Advanced Quantum Mechanics with Applications
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Lecture – 18
Teleportation, Quantum Teleportation for one spin

So, let us add to our ongoing discussion on Quantum Teleportation.

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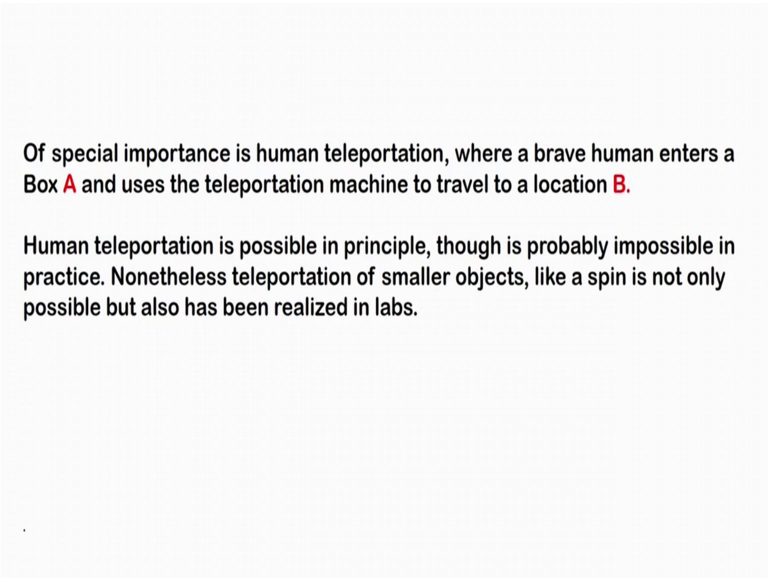

And in of particular you know, a usage or for discussion we will talk about transmission of information from one location to another at without actually having to move from one place to another. And of course, of special importance is human teleportation; that is, tele not completely getting teleported from one location to a remote location.

So, as it says that where a brave human being enters a Box A and uses the teleportation machine to travel to another location B. So, human teleportation is possible in principle though probably completely impossible in practice, or nonetheless the teleportation of smaller objects like spin, a single spin is not only possible, but has also been realized in labs.

The fact that a human teleportation is possible in principle has opened up a lot of research in that particular direction. In fact, even if it is not finally achieved it may open up various avenues for further research into the fields of quantum information quantum entanglement, teleportation, and finally to quantum computing.

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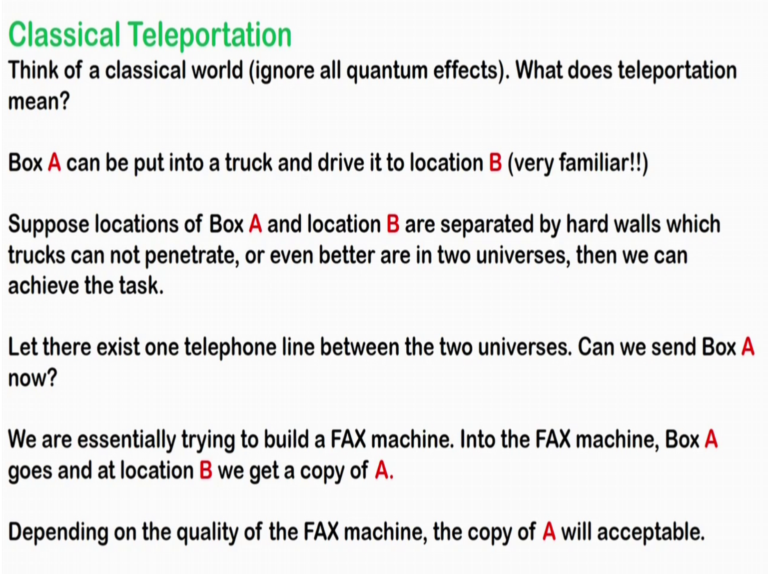

So, before we understand quantum teleportation let us see what classical teleportation would mean, in absolutely in live terms and non-technical terms. So, if you think of a world which is completely classical and ignore all the quantum effects, then a teleportation would mean that Box A which is what we have been discussing; that the Box A can be put into a truck and then driven it to the location B; which of course, is very familiar all packers and movers they do exactly the same thing; that is, putting things putting boxes inside containers, and these containers are being taken away by trucks and being delivered at the doorstep of the recipient.

Suppose now, the locations of Box A and the location B are separated by hard walls which the trucks cannot penetrate. Or say these Box A is in one universe and the Box B or rather the location B is in another universe, then can we achieve this task is what the question.

