Advanced Condensed Matter Physics
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Lecture – 07
Magnetism

So, we have learned magnetism, magnetic metal in the previous discussion in which we have seen that how spin only modeled in one dimension actually can be solved exactly in give raise to a magnetic metal. If you include the spin interaction in Z direction if the gap will open up and then will have a magnetic insulator depending upon what is the magnitude of the gap. So, let us now, look at more carefully magnetism.

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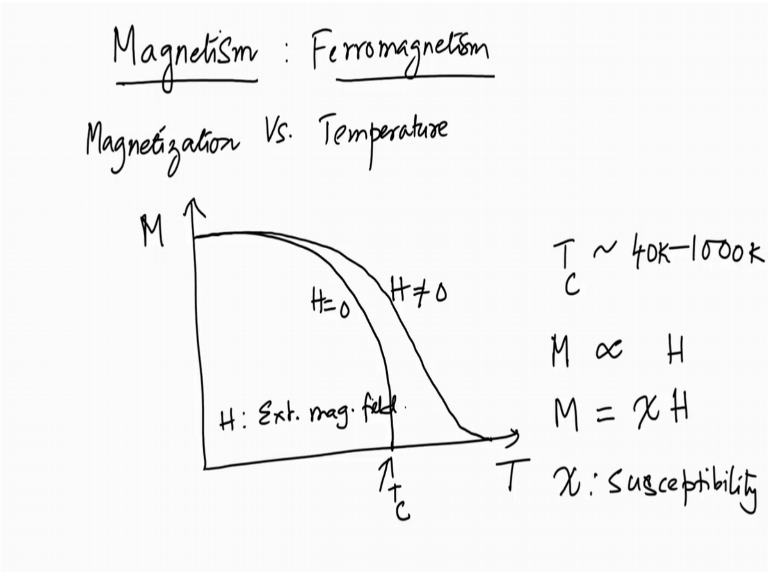

And we do it as an example of the on going discussion, but it also helps us to understand magnetism in particular and we will have going to talk about ferromagnetism to be more precise.

So, let us talk about ferromagnetism where by ferromagnetism what we mean is that the spins are all pointing in same direction. So, when the spins all of the all the spins are pointing in the same direction we get a magnetization of the system. So, there is a bulk magnetization that exists and this is put into use in various applications of magnets. So, the first thing that comes to our mind is that the magnetization verses temperature curve.

So, magnetization verses temperature and as I told that we are going to talk about ferromagnetism in particular.

So, that looks like, the magnetization is in the y axis and this is the temperature in the x axis, so it is transition which is like this for external magnetic field equal to 0 which we denote by H and this point is called as T C. So, when the bulk magnetization vanishes and the system loses its complete magnetization and it becomes a paramagnet and the same plot in magnetic external magnetic field in presence of external magnetic field looks like this. So, this is of the order of typically T Cs of the order of for some materials which will see such as nickel its of the order of it could vary from something like few Kelvin may be around 40 Kelvin to about may be even 1000 Kelvin and so 1000 Kelvin, if we do it more carefully.

And will list out various values of T C. So, what happens is that at T C the magnetization vanishes and the ferromagnet converts into or there is a transformation face transformation into a paramagnet where which results in random alignments of fence and we have shown to graphs, one is in the presence of external magnetic field which is a outer curve and the inner curve is a with the magnetic field equal to 0. So, now, what happens is at small values of field the magnetization is proportional to the field. So, this is the external field and we can write proportionality relation with this where chi is called as the susceptibility, and H as we said is the external magnetic field.

So, this is valued for low fields off course we have non-linear phenomenon which involves higher order of the external field where the magnetization ceases to be linearly depending on the field that very large fields, but we will only stay within this domain where the magnetization is proportional to the field and we can talk about it relation linear relation between magnetization and the magnetic field to be having this kind of relation. So, now, let us look at how the susceptibility which is an important measure of magnetization how that behaves with temperature.

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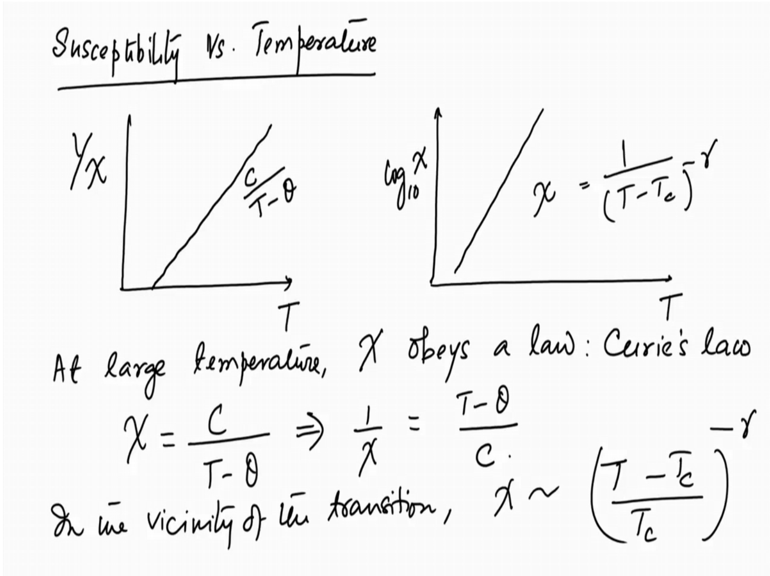

