Advanced Condensed Matter Physics

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Lecture – 21
BCS theory, Transition temperature

So, having laid the foundation of BCS theory that is how cooper established that there could be an electron attractive interaction mediated by phonons.

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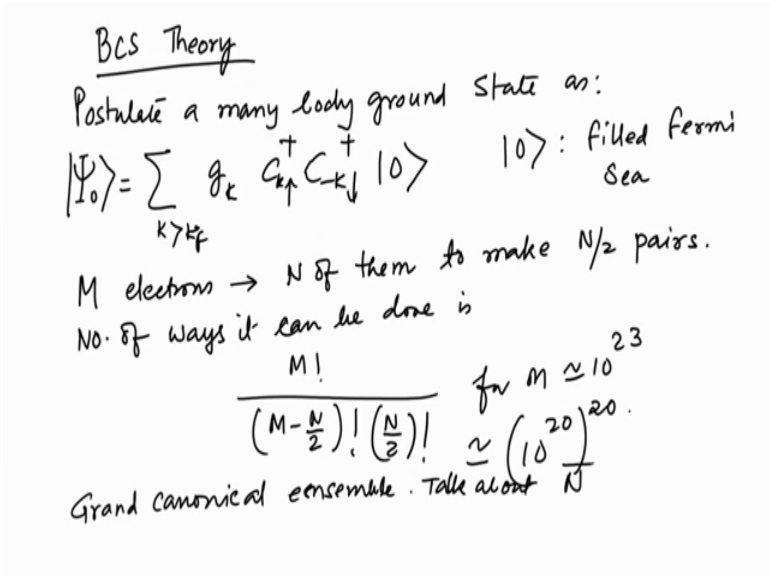

Let us go over to start BCS theory and before we start, let me tell that it involves some mathematics that is algebra which you should do because every step cannot be shown in the in the class. So, go through the steps yourself before you convinced that the results that we are coating are correct. So, postulate many body ground state as this is a sum over. So, this is like psi 0 and this is all K greater than KF and there is a g K, C K up dagger C minus K down dagger acting on the 0.

Here 0 is not vacuum, but it is the filled pharmacy the g K is some amplitude of the wave function and CK up dagger C minus K down daggers at 2 a pair is the pair created with momentum K and minus K with an up and down spins and this is a many minus body wave function superconducting many minus body wave function and then we have say m electrons and we want to choose N of them to make N by 2 pairs and the number of ways, it can be done is m factorial divided by m minus N by 2 factorial and N by 2 factorial.

For m equal to 10 to the power 23 this combination is equal to 10 to the power 20 to the power 20. So, in principle we have to solve for this many g K's in order to be able to write down a proper many body state and which is an impossible task. What can be done? Is that one can treat the problem statistically and in order to do that one can also take a grand canonical ensemble such that we do not keep the particle number of fixed and instead talk about an average number of particles.

So, we shall talk in grand canonical ensemble and talk about average noise, we call it as N bar instead of the number of particles.

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BCS ground state:

$$|\Psi_G\rangle = \prod_{\vec{k}_1, \vec{k}_2 \cdots \vec{k}_M} \left(u_{\vec{k}} + v_{\vec{k}}\, c^{\dagger}_{k\uparrow} c^{\dagger}_{-k\downarrow} \right) |\phi_0\rangle$$

$(\vec{k}\uparrow, -\vec{k}\downarrow) \longrightarrow |v_k|^2$

$(\quad " \quad)$ unoccupied $\longrightarrow |u_k|^2$

u_k & v_k are generally Complex

$$|u_k|^2 + |v_k|^2 = 1$$

$\vec{k}_1, \vec{k}_2$ $\quad u_{k_1}, u_{k_2} \to$ No pairs

$u_{k_1}, v_{k_2} \to$ 1 pair $(k_2, -k_2)$

$v_{k_1}, v_{k_2} \to$ 2 pairs $(k_1, -k_1)\,(k_2, -k_2)$

So, BCS ground state BCS rolled down the ground state as a psi g it is equal to a product of all these K one K 2 and K N or say m and this is u K plus v K C K up dagger C minus K down dagger and acting on a phi 0. So, this is the filled pharmacy as we have been talking about so, the probability of a pair to exist.

So, a pair formed off K up and minus K down to exist is given by v K square and that this is unoccupied is given by u K square that is right it has subscript and of course, the normalization says that the u K square plus v K square should be equal to one which means that the probability that a K up and minus K down would be either occupied or unoccupied and the total probability is equal to 1.

take 2 states K one and K 2. So, the amplitude with u K 1 u K 2 represent no pairs in these 2 states u K 1 v K 2 one pair K 2 and minus K 2 and v K 1 v K 2 clearly distinguish your use and v's there are 2 pairs for K 1 minus K 1 and K 2 minus K 2 both are occupied. So, these are the notations. So, a pair would be unoccupied with a probability u K mod square and a pair would be occupied would be given by v K square mod v K square.

