Advanced Condensed Matter Physics
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Lecture - 23
Quantum Hall Effect

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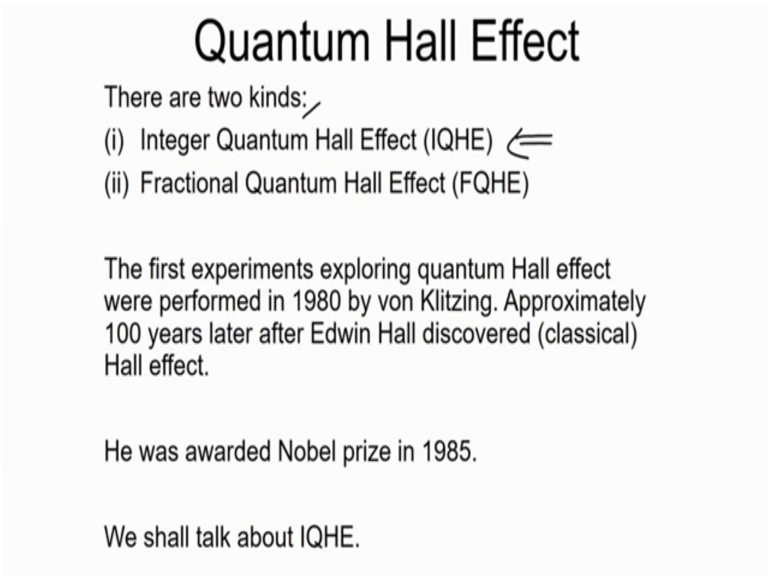

So, in this set of special lectures we shall do quantum hall effect to begin with and then we will see that how it gets connected to more exotic phenomena. So, we are familiar with hall effect, hall effect is an effect which was discovered in 1879 by Edwin hall in which the current is made to flow in a thin laminar sample in say the x direction and there is a voltage that is generated in the y direction because of the migration of the charge carriers abided by the Lorentz force equations and at equilibrium one measures transverse voltage which is known as the hall voltage.

So, this is the quantum version of that and in which will see that the resistivity versus magnetic field plot which we have seen even in the classical hall effect, here there are some strange features in this data and we will discuss that how it comes about.

So, there are in principle two kinds of hall effects which are integer quantum hall effect which is what we are going to do mostly and there are also fractional quantum hall effect I will just mentioned what they are, but will not get into the discussion because they are more difficult and they need interactions to be taken into account. So, the first experiments exploring quantum hall effect were performed by von Klitzing in 1918 and then approximately which is 100 years later after Edwin hall discovered the classical hall effect as we have just mentioned.

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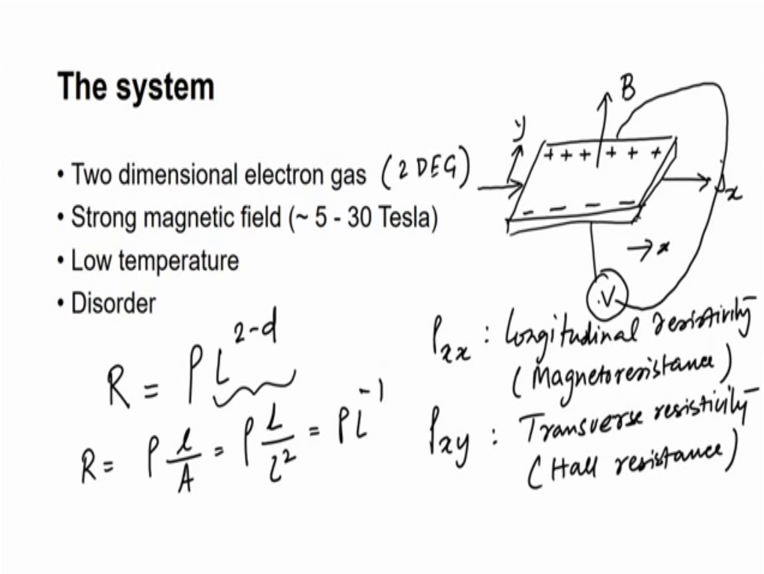

And Klitzing was awarded Nobel Prize in 1985 for this discovery and you will see that why this experimental discovery is so important so, as to you know warrant a Nobel Prize.

So, we shall talk about only IQHE as I just mentioned. So, what is the system? The system is a thin 2 dimensional material or quasi 2 dimensional material in which the electrons are restricted to move in a 2 dimension and the extent in the third dimension is fairly limited. So, the motion is confined in the in the plane let us call it as x and this as y direction. So, this is a 2 dimensional electron gas and it is written as 2 DEG in short and there is a strong magnetic field that is been applied which is perpendicular to it.

So, it is coming out of the plane and this is the magnetic field is of the order of 5 to 30 tesla, the experiments are conducted at very low temperature so, that the quantum effects become important. Disorder is an integral part of this of this effect or this experimental discovery and it plays an important role in a seeing the phenomena better. In fact, in the absence of disorder it can be shown that it reduces to classical Hall Effect.

