Lecture - 13
Non-Fermi Liquid Theory, Adiabatic continuity

So, just to remind you of the context, that, we were talking about; so, we are interested in the,

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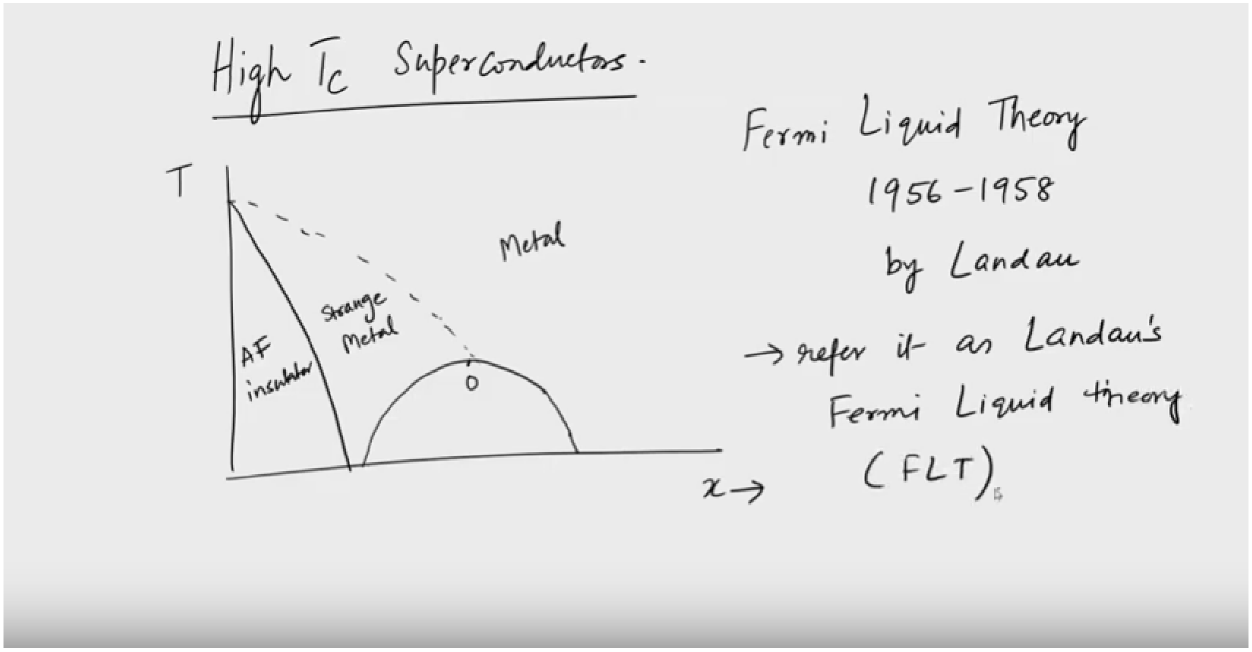

high temperature superconductor phase diagram, high TC superconductors, in particular, we are interested in this phase diagram, which is T versus, let's call it as a,’ Doping’. And it has, anti ferromagnetic insulator. So, this is in brief we'll call it,’ AF Insulator’. And there is a superconducting, dome that lies in the immediate vicinity and. Okay? And this is the position of maximum TC. So, this called as a,’ Optimally Doped Material’, anything other than this point let's call this point as something, let's call this point as O the left of O is called as the,’ Under Doped’, material and the right of that is called as,’ Over Doped Material’. So, as we go towards, the higher temperature that is we leave the dome and go to temperatures higher than that, we should get a normal state, but that's what does not happen, in fact there is an imaginary line, which can be drawn all the way till this. And to the left of, which we get, a strange metal and this part we write it as metal, where all the normal form a liquid characteristics are obeyed. So, we'll tell you what Fermi liquid theory is, very briefly and this strange petal seems to disobey, a lot of properties and characters of the Fermi liquid theory. So, a Fermi liquid theory is, not by for me, but it's more like by Landau and maybe other co-workers silly net etc. So, this was between 1956 to 1958 by Landau. So, we'll refer it as, Landau's Fermi liquid theory, in short we'll call it as FLT, let's see what are the essence of, the Fermi liquid theory and what are the points, in which they are disobeyed or rather they're found to be invalid, in the case of the strange metal. So, first of all what is the requirement, for forma liquid theory to be formulated what was wrong, with the free electron theory that we are all familiar, with. So, the free electron theory