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Average no. of particles $\bar{N} = \langle \sum_{k,\sigma} \hat{n}_{k\sigma} \rangle$

$$\bar{N} = \langle \hat{\Psi}_G | \sum_k c^\dagger_{k\uparrow} c_{k\uparrow} + c^\dagger_{k\downarrow} c_{k\downarrow} | \hat{\Psi}_G \rangle$$

$$= 2 \langle \hat{\Psi}_G | \sum_k c^\dagger_{k\uparrow} c_{k\uparrow} | \hat{\Psi}_G \rangle$$

$$= 2 \sum_k \langle \phi_0 | (u_k^* + v_k^* c_{-k\downarrow} c_{k\uparrow}) \underbrace{c^\dagger_{k\uparrow} c_{k\uparrow}} (u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}) \prod_{k \neq l} (u_l^* + v_l^* c_{-l\downarrow} c_{l\uparrow})(u_l + v_l c^\dagger_{l\uparrow} c^\dagger_{-l\downarrow}) | \phi_0 \rangle$$

$$\langle A\phi | \psi \rangle = \langle \phi | A^\dagger | \psi \rangle$$

So, average number of particles N bar this is equal to. So, the sum over K sigma and n K sigma which is an operator and this is equal to we can write this as psi g sum over K CK up dagger CK up that is the number operator for up spin and now we will add also some over spins.

Ah so, C K down dagger C K down and this expectation has to be taken between the ground state that we have written here. So, that is the ground state postulate of the VCS ground state. So, we have to take this thing here and this can be written as because there is no preference over one spin on another.

So, we can simply write this as twice of psi g sum over K C K up dagger C K up and psi g this I am writing it once, but then later on I will skip. So, this will be a phi 0 u K star plus v K star C minus K down C K up that is that is the psi g here the brass psi g and now I have a C K up dagger C K up and now I have a u K plus a v K C K up dagger C minus K down dagger and now I will have terms which are.

So, this is the same case as it is there in the operator n K sigma and now, I will also have all the other terms in which K is not equal to or rather K is not equal to l all the other indices which are not same. So, you l star plus a v l star C minus l down C l up and now I will have a u l plus v l C l up dagger C minus l down dagger and there is a phi 0 here ok. So, this corresponds to K naught equal to l and the top 1 is for K equal to l. So, in principle u K and v K are complex quantities.

So, let us write that u K and v K are generally complex that is why the stars are written separately and. So, what we have done here is that we have used a phi psi as a phi a dagger psi ok.

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look at $l \neq K$

$$|u_l|^2 + u_l^* v_l \underbrace{c_{l\uparrow}^\dagger c_{-l\downarrow}^\dagger}_{=0} + u_l v_l^* \underbrace{c_{-l\downarrow} c_{l\uparrow}}_{=0} + |v_l|^2 \underbrace{c_{-l\downarrow} c_{l\uparrow} c_{l\uparrow}^\dagger c_{-l\downarrow}^\dagger}$$

$\langle\phi_0| \quad |\phi_0\rangle = 0$

$$|u_l|^2 + |v_l|^2 = 1$$

look at $l = k$.

$$\underbrace{|u_k|^2 c_{k\uparrow}^\dagger c_{k\uparrow} |\phi_0\rangle = 0}_{\text{no states to annihilate for } k > k_F} \qquad u_k^* v_k \ \& \ u_k v_k^* \qquad |v_k|^2$$

$$\bar{N} = 2\sum_k |v_k|^2$$

2 : for the pair

$|v_k|^2$: probability of occupied states

Now, look at the term l not equal to K that is a the term that is written later now you can understand that this will have a u l mod square then you l star v l with a c l up dagger C minus l down dagger plus u l v l star and C minus l down C l up and plus a v l mod square C minus l down C l up C l up dagger C minus l down dagger now you can understand that.

So, this is coming from the product of these terms and so, they are four terms which are here now you can see that this term gives you 0 because it will create a pair and so, we will change the occupancy of pairs in the ground state. So, it will have 0 expectation value. So, this term is not equal to 0, but then when you take it between this then that is equal to 0 this is what I mean and similarly, this term will also yield equal to 0 when taken between the field for me see now this simply adds a normalization that is it creates a pair and then it annihilates a pair.

