Advanced Condensed Matter Physics
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Lecture – 19
London penetration depth, Type I and II superconductors

So, to continue with the discussion on superconductivity.

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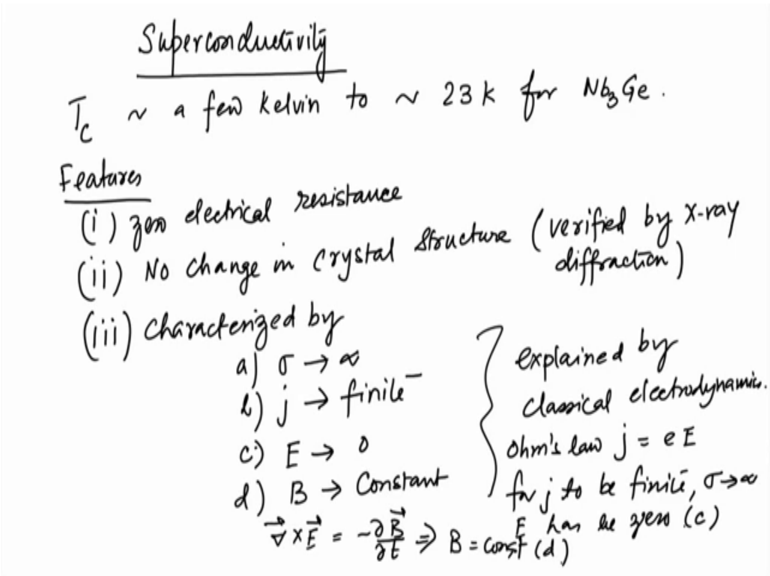

I will repeat a few things for your convenience and so, the sudden drop of resistivity below a certain temperature is called as a superconducting phenomena or this gives rise to superconductivity and the temperature below which the resistance vanishes is called as a critical temperature, which is a property of a particular material and so that T C the superconducting transition temperature is usually of the order of few Kelvin for the conventional superconductors.

Now, this we had discussed yesterday that there are classes of unconventional superconductors which are also known as high temperature superconductors and they are not as extensive knowledge about them as they exist for the conventional ones.

However the T C is really vary from ah few Kelvin to about twenty 3 Kelvin for this is for Nb 3 Ge and the features are one 0 electrical resistance or resistivity and no change in crystal structure and this is verified by x ray diffraction both below T C and above T C that is in the normal state and in the superconducting state and third is that it is the state the superconducting state is characterized by a conductivity to be infinite B the current density to be still finite c is that the electric field goes to 0 and B is that the magnetic field is constant and this cannot be explained by classical electrodynamics because ohms law says that j equal to sigma E for j to be finite j is the current density.

For j to be finite sigma has to go to sigma tends to infinity then E has to be equal to 0 0. So, this is the third there is a c condition and also that curl of E curl of E equal to minus del B del t that gives you B to be constant this is number d. So, these are some of the features of the superconducting state.

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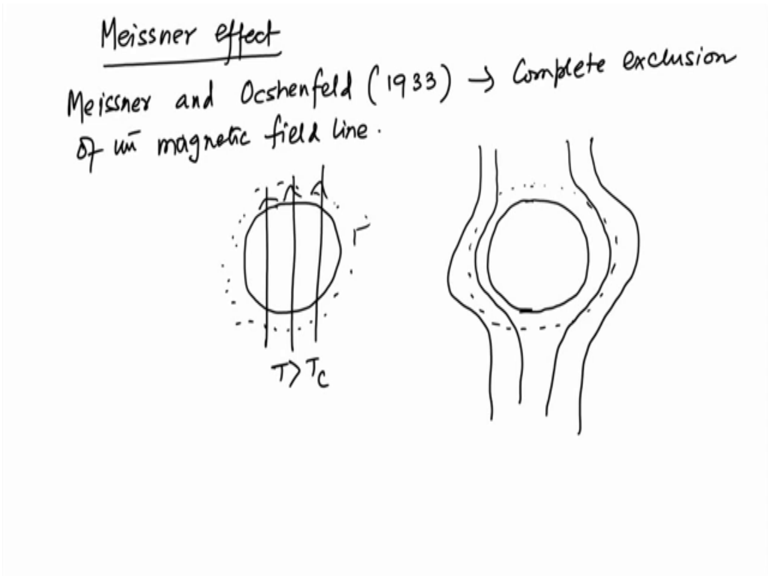

