Advanced Quantum Mechanics with Applications
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Lecture – 01
Introduction, Postulates of Quantum Mechanics

So, welcome to this course on Advanced Quantum Mechanics with Applications. In this course we will learn of course, the basic quantum mechanics and the various applications that have been developing since the last decade or so. And, various research topics that are associated with it. So, the applications part comes there, but of course, we have to also know the basic quantum mechanics which I am sure that all of you have studied in your undergraduate or graduate curriculum. So, this is a development on that I will of course, start with something simple and basic.

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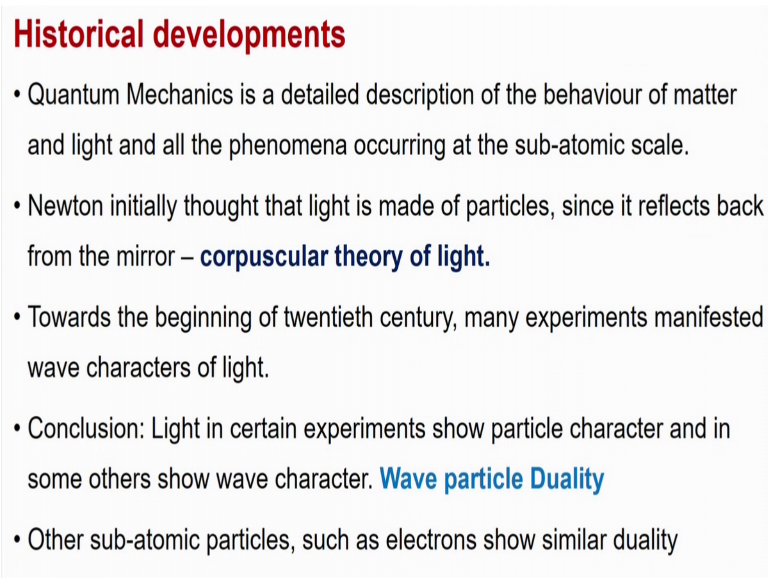

So, let us look at the historical development of quantum mechanics. So, it gives you a detailed description of the behavior of matter and light which means photons and all the phenomena that occur at the sub-atomic level. So, that is inside the nucleus or inside an atom whatever phenomena goes on they are governed by the quantum mechanics and the laws of quantum mechanics.

So, Newton initially thought that light consists of particles because it reflects from a mirror and this give rise to the corpuscular theory of light. However, towards the beginning of the twentieth century, there are lots of experiments done which have manifested the wave character of light; which means that they show interference refraction and reflection etcetera which otherwise especially the interference as a diffraction there otherwise absent in the case of particles.

So, this gave rise to this famous wave particle duality. So, the conclusion is that light in some experiments show particle character such as photoelectric effect and Compton Effect and so on. And in others they show wave character such as interference and diffraction. This gave rise to the wave particle duality of matter and it is not only the priority of photons, other particles sub-atomic particles such as electrons and protons etcetera they also show wave particle duality similar duality as seen for light.

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- The gradual accumulation of information about atomic systems produced curiosity and confusion.
- To a significant extent the confusion was removed in mid-1920s by Schroedinger, Heisenberg and Born who finally obtained a consistent description of matter at small scales.
- The most promising idea of sustaining a quantum world was based on probabilistic interpretation.
- The second pillar was the uncertainty principle related to the measurement which save a considerable amount of experiments done on sub-atomic particles.

So, there is a large amount of information that has started coming in for the atomic systems and they have raised lot of curiosity. And of course, have given rise to lot of confusion because of contradicting you know inference is coming out from various experiments. And, well a lot of people at that time they talking about the early 1920s and the mid-1920s between 20 and 1920 and 1930 where a lot of people have contributed to the birth and development of quantum mechanics. A notable among them are Schroedinger, Heisenberg and Born and they have enriched the field so much so, that there are you know consistent description of matter at very small lime scales that have emerged.

So, the basic pillars or the basic ideas that they are based upon mainly two of them one is called the probabilistic interpretation. So this has to be, this is distinct from the interpretation or the deterministic interpretation that we are familiar in classical physics. As a pose to that in quantum mechanics there is a probabilistic interpretation that needs to be incorporated which gives rise to a lot of satisfactory answers to a lot of experiments that have been done and phenomena that people have observed.

And the second one is the uncertainty principle which is related to the measure[ment] measurement of a particular measurement and it sort of saved, these two put together have saved a number of experiments that are done on subatomic particles. So, let us take this brief tour to how things have evolved and that will make the study of quantum mechanics more interesting. And of course, we will go to the applications part later on.

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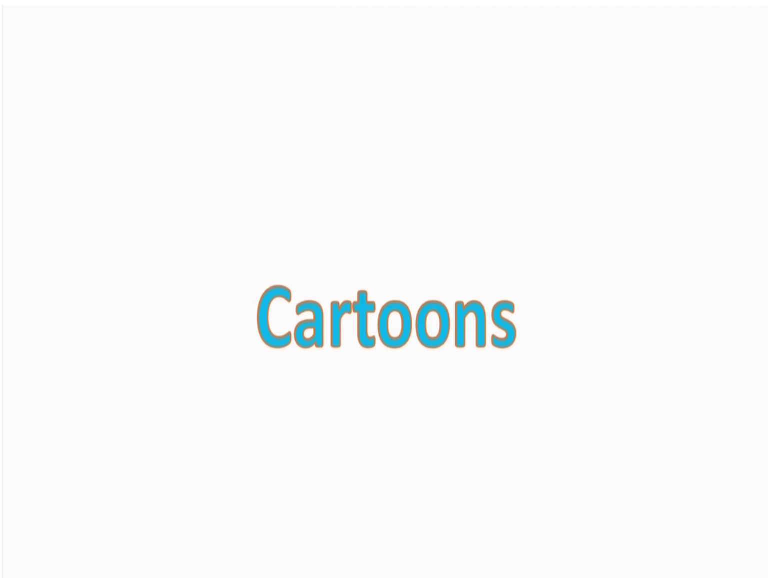

So, let us see some cartoons on a lighter note. So, that we understand things in graphics using graphics.