So, ultimately what happens is that. So, l not equal to K gives you u l square plus v l square equal to one and now let us look at the l equal to K this if you look at it carefully you will have a term which is a u K mod square CK up dagger C K up acting on phi 0 would give me 0 and u K v K because this tells you that there are no states to annihilate for K greater than K f and u kvk the cross minus terms both u K star v K and u kv K star will give 0 for the same reason as its told above.

So, the only term that contributes is a term with v K square. So, this N average which is equal to twice of K v K square. So, that is a result. So, 2 come because of the pair and v K square is the probability of occupied states.

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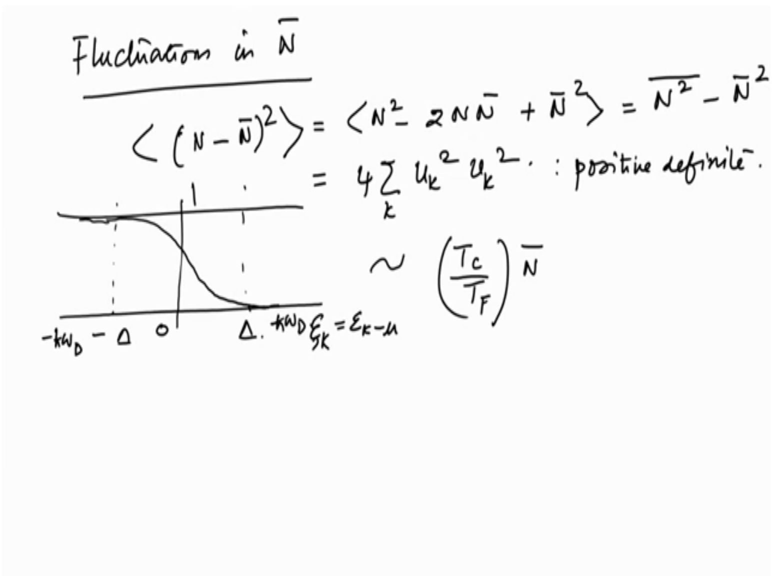

Now, second thing is a fluctuation in N bar. So, that is given by N minus N bar square which is equal to N bar square N square minus or minus twice of N N bar plus the N bar square which is equal to a N square bar minus a N bar square. So, this if you again repeat the same calculation and use the same logic to cancel out terms this is given by four K u K square v K square and this is positive definite.

In fact, v K as a function of K goes from one to 0 whereas, u K as the function of K goes from 0 to one. So, in a all happening in an energy range which is given by K t c. So, if you write down the variation of this. So, this is my. So, this is my epsilon K or psi K which is equal to epsilon K minus mu and the v K drops from 0 to 1 and this is my superconducting gap delta this is minus delta 2 plus delta and this is my.

So, this thing is my minus h cross omega d to this as h cross omega d. So, in this range v K becomes from one to 0 and so, the sum above goes as T over or TC over TF whole to the power N bar. So, this is the practically if you want to estimate the fluctuation in N. So, this goes as that. So, let us write down a many a BCS many minus body ground state.

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BCS many body Hamiltonian

$$\mathcal{H} = \underbrace{\sum_{k,\sigma} \xi_{k\sigma} c^\dagger_{k\sigma} c_{k\sigma}}_{KE} + \sum_{kl} V_{kl} \underbrace{c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} c_{-l\downarrow} c_{l\uparrow}}_{PE}. \qquad \boxed{\xi_k = \epsilon_k - \mu}$$

Do a variational theory with u_k and v_k are variational parameters.

$$\delta \langle \bar{\Psi}_G | \sum_{k,\sigma} \xi_{k\sigma} n_{k\sigma} + \sum_{k,l} V_{kl} c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} c_{-l\downarrow} c_{l\uparrow} | \bar{\Psi}_G \rangle = 0$$

Kinetic energy $\quad \langle \hat{K} \rangle = \langle \bar{\Psi}_G | \sum_{k,\sigma} \xi_{k\sigma} n_{k\sigma} | \bar{\Psi}_G \rangle = 2 \sum_k |v_k|^2 \xi_k$

Potential energy $\quad \langle \hat{V} \rangle = \langle \bar{\Psi}_G | \sum_{kl} V_{kl} c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} c_{-l\downarrow} c_{l\uparrow} | \bar{\Psi}_G \rangle$

$(|u_k|^2 + |v_k|^2 = 1)$ $\quad = \sum_{k,l} V_{kl} u_k^* v_l^* u_l v_k .$

Oh, rather, let us write down a Hamiltonian and the Hamiltonian is a single particle term epsilon K is psi K sigma C K sigma dagger C K sigma plus K l v K l CK up dagger C minus K down dagger C minus l down C l up that is the Hamiltonian and the of course, we know that psi K equal to epsilon K minus mu we have already argued that BCS theory cannot be obtained by doing a perturbation theory of any order.

