Advanced Condensed Matter Physics
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Lecture – 24
Spin Hall Effect, 2D topological insulator

So, in this part of the ongoing discussion we shall talk about spin hall effect and also introduced just introduced this Hamiltonian called as a Kene Mele Hamiltonian which is related to the spin hall effect. However, the level of rigor that will be used in this discussion will be far lesser than what we have done so far, we will simply make a few mentions and tell you the physics behind it, however, will not derive anything rigorously as we have done earlier.

So, here we are going to talk about spin hall effect. We have talked about hall effect which in principle means a charged hall effect where the charge migrate charges they migrate to different edges of the opposite edges of the sample.

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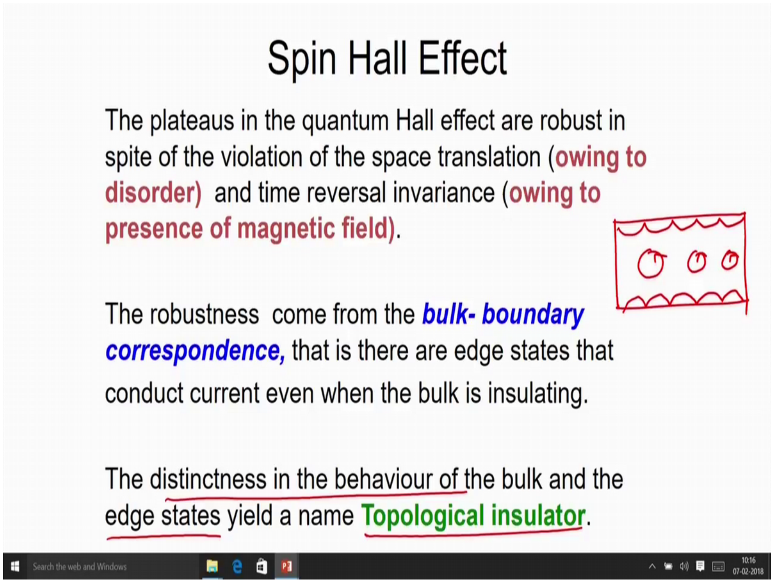

So, that there is a transverse field that develops. Here we are going to talk about the same effect with spins that is spins will segregate up and down spins will segregate on two edges of the sample and that will give raise to a spin hall effect. So, we have seen that the plateaus in the quantum hall effect are robust, very robust to the accuracy of 10 to the power minus 9 for their flatness because each of these plateaus take integer values of h over e square and these do not go away even in presence of disorder. And in any case the time reversal invariance is gone owing to the presence of the magnetic field a which is an essential component of the hall effect that we have seen.

Now, this robustness of the plateaus or the plateaus being a exactly occurring at integer multiples of h over e square or in the context of fractional quantum hall effect they occur at rational fractions of multiplied by h over e square. So, those come from the bulk boundary correspondence. What I mean is that, so the system will only conduct through the boundaries or the edges and the bulk remains completely insulating. So, physical picture is that this is a two dimensional electron gas and you will have the bulk where the electrons execute a cyclotron orbit while it is skips along the edges and where the entire you know the conductance takes place.

So, this distinctness in the behavior of the bulk and the edge that yield a name which is called as a topological insulator. So, it is a very important and a recent trend in the study of condensed matter systems and in which there are lot of classes of topological insulators that are being predicted and some of them have been discovered and they show unusual conductance properties which are very distinct from ordinary insulators or which are called as the band insulators.

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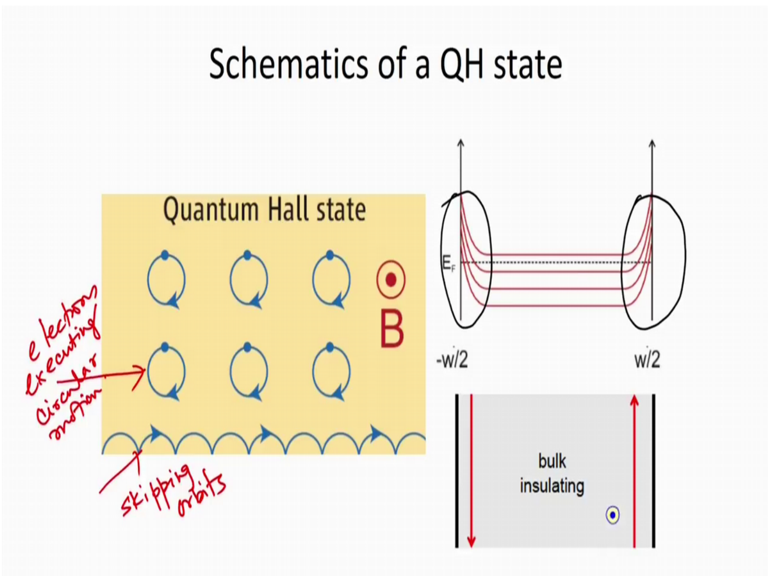

So, the schematics of a quantum hall state is just what we said that in the bulk we have electrons executing cyclotron motion. There is a magnetic field which is perpendicular to this which is coming out of the page and there are these skipping orbits. So, these are known as the skipping orbits, and these are electrons executing a circular motion.