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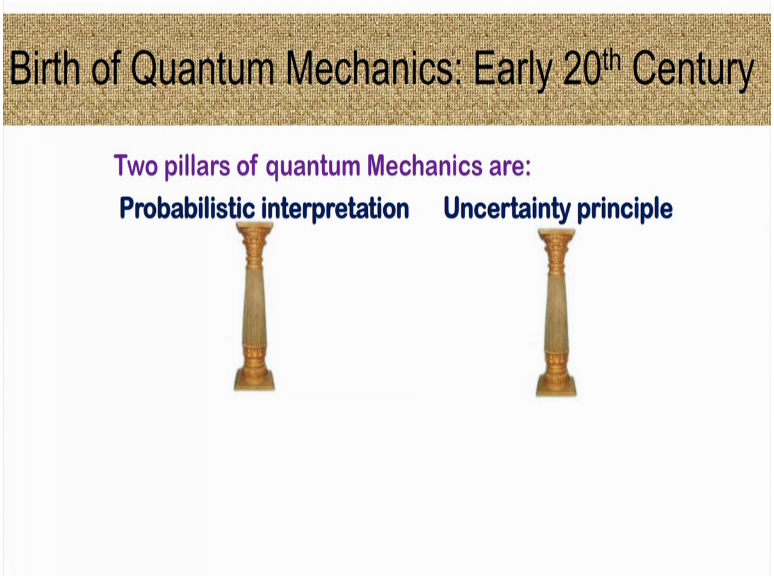

And there are some of them which are so, I have drawn two pillars the pillars as I said are one is the probabilistic interpretation, the other is the uncertainty principle of a quantum mechanics which is developed by Heisenberg we will go through that.

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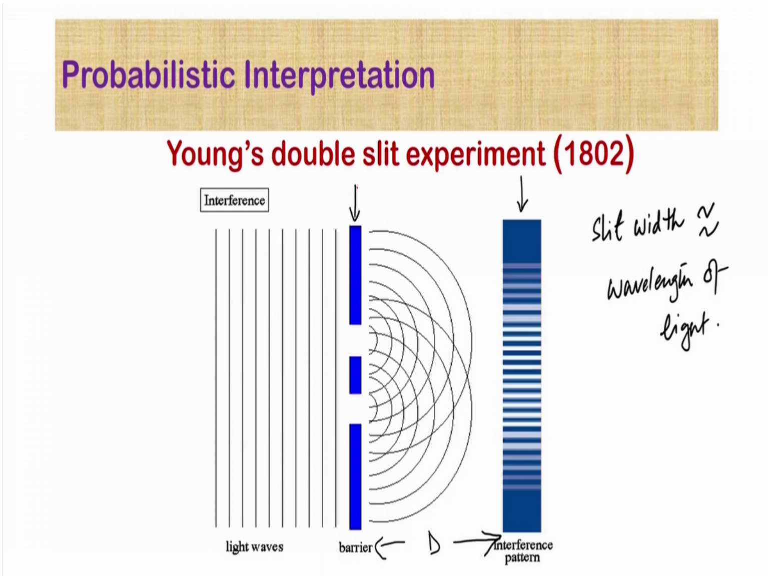

And, these say we go through one after another, the probabilistic interpretation was enriched by a simple experiment which is called as a double split experiment. And it is a Young it is called as a Young's double split experiment being performed in 1802 which is more than 200 years ago. So, there are light waves these vertical lines that you see are really the light waves that are coming, they are approaching these blue barriers like these ones, these blue barriers. These barriers have two slits and let us call this as a slit 1 here and this as a slit 2 here.

So, there are these two slits and there are these secondary these act as secondary sources and when light passes through them and they are allowed to fall on a screen that is kept here. So, this is the screen and when light falls on this light undergoes a phase difference from the coming out from the two slits. And this, phase difference gives rise to a interference pattern which means that there are bright bands and dark bands depending on the condition that these way this path differences or the phase differences that they satisfy and we get nice bands that are appearing on the screen as it is shown below.

And these are these are there in every undergraduate lab double street experiment which shows a interference patterns due to these two slits. One thing that should be remembered is that the slit width should be comparable to the wave length of light, which means that if wave length of light is of the order of 4000 or 5000 or 6000 Angstrom this should be of the order of a millimeter or so. So, then these or even less than a millimeter then these are the well-defined interference pattern could be seen.

And of course, the interference the distance between the dark and the bright bands that we see here depends upon the wave length of light and the slit width and also the difference between the barrier and the screen that you see here which I write as D. Now, these are matter of study in optics and in particularly in the interference chapter, we will not elaborate on that.

We have brought this for a particular reason of demonstrating how quantum mechanics can be derived or rather how the probabilistic interpretation can be inferred from this experiment. So, we will skip all the details that how what are the conditions for path difference or phase difference to have a constructive or destructive interference and so on. And, go on to the quantum mechanical aspects.