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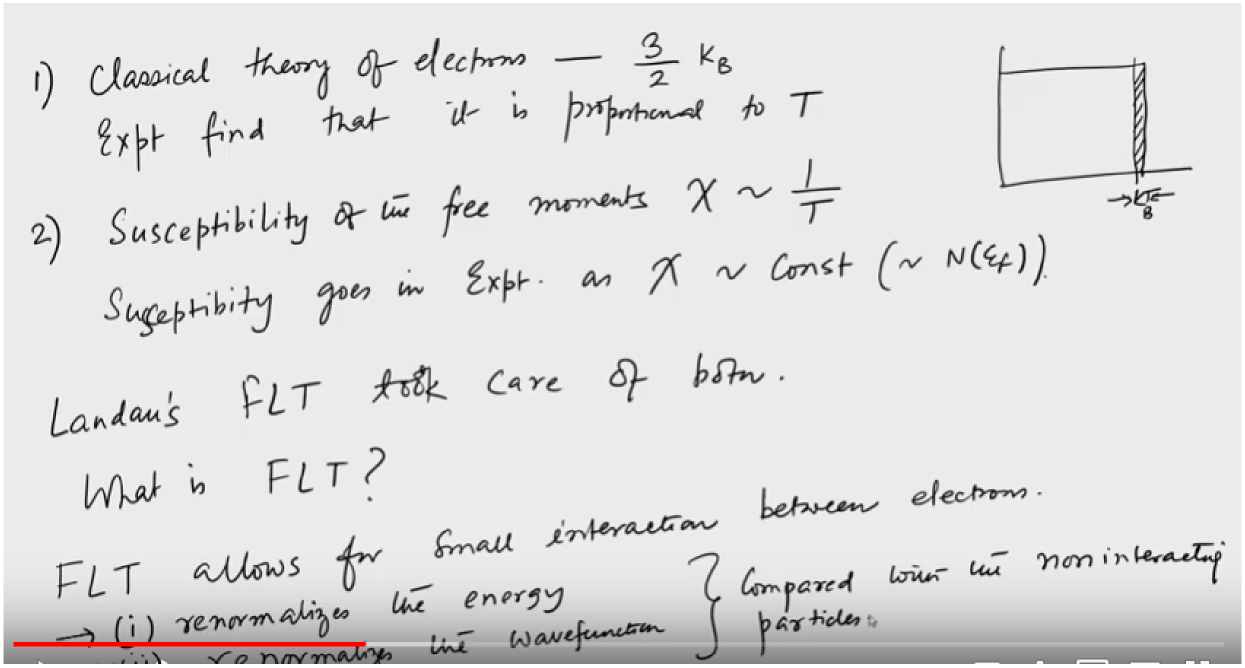

or rather classical theory of electrons, this gives a specific heat of 3/2 Bold man constant a per particle. And actually what is found, proportional to actually to tea and in a very simple words it can be understood that the electrons, which contribute to the specific heat, lie within. So, this is the Fermi Dirac distribution. So, they are within a small energy interval, which is given by K T. Okay? So, and they each one of them have an energy KT, at a temperature T. So, these energy the total energy, is the number of electrons, multiplied by the each the energy of each of the electrons. So, this KT into KT, which is K, T whole square and if the energy goes as T squared, the specific it should go as T. And this was something that was not captured by the classical theory, of these particles. Alright? The next thing is that, the diamagnetic, susceptibility or rather this is not the diamagnetic susceptibility, but rather this is the QD, susceptibility or rather the susceptibility of the free moments, moments was thought to be given, by 1 over T, which tells you that if you do a statistical mechanics, of n free moments, in a magnetic field and then write down the partition, function and then calculate the free energy and hence calculate the magnetization or the susceptibility, it goes as 1 over T, this is the famous Curie's law. However the it is found that these susceptibility, of free electrons, goes in experiment, as Chi being independent of temperature and it goes as the density of states, at the Fermi level. So, these were the drawbacks of the, classical theory, of, of particles or however, the Landau's Fermi liquid theory, took care of both. So, what is what is FLT? So, Fermi liquid theory, is allows, for small interaction, between electrons. Okay? So, it allows for small interaction, between the electrons. And so, this small interaction, 8 renormalizes, the energy and it also renormalize is the wave function, this is compared with, the with the non-interacting particles. So, let’s see how, this change, causes something that is significant for our discussion. So, we can write down, the time evolution of the wave function,