So, we will do a variational calculation instead. So, and the variational calculation with these small u K v k's which are the occupancies will be used as a variational parameter.

So, what we have to do is that we have to take this variation and it is a K sigma psi K sigma N K sigma plus K l a v K l CK up dagger C minus K down dagger C minus l down C l up that is the. So, we have to take a variation of this and put this equal to 0.

So, let us look at the kinetic energy term or the single particle the first term here. So, this is the kinetic energy and this is the potential energy and this is given by let us call this as kinetic energy operator which is a psi g sum over K sigma N K sigma this you should work out and get this thing as almost we have gotten this when we did the average number this comes out to be 2 sum over K v K square and a psi K and similarly for the potential energy.

We have v this is e equal to a psi g and then there is a sum over K l v K l CK up dagger C minus K down dagger C minus l down C l up and this and this comes out to be K l v K l u K star v l star u l v K. So, these are the kinetic energy and the potential energy these are the expectation values of those with of course, the constraint as u K square plus v K square equal to one now because of this constraint one can actually take a pair.

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$$(u_k, v_k) = (\sin\theta_k, \cos\theta_k)$$

$$\frac{\partial}{\partial\theta_k}\left[\sum_{k'} \xi_{k'}\left(1+\cos 2\theta_k\right) + \frac{1}{4}\sum_{k',l} V_{k'l}\sin 2\theta_{k'}\sin 2\theta_l\right] = 0$$

$$-2\xi_k \sin 2\theta_k + \sum_l V_{kl}\cos 2\theta_k \sin 2\theta_l = 0$$

Define, $\Delta_k = -\frac{1}{2}\sum_l \Delta_{kl}\sin 2\theta_l$

$$2\xi_k \sin 2\theta_k = \sum_l V_{kl}\cos 2\theta_k \sin 2\theta_l = -2\Delta_k \cos 2\theta_k.$$

$$\tan 2\theta_k = -\frac{\Delta_k}{\xi_k} \Rightarrow \frac{\sin 2\theta_k}{\cos 2\theta_k} = -\frac{\Delta_k}{\xi_k}.$$

And reduce so, a pair of variables like this and reduce the number of variables from 2 to one see it is always very difficult to do a variational calculation with 2 variational parameters then you have to look for the minimum in a space in a 2 minus dimensional space it is much easier to look for a minimum in a 1, on a 1 minus dimensional line and that is now given by this single variable theta K.

So, we can take u K equal to sin theta K and v K equal to cos theta K you can take the other combination that is u K equal to cos theta K and v K equal to sin theta k, but it seems that this combination works better. So, now, what we do is that we take a variation with respect to theta K of this. Now I will use a dummy variable K prime one plus cosine 2 theta K once again this algebra you should do because we have come from a v K square which we have written as 1 plus cosine 2 theta K and there is a K prime l. So, there is a v K prime l sin 2 2 theta K prime sin 2 theta l and put this equal to 0, in order to do a variational calculation and see that what is the extremum value of theta K which; minimizes the energy.

So, minus 2 psi K sin 2 theta K after you do this derivative. So, it is l and there is a v K l cos 2 theta K sin 2 theta l equal to 0 if we define delta K equal to minus half of sum over l v K l sin 2 theta l, then using this and putting it into this equation one gets a nice equation such as 2 psi K sin 2 theta K equal to sum over l v K l cos 2 theta K sin 2 theta l equal to minus 2 delta K cos 2 theta K plus 2 psi 2 psi K sin 2 theta K is equal to minus 2 delta K cos 2 theta K now this equation gives us tan 2 theta K if I divide or rather bring this below it is equal to a minus delta K by psi K.

So, this can also be written as sin 2 theta K cos 2 theta K which is equal to minus delta K by psi K.

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Use the definitions,

$$2u_k v_k = \sin 2\theta_k = \frac{\Delta_k}{\sqrt{\xi_k^2 + \Delta_k^2}}$$

$$v_k^2 - u_k^2 = \cos 2\theta_k = -\frac{\xi_k}{\sqrt{\xi_k^2 + \Delta_k^2}}$$

Δ_K assumes a form,

$$\Delta_k = -\frac{1}{2}\sum_l V_{kl} \sin 2\theta_k .$$

$$\boxed{\Delta_k = -\frac{1}{2}\sum_l V_{kl} \frac{\Delta_l}{\sqrt{\Delta_l^2 + \xi_l^2}}}$$

Now, if we use the definitions that 2 u K v K which is equal to sin 2 theta K this is equal to a delta K divided by psi K square plus a delta K square. So, this is my sin 2 theta K definition of sin 2 theta K which is also equal to twice of u K v K and also v K square minus u K square which is equal to cosine 2 theta K which is equal to psi K divided by root over psi K square plus delta K square, there is a little bit of understanding that needs to be done here is that this particular choice, we could have taken the other choice also that is a vice versa, but this choice fits all the definition and we have taken the cos 2 theta K to be negative because if psi K is large which means that epsilon K is much much greater than the chemical potential mu then v K should go off to 0 which is a apparent from this diagram.