And the band diagram for all these classes of topological insulators so to say, they actually cross at the boundaries of the sample. So, this is where the crossing of the Fermi levels occur and it is only at the edges the bulk is separated away from the Fermi level and hence they do not conduct. So, another schematic is that that the bulk remains insulating and there are edges where the charges propagate in two different directions and this gives rise to the conductance that we have seen in the quantum hall systems which are two dimensional electron gas with disorder.

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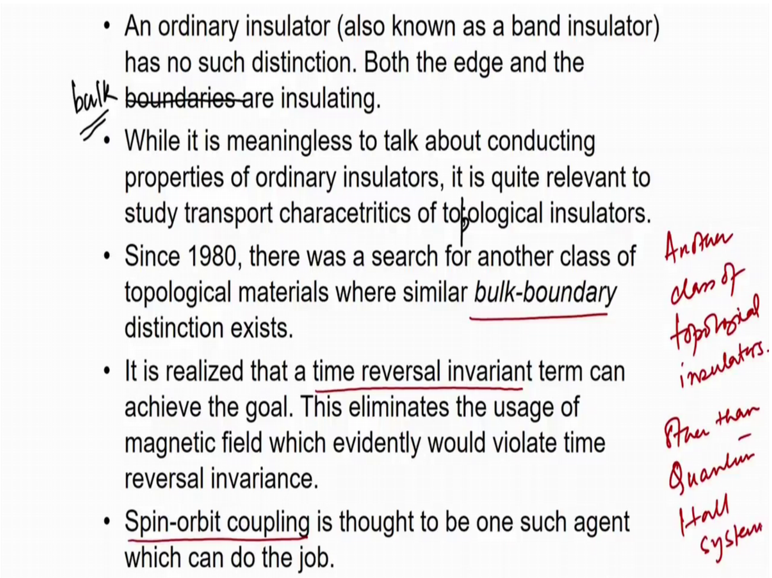

So, an ordinary insulator as I said that it is also known as band insulator has no such distinction. So, in which case they are both the edge and the boundaries are insulating, both the edge and the this should be bulk not boundaries edge means boundaries. So, both the edge and the bulk please make this correction are insulating a while it is of course, meaningless to talk about conductance properties of ordinary insulators, but it is quite relevant to study the transport characteristics of a topological insulators.

So, since 1980 that is the discovery of the quantum hall state there is a search for another class of topological materials where similar bulk boundary distinction exists. So, this bulk boundary distinction is an important component of the topological insulator which has to be present in order to be called as a topological insulator. So, it is realized that a time reversal invariant term can achieve the goal. So, you have to get a time reversal invariant Hamiltonian and this immediately eliminate the usage of the magnetic field which evidently would have violated the time reversal invariance.

So, in summary what we need is another class of topological insulators. Once again the word is coming; the word topology is coming because there is a distinction between the bulk of the sample and the edges of the sample. So, the topology is important and this topological insulators have only conducting states at the boundaries. So, we cannot use a magnetic field because magnetic field would immediately destroy the time reversal invariance. Now, what is understood is that a spin orbit coupling which is strong enough to give a visibility to this effect could be a thought of an agent which can do the job, that is get another class of topological insulators other than of course, which we have seen the the quantum hall systems.

So, this is what was what was thought about it at that point of time which is since 1980 onwards or rather 84-85 onwards when people have fully understood the quantum hall state and understood that the plateaus are actually protected by the edge states. So, as long as the edge states would continue to exist there will be transmission or there will be conductance facilitated by the boundaries even while the bulk remains completely insulated.

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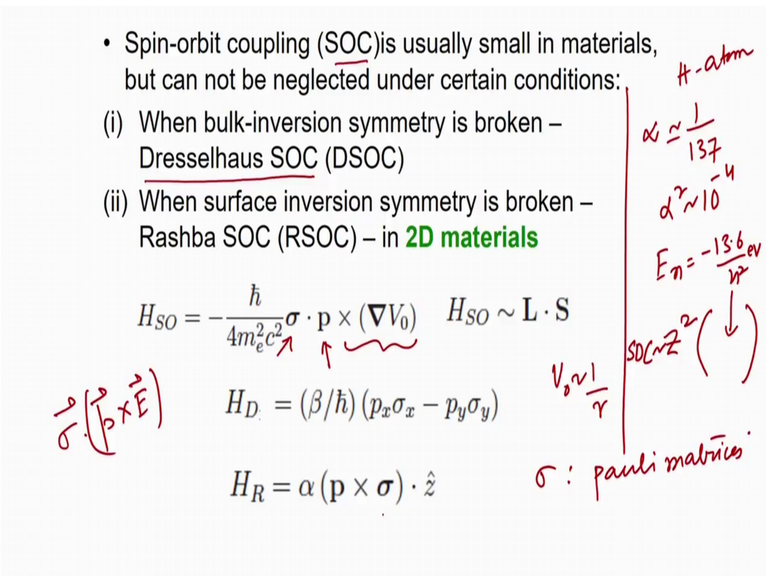