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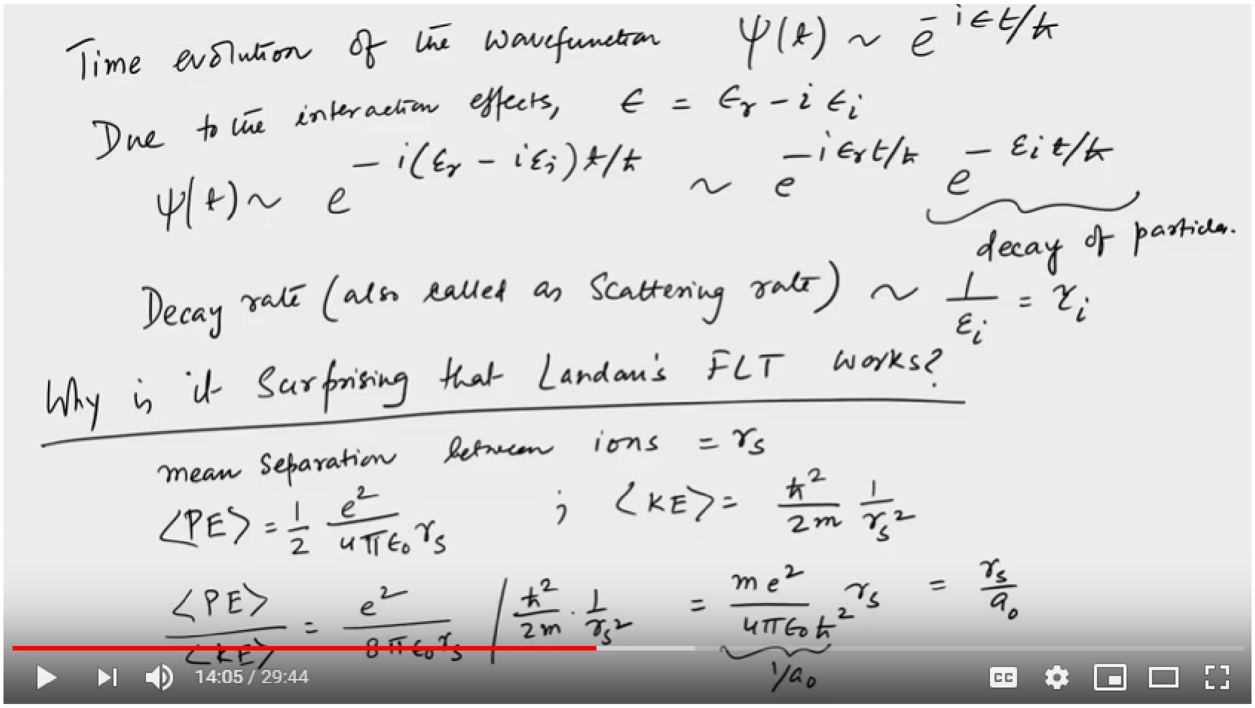

which is known from, the first course of quantum mechanics, is sigh of T, goes as exponential minus IET by H cross. Now due to the interaction, effects the energy can actually pick up, a small imaginary part, which we write it as, a real part a combination, of the real part and an imaginary part, which are given by epsilon RN, minus I epsilon I. So, if you plug it into this exponential minus I, epsilon R minus I epsilon I T over H cross. So, this actually looks like exponential minus epsilon IR T by H cross and exponential epsilon I T over H cross, this is related to the decay, decay of particles. And why is it related to the decay because you see that there is no item, which means it's not a propagating, part. So, it is related to the decay part and importantly there is a decay rate, which is also called as, scattering rate that goes as 1 by epsilon I and we call it as tau I.