At least for now, what we want is that a telephone line to exist between the 2 universes. So, via the telephone line can we send a Box A to the location B? So, what we are essentially saying is that, we are trying to build a fax machine a huge gigantic 3D fax machine into the fax machine the Box A will go, and at location B one will get a copy of A. So, depending on the quality of the fax machine the copy of A will either be acceptable or unacceptable. Of course, there will be some loss of quality, and it could be a serious impediment. But let us just assume that the quality of the fax machine is good so that the copy of A is acceptable.

At the same time, you can also think that the a at the origin that is the location from which it was teleported, that could be destroyed; that is it is shredded that information is shredded out.

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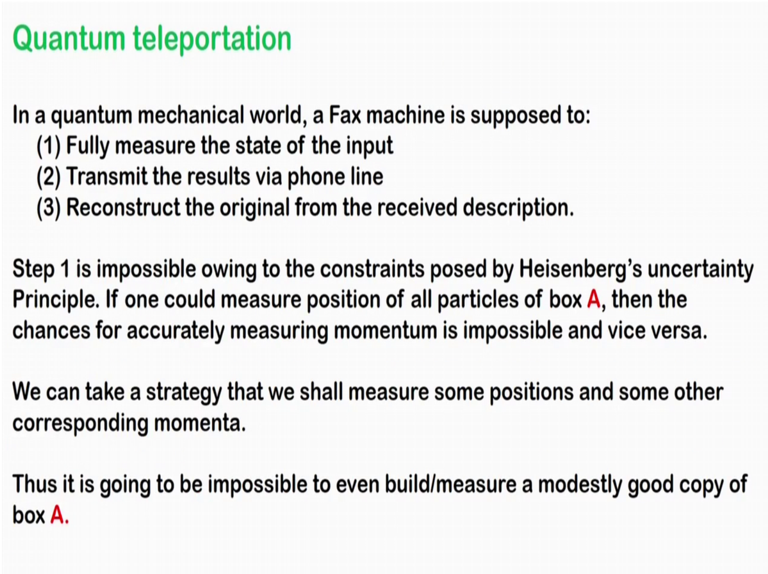

Now, to talk about quantum teleportation: so in a quantum mechanical world a fax machine is supposed to fully measure the state of the input, then number 2 is that the stage 2 of the process is transmit the results via the phone line that exists between the 2 universes. And then reconstruct or reassemble the original from the received description.

Now, it is quite a good possibility that one gets stuck at step one itself, because step one is impossible owing to the constraints posed by Heisenberg's uncertainty principle. So, it says that you cannot measure the conjugate variables with unlimited accuracy. So, if one could measure the position of all particles of Box A; that is assuming that the Box A is an extended object. So, all the points are then the chances of accurately measuring the momentum or the velocity is impossible and vice versa. So, if you know the momentum then the position of each of those points or the locations of the box becomes completely fuzzy, that is un measurable.

So, we can take a strategy that will compromise in some form that is will measure some positions and some other corresponding momentum. Thus, it is going to be impossible to even build or measure a modestly good copy of the Box A, because we are not taking all coordinates of A and measuring them. So, the transmitted information is certainly going to be having a deteriorated quality.

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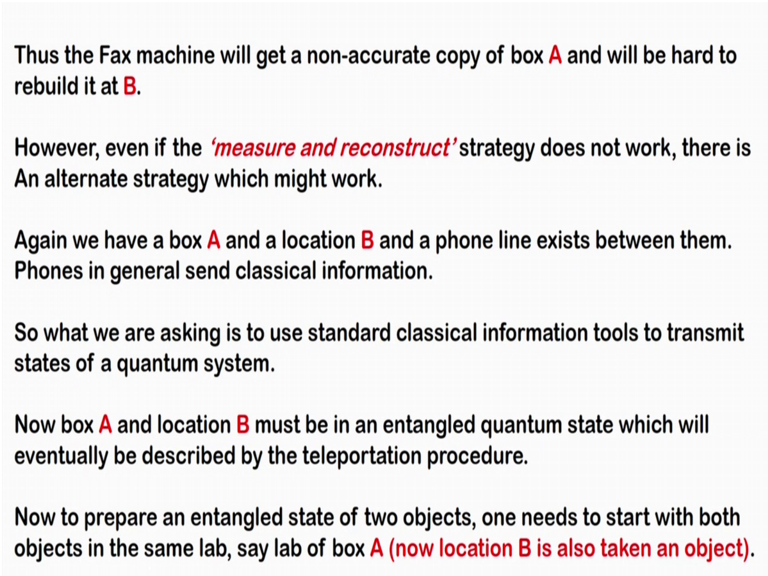

So, the fax machine that we are talking about will get a non accurate copy of Box A, and will be hard to rebuild it at Box B or the location B.