So, in order to understand spin orbit coupling this you might have been exposed to in the context of hydrogen atom or real hydrogen atom there could be L S coupling or which is known as the spin orbit coupling L is the orbital angular momentum and S is the spin angular momentum. However, in hydrogen atom you found it that it is alpha square times smaller than the unperturbed energy where alpha is called as a fine structure constant which has a value one over 137. So 137, alpha square is of the order of 10 to the power minus 4.

So, it is the unperturbed energy in hydrogen atom is of the order of this is minus 13.6 by n square electron volt. So, if we take a n equal to 1 then its minus 13.6 electron volt in its of the order of an electron volt whereas, this is 10 to the power of minus 4 times smaller. But when you also should realize that the spin orbit coupling term came with a Z square and Z for hydrogen atom equal to 1 which is called as the atomic number the number of a protons or a electrons that are present in the system protons present in the nucleus or electrons present in the system which is equal to 1.

However the spin orbit coupling can become important for Z to be large such as you know there are Z which is the uranium has Z equal to 92 etcetera. So, for them spin orbit coupling can still play an important role. So, this is the SO coupling let us call it as SOC this is what we have called this as a SOC is Z square times this number which is here. However, in real materials which are not atoms the spin orbit coupling can still be very significant and so much so that it cannot be neglected.

So, there are a number of reasons that spin orbit coupling arises in real materials, one of them being when there is a bulk inversion symmetry being broken. So, it is a three dimensional structure which is inhomogeneous and its somewhere at the middle of the bulk or the somewhere inside the bulk the environment that the electrons face in going from below that point or below that line or below that surface and above that surface. They are quite different that is called as a bulk inversion symmetry being broken and in which case add restless spin orbit coupling is an important quantity to consider.

Likewise, when the surface inversion symmetry is broken there is another spin orbit coupling goes by the name Rashba spin coupling its by a two people called by (Refer Time: 12:20) its popularly known as Rashba spin orbit coupling possibly a Japanese scientist. And it is called our soc and sulfus inversion symmetry is mainly broken in 2D materials. So, the environment at the surface is very different. So, above the environment is very different from below I mean above the surface the environment above the surface is very different from the environment below the surface and hence this is important in 2D materials.

So, as a spin orbit coupling if you open literature quantum mechanics book we cannot actually derive it from the first principle in non-relativistic mechanics it should come from Dirac equations. However, we are skipping those derivations and this is the spin which is interacting with the momentum and this is crossed with a delta V 0, where V 0 is of the order of one over r which is the ionic potential.

So, there is a ionic potential has a gradient at the surface and because of this gradient this is like an electric field. So, this is like a sigma dot P cross E and that is pretty similar to what we have in the Rashba spin orbit coupling. So, there are two of them as I told that is Dresselhaus which looks diagonal which is like p x sigma x, where sigmas are the Pauli matrices and p x sigma x minus p y sigma y this is the strength of that beta is the strength and h cross is the Planck's constant which we all know about and the Rashba spin orbit coupling is given by p cross sigma dot z cap, where p is the momentum and sigma are the two by two Pauli matrices.

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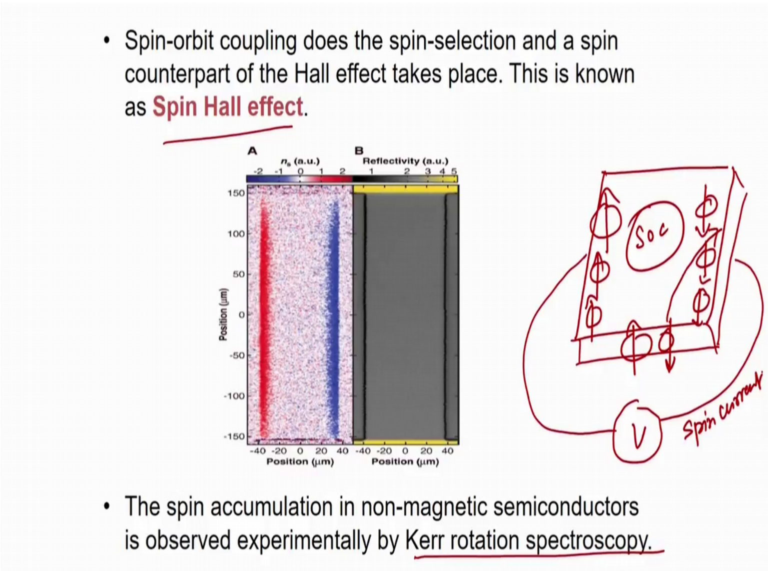