And so, susceptibility verses temperature. And we can have 1 over chi plot verses T which simply looks like a line like this a straight line like this which has the slope or rather it can be represented by formula which is like C divided by, where C is a constant C divided by T minus theta where T is the temperature and theta is some characteristic temperature which we are going to list out in a while. And also a very close to the transition that is a ferromagnet to the paramagnet transition we have the T actually or rather than chi actually depends on the temperature in an interesting way and which is often called as a universal behavior and which can be understood that if you look at the log of chi which is base 10 its chi verses temperature and then it looks like straight line again.

And this tells that the chi is actually 1 divided by T minus T C whole to the power gamma minus gamma and. So, this is the straight line if we simply you know erase this that it looks like a straight line and, so it is a straight line like this and where gamma is some coefficient or rather some number which is an indicator of the universality class of the transition and so at large temperature rather, at large temperature the susceptibility which is chi obeys a law called as Curie's law.

And so this can be written as what we are shown in the, so this is written as some it is, so chi is equals to C divided by T minus theta that tells us that 1 over chi goes as T minus theta divided by C. And also in the vicinity of T C in the vicinity of the transition which occurs at a temperature T equal to T C the chi actually goes as T minus T C divided by T C whole to the power minus gamma and the and this is will tell you as I told that it tells us about the universality class of the transition. So, let us list out a few values for known ferromagnets. So, we have let us, there are materials and there is a T C and this characteristic temperature theta.

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Materials	T_c (K)	θ (K)
Fe	1041	1093
Ni	631	650
Gd	293	302.5
$CrBr_3$	37	-

Some known Ferromagnets.

So, if you list out few known materials of which shows ferromagnetism which is iron which is about 1041, theta is 1093 nickel which is another known ferromagnet which is has an T C of 631 Kelvin, these are all in Kelvin. And this is the 650 Kelvin and a gadolinium has a 293 T C, 293 Kelvin which is almost room temperature and slightly higher value of theta. And a chromium Cr Br 3 so this has a T C of which is very low which is 37 Kelvin and there is a no data may be available for C r B r 3 for the value of theta.

So, these are some ferromagnets some known ferromagnets. So, before we go to the many body theory of understanding ferromagnets let us see how we can understand it in a simpler language of ferromagnetism and let us do models of ferromagnetism and write down a simple model and try to solve it if possible.

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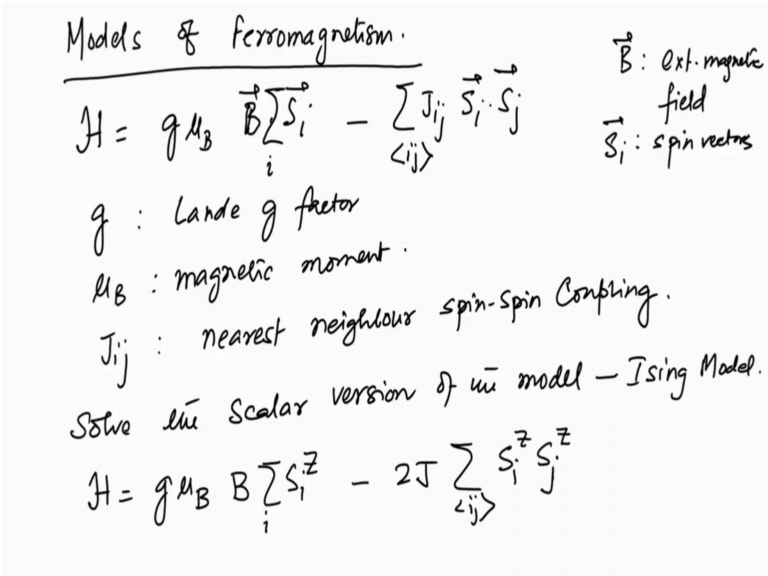

So, in presence of a magnetic fields let us call it as B. So, this magnetic field is offer interchangeably used by H and by B, B is usually called as magnetic induction and edge is called as magnetic field, but; however, sometimes a difference is not spelt out very explicitly. And one can write down Hamiltonian which is g mu B and B dot S i and some over i and a minus J i j a S i dot S j. So, first term is that a coupling between an external magnetic field B. So, this is the B is the magnetic field, a external magnetic field and S i are the spin vectors all right.

So, and off course we have to define other things also g is, it is it is called as the Lande g factor and mu B is the Bohr or it is the magnetic moment, it is the magnetic moment and J i j is a nearest neighbors spin spin coupling. So, this is Hamiltonian which this system of non interact or the system of spins which are placed in a external magnetic field. The first term denotes how an individual spin couples to magnetic field and the second term is that there is an interaction between 2 spins which are at neighboring sides i and j. So, when I write this form that is i and j in angular brackets they mean that i and j are nearest neighbors. So, there is a nearest neighbor spin interaction the amplitude or the magnitude of the spin spin interaction of the coupling is given by J i j which could be a constant for all i and j which may not be a constant for pairs of any pairs of i and j.