However, even if the measure and reconstruct strategy does not work there is an alternate strategy which might work; which is what we are going to discuss. So, again we have a Box A and a location B and a phone line exists between them, nothing has changed in that direction. Phones are in general they send classical information. They have the classical forms; I mean of course, they do not send any quantum information. So, what we are asking is to use standard classical information tools which deal with classical information to transmit states of a quantum system. How is that possible in any way?

So now Box A and location B must be in an entangled quantum state; which will eventually be described by the teleportation procedure. So, the way to do that is quantum entanglement, but even the entanglement does not solve the entire purpose. So now, to prepare an entangled state of 2 objects, one needs to start with both objects in the same lab say the lab of Box A now location B is also taken as an object. So, we have 2 objects

A and B. Say, Box A and Box B and we need to create this entangle states of these 2 boxes.

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Now we need to send one of the objects to location B.

In principle we could use quantum teleportation for this task.

But this would destroy one entangled state and create another entangled state with a net gain of nil.

Only solution being: sometime in the past the two locations (of box A and location B) must be together with no boundary walls in between. At that time people from the two locations met, created a large number of entangled states, and carried them to their respective locations so that they can teleport things back and forth.

The entanglement requirement poses a second issue, since we have mentioned that it gets destroyed when used. Thus it is a resource that it gets depleted when teleportation occurs. Thus it has to be replenished by meeting physically, create the entangled states and transfer without using teleportation.

Thus many entangled states are required at both locations for one teleportation.

So now we need to send one of the objects to location B; that is, that is say the one of the constituents of Box A. So, in principle we could use quantum teleportation for this task. But this would destroy one entangle state and create another entangled state. So, we basically gain nothing out of it. Because, in order to use quantum teleportation, you use you need to have an entangled state; where the properties of 2 systems are correlated. So, if in the process if you destroy one entangled state and then of course create another there is absolutely no net gain, because you need to send the whole object. So, we not you are just talking about sending maybe one point or one information of A to B.

So, one possible solution for this could be that sometime in the past the 2 locations of that Box A and the B that is the location B must be together with no boundary walls between them. At that time, people means the scientists, from these 2 locations met, they created a large number of entangled state. And they carried them to their respective location and then they got separated out, ok. So, this is what I mean the final scenario is that this A and B have no nothing no physical connection and they correspond to 2 different universes, ok.

So, these scientists would carry a lot of these entangled states along with them to their respective locations. And they would then teleport things back and forth. The entanglement required requirement thus we have just said poses another problem; a second problem.

Since we have mentioned that it gets destroyed when we used. This is the problem that a quantum entanglement gets destroyed when you once use it. So, it is a resource that gets depleted when teleportation occurs. Thus, it has to be replenished by meeting again physically create the entangle state and transfer without using teleportation. So, basically what we are trying to say is that there are many entangled states that are required at both the locations for one teleportation to occur.

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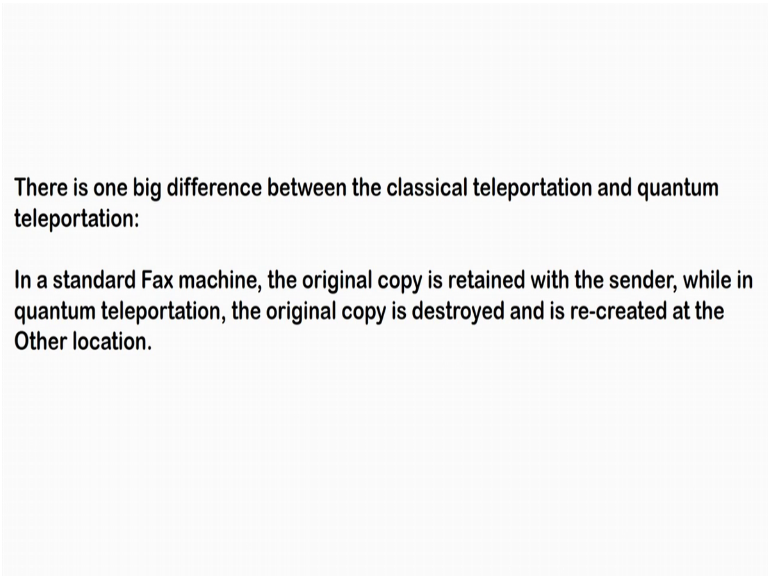

There is of course, one big difference between the classical teleportation and the quantum teleportation. In the standard fax machine, the original copy is retained with the sender. If you remember that just a while back I say it that, this original copy could be retained or it could be shredded, because in quantum teleportation this copy is never going to be there after the teleportation occurs.