So, there is a decay rate associated, with this landau quasiparticles, I will define what the quasi particles are in a moment, but let us see that why should we actually in fact the success of, Fermi liquid theories, also quite surprising, because it's almost starting from a non interacting picture, with minimal interactions included, as we seen how this theory can act explain, a very large number of metals, it of course fails also for a few metals, but by and large is quite successful in explaining the properties of a large number of metals. Why should it fail or why is it surprising. So, that's the question? And if we try to answer that question, we'll see that let us talk about, the mean separation, between, atoms or ions, is equal to say rs. So, the potential energy, of a solid, which is equal to e square over 4 pi. So, the average potential energy, which is given by, e square by 4 PI epsilon zero rs that's the Coulomb potential energy. And the kinetic energy, is the average kinetic energy is given by, H cross square over 2 m and there is a K square where k goes as 1 over rs. So, k square goes as 1 over rs square. So, if I take a ratio, of this potential energy, to the kinetic energy, I get a value, which is equal to, e square over, there is a factor of ½, which you can put because, to avoid double counting. So, there is a e square by 8, epsilon0. So, what I mean by double counting is that the potential energy, between the ion a and the ion B is given by this. And for the B and a is also given by this. So, there is a double counting so, to avoid that we can put a term, like this and then we have a H square, over 2 M and 1 over rs Square. And then this is equal to, M e square by 4 PI, epsilon 0, H cross square rs. Now this factor here, is actually 1 over a 0 where a is called as a,' Bohr radius'. So, this is equal to rs over a0. So, rs over a0 is the ratio, of the potential energy, average potential energy, by the average kinetic energy, for a representative metal say, where rs the mean distance between the ions or the atoms and a zero, is the Bohr radius.

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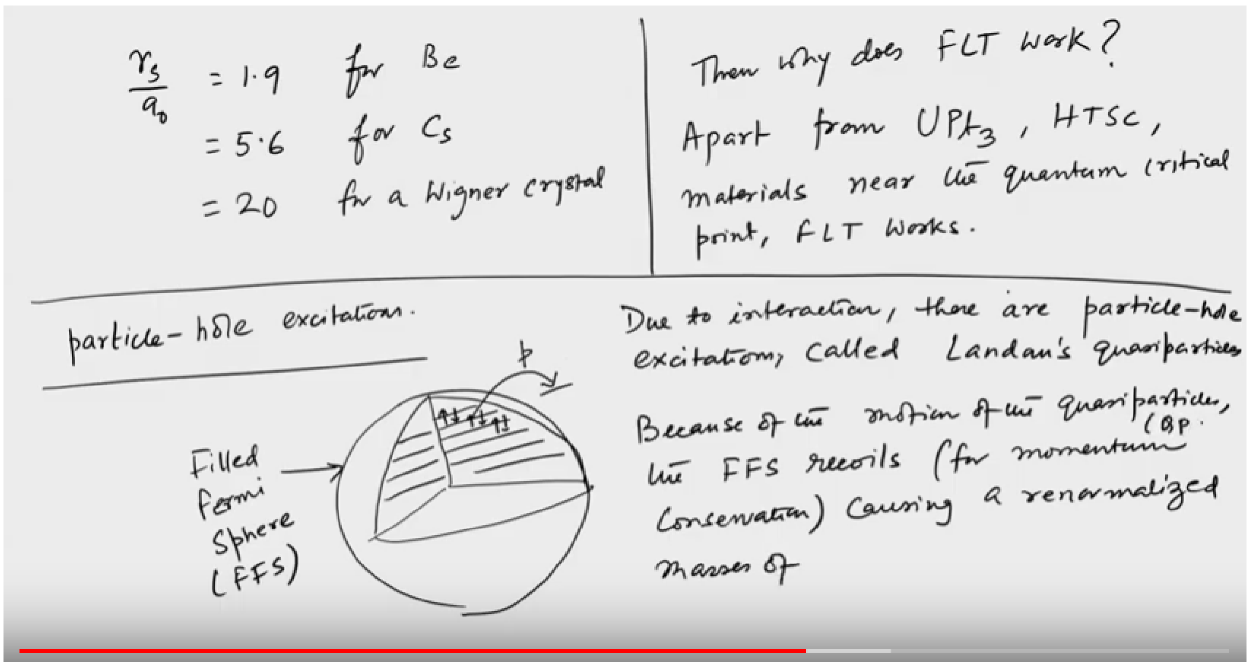

So, the important thing, is that for some materials rs by a0 equal to one point nine, for beryllium and it's equal to five point six, for cesium. And it's equal to about 20 for a Wigner crystal. So, this if it is true, then why does FLT work, if the potential energy is large, compared to the kinetic energy, then how are the interaction effects negligible, the free electron theory of course should not work. But even a land of for me liquid theory, which takes into account the interaction between the electrons, in a minimal sense that is there is a weak interaction, between the electrons why does it work, but it's just that it works. Okay? So, apart from, some quantum liquids such as, you PT 3 etcetera, high TC superconductors and materials, which are near the quantum critical point, FLT works. Okay? So, let us see the, what does FLT say or rather we'll talk about this what are called as a,' Particle Hole Excitations'. So, let us draw filled Fermi sphere let's see how good I can p draw it. So, this is so, this is like the field Fermi sea or field Fermi sphere. And they're these I've taken a quadrant or showing a quadrant here, where there are like as you open your fridge door and there are these stacks that are available, for keeping bottles or glasses there. So, there are they are filled with up-and-down electrons, in each one of the levels and so, on. Now because of interaction, they can actually make a transition and this is called as a,' Particle'. So, this one of the particles or electrons, can make a transition leaving, a hole behind and this is called as the,' Particle Hole