So, that is why the cosine 2 theta K is taken with a negative sign alternately we could have taken the sin 2 theta k, but that would not have satisfied the conditions or the boundary conditions that we have. So, hence the quantity delta K assumes a form that delta K equal to minus half of sum over l v kl sin 2 theta K and hence this is equal to minus half sum over l v kl delta K divided by putting the value of sin 2 theta K here is delta l square plus psi l square.

So, that is the definition of delta K so, a trivial solution. So, we have to solve for these in order to solve for delta K a priori let us say that the delta K is really the energy gap or the superconducting energy gap and we have to solve for it in order to find that what is the or how does the gap vary with different parameters especially say v and psi l or how does that enter into the expression of the gap.

In order to see that we can also look at this expression and see that the trivial solution is delta equal to rather this is delta equal to 0 is the trivial solution. Now you see delta is there on both the sides of the equation here it is just a standalone delta K and here it is sum over l delta l and K is not equal to l. So, for a given momentum value l delta l is the value and delta K has to be computed by summing over all those delta l, but since we are solving for delta l we do not know what it is. So, we cannot. So, the unknown quantity appears on both the sides.

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A trivial solution is $\underline{\Delta = 0}$

$$2u_k v_k = 0$$

$$v_k^2 - u_k^2 = -1 \qquad |u_k|^2 + |v_k|^2 = 1$$

$v_k^2 = 0$: No pairs $\Rightarrow$ Normal state

Cooper's proposition: $V_{kl} = -V$ if $|\xi_k - \xi_l| < \hbar\omega_D$.

$$\Delta_k = \frac{1}{2} V \sum_l \Delta_l \sin 2\theta_l \quad \text{for } |\xi_k - \xi_l| < \hbar\omega_D.$$

$$= 0 \quad \text{otherwise.}$$

$$\cancel{\Delta} = \frac{V}{2}\cancel{\Delta} \sum_l \frac{1}{\sqrt{\Delta^2 + \xi_l^2}}$$

So, a trivial solution as we said delta equal to 0 so, what does delta equal to 0 gives it gives that 2 u K v K equal to 0 because you see that 2 u K v K equal to delta k. So, if delta K equal to 0 2 K u K v K equal to 0 from this equation.

So, 2 u K v K equal to 0 and that tells that we also have v K square minus u K square equal to minus one, but then we have the normalization condition is that u K mod square plus v K mod square it is equal to one that it tells that v K square equal to 0. So, at delta equal to 0 implies that v K square equal to 0. So, there are no pairs and hence this should correspond to the normal state.

So, the trivial solution is important because it talks about the normal state, but at the same time it gives a meaning to delta now delta can be used as an order parameter for the superconducting transition because at normal state delta is equal to 0 and delta is not equal to 0 for the superconducting state. So, now, go to the original Cooper's proposition.

That v kl equal to minus v if psi K minus psi l is less than h cross omega d thus delta K equal to half v sum over l sin 2 theta l for psi K minus psi l is less than h cross omega d equal to 0 otherwise. So, this tells that as if delta does not depend upon K it simply depends upon it just a number. So, that is even more convenient because delta is now can we a truly thought as a number which when is nonzero will give a superconducting state; however, when it is 0 it will give rise to a normal state. So, then we have delta l which has to be put here. So, this says that if it is a sum over all le delta loses the K dependence.

So, it is just a just a number and has no K dependence. So, we can write this as delta equal to v by 2 delta sum over l one divided by root over delta square plus psi l square and we can cancel delta from both sides and this gives rise to an equation which is v by 2 l and a root over delta square plus.

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$$1 = \frac{V}{2} \sum_{l} \frac{1}{\sqrt{\Delta^2 + \xi_l^2}}$$

$$\frac{1}{V} = \int_0^{\hbar\omega_D} \frac{N(\xi)\, d\xi}{2\sqrt{\Delta^2 + \xi^2}} = \frac{N(\varepsilon_f)}{2} \int_0^{\hbar\omega_D} \frac{d\xi}{\sqrt{\Delta^2 + \xi^2}}.$$

$$\frac{2}{N(\varepsilon_f)V} = \int_0^{\hbar\omega_D} \frac{d\xi}{\sqrt{\Delta^2 + \xi^2}} = 2\sinh^{-1}(\xi/\Delta)\Big|_0^{\hbar\omega_D}.$$

$$\Delta = \frac{\hbar\omega}{\sinh\left(1/N(\varepsilon_f)V\right)}.$$

So, there is no delta l. So, it is equal to psi l square now this is the equation for delta you may not see delta on the left hand side to solve for, but there is delta in the right hand side and in the denominator and in the square root of a denominator that tells that it is a highly no linear equation and you have to solve it either by a root finding method or one of the root finding methods such as Newton, Rapson or bisection method if you want to do it using a computer that is numerically.