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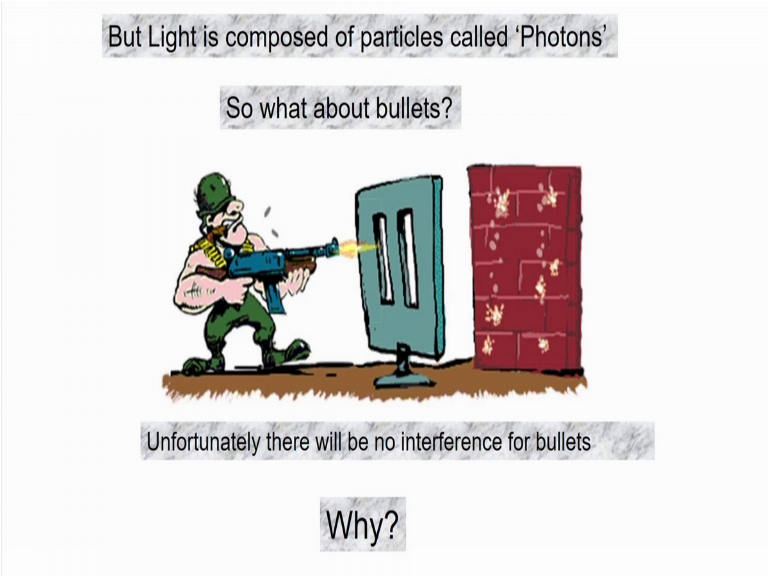

Now, light is of course, consist of particles called a 'Photons'. So, since we are talking about particles let us talk about bullets. You see this character which is firing guns through two slits, just like what we have seen in the last slide and these bullet marks are allowed to fall on a wall which is kept at a distance, again just like the photons were allowed to fall on a screen. Now, the bullets are allowed to impinge upon a wall. Now, do you think there will be interference pattern? Certainly not ok.

So, this if these character keeps firing bullets at the wall through the slits there will be bullet marks that will form on the along the two slits that are there and on the wall. And there will be no interference pattern, there will be no bright and dull bands that are going to form. So, the question is that if photons are particles the bullets are particles why does not why do not bullets give rise to interference pattern whereas, the photons do.

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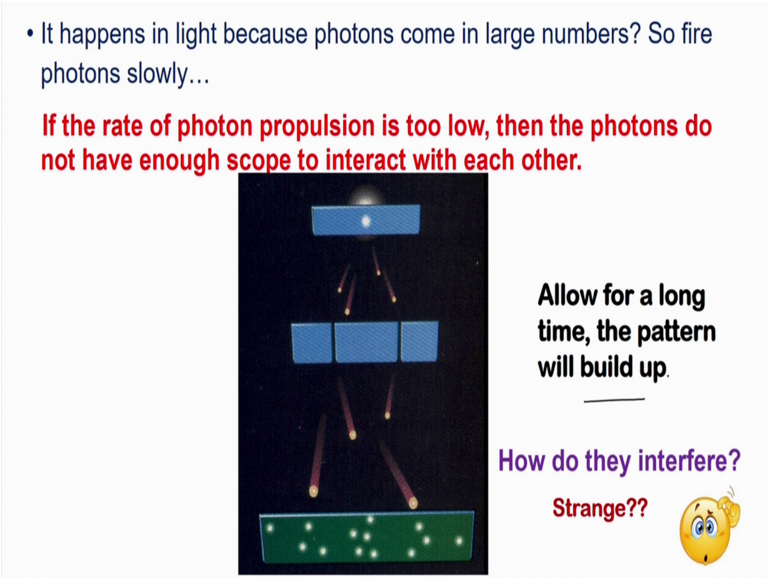

Could it happen because, the photons come in larger number because you are firing a light beam and the light beam has got a very large number of photons. So, is it that because they are coming in large number of photons. So, let us reduce the number. It is a thought experiment. We are not doing that experiment. Nobody is firing photons very slowly they are still at the velocity of light, but it is just a thought experiments you say that I am going to only release a few photons at a time. And they would go through the 2 slits, as shown here and fall on a screen and would ultimately give rise to an interference pattern as it is seen here.

So, if you allow for a long time then the pattern will build up and this is important thing the pattern builds up means that there is interference pattern. So, even if you reduce the number of photons by a large fraction the phenomena, the basic phenomena that happened earlier do not stop. So, if they are coming in small numbers the question is how does the interference pattern take place or rather it happens. So, how do they interfere that is the question.

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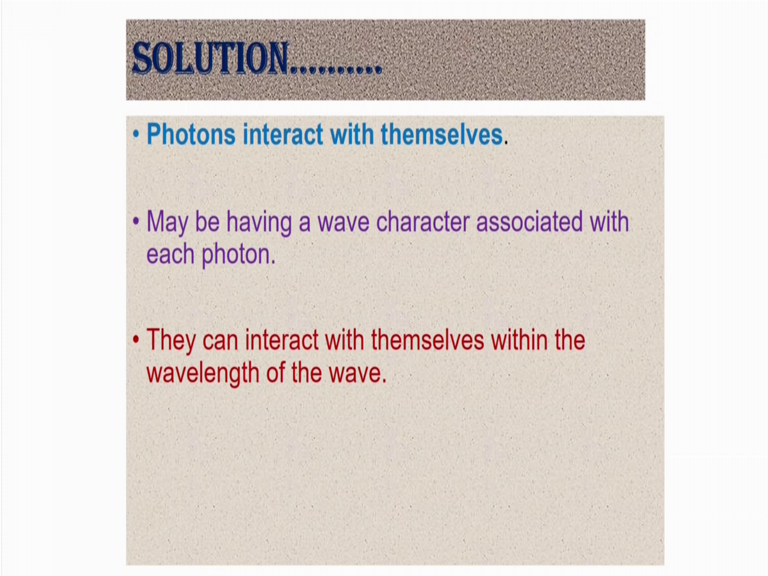

So, one solution is of course, that the photons interact with themselves. It is not very clear from the statement that we want to say, but they inter what all we mean to say is that they interfere with themselves. Or maybe they have a wave character associated with each photon which we know that certainly that is a possibility. And so, they can interact the coming back to the first point they can interact within the wavelength of the wave. So, they can actually interfere mutually interfere within the wave length of a wave and that could give rise to the pattern.