This model is quite complicated to solve in 3 dimensions and often there are now, solutions exact solutions unless you are ready to make some approximations. And let us solve the scalar version of the model. So, this is what we are going to do and scalar version of the model and which is we already know that is called as the ising model and so that ising model looks like, so my spin vector has only can only assume 2 orientations which is up and down that is called as the ising model. So, my Hamiltonian now, becomes g mu B and B and S i Z sum over i and minus 2 j sum over i and j S i Z S j Z i and j are still nearest neighbors, but now, it is scalar form of the spin that are considered.

And we have discussed this before that in order to avoid the double counting we have put this factor and this is the Hamiltonian that we need to solve.

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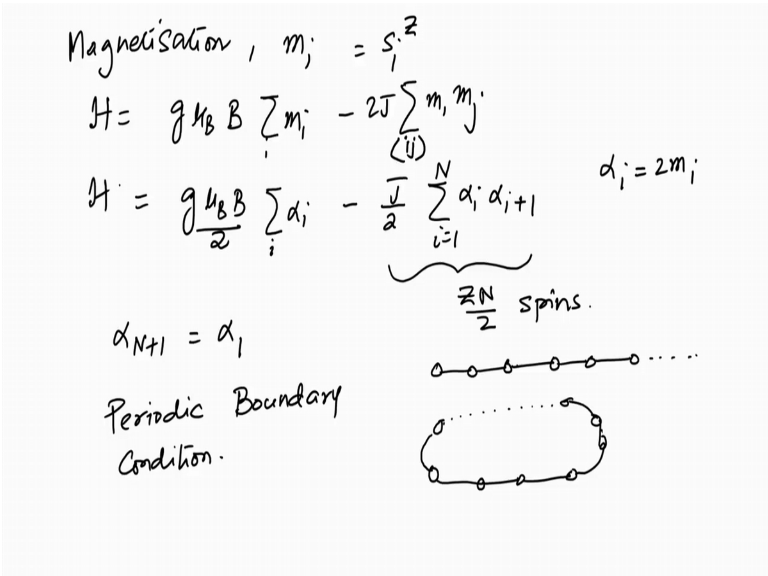

And we define the magnetization as let us call it with m i which is equal to S i Z S i z. Now, with this notation my Hamiltonian becomes equal to this g mu B b sum over i, m i and the 2 j m i m j and this is written as g mu B, B divided by 2 sum over i alpha i minus j by 2 sum i equal to 1 to N alpha i alpha i plus 1, where alpha i equal to 2 m i. So, basically these are the alpha i S will take values which are either plus 1 or minus 1 and if you notice that this sum or rather the second term contains a Z N by, so Z and N, N by 2.

So, these are the total number of spins that is there and if we add a term which is alpha N, N plus 1 we have to understand that alpha N plus 1 is same as alpha 1. So, the N plus 1 is spin is made as the same the as the first spin. So, that we have an open chain and so on and this open chain now, has been put into a closed form.

So, the last one and the first one are connected and we have a closed system such that my alpha N plus 1 equal to alpha 1 and this is called as the periodic boundary condition and this periodic boundary condition is enforced on the problem in order to solve it. So, its periodic boundary condition and this is the open chain. So, the open chain has been a converted into a periodic or rather close chain by applying a periodic boundary condition. Now, how to solve this Hamiltonian? One way of solving is the Hamiltonian is to write down the partition function and see that if the partition function has any closed form available.

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$$Z = \sum_{\{\alpha_i\}} e^{-\beta E_n\{\alpha_i\}} = \sum_{\{\alpha_i\}} e^{-\beta \mathcal{H}\{\alpha_i\}} \qquad \beta = \frac{1}{K_B T}$$

Transfer Matrix technique.

$$Z = \sum_{\alpha_1=\pm1}\sum_{\alpha_2=\pm1}\cdots\sum_{\alpha_N=\pm1} \underbrace{K(\alpha_1,\alpha_2)}\,\underbrace{K(\alpha_2,\alpha_3)}\cdots\underbrace{K(\alpha_N,\alpha_1)}$$

$$K(\alpha_1,\alpha_2) = \exp\left[-\frac{\beta g \mu_B B}{2}\left(\frac{\alpha_1+\alpha_2}{2}\right) + \frac{\beta J}{2}\alpha_1\alpha_2\right]$$

$\alpha_1=-1, \alpha_2=-1$

$\alpha_1=1, \alpha_2=1$ (1)

$\alpha_1=1, \alpha_2=-1$

$\alpha_1=-1, \alpha_2=1$

$$= \begin{pmatrix} \overset{\alpha_1=1,\alpha_2=1}{e^{(-x+a)}} & \overset{\alpha_1=1,\alpha_2=-1}{e^{-a}} \\ \overset{\alpha_1=-1,\alpha_2=1}{e^{-a}} & \overset{\alpha_1=-1,\alpha_2=-1}{e^{(x+a)}} \end{pmatrix}$$

So, the partition function is written as Z equal to alpha i and exponential minus beta E n alpha i. Now, I will repeat for convenience that this is the canonical partition function which means that it is partition function for a system which is in a contact with the heat bath at temperature T. And this is the form of the partition function where the energies or which is we could have written H as well, so we this is the same as what we mean by this is actually alpha i and exponential minus beta H alpha i and as you saw it here that H is a function of alpha i. So, this H is function of alpha i and so we are to exponentiate it multiplied with beta and then take the negative sign here and then sum over all alpha i S that are possible.