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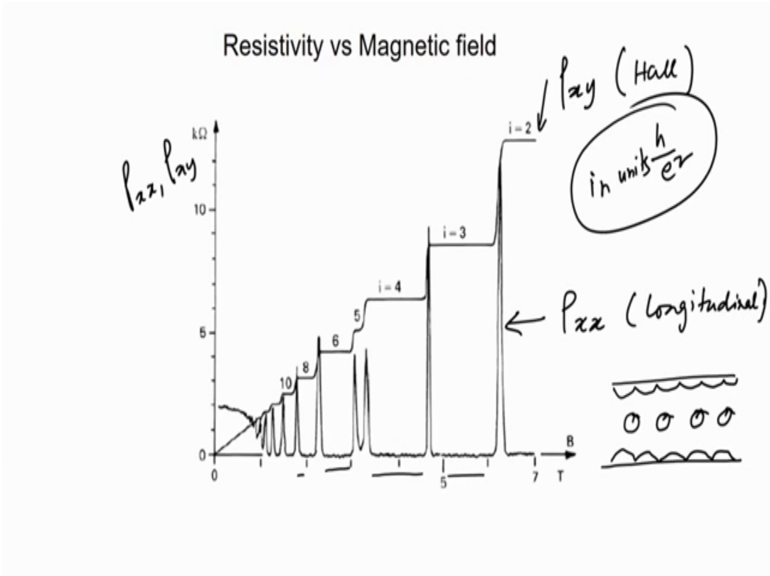

So, this is the plot of row x x and row x y so, let me point out which are the so, what is happening is that. So, this in this system a current is made to flow in this direction let us call it as j x and there is a voltage that is developed here in the transverse direction and because of the segregation of charges here and this voltage is called as hall voltage. So, there is also a resistivity in the direction of the flowing current which we will call it as rho x x which is generally called as the longitudinal resistivity or it is also called as the magneto resistance and there is the other one which is the called as the transverse resistivity and also called as the hall resistance or resistivity they are two different things because we know that the resistivity is a property of the sample while resistance has also the geometrical properties built into it.

So, in principle R which is the resistance which is what the experimentalist measure that has a relationship with the resistivity L into 2 minus d and this you can understand it well by noting down the definition of a resistance that you have read in school for wire of diameter d and length l. So, that goes as R goes as a row into l over A and. So, this has a dimension which is L and divided by L square. So, this is equal to row L to the power minus 1 and that is the story with d equal to 3 and.

Now, this difference between the resistance and the resistivity that go away a for d equal to 2. So, one can safely talk about either resistance or resistivity at d equal to 2, because the geometrical factor appearing here will not contribute anything to the formula extra.

So, R becomes numerically equal to d and that is why we can talk about either of either resistivity or resistance and so, these are the plots for. So, these step like plateaus are the plot for the hall resistance which will denote by a row x y and these spikes that you see are for the longitudinal resistance or the magnitude resistance. So, these are our plots.

in the classical hall effect, the whole resistance or their hall resistivity as a function of b was a straight line, here it is far from being a straight line, there are plateaus at some integer values will discuss what that is and at i equal to 2 and i equal to 3 and i equal to 4 and 5 and 6 and 8 and 10 and so, on and so, these are in units of e square over or rather h over e square. So, the resistivity the whole resistivity for this plateau is a to h over e square this is 3 h over e square and 4 h over e square and so, on.

Exactly that and these integers are independent of the material, independent of the amount of disorder present and is very very universal and so, much so, that these integers are correct up to ninth or 10 to the power minus 9 or 9 decimal places. So, the effects or the experimental observations at that robust and absolutely independent of the material used and hence this was really a discovery that has taken the scientific community by a storm and this lot of work since 1980 that have started and still going on in various fields.

So, this is another important thing about this resistivity that the longitudinal resistance is 0 most of the time, but it shows a peak as the hall resistivity makes a transition from one plateau to another. So, it is only when it makes a transition from one plateau to another, it shows a peak and you see the peaks are all there accompanied by the jump from one plateau to another. So, you see the rho x x which is the longitudinal resistivity of the magnitude resistance, it is 0 which means that resistivity 0 means it is a perfect metal or a perfect conductor and suddenly the thing the resistivity shoots up giving rise that there is a lot of resistance there. So, the system is undergoing a transition a series of transition from being insulated to metal to insulator to metal and so, on.

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Both Hall resistivity, ρ_{xy} and longitudinal resistivity, ρ_{xx} exhibit interesting behaviour. While ρ_{xy} has plateaus at integer multiples of h/e^2, that is,

$$\rho_{xy} = (h/e^2)(1/\gamma) \qquad (1)$$

where γ is an integer upto an accuracy of 10^{-9}.

h/e^2 is taken as the quantum of resistivity (also called as the Klitzing constant) having a value 25.8 K-Ohm. For γ=1, the above formula denotes one quantum of resistivity.