So, while in quantum teleportation the original copy is destroyed, and it is recreated in another location. So, these are the basic features of quantum teleportation, and as I said that that they do not require or rather endures superluminal communications, but they require of course, the classical communications as just being told that, the phone lines are required which are classical information tools.

So, with this we start studying how to teleport one spin, one single spin and then we go to more complicated systems. So, let us just start with teleportation of one quantum spin.

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Teleportation of one Quantum Spin.

Consider a $s = \frac{1}{2}$ particle which is in an unknown state,

$$|\psi\rangle = a|\uparrow\rangle + b|\downarrow\rangle$$

a, b are complex numbers satisfying $|a|^2 + |b|^2 = 1$

In a Stern Gerlach apparatus if this state is given as an input then $|a|^2 = |b|^2 = \frac{1}{2}$.

This, of course, corresponds to measuring spins in the z-direction.

It is also possible to measure the spins along x-axis.

So, to begin with consider a spin half particle; which is in an unknown state. So, that unknown states a call it as a up and b down. You could write that in the form of a qubit with a 1 and 0. This is also equally fine. So, what are a and b as, you know, that a and b are complex numbers satisfying a mod square plus b mod square equal to 1 ok.

So now in a stern Gerlach like experiment which we have discussed at the beginning of the course, in the stern Gerlach apparatus, let us call it an apparatus just to remind you that stern Gerlach apparatus is one in which there is a oven containing silver atoms. So, these silver atoms are vaporized and they come out from an opening in the oven, and then they passed through 2 collimated slits.

And such that they become a narrow beam, and then they are made to pass through a vegion of space; where an inhomogeneous magnetic field exists, which are created by the whole pieces where one of the whole pieces has got the shape of the roof of a hut. So, which creates a magnetic field gradient, and because of that there is a force that the magnetization of these silver atoms or the magnetic moments of the silver atoms they experience.

And because of this force they should actually they get deviated from their normal line of movement or line the path along with a move. And if you go by classical physics, then they would be from any value plus m 2 minus m or. So, they would be impinging on any value on a screen which goes from m b plus m b to minus m b; where m is a magnetic moment and b is the magnetic field.

But however, it is seen that as this experiment is performed the there are 2 bright spots which are obtained which says that they do not actually get smeared between plus m b and minus m b. Rather they actually focus on 2 points plus m b and minus m b and nothing in between. And these actually establishes without any doubt unambiguously that there is a quantization of spins, and these spins can only take values plus half and minus half in this case of silver atoms because of it is all filled shells and having one mt or rather known electron which is contributing to the spin.

So, this is the stern Gerlach experiment which we have learnt earlier. So, in this stern Gerlach experiment, if this state is given as an input, and input then of course, we know that a mod square equal to b mod square equal to half. So, there is a half possibility of having value plus half and other having a value minus half or having a value down is half as well.

So, this of course, corresponds to measuring spin in the spins in the z direction ; which is usually taken as a preferred direction in which the z component of the spin half operator which is a poly matrix z component is only diagonal the other 2 components are have of diagonal elements.

So, it is also possible to measure the spins along x axis instead of z direction. So, what are these spins along x axis?