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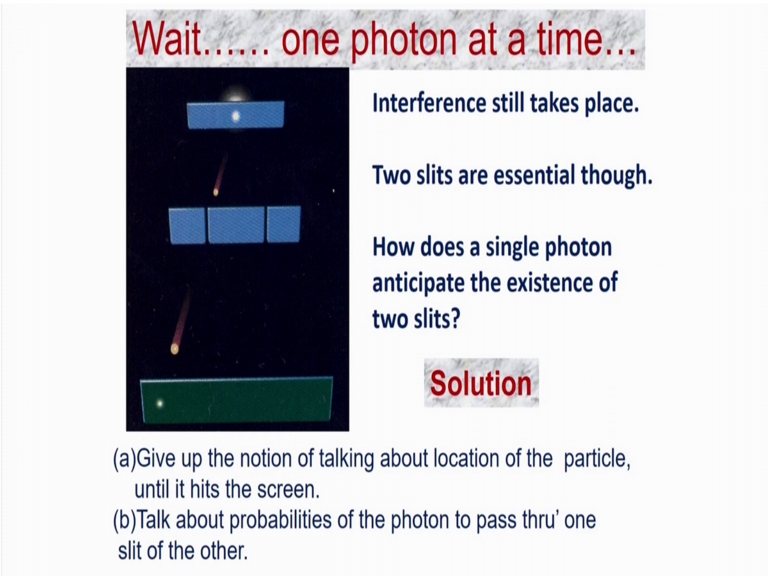

Now, let us reduce the stream even farther and send one photon at a time ok. Interference it seem that the interference still takes place, interference you one can still see an interference pattern if you wait for long enough, but there are two slits that are essential. So, the question is that that how does a single photon anticipate the existence of two slits. A photon coming through this slit that is left to mine is the photon that is passing through that is slit, it when it is passing through this it of course, does not know the existence of the other slit. So, how does the interference still take place if you reduce the stream to such an extent?

And the solution is that give up the notion of talking about location of the particle until it hits the screen. So, this picture is this middle picture where we actually see that its passing through one of the slits is misleading. Do not talk about measuring the position of particle or the location of the particle, until it hits the screen. If it hits the screen it will leave a mark and then you know that it has passed through either of the slits ok.

So, talk about the probabilities of the photons to pass through one slit or the other. So, we do not know we simply say that the photon must have passed through either the left slit or the right slit or call it slit 1 and slit 2, but we really do not know. And, if you allow that to happen or allow that at or rather you assume that then the interference pattern can be understood. So, that tells you that is the properties of quantum objects are linked to the measurement.

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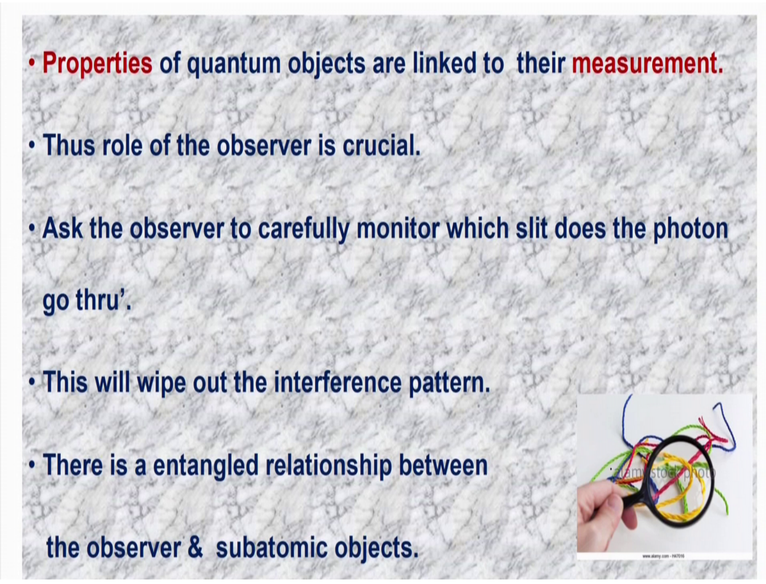

So, when you do the measurement, when you talk about the measurement that is important. So, do not talk about the measurement of the particle when it is passing through one of the slits; you really do not know which slit it is passing through. Similarly, the role of the observer is also crucial. So, if there is an observer to carefully monitor so, suppose there is a robot sitting at one of the slits or at both the slits. And, this robot actually knows exactly where it passes through which slit it passes through or say that this is one robot which counts the number of photons passing through that slit.

So, then immediately you are making a deterministic and if you do that then the interference pattern will go away. So, if the observer carefully monitors which slit does the photon go through will wipe up the interference pattern. So, there is an entangle relationship this is what we talk about entanglement in daily life that everything is you know knotted. And so, there is a nice entangled relationship between the observer and subatomic objects. This is what one of the basic principles of quantum mechanics.

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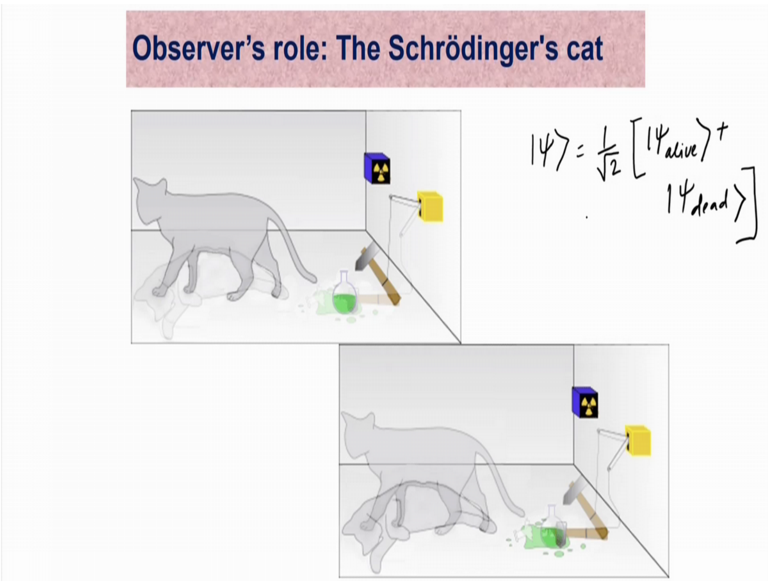

Let us now, assess the observer's role. So, this is called as Schrodinger's cat. So, there is a box in which this cat is enclosed and there is a hammer which is hanging or rather which is tied to a device or a pulley or some kind of a sort of lever. And, this is right over a green colored liquid in a bottle or in a vessel and this is assumable the poison. So, if you leave the cat inside the room and lock from outside and you do not know what is happening to the cat.