But look at this simplicity of this formula that alpha i S can only take values which are plus 1 and minus 1 because of the fact that the S i S or the spins are only the Z components of the spins and Z components take a value which is the Z component of the spins take values which are plus 1 and minus 1.

So, even it looks like a complicated formula to solve or rather summation to compute it is not so will see that here and we will use a technique which is called as the transfer matrix technique to solve this. So, in order to do understand ferromagnetism from very simple considerations we are written now, a general Hamiltonian where the spins are interacting with the neighboring spins and they are in presence of a magnetic field and there is a Zeeman coupling as it is called as a B dot S coupling at each site B is uniform magnetic field external magnetic field is uniform.

And that problem has very complicated solution and in 3 dimensions it is not possible to solve unless you do some approximations you can solve with numerically by doing exact diagonalization of finite number of spins, but that is not the focus here focus is to actually solve this model by using analytic techniques.

Then that Hamiltonian was converted to a one dimensional Hamiltonian which is like an ising modeling in presence of a magnetic field, and we are trying to solve the ising model in presence of a magnetic field and in one dimension. And this requires us to compute the canonical partition function which is written as this. So, this Z can simply be written as alpha i equal to plus minus 1 and these are there are N of those, so this is alpha. So, this alpha 1 and there is alpha 2 equal to plus minus 1 and then there are sums of this kind and alpha N equal to plus minus 1 and we can write down terms which are like k alpha 1 alpha 2, k alpha 2 alpha 3 and all the way up to k alpha N alpha N plus 1, but as I told that because of the periodic boundary condition alpha N plus 1 is same as alpha 1.

So, the advantage of this expression is that each of those keys will see what these keys are, each of these keys involve 2 spins at a time. So, this term involves the first and the second spin where for alpha equal to 1 alpha 1 equal to plus 1 and minus 1 alpha 2 equal to again can be plus 1 and minus 1, similarly for alpha 2 and alpha 3 and then alpha 3 and alpha 4 and continuing all the way for N spins and this term is equal to alpha N alpha 1. So, each of these k alpha 1 alpha 2 you should work it out carefully from this formula that formula for the partition function that is written over here. So, this is equal to a minus beta g mu B B. So, I should write here the beta equal to inverse of temperature or its 1 over K T, where K B as its written here is called as the Boltzmann constant which has the value which is often you will find it any book.

So, this divided by 2 and alpha 1 plus alpha 2 divided by 2 and plus a beta j by 2 alpha 1 alpha 2 and this is the form of each one of those K expressions that are written inside the sum. So, and this can be a summed over each one of those case can be summed over for values of alpha 1 and alpha 2 or alpha 2 and alpha 3 or alpha 3 and alpha 4 each having value plus 1 and minus 1. So, this requires us to solve a matrix problem, where a this k actually looks like matrix written like this, where this is for alpha 1 equal to 1 alpha 2 equal to 1 and this is alpha 1 equal to minus 1 and alpha 2 equal to minus 1. In this basis one can write this as exponential minus x plus a I will tell you what x and a are, and exponential minus a and this is exponential minus a and this is exponential x plus a.

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$$x = \frac{g\mu_B B}{2k_BT} \qquad a = \frac{J}{2k_BT}$$

$$Z = Tr(K^N)$$

K has 2 eigenvalues. $\rightarrow \lambda_1, \lambda_2$.

$$Z = Tr(K^N) = \lambda_1^N + \lambda_2^N = \lambda_1^N\left[1 + \left(\frac{\lambda_2}{\lambda_1}\right)^N\right]$$

$$\lambda_1 > \lambda_2$$

$$Z = \lambda_1^N \; ; \quad \lambda_{1,2} = e^a\left[\cosh x \pm \left(\sinh^2 x + e^{-4a}\right)^{1/2}\right]$$

$$\lambda_1 = e^a\left[\;\text{"} \; + \; \text{"}\;\right].$$

So, where my x is equal to g mu B B divided by 2 K T and a equal to J over 2 K T. So, what is shown here is the following that each of those keys written in the alpha basis which are, there is one term for which alpha equal to 1 and alpha equal to alpha 1 equal to 1 and alpha 2 equal to 1. So, that is is given by; so this should be more carefully written as, we have a 1 1 and that is equal to. So, this is the 1 1 element and alpha 2 equal to minus 1 here. So, we have to write it as if we have these things. So, there are 4 combinations alpha 1 equal to 1, alpha 2 equal to 1 alpha 1 equal to 1 alpha 2 equal to minus 1 alpha 1 equal to minus 1 alpha 2 equal to 1 and alpha the other term is alpha 1 equal to minus 1 and alpha 2 equal to minus 1.