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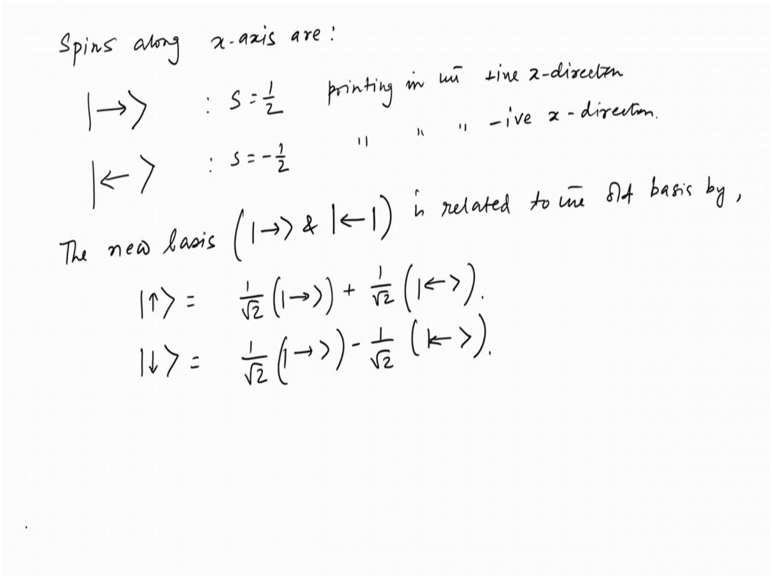

So, the spins along x axis are; so instead of up and down now they will be written as this. So, this is along x axis; which corresponds to say now spin equal to half, and there is also a down which corresponds to s equal to minus half. So, this is pointing in the positive x direction, and this is pointing in the negative x direction ok.

So, now if we want to write this up and down spins in the z basis, in terms of the spins in the other of the basis kept vectors in the x basis, then the new basis is related to the ; which means this is a new basis. So, I can write down up by a 1 by root 2 plus a 1 by root 2 ok. And the minus or rather the down is written as 1 by root 2 up minus 1 by root 2 ok. So, these are the transformations between the upstate downstate with the right state and the left state.

So, we will call them as up and down, and them as right and left. So, these are up down and then right state and then left state, right state and left state ok. So, these are the things that will follow. So, what happens to that state?

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$$a|\uparrow\rangle + b|\downarrow\rangle = a\left(\frac{1}{\sqrt{2}}|\rightarrow\rangle + \frac{1}{\sqrt{2}}|\leftarrow\rangle\right) + b\left(\frac{1}{\sqrt{2}}|\rightarrow\rangle - \frac{1}{\sqrt{2}}|\leftarrow\rangle\right)$$

$$= \frac{(a+b)}{\sqrt{2}}|\rightarrow\rangle + \left(\frac{a-b}{\sqrt{2}}\right)|\leftarrow\rangle$$

When measured in the x-basis.

$\frac{(a+b)^2}{2}$ is the probability to be seen in $+\frac{1}{2}$ (right)

$\frac{(a-b)^2}{2}$ is the probability to be seen in $-\frac{1}{2}$ (left)

So, that state that we have initially taken, or a up plus of b down can now be written as a and then 1 by root 2 right plus left plus b b 1 by root 2 right minus 1 by root 2 left. So, this is that state. If you simplify this looks like a a plus b by root 2 and this plus a minus b by root 2. And this right and left.

So, when measured in the x basis. So, a plus b whole square by 2 is the probability to be seen in plus half state which is called as right state, and a minus b whole square by 2 is the probability for it to be seen in minus half; which is also called that as left state. So, these are the measurements in the x basis.

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In particular case, had we started with $|\uparrow\rangle$ state $(a=1, b=0)$ or a $|\downarrow\rangle$ $(a=0, b=1)$ then there is equal probability of observing $|\rightarrow\rangle$ or the $|\leftarrow\rangle$.

IMPORTANT

After any measurement, the state of the spin is changed to the outcome of the measurement. That is, if we measure in x-basis and observe the outcome of $-\frac{1}{2}$ or the $|\leftarrow\rangle$ state, then the state of the spin is $|\leftarrow\rangle$ irrespective of what state the spin was in before the measurement.

So, in particular we take a particular case had we started with a spin up state, then that would mean a equal to 1, b equal to 0; or down state which would mean a equal to 0, b equal to 1, then there is equal probability of observing this right state or the left state. So, what is important? Let me write this with red color. So, after any measurement, the state of this spin is changed to the outcome of the measurement. Pay extra attention here because this is very important.

So, basically what we are trying to say is that if we measure in x basis and observe the outcome of the minus half which is or the left state, then the state of the spin is this irrespective of what state the spin was in before the measurement. So, this is an important thing about measurement. So, of course, there are various measurements that can be done. In principle the spin can be measured along any arbitrary axis. And so, each of these measurements will give us some information about the coefficients a and b but not the full knowledge of a and b.

So, let me write this because it is important.