So, these are some of the interesting features of the quantum hall effect which are which are very important and for us to understand that what is going on. So, just to summarize that we have said this the hall resistivity and the longitudinal resistivity, the hall resistivity is given by row x y and longitudinal resistivity by row x x exhibit very interesting behavior as we have just seen, a while rho x y has plateaus at integer multiples of h by e square that is a rho x y equal to h by e square and 1 over gamma where gamma is an integer up to an accuracy 10 to the power minus 9. And because of this robustness of these plateaus and remember that we have said that disorder is an integral part of the system and since disorder does not do anything which means the fact that translational invariance is broken has does not have any effect on this.

So, there must be some other mechanism that is protecting the flatness of the plateaus h over e square is thus taken as a quantum of resistivity it is also called as a Klitzing constant by the name of the discoverer it has a value approximately 25.8 kilo ohm and you can see that for gamma equal to one here this denotes exactly one quantum of resistivity. So, it becomes 1 over 1 which is just simply h over e square.

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The center of each plateau occurs as the magnetic field assume a value

$$B = hn/\gamma e = (n/\gamma)\Phi_0 \qquad (2)$$

n : electron density.

$\Phi_0 = h/e$ is the flux quantum.

These are precisely the values of the magnetic field at which the γ number of Landau levels are filled and the Hall resistivity takes values as in Eq (1).

The quantization persists over a range of the external magnetic field.

Another striking feature is that the plateaus are robust to disorder (sounds odd!!)

There are more features which we unfold 1 by 1, the center of each plateau occurs at the magnetic as a magnetic field assumes a value which is h n over gamma e where h over gamma rather n over it is n over gamma and h by e, where h by e is called as a flux quantum. So, as it takes a fractional multiple of this phi 0 or rather this sum coefficient multiplied by phi 0 then that is where the plateau occurs. So, it could happen that the value of the magnetic field and so, n is the density of the material hence n is the electronic density.

So, what it is saying is that the value of the magnetic field and the electron density have to conspired to give rise to a plateau. So, this is the flux quantum phi 0, which is equal to h over e. So, these are precise the values of magnetic field at which the gamma number of landau levels are filled at this moment it is an unknown word, but we will see what landau level spin and the hole resistivity takes a value as we have given in equation 1 which is this.

The quantization also persists over a range of the external magnetic field and another striking feature which is we have already told is that there robust to disorder and it may sound odd in the context of condensed matter physics because a large number of phenomena gets adversely affected by presence of disorder while this is not.

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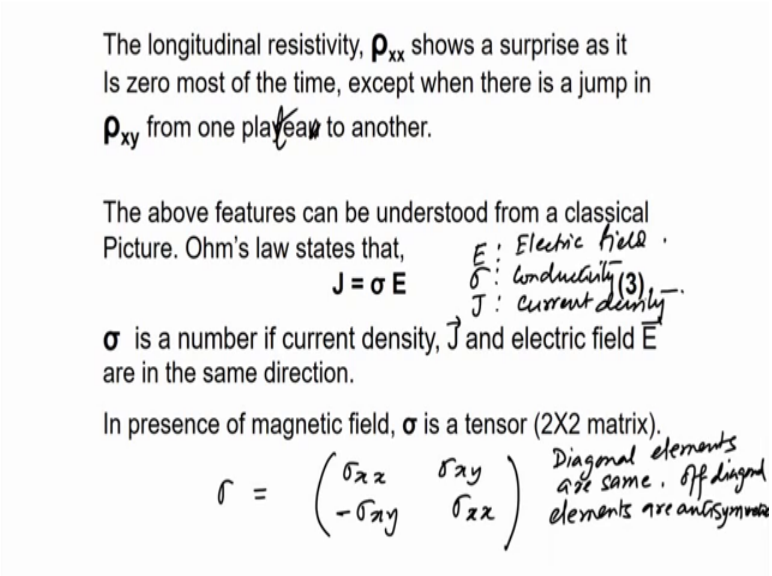

So, the longitudinal resistivity which is rho x x it shows a surprise as it is 0 most of the time as we have told. So, it is 0 here, 0 here, 0 here and so, on except when there is a jump in rho x y from one plateau to another. So, the above features can be understood from a classical picture. So, let us see what the classical picture unfolds in this particular case, it has got nothing to do with quantum mechanics, but these will help us in understanding.

So, ohms law states that J equal to sigma E, this is another way of writing V equal to IR, everyone knows sigma is a is a conductivity. So, sigma is conductivity, J is current density and E is the electric field. So, sigma is a number if the current density J and the electric field E are pointing in the same direction. So, J x equal to sigma E x and so, on and in which case sigma is a number, but in presence of a magnetic field sigma is a tensor, it is a 2 by 2 matrix having a form. So, sigma is equal to sigma x x, sigma x y minus sigma x y, sigma x x. So, it tells you that diagonal elements are same of diagonal elements are anti symmetric.

So, these are the features of this matrix and so, if we write the conductivity tensor we can also write the resistivity tensor.