Excitations'. So, due to, interaction there is or there are particle hole excitations. And because of this motion of the quasi particle. So, these particle hole excitation, these are called as,' Quasi Particles'. Or Landau's Quasi Particles'. And these are these, because of the motion of the quasi particles, the this will call it as ,'FFS', filled Fermi sphere, the filled Fermi sphere recoils, for momentum conservation, causing renormalized masses of the, quasiparticles. We call this quasi particles as QP of the quasi particles. Okay? Just like, if you have a gun on your shoulder and the gun fires they'll be recoil due to the gun, which you would understand, because you need to tend or move back and that's why these in these Olympics or in these games, when these a pistol or these shooting, competition they actually get hold of the, arm by which they're shooting.

So, that it doesn't recoil, because recoil would the they could miss the target. So, that they don't miss the target and this that's why they actually, either put their, hand on a stand or they actually hold their hand, by which they are the firing in the pistol or the revolver. Alright? So, this is similar to that, because of the conservation of momentum, there are these the Fermi sphere recoils, actually this whole the quasi particle, has to be pushed away and this quasiparticles because of this pushing, away there is a renormalized mass of the quasi particles and then, this is what causes the so, M becomes M star. Okay? So, in the Landau's Fermi liquid picture that's the only thing that happens that is. We get a renormalized mass, instead of the bare mass as was there in the, usual non interacting electron theory. So, in the Landau's form a liquid theory, these bare masses are changed by or they're replaced, by the masses of this the quasi particles or this M star and so, on. Now what is important is that that how is this taking into account, small interaction between the electrons, you're still able to carry on, similar description, very similar description, as the non-interacting, electrons themselves. Okay? And this is the,

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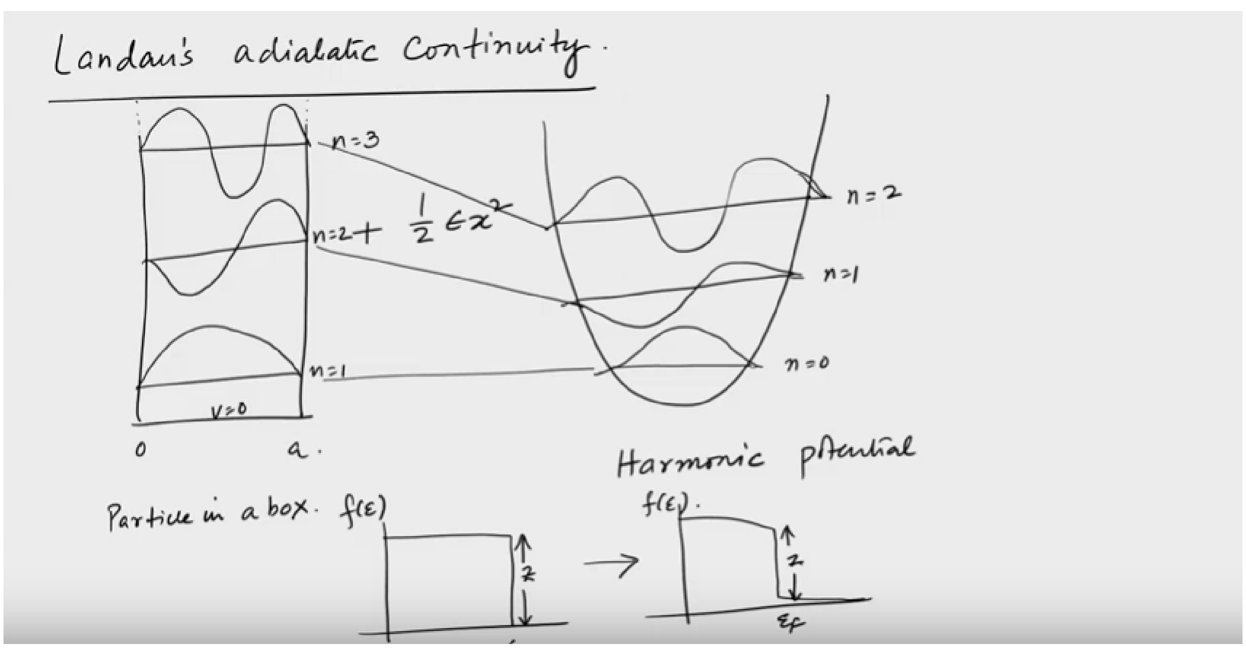