So, this spin orbit coupling does the spin selection which otherwise would have been done by the magnetic field in terms of. So, the magnetic field when they act on the system of charge carriers there is a Lorentz force which segregates the charges from into the two different you know ages of the sample, same kind of things would take place in system in presence of a spin orbit coupling. So, this is like a two dimensional system where these are up and down and in presence of the spin orbit coupling that down would go here and the up would go here. And so there is an accumulation of up spins and there is an accumulation of down spins here. So, if you have a voltage measuring system which measure the spin current or the spin voltage then they could detect that there is a significant accumulation of up spins on one edge and down spins on another engine this is called as a spin hall effect.

So, it is a counter part of the charge hall effect and here we show an experimental figure picture rather where you have a band red band on the left corner, and a blue band on the right corner and these are also shown in black and white where there is a accumulation of spills being shown in a across the edges of the sample. The only problem is about the detection of this voltage the voltage is a usually very small even less than a micro volt and usually of the order of nano volts. And this spin is this accumulation is detected in non magnetic semiconductors by optical methods which are a known as the Kerr rotation spectroscopy.

We will not go into details of that a just know that there are systems which in presence of the spin orbit coupling SOC is being present here the spins will actually get lined up on the opposite spins will get lined up in two different transverse edges of the sample and there will be a spin current that is going to be generated. So, this is that spin current detection, and a multi meter or an ammeter which measures a spin current or a voltage that is being measured to spin hall voltage.

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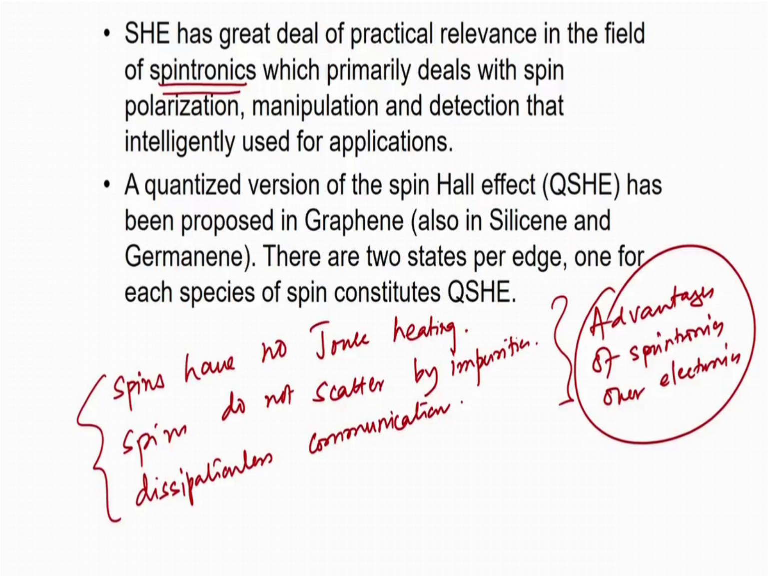

This spin hall effect has a great deal of practical relevance in the a field of spintronics and it is even now it is growing at a rapid pace to this field of study which deals with spin polarization, manipulation and detection of spins that are intelligently used for applications. So, what we used to do with charge will do it with the spins, the spins will carry the information will process the information and will do the same electronics that the electron used to do and now, it will be done via the spin. And a large number of a advantages such as spins have no joule heating, spins do not scatter by impurities, and so there is a dissipation less communication. So, these are the advantage of spintronics over electronics.

So, a meanwhile a quantized version of the spin hall effect or rather quantum version of the spin hall effect which is called as a Q quantum spin hall effect has been proposed in grapheme. We have seen graphene when we calculated the band structure of graphene and we saw that graphene has a band structure which is linear close to the Fermi level or there are some points which are called as the Dirac points in the vicinity of which we have a linear dispersion and there are only two distinct points in the Brillouin zone. So, the dispersion the two bands of graphene the type binding graphene that touch at 6 points.

However, there are only two distinct points which are called as k and k prime and at these points are important and these are called as the valleys, so there are these k and k prime points which we have seen when we are talking about a applications of second quantization and we were talking about the study of tight binding Hamiltonian in different systems. So, there it was proposed that this quantum spin hall effect could be observable in graphene.