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The resistivity Tensor is written as,

$$\rho = \sigma^{-1} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ -\rho_{xy} & \rho_{yy} \end{pmatrix}$$

Components have the relation,

$$\sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2} \; ; \; \sigma_{xy} = \frac{-\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2}.$$

1) If $\rho_{xy} = 0$, we get $\sigma_{xx} = \frac{1}{\rho_{xx}}$ (familiar),
$\sigma_{xy} = 0$.

2) If $\rho_{xy} \neq 0$, σ_{xx}, σ_{xy} both exist.

So, the resistivity tensor is written as rho which is equal to sigma inverse this preserves the similar properties with an of diagonal elements being anti symmetric. So, the it is easy to see that the components have the relation that sigma x x equal to rho x x divided by rho x x square plus rho x y square and the off diagonal elements have minus rho x y divided by rho x x square plus rho x y square.

Now, if rho x y equal to 0 then we get sigma x x equal to 0 so, sigma x x equal to sorry the sigma x x is not equal to 0, but sigma x x is equal to 1 over rho x x. So, this is familiar and sigma x y is equal to 0. So, this is 1, 2 is that if rho x y is not equal to 0 then sigma x x and sigma x y both exist. So, this is clear to easy to see that if rho x y is not equal to 0 then both of them will exist and provided of course, your rho x x is also not equal to 0.

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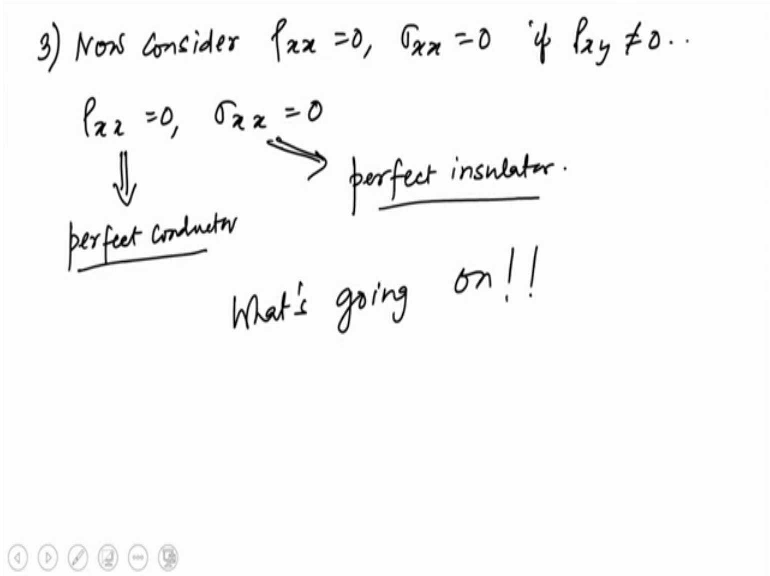

So, now consider rho x x is equal to so, sigma x x equal to 0. So, that is clear from here if rho x x is 0 then sigma xx is equal to 0 if of course, rho x y is not equal to 0. So, what do we want to call the system with rho x x equal to 0? So, we have rho x x equal to 0 sigma x x equal to 0, on one hand this tells that it is a perfect conductor and this says that it is a perfect insulator. So, one is implying the other, but one inference is that it is a perfect conductor the other inference is that it is a perfect insulator.

So, the question is that what is going on? And in order to understand this let us just look at the Drnde's model.

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Drnde Model

$$\sigma_{xx} = \frac{\sigma_0}{1+\omega_c^2\tau^2} \qquad \tau = \frac{l}{v_F} = \frac{\text{mean free path}}{\text{velocity}}.$$

$$\sigma_0 = \frac{ne^2\tau}{m} \qquad n: \text{density} \quad m: \text{mass}.$$

Thus $\sigma_{xx} = 0$ imply that $\tau \to \infty \Rightarrow$ absence of scattering.

Current is flowing perpendicular to the field.

$$\vec{E} = \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} 0 & \rho_{xy} \\ -\rho_{xy} & 0 \end{pmatrix}\begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} \rho_{xy} j_y \\ -\rho_{xy} j_x \end{pmatrix}$$

$\vec{E}\cdot\vec{J} = 0$. Remember $\vec{E}\cdot\vec{J}$ is the work done in accelerating the charges.

So, in the Drnde model this is in the preliminary solid state physics course were are in the free electron theory of studied Drnde a model, which gives you the assumption that the electrons are non interacting and they only interact when they collide with each other which happens after every time scale given by the relaxation time. So, a between these relax 2 scatterings the electron propagates as free particle. So, that is the assumption and the conductivity is given by a 1 plus omega c square tau square where tau is the relaxation time given by l by v F and omega c some cyclotron frequency where l is the mean free path and this is the velocity and sigma 0 is taken as n e square tau over m with n as the density m being the mass.

So, thus sigma x x equal to 0 imply that the relaxation time becomes extremely large so, that tells that no scattering. So, in this particular case the current is flowing perpendicular to the field and it has a form which is we are talking about 2 dimensional transport. So, this is like 0 rho x y because rho x x is equal to 0 minus row x y plus 0 and j x j y and this is equal to rho x y j y and rho x y j x with a minus sign and so, on. So, it is easy to see that that the e x is the so, g y is in the direction of e x and j x in the direction of opposite to the direction of e y and that tells that e and j are perpendicular vectors.