Either the cat could get very inquisitive or very curious about what is this green colored liquid and it would sort of disturb this whole apparatus and this hammer could drop down and break the poison. And if it breaks the poison then the cat dies and if the cat dies then of course, we would open the door and find the cat to be dead. And suppose, it could also happen that the cat is in such strange ambience for the first time and it does not want to do any mischief with this green colored liquid and it stays away from it nothing happens, open the door and you find the cat to be alive.

So, let us hope the cat to be alive, but there is all always the possibility that cat could be dead because of the reason that we just say. So, once you are out of the room and not watching it then in your mind there is a 50-50 possibility of the cat to be dead or the cat to be alive. And, as soon as you open the door the whole state of the room collapses into one of the possible states said to the say the cat is alive, which is an optimistic scenario. So, it is the observer's role is important observer's role only comes when you make an observation or you make a measurement. And, the system actually collapses into one of its available states these are called eigenstates of the system.

So, the eigenstates it is the cat is either its dead or alive. So, if you want to write it mathematically then we could write it as it is a 1 by root 2 is a normalization factor and for the psi of the cat to be alive and its psi dead. And the moment you made the measurement it collapses into one of the states. So, the observer's role is very important here.

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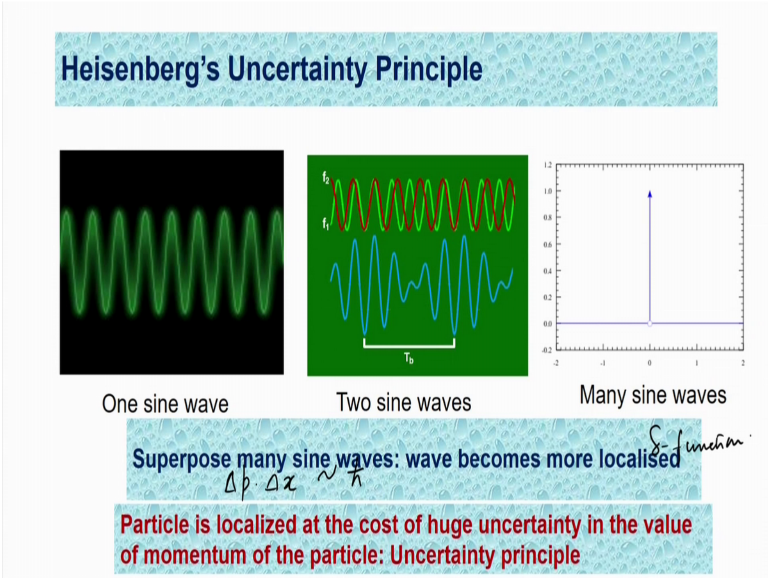

Let us now go to the other pillar that we have talked about of quantum mechanics that is Heisenberg's Uncertainty Principle. How we can understand Heisenberg's uncertainty principle simply, this is what a simple sine wave that you see. So, it is a sin x versus x so, this is your y axis in which we plot sin x and this x axis you plot x. So, it is a one sine wave, these are two sine waves. So, the top panel shows two sine waves with frequencies f 1 and f 2 with the green and red line and below that is a superposition of these two sine waves names which consists of f 1 and f 2.

And what happens we when we superpose many sine waves? It you see by superposing two sine waves the there is getting shrunk on, these period that is the periodicity is getting shrunk. And so, these two are at the same phase and of course, we are we are getting that this that the particle is getting localized. And, if you superpose many such sine waves it will be absolutely localized and we will see what is seen here which is like a delta function.

So, if you take many sine waves sin k 1 x plus k 2 x plus sin k 3 x plus sin k 4 x and so on and then try to plot them where, k is equal to you know 2 pi over lambda and lambda is related to the frequencies and so on. So, these are will give rise to a delta function type structure where, the particle is a if the particle is associated with this sin of this wave packet which we call it as wave packet where we superpose many sin waves, these are these will look like this. So, what it means is that this is the essence of uncertainty principle that if a you try to localize the particle, if you try to do precise measurement of its position you have to give up upon the momentum uncertainties; as here the momentum uncertainty is really infinity, because, a very large number of sine waves have been superposed and they are over range over a spectrum so that the delta p is very large at the expense of delta x to be very small, so the relationship is that the delta p and delta x this should be order of h cross or h we will see in a in just a short while that what h means or this is h cross is equal to h over 2 pi where h is known as a Planck's constant.

So, this is the essence of the uncertainty principle I will show an interesting picture. So, this is written there that superpose many sine waves and the wave becomes more localized and this localization leads to a very large uncertainty in the momentum. So, particle is localized at the cost of huge uncertainty and the value of the momentum and this is what the statement of the uncertainty principle is.

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So, this is a famous joke which was so, apparently these are two cars. So, there is a police car which is chasing this black car, the police car is in red and white. And, apparently this is been driven by Heisenberg and the police apprehension for speeding. And when he is caught speeding he says that the police asks him that do you know how fast are you driving because, you have broken all those speed limits.

Heisenberg says no, but I know exactly where I am. So, this he says my delta x is equal to 0. However, since I do not know how fast I was driving so, my delta v or my delta p is unknown to me or it is large. But, I know exactly where I means my delta x is very small between which means my position uncertainty is very small. This is just a cartoon or just on a lighter note what it is what is essence of uncertainty principle. Now, let us ask a very pertinent question. So, we are talking about quantum mechanics right from the beginning it is applicable to whom. And, the answer is that to everything in the universe.