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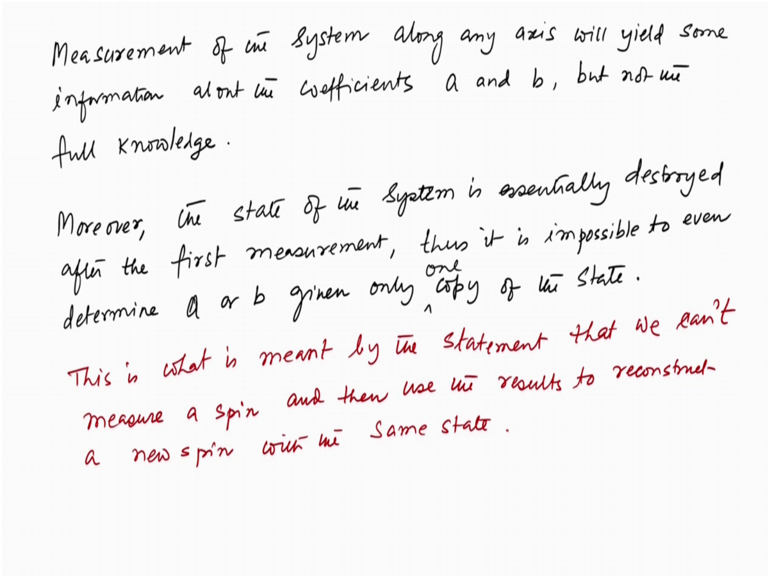

Will yield some information about the coefficients a and b, but not the full knowledge; moreover, the state of the system is effectively destroyed after the first measurement. So, it is impossible to determine a or b given only one copy or a single copy of the state. After the first measurement this a or b if we are given only one copy of the state.

In fact, this is what is meant by the statement that we cannot measure a spend, measure a spend means the state of a spin and then use the results to reconstruct a new spin with the same state. So, essentially when you make a measurement you destroy that state. So, having a full knowledge of either of a or b is a becomes out of question.

So, what is our motive? Our motive is to achieve teleportation.

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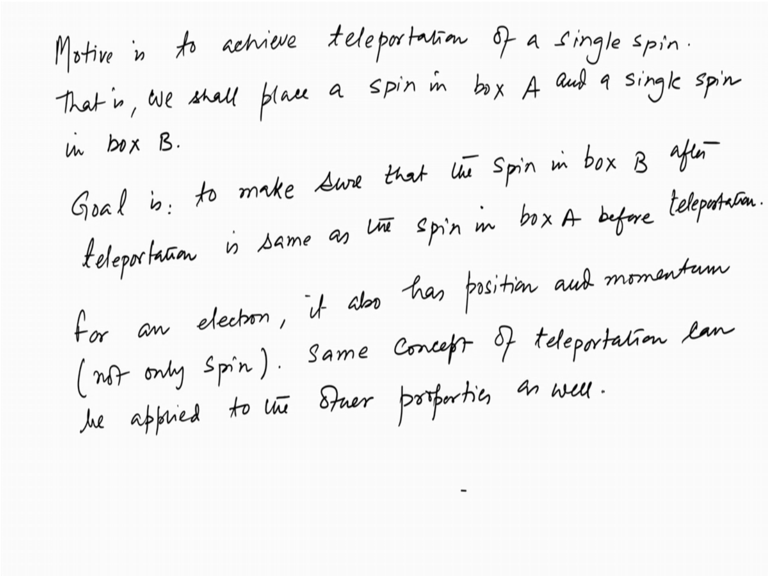

Portation of a single spin; that is we shall place a spin in box A, we talking about spin half particle in box A and a single spin; you can also say it is an electron because the spin is carried by the electron, but we are just thinking that spin as a particle.

So, the goal is to make sure that the spin in box B after teleportation is same as the spin in box A before teleportation. So, that is what teleportation would mean that the state of a goes to B and. So, the B will be in a state of as in the same state as that in a. It of course, would look like a much weaker requirement because we are only talk about spin and if we talk about the full electron it has position momentum and spin, but the same protocol actually can be extended to positions and momentum and so on.

So, for an electron it also has position and momentum and not only spin which we have talked about same concept can be applied to the other properties as well. Needless to say, that the spin has become an important property, because the spin with it is possibility of being up and down or being right and left they correspond to the states of a qubit which is one and 0.

So, we have talked about just a spin single spin how it can be teleported or rather what are it is properties and so on. We will see little more complicated situations, and then finally I get on to a discussion of the quantum teleportation phenomena.