We can also solve this problem analytically by converting this sum over l to an integral which is of this form remember the v is the strength of the attractive potential. So, it is the once when we convert a summation into an integral we need to bring the density of states or we can write down the density of states like this and do this integral such as.

So, there is a 2 plus delta square plus psi square by psi is the variable. Now, the integral will be from 0 to h cross omega d this is what we have said earlier and Cooper had explained that how the pairs have to be formed within an energy shell of h cross omega d from the Fermi surface. So, it is measured from the Fermi surface and the now N of psi the detailed feature of N of psi is not required because we know that this whole phenomena is occurring at the Fermi level.

So, we can write this as N of epsilon f by 2 0 to h cross omega d d psi and a root over delta square plus psi square and that tells us that this is equal to 2 by N v and this is 0 to h cross omega d d psi root over delta square plus psi square and this is equal to 2 sin hyperbolic inverse psi by delta and from 0 to h cross omega d if we put these values and rearrange then we will get delta equal to h cross omega divided by sin hyperbolic one divided by N epsilon f v.

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for weak coupling superconductors,

$$N(\varepsilon_f)V << 1$$

$$\boxed{\Delta = 2\hbar\omega_D\, e^{-1/N(\varepsilon_f)V}.}$$

$$\Delta \sim k_B T_c$$

$$k_B T_c \simeq \underbrace{2\hbar\omega_D}_{1.14}\, e^{-1/N(\varepsilon_f)V}$$

$$|v_k|^2 = \frac{1}{2}\left(1 - \frac{\xi_k}{E_k}\right) = \frac{1}{2}\left[1 - \frac{\xi_k}{\sqrt{\Delta^2+\xi_k^2}}\right]$$

$$|u_k|^2 = 1 - v_k^2 = \frac{1}{2}\left(1 + \frac{\xi_k}{E_k}\right).$$

Now, since we are talking about for week coupling super conductors N epsilon fv is much smaller than one thus delta assumes a form which is 2 h cross omega d exponential minus one by N ef of V.

So, we are getting a similar expression for the energy gap as we have done by solving the 2 particle Schrodinger equation this tells that this much of energy has to be supplied in order to break a pair and go from a superconductor to a normal state and in BCS theory this energy gap is a scale which is given by the temperature scale. So, delta is of the order of K T C and. So, a K T C becomes equal to 1.14.

It is actually 2 here as we have written there and very accurate calculation shows that this is equal to 1.14 and so, the TC expression is obtained from here we will just do it in a minute. So, this gives the how the energy gap depends upon the phonon energy spectrum and how the density of states at the Fermi level come into the picture and the strength of the attractive interaction is also there which is v and we also have an epsilon f multiplied by v is much smaller than one which is relevant for a weak coupling superconductor.

So, let us give you all these occupation probabilities or what are also called as coherence factors that we have a v K square equal to half of one minus psi K by EK which is equal to half of one minus psi K divided by root over delta square plus psi K square. So, that is v K and u K square is simply equal to one minus v K square which is equal to half one plus psi ky K. So, these are important because these decide the behavior of the gap the K dependence of the gap now let us go to the finite temperature.

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Finite temperature

$$H_{BCS} = \sum_k \xi_k c^\dagger_{k\sigma} c_{k\sigma} - \underbrace{\sum\left(\Delta_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} + \Delta^*_k c_{-k\downarrow} c_{k\uparrow}\right)}_{\text{Mean field decoupling of PE term.}}$$

→ Diagonalized using a Bogoliubov Valatin transformation.

$$\left.\begin{aligned} c_{k\uparrow} &= u^*_k \gamma_{k_0} + v_k \gamma_{k_1} \\ c^\dagger_{-k\downarrow} &= -v^*_k \gamma_{k_0} + u_k \gamma_{k_1} \end{aligned}\right\} \gamma\text{'s are quasiparticle operators}$$

$\{\gamma_1, \gamma^\dagger\} \rightarrow$ usual anticommutation relations

So, we have seen that lets. So, let us write down the BCS Hamiltonian once more which is a mean field BCS Hamiltonian. So, this is equal to psi K C K sigma dagger C K sigma and in the mean field picture we have a minus delta K CK up dagger C minus K down dagger plus a delta K star a C minus K down CK up this would be obtained if you take the mean field decoupling of the p e tem of the potential energy term.