idea is Landau's adiabatic continuity. Okay? So, what it says is that the electrons are or the interaction between, the electrons, is switched on adiabatically, which means it switched on, slowly and the by the time the quasi particles remain a valid description, during this process of switching on the interaction. Okay? So, there are some properties, which remain completely, unchanged as you go from a totally non interacting system, to a system which is weakly interacting, in order to understand that let us talk about, this particle in a box, which you all know in quantum mechanics. So, V equal to zero, here and V of course goes to infinity, along these things. And let's just write it here, V equal to zero, if you solve for the Schrodinger equation, you would get the sine functions. And which are and the energy levels, of course go as case n square and so, these are these have nodes and so, on and then they are like and so, on. Okay? So, this is from zero to a, there is a known problem particle in a box, let us superpose, a small harmonic potential, of the form, half epsilon X square. So, here the ground state or the zero-point state, is this whereas we have the wave function, of course is you know that it it's a Gaussian multiple, hermite polynomial a polynomial, which has a property that it's even for even n that is if you change X 2 minus X its remains unchanged, for even N and it is it changes sign or it becomes, odd as you change it, as you have ordained. So, this is don't mind this crossing, there's not intended. Okay? And so, on. Alright? So, these are Gaussian of course, Okay?

So, they got the decay like this and so, this is equal to again this is n equal to 1, this n equal to 2 and n equal to 3, this n equal to 0, n equal to 1, n equal to 2 and so, on. Okay? So, if you do that, if you add such a small harmonic potential, to this in fact all your eigenstates, these clear demarcation of the eigenstates, they go away and what happens is that? You get a sort of superposition of states, for the resultant problem that's for the combined problem. So, no longer you have these n equal to 1, to be sine PI x over L and so, the N equal to 2 to the sine 2 pi X over L and so, on. But you will have a combination of that with some coefficients that is going to happen. However even that happens, there could be a 1 to 1 correspondence such as, this is connected to this and or this is these are these correspond, to this and so on. So, each of the levels can correspond to this and they as something remains, invariant if you look at it carefully, the number of nodes become R they remain invariant. So, if we try describing the combined system, by the nodes or number of nodes, of the Pretender of the wave function, then the number of nodes remain same, in both the, the initial problem and the initial problem plus the when it's compared with the combined problem, through the initial one which has small harmonic part, added to it. This is the essence of Landau's form a liquid theory that there should be a one description or there should be a description that remains invariant, as we go from, now non-interacting description, to a weakly descripting, interacting description, this harmonic potential and there could be, you know there could be some order parameters that are that could be defined and so, on and then I mean what can happen is that that you know that in super conductivity, there is a sudden vanishing, of electronic states, across the Fermi energy. And so, basically the there is no, the states after that but in an interacting system, this could still remain, in the sense that this could be slightly and it could be like this. So, if we call this as Z is a jump or the discontinuity, which is related to the, imaginary part of the self energy or imaginary part of the energy that we talked about, because of interaction. So, this remains, invariant as you go from, one from the totally non interacting description, to a weakly interacting description.

So, this adiabatic, continuity or this adiabatic connection, is key to the to the Landau's for me liquid theory. And of course I mean retaining the labels, of the non-interacting system, means that the configuration entropy, which is given by Sigma P I log P I. So, that remains unchanged, in the interacting system, which means the of course the quasi particle distribution, which means this the quasi particle distribution, remains unchanged, even if there is a tapering off of the distribution. But still it retains that gap or this discontinuity that we see in the distribution function. So, these are some of the important, concepts of Landau's adiabatic continuity, will see in a in the next discussion that how some of these adiabatic, discontinuity or the fact that the quasi particles are long-lived, at the Fermi surface these notion, these breaks down, in case of the high TC superconductors or the couplet superconductors, at least in the normal state, giving rise to a lot of anomalous properties, which are very inherent or rather very core to the understanding or rather the lack of, understanding therein in these high TC, superconductors .