So, in principle it is possible; however, the spin orbit coupling in graphene unfortunately is way too small it is only of the order of a few Kelvin for spin hall effect or quantum spin hall effect to be observed. Nevertheless it remained as a good theoretical proposal and if that would have been possible suppose one can a manipulate which in a principle it is possible to manipulate the spin orbit coupling the strength of the spin orbit coupling in materials. So, in graphene it should be possible and there are a certain add atoms heavy add atoms which are often used in order to induce a strong Rashba in orbit coupling in graphene then it would have been I mean it should be in principle possible and probably the future of such a discovery.

So, this has given rise to a theoretical studies and have given rise to a model Hamiltonian which predicts in principle the quantum spin hall effect in graphene. We will have a glimpse of that in just a while.

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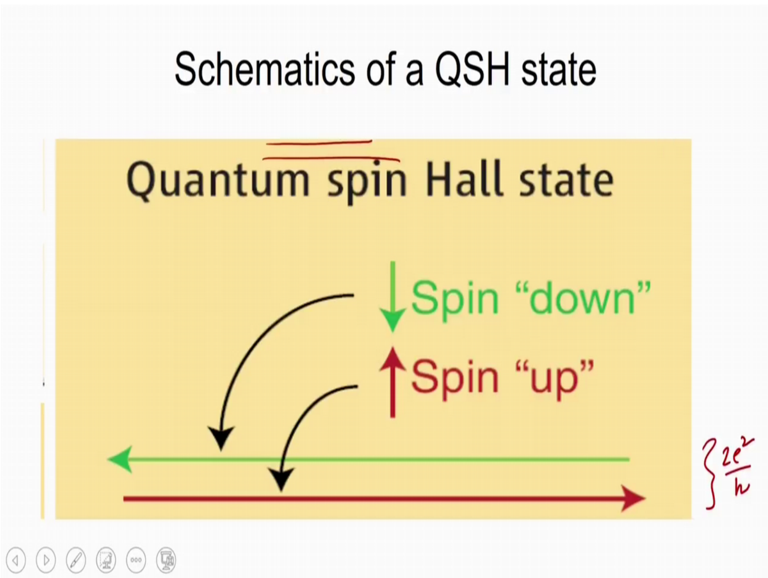

So, just to distinguish between a spin hall and a quantum hall, now quantum hall had only one edge. So, this edge I mean one state per edge and here we are talking about n equal to 1 or mu equal to 1 rather. Here there are two of them and one for each spin and the spins are propagating in opposite directions. So, if the green denotes a down spin then its opera propagating from right to left and if red denotes an up spin then it is propagating from left to right and same happens in the other edge as well ok.

So, these are called as helical edge states as compared to the Chiral edge states what we have seen in the quantum hall state. So, this defines or schematically it defines a quantum spin hall state where the there will be quantized a conductance which is in; so if the resistance is H over E square. So, there are the conductance will be e square over H. So, this will give rise to 2 E square over H conductance even at 0 biasing.

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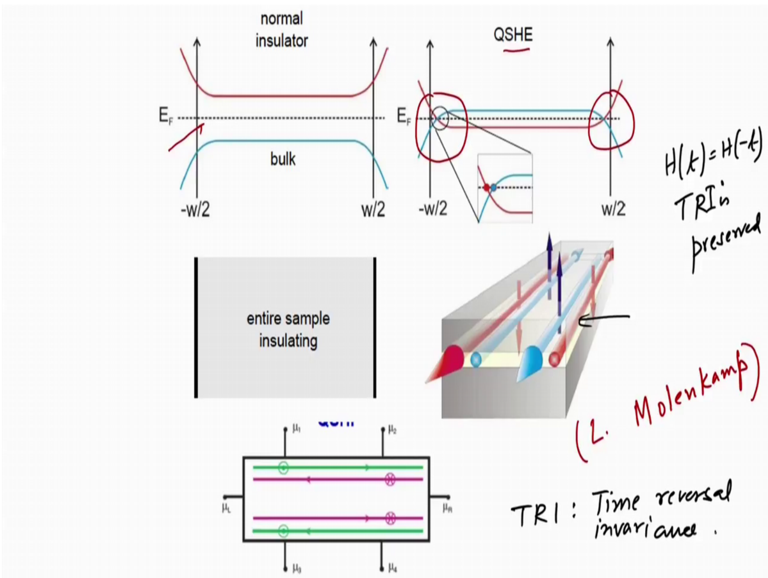

So, there are some more cartoons which will help you to understand better. See the normal insulator we have a the blue one is the valence band and the red one is the conduction band and the Fermi level is the dotted line that is shown here. So, the there is a distinct difference between the red and the blue with regard to the dotted line which means they never touch each other and hence there are no conduction and this is called as an ordinary insulator or a band insulator you can call it a normal insulator.