These kind of d couplings we have done earlier and this equation or rather, this Hamiltonian can be diagonalized using a Bogoliubov Valatin transformation where the C operators are transformed into quasi particle operators of this form C K up equal to u K star gamma K 0 plus v K gamma K one and the C minus K down dagger equal to minus v K gamma K 0 plus u K gamma K one and. So, gammas are quasi particle operators.

So, they have. So, gamma dagger they have usual anti commutation relations as the C K daggers now a generic form of the gap.

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A generic form for the gap.

$$\Delta_K = -\sum_l V_{kl} \langle c_{-l\downarrow} c_{l\uparrow} \rangle$$

$$= -\sum_l V_{kl}\, u_l^* v_l \langle \underbrace{1 - \gamma_{l_0}^{\dagger} \gamma_{l_0} - \gamma_{l_1}^{\dagger} \gamma_{l_1}}_{1 - f(E_l) - f(E_l)} \rangle$$

$$\langle 1 - \gamma_{l_0}^{\dagger} \gamma_{l_0} - \gamma_{l_1}^{\dagger} \gamma_{l_1} \rangle = 1 - 2f(E_l)$$

$$\Delta_K = -\sum V_{kl}\, u_l^* v_l \underbrace{\left(1 - 2f(E_l)\right)}_{\tanh(\beta E_l/2)}$$

with $V_{KK'} = -V$

Now, we call delta as the gap because we have established that delta is nonzero for the superconducting state and 0 for the normal state. So, this is equal to a minus you can call it l and a v K l and a C minus l known C l up. So, a little bit of algebra in terms of this gamma operators will yield v K l u l or u K star v K and one minus gamma l 0 dagger gamma l 0 minus gamma l 1 dagger gamma l 1 and this at finite temperature is given by each one of them will be given by a Fermi distribution function which is or this is E l and the E l.

Thus one minus gamma l 0 dagger gamma l 0 minus gamma l one dagger gamma l one it is equal to one minus twice of l. So, to say. So, delta K putting it back into this equation the gap equation it is equal to minus of v K l u l star u l. So, this should be l actually ul star v l u l star vl and the one minus 2 f E l and this is nothing, but tan hyperbolic beta E l by 2.

So, with v K prime equal to minus v which is coopers assumption I get this Equation as one over v equal to half sum over K tan hyperbolic beta E K by 2 by E K again converting that sum into the integral and using the density of states to have a value that is at the Fermi level.

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$$\frac{1}{V} = \frac{1}{2}\sum_{K} \frac{\tanh\left(\frac{\beta E_K}{2}\right)}{E_K}.$$

$$\frac{1}{N(\epsilon_f)V} = \underbrace{\int_0^{\beta_c \hbar\omega_c/2} \frac{\tanh x}{x}\,dx}_{\ln\left(\frac{2e^{\gamma}}{\pi}\beta_c \hbar\omega_c\right)} \qquad x = \frac{\beta E_K}{2}$$

γ : Euler's constant $= 0.577$

$$\frac{2e^{\gamma}}{\pi} = 1.14$$

$$\boxed{k_B T_c = 1.14\,\hbar\omega_c\, e^{-1/N(\epsilon_f)V}}$$

So, this is equal to a one N this and a 0 to a beta C h cross omega C by 2 a tan hyperbolic x by x dx where x equal to beta e K by 2 this integral as a standard value which is given by log of 2 e to the power gamma by pi beta C h cross omega C where beta C is equal to one over K T C and gamma is the Euler constant which has a value 0.577 and hence this 2 E to the power gamma by pi has a value 1.13 or 14 around 14 and that if you simplify this it comes out as K T C equal to 1.14 h cross omega C exponential minus 1 by N epsilon f v, this is exactly the formula for TC that we have got.

So, this is the how TC varies with the phonon frequency and the density of states at the Fermi level and v so, thus delta 0.