Let us now look at Meissner effect. So, the expulsion of the magnetic field from the bulk of the superconducting material is called as Meissner effect. So, this complete and sudden vanishing of the field or rather as the system goes into a superconducting state is really something that is that distinguishes it from an ideal or a perfect metal, let us see that.

So, Meissner and it is usually known as Meissner effect, but it is also said as Meissner and Ocshenfeld in 1933 they discovered that there is a complete exclusion of the field lines magnetic field lines. So, the way the experiment can be done is that one can take a superconducting sample and then put a lot of iron filings around that sample and then apply a magnetic field and along with that cool the temperature of the specimens.

So, that it enters into a superconducting state as it enters as it is in a superconducting state then the magnetic fields will be pushed outside the sample and then the iron filings would get lined up in a regular fashion outside the sample. So, let us think that this is what the sample is like. So, the flux lines will go through this if it is t is greater than pc and as t becomes. So, the iron filings are all scattered in around this superconducting sample and now as t goes below T C, then these flux lines the magnetic field lines are pushed out like this and because of that the iron filings will nicely aligned around the superconducting sample.

So, this is the tabletop experiment for seeing Meissner effect and it is seen for superconductors such as lead and team and so on. So, now in certain materials it is only above a certain critical magnetic field which is of the order of a few or states there is no expulsion of the magnetic flux.

So, whence the superconducting super conductivity disappears the material reverse into the normal resistive state and the magnetic field fully penetrates through it and so, there are materials in which magnetic field can penetrate up to a certain extent beyond that if you increase still the value of the strength of the magnetic field then it gets pushed out and this is called as a type 2 superconductors which allow some magnetic for some range of magnetic field for the magnetic field to penetrate inside the sample.

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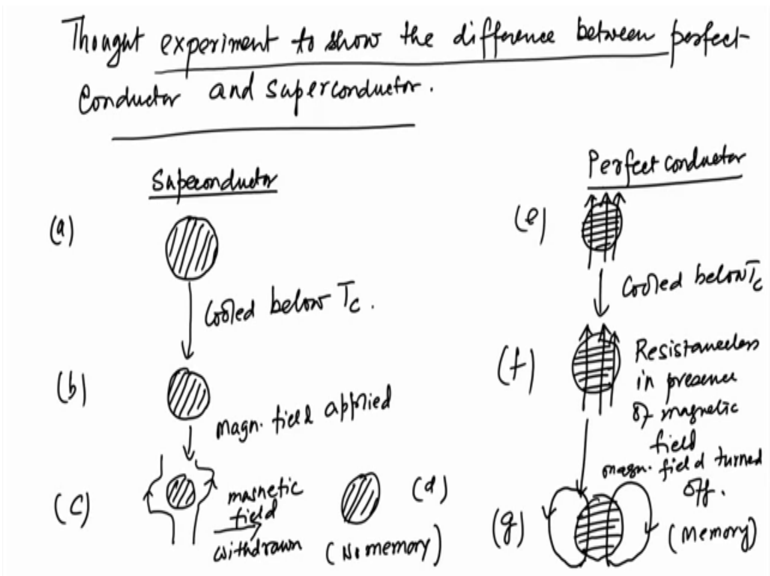

Now, let us do a thought experiment of to show the difference between perfect conductor and superconductor this has been told yesterday or rather in the last discussion through slides I just wanted to make it a little more clear by drawing the diagram. So, this is a typical superconductor and this is a perfect conductor and so, say the state a is that this is a superconducting materials and this is cooled below T C and then it becomes superconducting and then magnetic field is applied and then there are you know the expulsion of the magnetic field as we have said and then magnetic field is withdrawn field withdrawn superconductor goes back to its original state no memory and what happens to a perfect conductor.