So, if e and j that is the electric field and the current density of perpendicular then e dot j equal to 0, but remember that e dot j has the meaning of work done in accelerating the charges. And so, the fact that a dot j equal to 0 it means that there is a steady current.

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$\vec{E}\cdot\vec{J}=0$ implies that there is a steady flowing in the sample without requiring any work, and hence no dissipation.

$\sigma_{xx}=0$ tells us that no current is flowing in the longitudinal direction (like an perfect insulator), while $\rho_{xx}=0$ tells that there is no dissipation of energy. (like a perfect conductor)

$\sigma_{xx}=0=\rho_{xx}$ is accompanied by.

$$\sigma_{xy} = -\frac{1}{\rho_{xy}} = \nu e^2/h$$

ν: an integer

$\sim 10^{-9}$ accuracy

Implies that there is a steady current flowing in the sample which does not require any work to be done and hence no dissipation.

So, sigma x x equal to 0 tells us that no current is flowing in the longitudinal direction. So, this is like an insulator, while row x x equal to 0 tells that there is no dissipation of energy. So, this is the like a perfect conductor when we add the word perfect he also like a perfect conductor. So, so, sigma x x equal to 0 equal to rho x x is accompanied by by that sigma x y equal to minus of rho x y equal to mu e square over h and mu being and integer. So, this quantization of mu being an integer to the accuracy of 10 to the power minus 9 is independent of the details of the system and hence the effect is universal.

So, this goes back to this plot so, this plot tells exactly that the row x y will become a quantized number in units of h over e square and it will make a transition from one plateau to another and during which the longitudinal or the magnet or resistance longitudinal resistivity of the magnet or resistance will show a sharp peak and this at least qualitatively explains that these are universal features of a 2 dimensional electron gas and persists even in presence of disorder. An important thing is that since disorder is not protecting the plateaus, that is it is getting or rather it is contributing to the robustness of the plateaus as we have seen and a magnetic field already has broken time reversal symmetry. So, there has to be a another quantity or another you know sort of preserving factor that preserves this plateau and which is what has been attributed to the topological properties of the system that is protecting these plateaus.

So, what happens is that in presence of the magnetic field ah. So, their electrons go round in such circular orbits and the radius of this or rather radii of this orbits are proportional to the magnetic field. So, this does not lead to any transport, but; however, the transport occurs along the edges, where the electrons actually skip and these 8 states are robust no matter whatever a kind of perturbations you put. So, these 8 states will be robust and will carry current and will give rise to this resistivity curves a while the bulk of the material that is interior of the 2 dimensional electron gas the electrons will not contribute to any transport of charges. Let us try to understand these features better and let us talk about a simple language and non-interacting.

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Landau levels: Electrons in a magnetic field ($\vec{B}$).

$\vec{p} \to \vec{p} - q\vec{A}$ $\qquad q = -e$ for electron

$\to p + e\vec{A}$ $\qquad \vec{A}$: vector potential.

$\vec{\nabla} \times \vec{A} = \vec{B}$

Schrödinger equation that we have to solve:

$$\frac{1}{2m}\left(\vec{p} + e\vec{A}\right)^2 \psi = E\psi$$

$$\vec{A} \to \vec{A} - \vec{\nabla}\chi(\vec{r}) \qquad \psi(\vec{r}) \to \underbrace{e^{-(ie/\hbar)\phi(\vec{r})}}_{\text{phase}}\psi(\vec{r})$$

$|\psi|^2$ remains invariant.

So, we want to talk about landau levels so, electrons in a magnetic fields. So, these are non interacting electrons in a magnetic field. So, remember what happens in for a charged particle e to be put in a magnetic field the canonical momentum p gets replaced by p minus q A over well c will drop those are old units we can work in new units and now q is equal to minus e for electron. So, we can write this as p plus e A, where A is the vector potential and it has a relationship with the magnetic field given by curl A equal to B, that is the magnetic field we are talking about ok.

So, the Schrodinger equation that we have to solve is 1 over 2m p plus e A square psy equal to e psy. So, that is the time independent Schrodinger equation where the p has been replaced by p plus e A, now Schrodinger equation is gauge invariant. So, even if we change A to A minus A minus gradient of a scalar function call it chi an arbitrary scalar function which is a function of r only, then size simply picks up a sign which is given by and because if psy picks up only a sign only a sorry face not sign it is a phase this quantity which is a physical quantity remains in variant.

So, just to revise it once again that we write down the Schrodinger equation, the A is a gauge invariant and because of the gauge invariance psy picks up a phase of this form, because I only picks up a phase and not anything else, the psy mod square will not have this phase because it is psy mod square is psy star psy and you will have a simple you know these will be same and hence we can do a gauge transformation on this and still get the same result.