So, if you write it that basis without writing it like this then it looks like each of these K's will look like. So, one term is this, this term is the first term. So, this is the first term and there is another term which is alpha 1 equal to 1 alpha 2 equal to minus 1. So, which tells that this term cancels out because one of the term is plus 1 and the other is minus 1; however, this becomes exponential minus beta J by 2 which is written by a, I will show you that.

So, this terms are first terms when one of them is plus 1 and the other is minus 1 and this one of them is, this one for alpha 1 equal to 1 alpha 2 equal to 1. This is alpha 1 equal to 1 alpha 2 equal to minus 1 this term is alpha 1 equal to minus 1 alpha 2 equal to 1 and this term is for alpha 1 equal to minus 1 alpha 2 equal to minus 1. So, this is these are the 4 terms. So, do not. So, this is written here you should do it and convince yourself to get matrix of this kind for these 4 a unique combinations.

And this is the matrix that we have to solve and. So, x is equal to g mu B by 2 K T and a equal to J over 2 K T and so we have the full Z the total partition function can be written as the tress of K and N to the power N because if you see this is not only true for alpha 1 alpha 2 it is true for alpha 2 alpha 3, it is also true for alpha 3 alpha 4 is true for alpha 5 and alpha 6 and so on and alpha N and alpha 1. So, we have similar k matrices coming and so the entire thing entire partition function will be the product of all these matrices and since we are doing a sum it will convert into a trace. So, trace of those K's raise to the power N for N spins and this is the form of the partition function.

Now, should understand that all the internal spins or all the internals alphas that is what are the internal alphas alpha 2, alpha 3, alpha 4 , alpha 5, and alpha N minus 1 there are all summed over they are all summed over. And we have the only alpha 1 and alpha 2 sorry alpha 1 and alpha N are not summed over or rather they remain and that is what give raise to this formula. So, it is a trace of that and because it is a 2 by 2 matrix the key has 2 Eigen values, and those Eigen values let them be lambda 1 and lambda 2. So, my Z since it is equal to trace of K to the power N.

So, this is same as lambda 1 to the power N plus lambda 2 to the power N. So, this is the general thing that you should keep in mind because I am taking a tress of N 2 by 2 matrices. So, finally, the this is going to be 2 by 2 matrix and those will have each one will have 2 Eigen values lambda 1 and lambda 2. So, the whole thing will come as lambda 1 to the power N plus lambda 2 to the power N which can be simply written as lambda 1 to the power N and a 1 plus lambda 2 by lambda 1 to the power N.

Now, you can see that if my lambda 1 is a bigger Eigen value this term goes to 0 for N going to be very large, because if this is the leading Eigen value. Lambda 1 is a leading Eigen value lambda 2 by lambda 2 goes to smaller with each power of N that is the spins and when N goes to very large and left with only. So, if my lambda 1 is greater than lambda 2 among the 2 Eigen values my Z becomes only 1 and I need to solve only one of the Eigen values of this matrix which is shown here, and that the bigger one and then I raise it to the power N. So, I can one can easily do that and one gets a lambda 1 2 which is equal to e to the power a cosine of hyperbolic x and the plus minus sin of hyperbolic square x plus exponential minus 4 a whole to the power half and that is it.

So, these are the lambda 1 and lambda 2 where the lambda 1 corresponds to the plus sign and the lambda 2 corresponds to the minus sign and we can a simply take lambda 1 which is equal to exponential a and this term plus this term that is what is going to be used in calculating the partition function. So, what we do after calculating the partition function?

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Free energy $F = -k_B T \ln Z = -N k_B T \ln \lambda_1$

Magnetisation $M = \lim_{B\to 0} \frac{-\partial F}{\partial B} = \lim_{B\to 0} \frac{N g \mu_B}{2} \left\{ \frac{\sinh\left(g\mu_B B / 2k_B T\right)}{\left[\sinh^2\left(\frac{g\mu_B B}{2k_B T}\right) + e^{-2J/k_B T}\right]^{1/2}} \right\}$

$M = 0$ as $B = 0$.

$$M = \left[\underbrace{\frac{N g^2 \mu_B^2}{4 k_B}}\ \frac{e^{J/k_B T}}{T}\right] B. \checkmark$$

$$\chi = \frac{\partial M}{\partial B} = \frac{e^{J/k_B T}}{T}.$$

$$\frac{1}{\chi} = T e^{-J/T} = T\left(1 - J/k_B T + \cdots\right)$$
$$= T - J/k_B. \Rightarrow \chi = \frac{C}{T - \theta}. \quad \theta = J/k_B.$$

We calculate the free energy and the free energy is calculated using F equal to minus K T log of the partition function which is simply equal to minus N K T log of Z 1, sorry log of lambda 1 not Z 1. It is because we are decided to take the larger Eigen value it is the log of lambda 1 and the magnetization can be derived from the free energy and which is equal to minus del F del B off course in the limit B going to 0 that is what the definition says.