So, this is a perfect conductor and. So, there is an external it is in an external field. So, it is cooled below T C the flux still penetrates, but it is resistance less in presence of magnetic field now when field is withdrawn it goes into a state which is. So, this is magnetic field turned off of and it has a memory. So, because the perfect conductor does not have Meissner effect this is what happens.

So, if you compare. So, this is a, this is b, this is c, and this is d, and now we have this let us call it as e, this is f and this is g. So, if you compare d and g, it is easy to see that a perfect conductor depends on its history and Meissner effect does not happen in perfect conductor and that is what distinguishes perfect conductors from superconductors. So, the total exclusion of the magnetic field from the inside of the superconductor is a property which is called as diamagnetism and you can understand this as follows.

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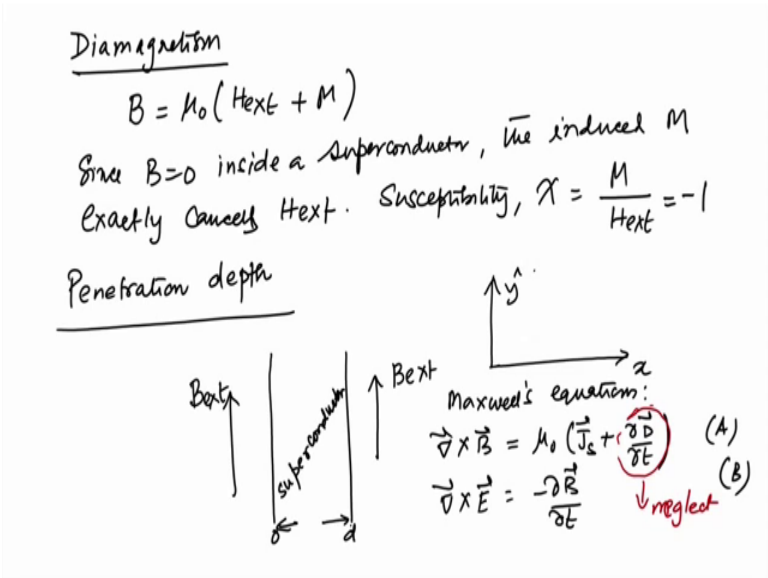

So, we have B equal to mu 0 H external plus M H external is the externally applied magnetic induction, we have not written in vector form, but they are vector equations. So, since B equal to 0 inside a superconductor the induced M M exactly cancels H external. So, susceptibility chi equal to M by H external becomes equal to minus 1 and the as we have said earlier that nothing is more diamagnetic than a superconductor.

In fact, in best of the diamagnetic metals have susceptibility which is extremely small of the order of 10 to the power minus to 10 to the power of minus 5 to 10 to the power minus 6. Now, let us a look at the electrodynamics on superconductors and particularly we are going to talk about the penetration depth.

So, let us look at penetration depth and before we do any calculation let us say that ah. So, we have claimed several times the magnetic field is totally expelled, but the fact is that that it enters only up to a certain distance which is called as the penetration depth now consider the following geometry to compute the distance through which it it penetrates. So, this is the superconductor and one has applied an external magnetic field here and lets call this direction B x and this direction B y ok.

So, this is the geometry of the sample now the Maxwell's equations can be written as curl of B equal to mu 0 j s plus del d del t and curl of B these are the last 2 Maxwell's equation minus del v del t ok. So, this is the equation Maxwell's equation that we all are aware of this is classical electrodynamics we are simply doing classical electrodynamics and on super conductors and trying to get some information on how much magnetic field can penetrate inside a sample.

Before it gets completely ex expanded. So, there is a certain critical depth up to which the magnetic field can penetrate and this by no means contradicts the statement that we have made earlier that in superconductor the classical electrodynamics laws are not strictly valid. We are simply trying to get some information out on a quantity called as penetration depth and to tell you a priori the reason is that that this is one of the important length scales of the problem and this length scale along with another one called as the coherence length determines the material to be of type one or of type 2. So, what happens is that since the displacement current.

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Since the displacement current << Supercurrent.