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Choose a particular gauge :

$A_x = -By$, $A_y = A_z = 0$. (Landau gauge)

$\vec{\nabla} \times \vec{A} = \vec{B} \rightarrow \vec{B}$ is in the z-direction.

Sch. equation becomes,

$$\left[\frac{1}{2m}\left(p_x - eBy\right)^2 + \frac{p_y^2}{2m} + \frac{p_z^2}{2m}\right]\psi(\vec{r}) = E\psi(\vec{r})$$

$\mathcal{E} = \frac{p_z^2}{2m} \rightarrow$ Motion in z-direction is that of a free particle.

$f(z) = e^{ikz}$.

So, now choose a particular gauge for calculation and that gauge is let us call it A x equal to minus B y. So, that component x component of A is equal to minus B y, while the y component and z component to be equal to 0.

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$$\Big[\underbrace{\frac{1}{2m}\left(p_x - eBy\right)^2 + \frac{p_y^2}{2m}}_{H(x,y).}\Big]F(x,y) = \mathcal{E}F(x,y) \quad (2)$$

$$[p_x, H(x,y)] = 0 \Rightarrow p_x \text{ is a constant of Motion}$$

$$p_x = 2\pi\hbar \frac{n_x}{L_x} \qquad n_x = 1, 2, 3 \ldots\ldots$$

$$\text{Eq. (2) as, } \left[\frac{p_y^2}{2m} + \frac{1}{2}m\left(\frac{eB}{m}\right)^2(y-y_0)^2\right]f(y) = \mathcal{E}f(y)$$

$$y_0 = \frac{p_x}{eB}.$$

This is called as the landau gauge that is the definition, this is fine because curl A is B and so, B is in the z direction as is a requirement of the hall effect. So, we have taken in principle we have taken a 3 dimensional Schrodinger equation, but; however, very soon you will see that that it boils down to actually a 2 dimensional motion and the B is actually appointing the external magnetic field is pointing in the z direction and if that is the case then in this gauge Schrodinger equation I am writing it in short. So, it becomes equal to 1 over 2m p x minus e B y whole square plus p y square over 2 m plus a p z square over 2 m and a psy of r which is equal to E psy of r. So, as any quantum mechanics problem we have to find out the Eigen values and the Eigenvectors for this particular equation.

So, you can see that the z direction the particle motion is like a free particle. So, we have the energy in the z direction is like a free particle and there is no other term which involves z. So, the motion is that of a free particle. So, now, we can now neglect the z direction because it is like a free particle and see what happens in the x and y direction.

And to see that let us write down p x minus e B y square plus p y square over 2 m and now the wave function let us call it as F x y is no longer a function of z because in the z direction the wave function is like let us call it as say F of z is some with some normalization it is like exponential I k z.

So, we are only concerned about the 2 dimensional motion and this is what we were talking about earlier that even if you have taken 3 dimensional case in presence of a magnetic field for a non interacting electrons it boils down to a problem of 2 dimensional motion. And this is equal to some epsilon and F x y so, this is like the Hamiltonian in 2 dimension H x y ah. It is easy to see that p x is a constant of motion y p x is a constant of motion because nowhere in this Hamiltonian we have x the variable position variable x and p x is equal to this is equal to 0. So, that tells that p x is a constant of motion, remember p y is not a constant of motion because of the gauge chosen where we have a y remaining so, p y does not commute with y and hence it is not a constant of motion.

So, there is no term because there is no term linear in x this is what happens and that also as I told because of the gauge, but it does not matter even if you can taken term A to B B x then the x should have been p x would not have been a constant of motion while p y would have been so, x and y simply gets interchanged. So, if p x is a constant of motion then p x is actually quantized by this form where n x is of this quantized in this form which is which takes values which are like 1, 2, 3 and so, on.

So, one can write this equation let us call this the original equation to be 1 and this equation to be 2. So, one can write equation 2 as p y square over 2 m plus half m e B over m square y minus y 0 square f of y which is epsilon f of y. So, this is the equation, where y 0 is equal to p x over e B. So, that is the equation that is the Schrodinger equation that we have to solve and it is very clear that this equation is the equation for the same equation same Schrodinger equation, when written for harmonic oscillator in the y direction and oscillating about this point is 0 given by p x by e B and then of course, we know that we have solved the problem.

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Simple Harmonic oscillator with frequency $\left(\frac{eB}{m}\right) = \omega$.

The eigenvalues are given by,

$$\mathcal{E} = \left(n+\frac{1}{2}\right)\frac{eB}{m}\hbar \rightarrow \text{Landau Levels}$$

Eigenfunctions

$$f(x,y) = \frac{1}{\sqrt{L_x}} e^{ik_x x} N_n e^{-\frac{eB(y-y_0)^2}{\hbar}} H_n\left(\frac{eB(y-y_0)}{\hbar}\right).$$

p_x is a Constant of motion $\Rightarrow$ all n_x that satisfy

$$k_x = \frac{2\pi n_x}{L_x} \quad \left[y_0 = \left(\frac{h}{BL_x}\right) n_x \right]$$

$\Rightarrow$ Hugely degenerate Landau levels.