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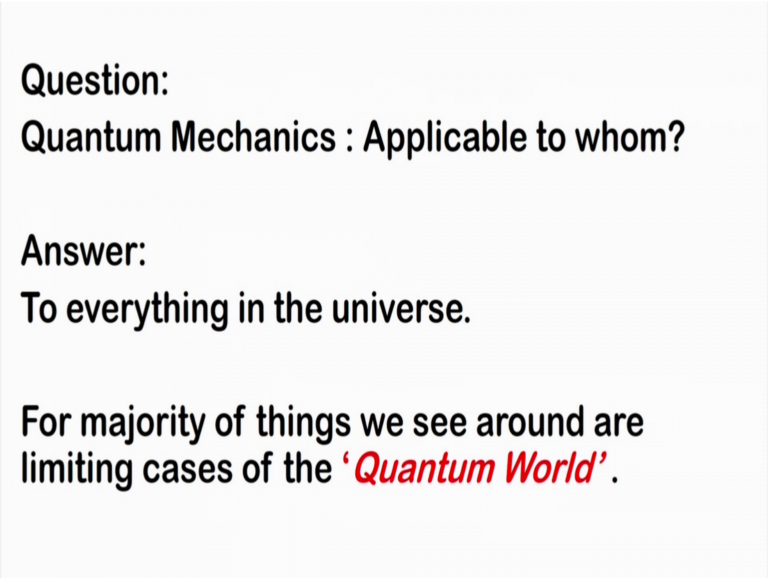

And so, for majority of the things that we see around are really the limiting cases of the quantum world that we live in ok. So, we will see that that how so, what is the meaning of then studying classical physics at all levels. I mean in the sense starting from your school level and then your undergraduate level and then your graduate level and so on; the classical mechanics, classical physics, classical electrodynamics that we talk about then what is the existence of that or rather what is the utility of that. And, that we will show that these are the classical world is really a limit of the quantum world.

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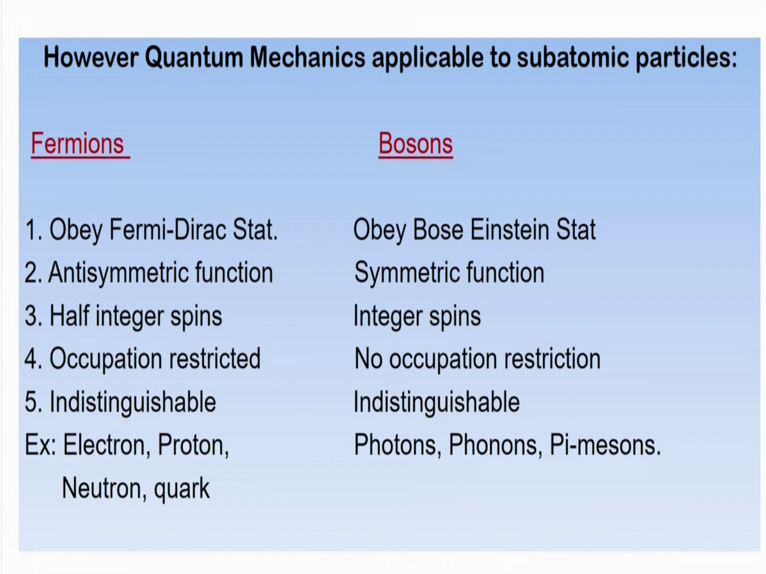

So, a quantum mechanics as said earlier number of times that it is applicable to subatomic particles. And, most familiar subatomic particles are fermions and bosons and we do not want to get into details about the properties, but just for your information they are the fermions. So, the fermions of statistics they have anti symmetric wave function, they have half integer spins. The occupation is restricted and basic they obey highly exclusion principle and then they are indistinguishable and the examples are electrons, proton, neutron, quark etcetera, whereas, bosons obey Bose Einstein statistics.

They have symmetric function, the integer spins, there is no restriction on occupation of bosons in a given energy state or a given quantum level. They are of course, also indistinguishable just like the fermions and the example of photons, phonons, pi-mesons etcetera.

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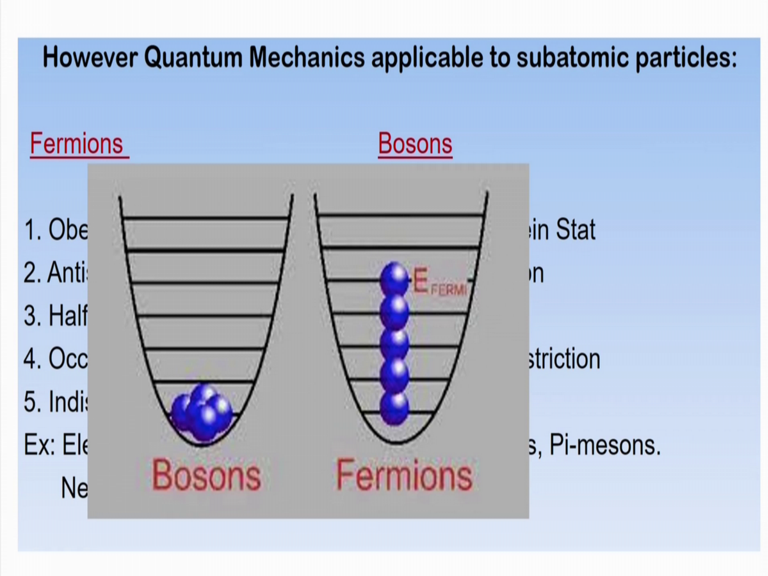

So, just another cartoon of showing that bosons can actually be in any of the states, a here it is shown as the ground state that phenomena has a particular name which is called as Bose Einstein condensation, whereas the fermions two fermions cannot occupy the same level. So, they are distributed over all these excited state energy levels as well or rather all the energy levels that appear here.

