle lifetime becomes small and then the, the Quasiparticle are not very, long-lived and they decay, before the interaction must be I mean, during the time the interaction, is switched on. So, this one way: that the Fermi liquid theory may be, not holding good or may not be valid, in the normal state that we are trying to address. So, it, it could be happening: that by the time that one is actually switching on the interaction term, all these possible articles decay and no longer their valid descriptions of the system. Okay. And if they are not valid description of the system Landau's from illiquid Theory cannot be, applied. There is another way to understand this, is the following.

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That if you remember that we have drawn this, picture and plus a weak, term like this, these are the energies and so on and whereas these of course energies are equally spaced. And what we have shown is that? That there is a one-to-one correspondence, between these energy levels, in order for the form a liquid theory to hold code and why one-to-one? Because these are, the this should not go out, of course you understand the wave function has to vanish, I'm drawing a very rough sketch, so you should. Okay? And so on and then of course you have this and these are, the correspondence, now if it happens: that the correspondence can get messed up, if we have, a level crossing that happens somewhere. Okay? And if there's a level crossing of this, kind so while you try to connect or rather establish a one-to-one correspondence, there is a crossing of levels. So, if there's a level crossing then at this point, it is not difficult to make a decision whether we will go, we take this direction or we'll take this direction and this level crossings would invalidate, the Fermi liquid theory the assumptions, of the Fermi liquid theory or the quasi particle description. So, in a nutshell: that is what must be happening, in the normal state of the cuprates, which has created problems, in understanding it in a lucid manner and then making predictions or rather forming a complete theory, out of it. There is another factor that has, contributed even more seriously to the understanding of these normal state. the whole, current as we said that the, decay rate actually, goes as the resistivity itself. So, if the decay rate dependence is like T , which is, I think, what we have written there, the decay rate dependence is, as the resistivity.

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$$\begin{array}{lcl}
 \text{Hall current} & \rightarrow \rho_H \rightarrow T^2 & \\
 \text{usual " } & \rightarrow \rho \rightarrow T &
 \end{array}
 \left. \vphantom{\begin{array}{lcl} \text{Hall current} & \rightarrow \rho_H \rightarrow T^2 \\ \text{usual " } & \rightarrow \rho \rightarrow T \end{array}} \right\} \text{Are there two kinds of Quasiparticles?}$$

FLT breaks down in the vicinity of the Quantum critical point. (QCP)

$$\begin{array}{lcl}
 C_v & \sim & T \ln T \quad \text{near QCP} \\
 \chi & \sim & \ln T \\
 \rho & \sim & T^\alpha
 \end{array}$$

And now there's also, what is called as a hall current? And this hall current, is in presence of a perpendicular strong, perpendicular magnetic field and this hall current, from the hall current the resistivity, the hall resistivity, let's write it with a row H, the resistor Hall resistivity goes as T Square, as a Fermi liquid character or rather the signature of a Fermi liquid theory, whereas, usual charge current: that goes us, as we just said that, it goes as T. So, which means that, are there, two kinds of Quasiparticle and it's very, easy to understand that, the Fermi liquid theory completely, excludes, such exotic, ideas that there could be two kinds of quasi particles, one would have a different temperature dependence than the other or rather one, would have properties, which are different than the other and then of course, all these basic assumptions, of Fermi liquid theory, they break down. Okay? And we're all Fermi liquid theories or the only place in the normal state or the under dopes normal state of the high TC superconductors that, the this thing, Fermi liquid theory breaks down, there are other places as well, such as, quantum critical point, in the vicinity of the quantum critical point. So, what's the quantum critical point? We have shown you, in the context of these iron based superconductors there are, quantum critical point, so basically, usually, all these phase transitions: that occur, due to a phase transition, from a disordered state to an ordered state, such as a metal to insulator or metal to superconductor transition or a paramagnet to a ferromagnetic transition. So, these are second-order phase transitions, where of course we know that, these fluctuations, of the order parameters become large and which can actually, give impart large matrix elements, of the interaction term between initial and final states, which is why? We have proposed that or rather conjectured that, one of the reasons we're Fermi liquid theory could be going down. But, there are other kinds of phase transition classes of phase transition, which happens at zero temperature, which are not different, driven by thermal fluctuations and purely driven by quantum fluctuations. So, quantum critical point is actually, going from one ordered state, to another order state and in that kind of, phase transitions, these matrix elements, between the initial and final states, can actually become very large. Okay? And this would give rise to like there are known results that, how Fermi liquid Theory breaks down, your C_v goes as, $T \log T$, in some near this called, 'Quantum Critical Point' we will call it as, 'QC P' and there are, χ that goes as, $\log T$ and so on and then ρ goes as some power of α , which the Alpha is certainly not equal to one, but, could be five by three or could be two as we have seen, in various – sorry, it's one, in these so, high TC superconductors. So, as I so just to summarize the whole issue is that, we have, talked about the high temperature superconductors, which are not conventional superconductors, which are unconventional superconductors, they have a different pairing symmetry. So, let me write down all these briefly.

Summary

High- T_c Superconductors are characterized by

- (1) d-wave pairing symmetry.
- (2) Strong electron electron correlation.
- (3) Unusual undoped state (AFI)
- (4) unusual underdoped normal state.
- (5) $\rho \sim T$ (as opposed to T^2 which is a prediction of FLT)
- (6) $\rho_H \sim T^2$: suggesting two kinds of quasiparticles.

Mostly a d-wave, $DX^2 - Y^2$, d wave bearing symmetry, at least most of them, so we are talking about the $d_{x^2-y^2}$ mainly, strong very strong electron, electron correlation, which is understood from the parent compound being, an anti-ferromagnetic insulator, because usually, for a metallic case, it's a half-filled metallic band would have been observed, but, because the undoped state, is an anti-ferromagnetic insulator, this confirms that there are, a very, large in electron, electron interaction effects: that are prevalent in this material, then of course many things such as, unusual undoped state, which is, what we just said, of that of an anti-ferromagnetic insulator, unusual underdoped, normal state, the resistivity goes as T which is as opposed to T^2 , which is a prediction of Fermi liquid theory, ρ_H goes as T^2 , suggesting two kinds of Quasi particles and in general violation of the Landau Fermi liquid theory, in most of the aspects, such as even this scattering rate, which can be, determined from the optical conductivity measurements, we haven't discussed that, but these, scattering rates, actually can be calculated from, the optical conductivity measurements, they show, anomalous dependencies on temperature and various other, things which are, have contributed to the to this confusion, about the anomalous as normal state and because, the normal state is poorly understood, one cannot actually make up a complete theory, for these class of superconductors. And it nearly went on for about, thirty years or three decades, probably a little more than that and the research had enriched, of course the understanding of all these superconductors, both theoretically and experimentally, But, yet, there is no complete understanding of the entire, phenomenon of superconductivity, in these classes of cuprates superconductors.