Whereas in the quantum spin hall insulator you see they are crossing only at the edges. So, there are crossing here and the crossing is being shown nicely here and so there are of course, one line is shown, but there are two lines correspond to each spin which would be shown here. So, again schematically the entire sample would be insulating while the edges would be conducting as we have seen, and a similar picture is being shown here and so these are the red one is. Now, representing a down and the blue one is the up and so on, so there are these two edges which are each having this two a states one for up and one for down and propagating in different direction. This has been taken from some presentation of a Lorentz Molenkamp, who is one of the first person to talk about this quantum spin hall effect and the same thing is shown here in a 6 terminal geometry say for graphene.

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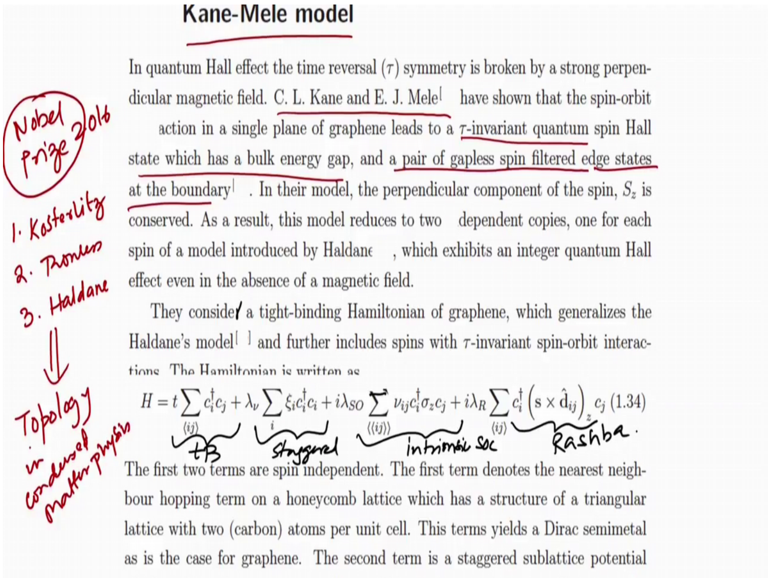

So, as we were talking about the theoretical prediction spade and this goes by the name a Kene and Mele. In fact, they have built upon this model on what Holden proposed at least about 20 years back in 1988 and they have built upon the model and they have added another degree of freedom which will just see.

And by the way 3 people got Nobel Prize in 2016 and all for these topological implications in condensed matter physics one of them is called Kosterlitz the other is called Thouless and the third is Haldane so, for this is in 2016 and for study of topology in condensed matter physics.

So, let us see what the Kene mele model is, we will simply introduce the model and will not derive it; however, it can be derived; however, that will be a long exercise now, which we do not want to get in. So, in the quantum Hall Effect the time reversal symmetry as we have said is broken by a strong perpendicular magnetic field. Now, Kene and Mele they have shown that the spin orbit action in a single plane of graphene it leads to a time reversal invariant quantum spin hall effect which has as we know now, that a bulk energy gap and a pair of spin filtered edge states been filtered means spin resolved edge states at the boundary. And in the model the perpendicular component of the spin S Z is conserved that simplifies discussion because now, the model reduces to two copies and one for each spin of which was introduced by Haldane as I was telling that it was actually adding another degree of freedom which is this spin degree of freedom and which in, so this model of Haldane which exhibits integer quantum hall effect in the absence of a magnetic field.

So, all this work had been motivated by a seminal paper of Haldane in 88 in which it said that you can also have quantum hall effect without the presence of a external magnetic field, which otherwise we have been so familiar to. So, they consider a tight binding model of graphene which generalizes Haldane model and further includes spins with time reversal invariant spin orbit perturbation. The Hamiltonian is written as, so there is a tight binding term there is an onsite term, staggered onsite term, there is an intrinsic spin orbit term, and there is a Rashba term.

So, it consists of, so they wrote down this Hamiltonian and it can be shown rigorously that this Hamiltonian does not break time reversal invariance. So, it is there they are time reverse I mean all the terms put together they preserve the time reversal invariance and. So, these states that you see here they cannot be destroyed by any perturbation which is a time reversal invariance. You have to break the time reversal invariance in order to the break this. So, what I mean by time reversal invariance is that as soon as H of t equal to H of minus t time reversal invariance is preserved, time reversal invariance is let us call it TRI is preserved and if this is not true then of course, the time reversal invariance is gone. So, TRI means time reversal in variance.

So, the first two terms are spin independent. First term we have already been exposed to when we did the tight binding model nearest neighbor hopping them on a honeycomb lattice which is a structure of a triangular lattice with two carbon atoms per unit cell this you already know. The term yields a Dirac semi metal as is the case for graphene and then the second term is the sub lattice potential and which has the asymmetry between the A and B sub lattices with xi i equal to plus minus 1. So, what it means is that for the i to be on the A sublattice xi i equal to plus 1 and for i to be on the B sublattice it will be a minus 1. So, this is your A sublattice here and your B sublattice here. So, these will take this thing.