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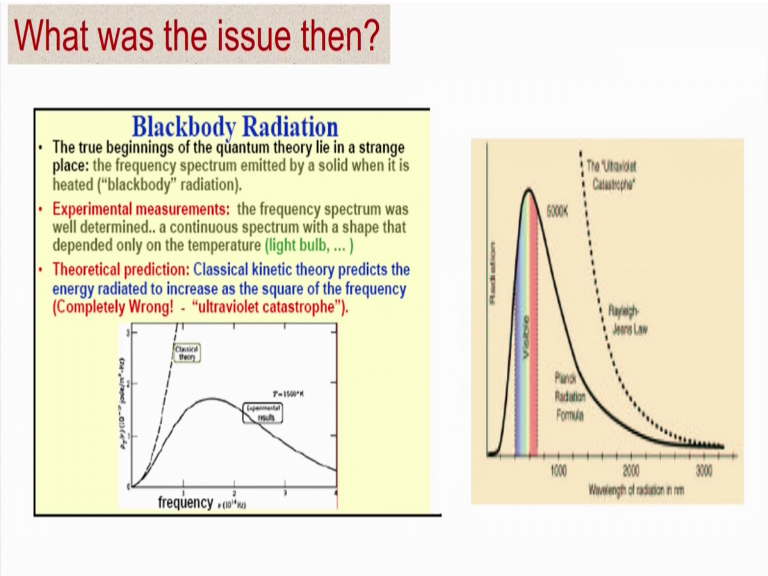

So, how the quantum mechanics was thought to be you know you invented or people have started talking about it. How did that happen? So, it happened with a simple experiment which is called as a Blackbody radiation. Now, again I will not go into what the what is the definition of a blackbody but, a blackbody is a body which absorbs all the radiation that incidents on it. So, we are talking about a the radiation from the sun and radiation from other stellar bodies. So, suppose you have a body which absorbs all the radiation and emits nothing then that is called as a perfect blackbody.

So, such blackbodies one was interested in seeing that how the intensity of the spectra emitted. So, we are talking about also emission spectrum. So, it is it emit some radiation so, does not absorb everything, so the emission spectra and as a function of frequency and at a given temperature ok. And, what was shown is that it shows the nice; I will use another color to give a guide to your eye. So, this is that experimentally it is found that it has a non-monotonic dependency. Whereas, the classical theory predicts that it should go like this and then there was another theory that was made by Rayleigh and Jeans which says that it goes like this.

So, nobody actually predicted that it goes up and then goes down and this was a problem at hand with Planck. And Planck almost like as he explained later that as a matter of desperation, he had proposed something which we will just see in a while. And so, this is called as the ultraviolet catastrophe that this classical kinetic theory re predicts energy radiated to increase the square of the frequency and so on. So, these are all wrong and what you see this non-monotonic thing or these thing here that you see here are the once that are correct and experimentally observable.

And this is shown as a function of frequency. This is shown as a function of wavelength. However, they all carry similar meaning or rather it just says that at a given temperature, if you look at the intensity of the radiation a then either the function of frequency or wavelength one sees a non-monatomic nature which cannot be explained by classical theory.

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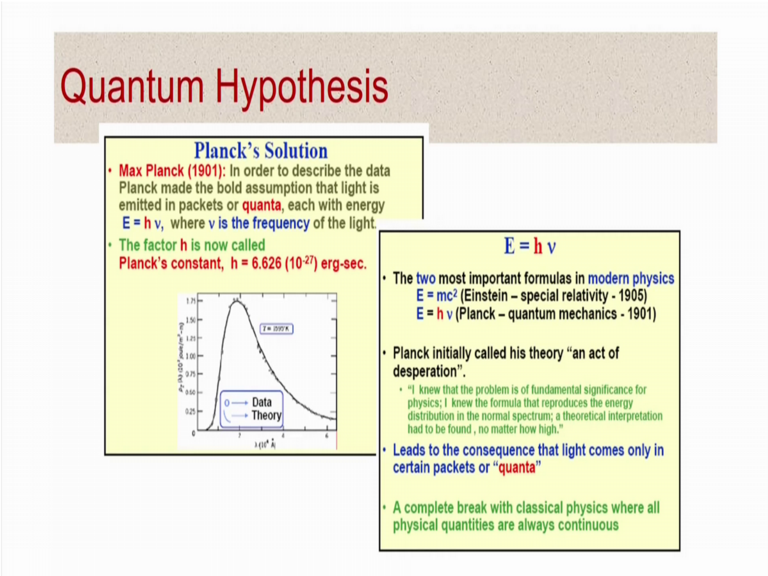

So, Planck solution was that that if said that look he made a bold assumption that light is emitted or absorbed in a some discrete packets or quantum which is given by E equal to h nu. When nu is the frequency of the light and this h is called as the Planck's constant. We will just detail these little later and this h has a value which is 6.62 7 into 10 to the power minus 27 erg-second or equivalently it is 6.6 into 10 to the power minus 34 joules second and so on.

So, E equal to h nu; so energy of this emitted radiation is not a continuous stream, this happens in packets and each packet has an energy which is h nu; nu is the spectral frequency and h is of course, here looks like a constant of proportionality that is what we will call, but this is known as Planck's constant. Planck initially called this theory "as an act of desperation". He says that "I knew that the problem is of fundamental significance for physics; I knew the formula that reproduces the energy distribution in normal spectrum; a theoretical interpretation had to be found, no matter how high".

So, it leads to the consequence that light comes only in certain packets of quantum quanta. So, this a complete breakdown on classical physics of all the physical quantities are known to be always continuous. This Indian scientist S. N. Bose had a large contribution here, what Planck did on an ad hoc basis. He actually wrote down the correct distribution for the photons which is known as the Bose Einstein distribution.

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- To make further advances into the subject of quantum theory, it is required to look at the origin of quantum of action, 'h', which can be thought of as proportionality constant of the equation,

$$E = h\gamma$$

Where γ is the spectral frequency.

Example

Consider a macroscopic oscillator of mass 0.01 kg, the maximum velocity = 0.1 $m s^{-1}$ and amplitude = 0.01 m.