In the sense that we know that it is of the harmonic oscillator form. So, it is a simple harmonic oscillator with frequency e B over m. So, the Eigen values are simple epsilon equal to n plus half e B by m and h cross.

So, that is our so, it is n plus half h cross omega and omega is given by this. So, this is the eigenvalue for this problem and these are called as the landau levels. Not only we know about the energy Eigen values we also know about the Eigen functions so, the 2 dimensional Eigen functions. So, it is a free particle we told that the k x h cross commutes with p x or k x. So, it is a free particle in that direction in the k x direction and some normalization, this is the Gaussian which is e B by y minus y 0 square divided by h cross and now the hermite polynomial you must be knowing the properties of the hermite polynomial when n equal to even the polynomial is even when n equal to odd the polynomial is odd and it is given by y minus y 0 divided by h cross and that is the complete solution of the problem.

There are more things to understand here is that so, these are landau levels and these are the landau level energies and now you see that a p x is a constant of motion. So, that tells that all n x that satisfy all n x that satisfy this quantization condition 2 phi n x over L x, where y 0 equal to h cross h sorry h over B L x h n x. All n x values are valid solutions and since all n x values are valid solutions the problem is huge legion degenerate and gives rise to the enormous degeneracy of the landau levels because each n x value that satisfy this relation L x is the dimension of the sample in the x direction will be a valid solution and hence it is a enormous this degeneracy will be there.

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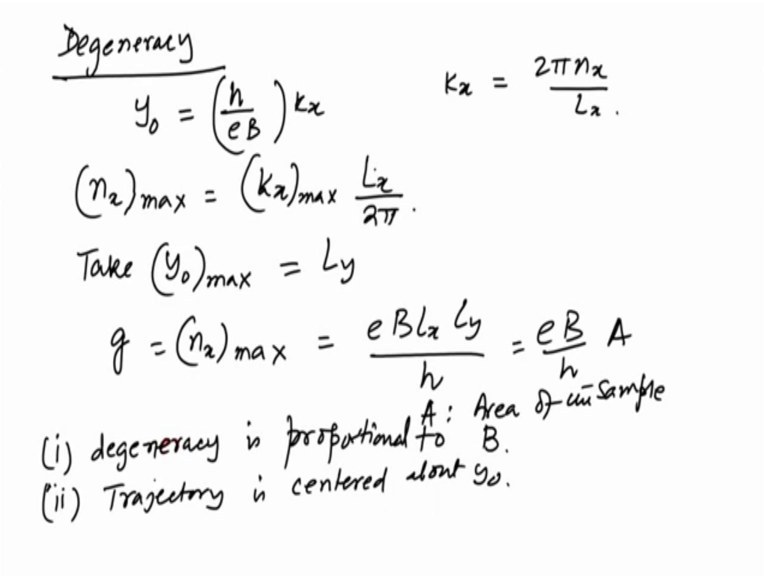

Now, what is the extent of degeneracy? So, let us try to understand. So, let us take this energy value so, your y is y naught. So, this is the point about which the particle is undergoing a simple harmonic motion. So, this is given by h over e B into k x and of course, as I told that k x is written as 2 phi n x by L x. So, my n x max is given by k x max and divided by I mean k x max into l x by 2 phi.

Now, if we put that these k x max here and take y 0 which is the point in the y-direction about which the simple harmonic motion takes place to be the. So, take y 0 max equal to L y which is the size of the sample in the y dimension a y direction so, it cannot y 0 cannot be greater than L y. So, the particle is undergoing a simple harmonic motion about y 0 and y 0 the maximum of y 0 is the sample dimension in the y direction.

So, the degeneracy is then given by g is equal to n x max which is equal to e B L x L y divided by h which is equal to e B by h into A, where A is the area of the sample. So, let us make 2 comments here one is that the degeneracy. So, if we put all these if y 0 max to be equal to L y and put k x max in terms of n x max one, one gets this degeneracy. So, degeneracy is proportional to B that is a magnetic field. So, as you increase magnetic field the degeneracy will increase and the second is that the trajectory is centered about y 0.

So, this is the story about that electrons non interacting electrons are put in a magnetic field we have taken a simple gauge called as a landau gauge and have solved the problem. The problem says that the motion in the z direction is that of a free particle so, one can neglect that. So, now, the Hamiltonian becomes 2 dimensional and so, the motion is constrained in a plane in the x direction because of the choice of the gauge the particle moves as free particle with the momentum vector quantized in terms of 2 pi over L, some integer multiple of 2 pi over L and in the y direction it executes a simple harmonic motion.

So, we can find we can we get this effective Schrodinger equation is a 1 dimensional a simple harmonic oscillator problem we know what the energy Eigen values are, we know what the energy Eigen vectors are. Because, any a quantum number in the x direction is provides a valid solution for the problem, the problem is enormously degenerate and this degeneracy is found and this degeneracy is proportional to B and it is also proportional to the area of the sample and the trajectory is a simple harmonic which is centered at y 0.