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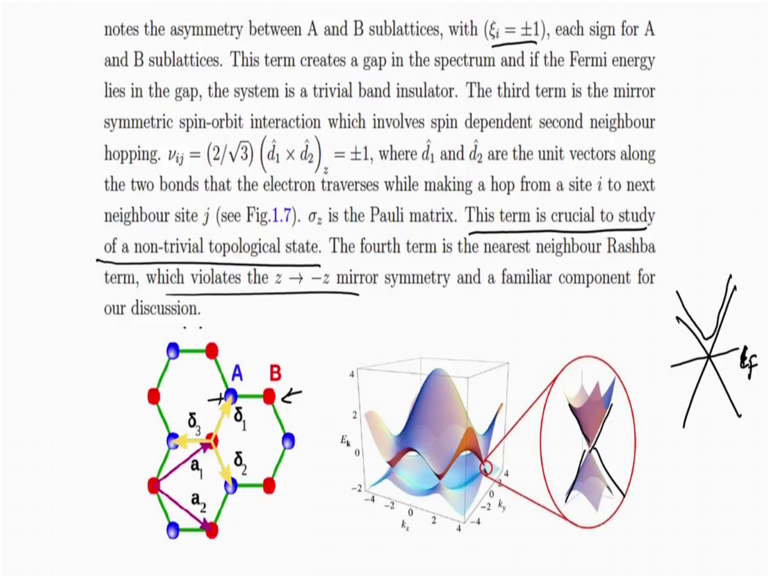

So, this term creates a gap in the spectrum. So, what was told is that this is the Dirac spectrum. So, close to one of the points, one of the Dirac point you see that this is linear. So, this is like this at the. So, this is your Fermi level and this is linear; however, this term splits that. So, if there is a gap that is created, so there are you know gaps that is that are created.

So, the third term is the mirror symmetric spin orbit interaction which involves spin dependence second neighbor hopping with the complicated forms for nu i j not to complicate I will show what it is which also could be plus or minus 1 depending upon this d 1 and d 2 vectors which are unit vectors going from site i to site j, but site j is not the nearest neighbor is the next nearest neighbor term sigma Z is of course, the Pauli matrix. And this term is the most crucial term for getting a non trivial topological state which we are always aiming at.

The fourth term is the nearest neighbor Rashba term which violates the Z going to minus Z mirror symmetry and the familiar component for our discussion.

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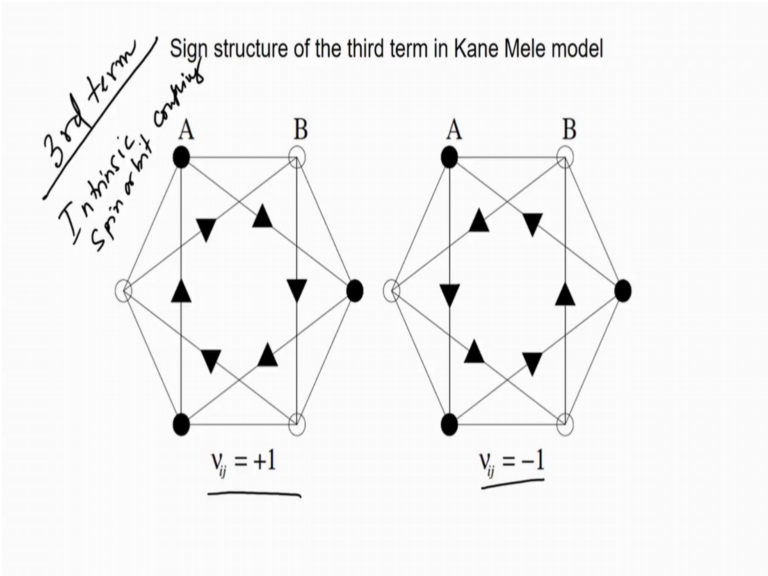

So, the third term is important. So, this is that intrinsic spin orbit coupling and. So, there are A and B sublattices being shown is shown with A filled circle and B is shown with an empty circle. So, as long as you have a clockwise hopping from the neighbor to from from one side to its next year a neighbor.

So, from this to this, it is a clockwise hopping. So, all these things correspond to in nu i j equal to plus t and when you take the anti clockwise hopping. So, then it is equal to minus 1. So, this is an important term that is this condition is very important for having the time reversal in variance and the gap is surely to open up in the bulk but they will be still 8 states and this is what is shown in this that now, you see that there is gap that is created in the bulk because of all these other terms only tight binding would have given you this kind of dispersion with no gap.