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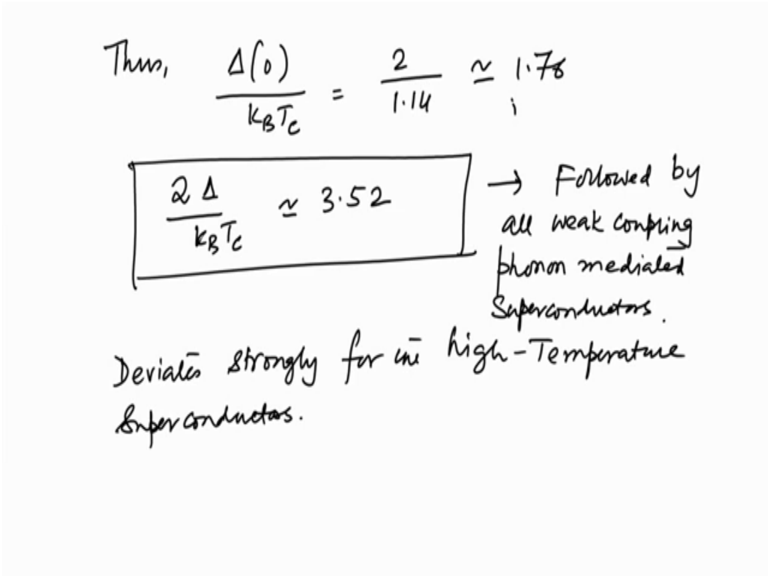

So, that is a value of the gap at t equal to 0 it is equal to 2 divided by 1.14 which is equal to 1.67 nearly and that tells you that this is a feature of BCS super conductivity that twice of delta divided by K TC it is equal to 3.52 this is followed by all weak coupling phonon mediated superconductors ok. So, this is the a feature or rather a property of this that the twice of the energy gap verse divided by TC or K TC should be a number which is three point five 2 it deviates strongly for the high temperature superconductors.

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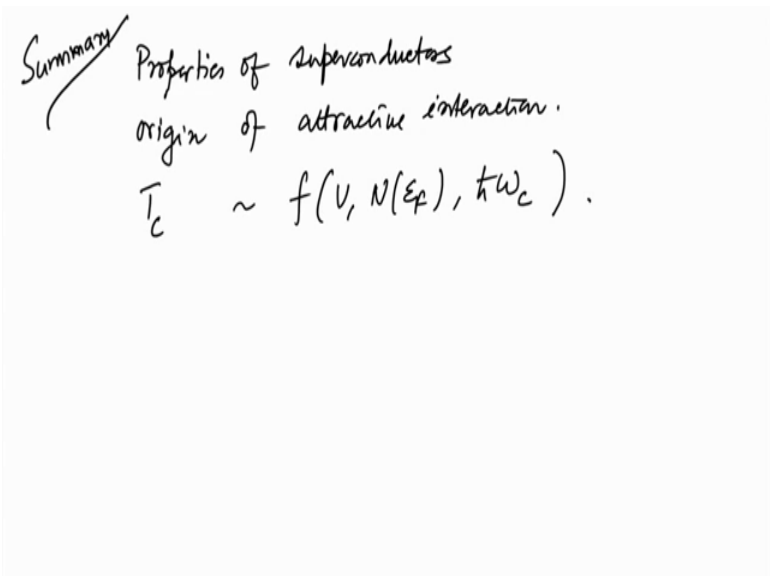

So, let us summarize what we have seen so far we have generically talked about the properties of superconductor and those properties include the Meissner effect that is the electromagnetic response then the thermodynamic response and the conducting properties or rather the resistivity how it suddenly drops to 0 below certain critical temperature or threshold temperature and so on.

And then we have gone on to talk about what is the origin of attractive interaction and this tells explicitly that there is a phonon part involved the role of phonon is very apparent because of the isotope effect that we have seen because the Debye frequency actually scales as the ionic mass. So, the lattice is or rather the TC scales with the ionic mass. So, the involvement of the lattice is very clear and there we can actually for us narrow energy range we can get an attractive interaction between the electrons.

So, the wave function of the electrons will have to actually select this energy range to form a bound pair and then we have written down a many minus body ground state and have taken a Hamiltonian which is a generic Hamiltonian where the an interaction is taking place between 2 particles for a particular form of this interaction term that is v K prime equal to a minus v for the epsilon psi K minus i K prime to be falling in this energy interval h cross omega d.

We find that the gap or rather we have been able to write down the equation of a gap and also at finite temperatures have able to write down the temperature dependence or rather the TC how the TC that threshold temperature the critical temperature depends on these quantities such as v that is a attractive interaction the density of states and the phonon frequency call it omega d or omega C.

So, this is in a nutshell what is super conductivity all about and. So, as you either destroy superconductivity by using temperature or thermal effects or you destroy superconductivity by applying magnetic field the state goes on to a normal state a metallic state; which is apparently not the case for the high minus temperature superconductors and they have many other complications there are very little consult consensus. Whether phonons are involved into this pairing mechanism or there is something else that is involved in any case.

And also these behavior or rather this 2 delta by K T see that we have found to be 3.52 is a value there is much higher maybe 5 between 5 and 6 and which are not at all called as the weak coupling superconductors and these high minus temperature superconductors they belong to a class of a non-weak coupling rather strong coupling superconductors.