Solution $x = A \sin\omega t \Rightarrow \frac{dx}{dt} = A\omega \cos\omega t = A\omega\sqrt{1-\left(\frac{x}{A}\right)^2}$

$$\frac{dx}{dt} = \omega\sqrt{A^2 - x^2}$$

So, they more advances into the subject of quantum theory is required to look at this origin of this quantity called h. It is also called as a quantum of actions popularly of course, known as Planck's constant and it is as I told that it can be thought of as a proportionality constant of this equation where, gamma is a spectral frequency. So, let us see that when is this h important. So, let us take an example. So, consider a macroscopic oscillator, a classical oscillator that is I mean of mass 0.01 kg and the maximum velocity of the oscillator is equal to 0.1 meter per second and the amplitude. And so, basically the amplitude of oscillation is amplitude equal to 0.01 meter.

Now, of course we know that this does not fall into the real mof quantum physics, but we will show that explicitly. So, what is the solution of of a classical oscillator? It is x equal to A sin omega t A sin or cosine does not matter, which gives you a dx dt equal to a A omega cos omega t; which tells you that it is equal to A omega into 1 minus x by A whole square which is equal to omega into A square minus x square. So, that is the velocity of the oscillator, the maximum velocity of the oscillator is when x equal to 0 that is when the oscillator is at the mean position.

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$$\left(\frac{dx}{dt}\right)_{max} = v_{max}\Big|_{x=0} = A\omega = 2\pi A \nu .$$

(mean position)

$$\nu = \frac{v_{max}}{2\pi A} = \frac{0.1}{6.28 \times 0.01} = 1.6 \text{ Hz} .$$

$$\text{Time, } T = \frac{1}{\nu} = \frac{1}{1.6} = 0.63 \text{ sec} .$$

$$\text{Energy } E = \frac{1}{2} m v_{max}^2 = 5 \times 10^{-5} \text{ J}$$

$$h \sim 10^{-34} \text{ J-s.}$$

Product

$$(ET) \approx 10^{-5} . \qquad (ET) >> h .$$

$$\boxed{\text{Quantum effects are negligible}}$$

And so, dx dt max which is equal to v max happens when x equal to 0 which is the same as the mean position and this is equal to omega into root over. So, we just put this equal to 0 and this becomes equal to A omega. So, at x equal to 0 x equal to A omega which is equal to a 2 pi A nu where, nu is equal to v max divided by 2 pi into amplitude. So, if you do this 0.1 2 pi 6.28 into 0.01 which is the amplitude this comes out to be 1.6 hertz. So, that is the frequency of oscillation of this thing and the time that is associated with this frequency which is equal to say T equal to 1 over nu, which is 1 divided by 1.6 which is equal to approximately 0.63 second ok.

So, what is the energy associated with this oscillator energy E we are talking about the total energy which is of course, the kinetic energy, but at the potential energy. But, when it is at the mean position this is what the total energy is; if you put in all these values of m etcetera and v max that we have written earlier it comes out as 5 point 5 into 10 to the power minus joules. So, this energy E into T which is so, the product the product of the E and T that is 10 to the power of the order of 10 to the power minus 5 or 10 to the power minus whatever 4 or something. So, this E T is certainly much greater than h.

So, these canonical variables such as energy and time they are when they are multiplied they give values which are much much greater than h. Because, h is of the order of 10 to the power minus 34 joule second and this is only coming out to be 10 to the power minus 4 or 10 to the power minus 5. So, the conclusion is that the quantum effects are negligible. So, this quantum effects are negligible and so we are in the classical regime and we say safely employ formula of classical physics. So, that is the importance of h that we want to bring out.

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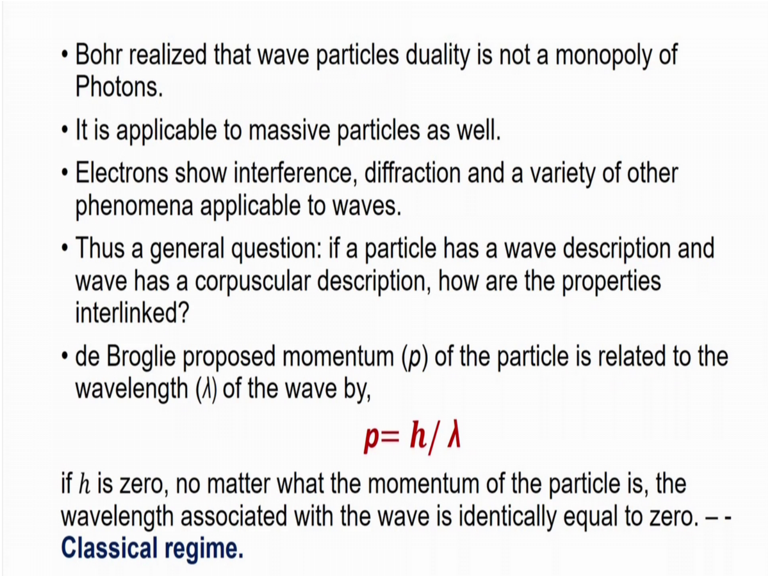

Now, Bohr realized that this wave particle duality not a particles it is wave particle duality is not the monopoly of photons. It is applicable to massive particles as well, what we mean by massive is that which has a mass, it necessarily does not need to be heavy. Such as electrons they show interference diffraction and a variety of other phenomena which are applicable to waves.

Thus, there is a general question: if the particle has a wave description and wave has a corpuscular description means particle description, how are the properties interlinked? And de Broglie solved this problem; he said that look the momentum of the particle is related to the wavelength of the wave. So, single thing or a single particle or a single object can have both the descriptions, both particle and wave description. The wavelength of the wave is related to the momentum of the particle and this is by this relation p equal to h over lambda, this called as de Broglie relationship.

So if h is 0, no matter what the momentum of the particle is, the wavelength associated with wave should identically be equal to 0. And, we are should be in the classical regime as we have just seen in the example or the illustration just in the last slide or the earlier slide.