So, will have limit B going to 0 here and this is equal to N g mu B by 2 you should do it and check it is a very simple algebra g mu BB by 2 K T and we have sin hyperbolic square g mu B by 2 K T and a plus exponential minus 2 j by K T whole to the power half. So, this is has to be written with. So, this whole to the power half and let us put that is the form for the magnetization. And interestingly it can be seen that M goes to 0 as B goes to 0 ok.

So, if there is no external field one dimensional such spin model does not show any magnetization this is called as the spontaneous magnetization. So, there is no spontaneous magnetization of a 1D ising model if we only put the external magnetic field then there will be magnetization there and for B going to 0. So, we have not still done the B going to 0 limit. So, which if we do that my magnetization looks like g square and a mu B square divided by 4 k and exponential J over K T by T and B.

So, that is the form for the magnetization as a function of a temperature and also. So, this is that chi which one can rather this is not the chi because you have a temperature which is I mean. So, this is basically the chi because the chi is nothing, but del M del B and this is equal to exponential J by K T divided by T and apart from this factor. So, this is equal to, so over chi that goes as T exponential minus J by T which can be written as T 1 minus J by K T and plus. So, I have a expanded the exponential and kept only the first order term and this is equal to T minus J over K B.

Now, you see that this is very similar to chi equal to C divided by T minus theta. So, our intension was to derive the curies law from a Hamiltonian from a calculation and this gives the derivation of the curies law where the 1 over the chi looks like T minus some characteristic temperature which is J over K Boltzmann's constant. And if you simplify this then chi looks like formally looks like as if some constant divided by T minus theta theta is some characteristic temperature and which is here in this case its J over K. So, this is the simple discussion of ferromagnetism in model Hamiltonian which arises out of spin spin interaction.

And once again point it out that this model or has no spontaneous magnetization which mean that as you switch off the external field the magnetization would vanish. At small a field the magnetization looks like this edges shown here and the susceptibility looks like it has a form which is like this we have propped this the initial factor here just kept only the T part, and associated the energy or rather the which depends on J over K is has the characteristic temperature which is theta and then we have derived how chi depends on temperature which is simply a 1 over T minus theta where theta is some characteristic temperature. So, this is called as Curie's law.

Now, let us see the same thing or rather ferromagnetism from Hubbard model.

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Ferromagnetism in Hubbard model : Stoner Criterion

$$\mathcal{H} = -t\sum_{\langle ij\rangle,\sigma} c_{i\sigma}^{\dagger} c_{j\sigma} + U\underbrace{\sum_i n_{i\uparrow} n_{i\downarrow}}$$

Mean field approximation — Hartree fock approximation.

$$U\sum n_{i\uparrow} n_{i\downarrow} \rightarrow \underbrace{U\sum_i \langle n_{i\uparrow}\rangle}\, n_{i\downarrow} + \underbrace{U\sum_i \langle n_{i\downarrow}\rangle}\, n_{i\uparrow}.$$

$\langle n_{i\uparrow}\rangle >> \langle n_{i\downarrow}\rangle$: one species outnumbers the others — ferromagnetism.

Magnetisation, $m_i = \langle n_{i\uparrow}\rangle - \langle n_{i\downarrow}\rangle = m$

$$m = \int_{\varepsilon_A}^{\varepsilon_F} dE\, N_{\uparrow}(E) - \int_{\varepsilon_B}^{\varepsilon_F} dE\, N_{\downarrow}(E)$$

And in particular we are going to derive what is called as Stoner criteria all right. So, what was the interaction term? So, will write down the Hamiltonian the Hubbard Hamiltonian once more which is minus t c i sigma draggers c j sigma there is a sum over i j which are nearest neighbor and there is a spin sigma and there is a U n i up and n i down there is a sum over i.

So, this is called as the onsite interaction between the electronics spins and since exclusion principle prohibits up spins to be present the same site, but there could be a up and down spins present and which would have interaction of this for. It is very clear that if U is switched off that is which is called as a atomic limit will have a metal given by the only a kinetic energy term. And if you use a much larger than T will have an insulating term which means that the electrons will not be able to hop from one side to another and the spectrum will be a large gap opening up due to U if U is much greater than T ok.

So, this as we said that this has no exact solution in 3D and we are even in 2D we are going to make some approximations and this approximation is called as the mean field approximation or in this particular case it is called as the Hartree Fock approximation I will tell you what it is. So, in order to solve it will have to use this approximation which is also called as the Hartree Fock approximation. We will see this more elaborately when we talk about greens function which we are going to start very soon; Hartree Fock approximation.

And suppose you are sitting in class and you have friends who are sitting right beside you and there are some friends who are sitting very far away from you you cannot communicate in any way while the class is going on, but however, the people who are sitting right in the vicinity that is in your just near neighborhood, they can actually speak to you or they can point out they can communicate in some form with you. This is a really a many body phenomenon because you have some interaction with the people who are right at the vicinity and you have a lesser interaction with people who are far off and even lesser interaction with the people who are farther off. However, if I make an approximation that you are sitting in the class and the rest of your friends who all the other friends other than you can be considered as as if you are in an external or an effective field because of all the students present all the friends present in the class.