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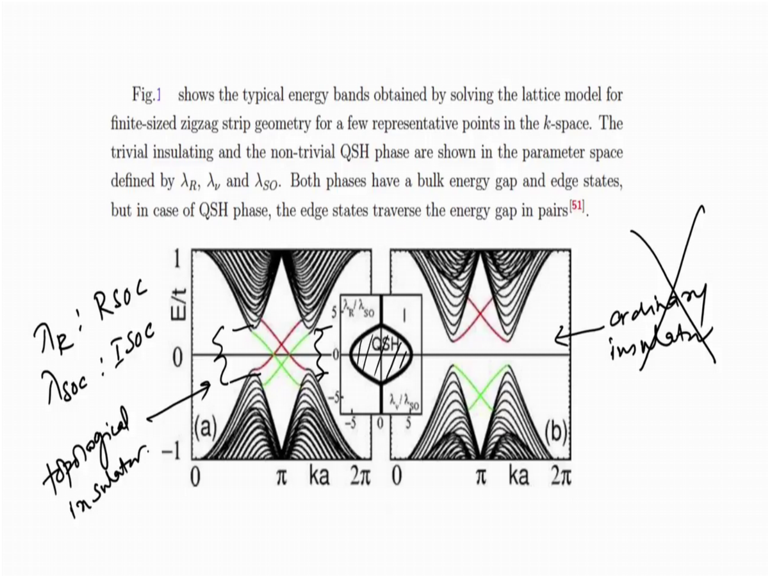

Now, there are these are spin result spectrum. So, there are these red lines and green lines correspond to up and down and now, the two spectra has separated spectra have separated because of the presence of spin orbit coupling. So, this gap is there so without these greens lines joining the conduction band and the valence band we would have had a normal ordinary insulator or a band insulator.

However; because of this we have a topological insulator and exactly for some lambda R and lambda SO. So, lambda R is for the Rashba spin orbit coupling and lambda SO is for the intrinsic spin orbit coupling. And only we see the quantum spin hall effect the QSH only in this regime and if you go beyond this that is lambda R by lambda SO, if it exceeds these value that is within this you know the kind of structure, that a circular structure that you are seeing here you will have a band insulator or an ordinary insulator. So, this is a topological insulator and that is a an ordinary insulator.

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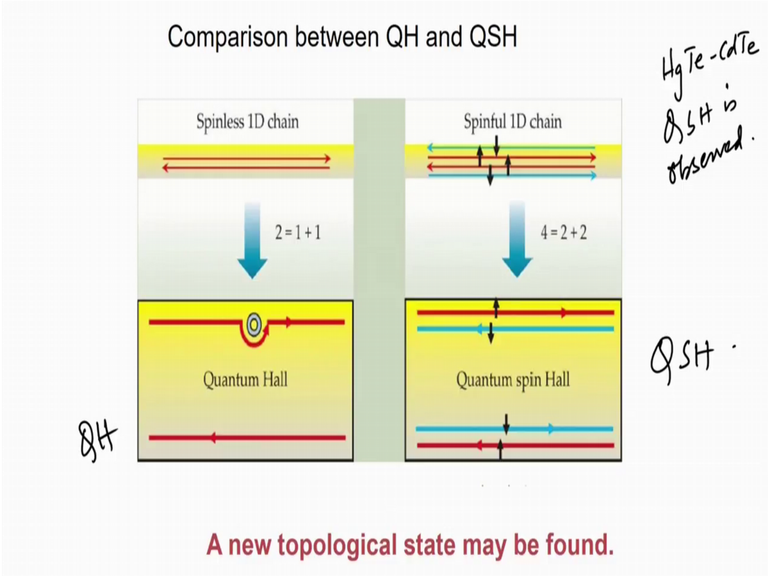

And this is the comparison between the quantum hall state and the quantum spin hall states. So, in a quantum hall state this is shown particularly you know with an impurity. So, there is no phase space for back scattering. So, an impurity will not do anything to a quantum hall state and that is why its robust to presence of impurities. And the same thing I just you know the dimension just goes up to from rather 1 to 2 or 2 to 4 that we have these spin full conditions coming by where there will be helical edge states that are going to propagate on two edges of the sample. So, there are only one state at the edge there are while there are two states in quantum spin hall. So, quantum hall has 1 per H and quantum spin hall at 2 per H.

So, we will stop this discussion this is very technical and it will require a lot of time and effort to completely understand. Even if one had not seen experimentally a quantum spin hall phase in graphene, there are other material strain semiconductors where a spin hall the quantum spin hall phase has been observed which depends on the band inversion properties and we do not sort of discuss it here. But that is in other thing that has been done by 3 people every Huges and Junk where experimentally that is a theoretical prediction of seeing quantum spin hall effect and Molenkamp as said earlier. Molenkamp has actually seen that experimentally in strain semiconductors which are given by the Hg Te Cd Te which are mercury telluride, and cadmium telluride, sandwich structures is a quantum well structures where a quantum spin hall effect is observed experimentally.

And also there is a theoretical prediction which has finally, you know been realized in actual experiments.

Thank you.