Now, with this so, having discussed this landau levels and degeneracy let us talk about how this problem, now you should be finding a similarity between this problem and the quantum hall effect. There are also the electrons were put in a strong magnetic field in the z direction here also we are talking about a single electron and finding that how the motion is a in a presence of a magnetic field.

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Hall voltage is defined by,

$$V_H = \frac{BI_x}{ne}.$$

Hall coefficient $R_H = V_H/I_x = \frac{B}{ne}$ (classical result).

$g = \frac{eB}{h} A. \rightarrow$ degeneracy

$$\frac{\text{degeneracy}}{\text{Area}} = g/A = \frac{eB}{h} = N \text{ (say)}$$

$$R_H = \frac{eB}{ne^2} = \frac{Nh}{ne^2}$$

n: current density

If $n = \nu N$ ν is an integer $= 1, 2, 3 \ldots$

$$\nu = n/N$$

So, now the hall voltage is defined by V H which is equal to B I x over n e and hence the hall coefficient R H which is equal to V H by I x this is equal to B by any. So, this is a classical result which is well known.

So, the degeneracy of the landau levels is given by e B by H into A so, the degeneracy so, this is the degeneracy so, degeneracy per unit area which is g over A, that is equal to e B by H let us call that is equal to n. So, R H becomes equal to e B over any square I multiply by the both the numerator and denominator here by e and this becomes equal to N h by n e square. So, N is the current density, if n becomes equal to some mu into N, where mu is an integer equal to 1, 2, 3 etcetera then R H, let us do it in a new page.

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$$R_H = \frac{Nh}{\nu N e^2} = \frac{h}{\nu e^2}$$

$$\nu = \frac{n}{eB/h}$$

Thus a quantized Hall resistance is always expected if a carrier density, n and the magnetic field B are adjusted in such a way that the filling factor (ν) of the Landau Levels becomes an integer.

Satisfying, $\nu = \left(\frac{n}{eB/h}\right)$

Then R H becomes equal to N h divided by mu N e square it becomes equal to h by mu e square. So, e becomes equal to n by e B over h. So, this was the quantization that was discussed at the beginning of the lecture. So, even starting with a classical picture if you think that this the assumption is that if the degeneracy per unit area is a quantity which is like you can take it as a number as n and then the hall resistivity it is quantized in terms of h over e square or rather it is a where N is the electron densities N h over n e square and if n is taken as the gamma into this capital N which is taken as the degeneracy per unit area then it is quantized actually as h by e square and L over mu.

So, thus a quantized hall resistance is always expected this is important, if a carrier density n n and the magnetic field B B are adjusted in such a way that that the filling factor. So, this called this mu is called as a filling factor, mu of the landau levels satisfies mu equal to n by e B h becomes an integer. So, it is satisfying this becomes an integer.

So, just to rephrase this that even from a classical picture we got this quantization of the plateaus or the hall resistivity or the resistance, here what we have taken is that this is called as a filling factor where the total electron density is taken as a filling factor multiplied by the degeneracy per unit area. So, your gamma is simply equal to n by N. So, that is the electron density divided by this N, which is the degeneracy or the degeneracy per unit area if these two things and the b that can be adjusted in a manner such that this quantity becomes an integer then there will be quantization of the hall resistance.

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Under this condition, σ_{xx} (longitudinal) becomes zero since the electrons are moving like free particles exclusively perpendicular to the electric field $\rightarrow$ no scattering in the direction of the electric field is possible.

So whenever ρ_{xy} acquires a plateau, ρ_{xx} (or ρ_{yy}) will drop to zero.

So, what happens is that under this condition the magneto conductivity let us call it as sigma xx so, it is a longitudinal becomes 0. Since, the electrons are moving like free particles this is what we have got that the electrons in the x direction that it moves like free particles exclusively perpendicular to the to the electric field. Thus there is no scattering in the direction in the direction of the electric field is possible. So, whenever rho x y acquires a plateau rho x x or rho y y in this particular direction will drop.

So, this discussion on a quantum hall effect at least in a as a you know new person or introductory sort of discussion of this quantum hall effect to you should be sufficient to make you understand the importance of this particular experiment and the universality of this experimental observation which is independent of the nature of the material or details of the material and is independent of the disorder is a very important thing. And the hall effect where these all these labs were very sophisticated measurement of resistivity is by is there, people do the hall effect measurements almost on any sample that comes because it gives rise to not only the magneto resistance of the magneto conductance also the hall resistance and helps us in understanding the quantization of the plateaus.

Remember this thing that we have said that this has to be a number depending on the value of B, because the degeneracy is very high the B has to be large and. So, in classical Hall Effect this was missed and one really saw this as a straight line which is a row x y versus B or r h versus B was seen as a straight line, which is having a constant slope which is given as 1 over any. However, in large values of the magnetic field, this is not the case where hall resistivity has intermitted plateaus at integer values of h over e square, the same thing in a, happens in fractional quantum hall effect where the quantization occurs at rational fractions of in units of h over e square.

Thank you.