That is I am not making any distinction between the friends who is right in neighborhood and the friend who is farther off, I am treating both of them to be equally or giving equally importance to them and saying that you are freely and average field due to all your friends in the class. So, if you want to go beyond that want to make a better approximation then you would say that somebody who is in the neighborhood has has more interaction with you than someone who is at the farther of positions and there could be fluctuations or there could be you know corrections to these average field picture or the mean field picture.

So, we will go with the mean field model where you are considered as if you are facing or rather feeling an effective field from all of your friends. So, we would particularly concentrate on one spin in the system and this that spin as if that spin is facing an effective field from all other spins in the system. So, to do that what we need to do is that we can split this term let us write it. So, I will write this and n i up n i down and U n i down n i up. So, you see here the the up the downs spins are facing an average field which is U into the average up spins density. So, as soon as I take an average or an expectation of this between the see the state of the system of the ground state of the system this becomes a number.

So, this is the field that each of those spins down spins will be freely and similarly without making any bias towards the up spins the up spins will be facing this field. So, this is the average field or the mean field that each of those spins will be facing or feeling. Now, understand that my n i up one kind of spins is much much greater than the other kind of spins because of the ferromagnetism. So, this is saying that one species out numbers the others and this is the essence of ferromagnetism. So, you know that in a ferromagnet we have predominantly or majority spins are pointing in the in one particular direction. So, in this model we do not have to have external magnetic field to be present and now let us simply define magnetization.

We will not define bulk magnetization rather we will define this the what is called as the sub lattice magnetization which is at a given site m i this is equal to n i up minus n i down ok. And, so the magnetization is same at all sites because of the that all the up spins are facing a potential which is U into the sum of all the down spin densities. So, how do we calculate this magnetization? This magnetization m is same as m which is for all sites. So, m equal to epsilon I mean epsilon F let us call it as epsilon delta to epsilon F and there is a dE into n up minus this is an epsilon ok. Let us write this as you can more clearly write this is epsilon A, I will tell you what epsilon A is, some energy and to epsilon F and dE and N down E.

So, we are simply computing the average up spin density which is the the density of states of the up spin and integrated over energies from epsilon A to epsilon F and I will just tell you what epsilon A is and minus the down spin density integrated over epsilon B 2 epsilon F.

So, this model here as given rise to a band picture which looks like this. So, there is an up spin band. So, this is the down spin band and there is an up spin band like this and my Fermi energy is here.

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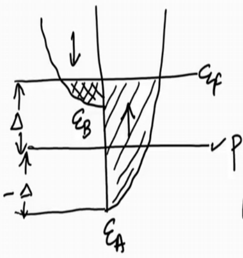

$$m = \int_{-\Delta}^{\varepsilon_f} dE\, \underbrace{N(E+\Delta)}_{N_\uparrow(E)} - \int_{\Delta}^{\varepsilon_f} dE\, \underbrace{N(E-\Delta)}_{N_\downarrow(E)}$$

$$\left.\begin{aligned} N(E+\Delta) &= \frac{dN}{dE}\Delta \\ N(E-\Delta) &= \frac{dN}{dE}(-\Delta). \end{aligned}\right\} \quad \Delta = \frac{Um}{2}$$

$$m = 2\Delta \int_{\Delta(=0)}^{\varepsilon_f} \frac{dN}{dE}\, dE \qquad \varepsilon_f >> \Delta, \quad \Delta \simeq 0.$$

$$= 2\Delta N(\varepsilon_f) = mU N(\varepsilon_f)$$

$$N(\varepsilon_f) = \frac{1}{U} \quad : \text{Stoner Criterion.}$$

So, this is my Fermi energy and my; so this is the down spin this is the up spin and this is my Fermi energy is and my epsilon A is right here or epsilon up if you want. So, that is the bottom of the band for up spin and this is the bottom of the band for down spin. Now, you understand that if I have to know what is my total up spins or average number of up spins then I have to take the density of states of up spins and integrate those from here this point which is known as epsilon A to epsilon F that is I will have to integrate over all these place where the up spin is there and in order to do it for the down spin I will integrate it from epsilon B to epsilon F which is all these down spin.

So, I will try to. So, the magnetization will be the shaded area on the right minus shaded area on the left. And so what is my; so if I take a middle point here. So, this is my delta let us call it as delta I will define what delta is and this is my minus delta. So, this is also delta, but this is this is for delta equal to 0, there is a reference line this line is reference line and you have delta. So, from that reference line all the way are up to the Fermi energy the gap is known as delta and from there again the reference line to all the way to the epsilon A which is a bottom of the up spin is called as delta.

So, if you have this nomenclature then my m can be written as minus delta to epsilon F which is epsilon A to epsilon F. So, this is equal to minus delta and N of so dE and we have a N E plus delta. So, the density of the states is at energy E plus delta minus a delta to epsilon F. So, that is this point marked by say let us call is as P. So, the from P to epsilon F, so that is my the second one which is dE N E minus delta. So, this is my N up E as written earlier and this is my N down E you know that N up is much greater than N down which is by the magnetization is large.

And so now, since E is greater than delta E is much greater than delta because delta is the gap. So, we can do a tailor expansion of the density of the states and each of those N E plus delta can be written as dN dE delta and N E minus delta can be written as dN dE minus delta. And then I can write down skipping one step which you should fill up which is equal to 2 delta where I have not defined delta. So, delta is equal to delta equal to by 2. So, M equal to 2 delta and I have delta to E f d N dE and a dE and now, this delta is much smaller than epsilon F. So, we are talking about epsilon F which are of the order of few electron volt delta is very small.

So, epsilon F B is much greater than delta we can approximate delta to be equal to 0. So, we can or rather change this the lower limit of the integration to 0 and then this m becomes equal to 2 delta and the density of the states computed at the Fermi which is nothing, but equal to m U N epsilon F. So, this will tell you since magnetization is not equal to 0, we get a condition for ferromagnetism to occur which is equal to 1 over U. So, this is called as the Stoner criteria. So, let us see what Stoner criterion is.

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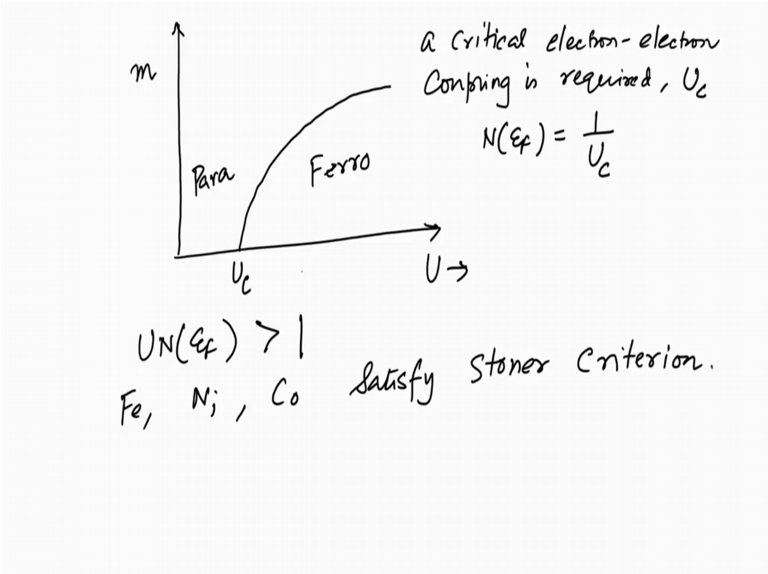

And so this tells the ferromagnetism will occur in a system in which if I have m verses U which is the electron electron interaction, see the difference between this problem and the problem that we have done earlier that is writing down Hamiltonian in a spin only Hamiltonian and solving it via computing the partition function is that that model did not have any electron, if it is a only a spin model. Now, we are having an electronic model. So, there are electrons which can hop from one place to another which is a more realistic model because the spins have to be carried by somebody, spins are not they do not exits on their own.

So, the electrons carry their spin. So, at spin only or rather the electronic model is more reliable and more realistic than a spin only model. So, here a Hubbard model is a electronic model. So, this electronic model is we are trying to find how ferromagnetism can be derived from the from this electronic model and in process we have found out a criterion called as the Stoner criterion. And which tells you that the density of states for the electrons at the Fermi level has to be inverse of this electron electron interaction which is the very surprising condition that we have found and. So, so ferromagnetism, for ferromagnetism to occur we need a critical a critical electronic coupling, a critical electron electron coupling is required which is you see some critical coupling, so that the N e Ff is equal to 1 over U C.

So, it is inbuilt in the problem what is the density of the states at the Fermi level. So, the electrons contribution to the density of the states of the Fermi level is decided by the system itself. However, that has to match with the critical electron electron coupling for ferromagnetism to occur. So, that gives you a more realistic picture about ferromagnetism and what happens is that, this is like this. So, you have a U equal to U C here. So, below that there is a paramagnet and above that this is a ferromagnet and we have this as the; so this is the Stoner criterion. So, in order for ferromagnet to happen U your N epsilon F has to be greater than 1.

And this is a very stringent condition that is the electron electron interaction multiplied by the density of the states of the Fermi level will have to exceed unity that is the value has to be more than 1 in order to have ferromagnetism. And is quite a stranger criterion and iron, nickel and cobalt satisfy well this criteria. So, this is called Stoner criterion.

So, to summarize the discussion that we had just now, is that we wanted to understand magnetism in particular ferromagnetism we have written down a spin only model and have solved it in one dimension and with ising kind of interactions. And we found that it gives a ferromagnetism where it is same as the Curie’s law and it gives you characteristic temperature for the ferromagnetic transition which is given by theta or T C which we have called as T C at times we have called theta at times.

However, if we revisit the same problem from an electronic model then we get a Stoner criterion and Stoner criterion tells that the electron electron interaction multiplied by the density of the states at the Fermi level has to exceed 1 for ferromagnetism to occur.