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}_s \qquad (1)$$

$$\vec{J}_s = n_s e \vec{v}_s \qquad (2)$$

$$m\dot{\vec{v}}_s = e\vec{E} \qquad (3)$$

Differentiate (2) wrt time,

$$\dot{\vec{J}}_s = n_s e \dot{\vec{v}}_s \qquad \left(\text{frm } (3)\right) \text{ — } (4)$$

$$\frac{dJ_s}{dt} = \frac{n_s e^2 \vec{E}}{m}$$

Putting (4) in (B) $\quad \vec{\nabla} \times \left(\frac{\partial \vec{J}}{\partial t}\right) = -\underbrace{\frac{m}{n_s e^2 \mu_0}}_{\alpha} \vec{\nabla} \times \left(\vec{\nabla} \times \frac{\partial \vec{B}}{\partial t}\right)$

$$\frac{\partial \vec{B}}{\partial t} = -\frac{m}{n_s e^2} \vec{\nabla} \times \left(\frac{\partial \vec{J}}{\partial t}\right) \qquad (5)$$

So, let me go through it again curl of B is the curl of the magnetic field this is mu 0 is the permeability J s is the super current density and del D del t is the variation of the displacement current density or displacement current and this is that E and B are the electric and the magnetic fields respectively.

Now, the displacement current variation or displacement current rather this is a, which is the time derivative of the displacement vector. So, the displacement current is far smaller than the super current. So, in this term you can neglect this one. So, neglect this and write the Maxwell's equations as a curl of B equal to mu 0 J s and J s is nothing, but it is equal to ns E v s which is the super current density is the density of the super electrons multiplied by the electronic charge.

And the velocity of these super electrons and the equation of motion of these super electrons can be written as in an electric field to be mv s dot which is nothing, but Newton’s law mv s dot equal to charge times the electric field in an external electric field that we are talking about. So, if we call this as one this as a 2 and this as 3 and call these equations may be as A and B. So, what happens is that if you put 3 or rather if we have if we take differentiate 2 with respect to time. So, J s dot equal to ns e v s dot and I can replace v s dot from this equation 3 so, n s E square E over m from 3.

So, that is the equation for j s dot. So, this is equal to dj s dt this is how the super current varies with time now if I put 4 let us call this as 4 if I put 4 in B putting 4 in B, I will have del v del t which is equal to minus m by n s e square, curl of del j s del t and this is nothing, but now I will use 4 to replace dj s dt as this is equal to minus m over ns E square mu 0 curl of curl of del v del t.

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$$\frac{d\vec{B}}{dt} = -\alpha\, \vec{\nabla}\times\left(\vec{\nabla}\times\frac{d\vec{B}}{dt}\right) \qquad \alpha = \frac{m}{\mu_0 n_s e^2}$$

$$\vec{\nabla}\times\left(\vec{\nabla}\times\frac{d\vec{B}}{dt}\right) = \vec{\nabla}\underbrace{\left(\vec{\nabla}\cdot\frac{d\vec{B}}{dt}\right)}_{=0} - \vec{\nabla}^2\left(\frac{d\vec{B}}{dt}\right)$$

$$\frac{d}{dt}\left(\vec{\nabla}\cdot\vec{B}\right), \quad \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\left(\vec{\nabla}\times\frac{d\vec{B}}{dt}\right) = -\vec{\nabla}^2\frac{d\vec{B}}{dt} \qquad (6)$$

Putting (6) in (5).

$$\frac{d\vec{B}}{dt} = \alpha\,\vec{\nabla}^2\frac{d\vec{B}}{dt} \Rightarrow \boxed{\vec{B} = \alpha\nabla^2\vec{B}} \qquad (7)$$

Since $\frac{\partial B}{\partial y} = \frac{\partial B}{\partial t} = 0$.

So, that is curl and curl off for this and hence what we have is that the dB dt. Now, I can write it as a complete differential is equal to minus alpha times curl of curl of dB dt. So, this is the equation for dB dt and alpha is given by m divided by mu naught ns E square that is what this is defined as alpha. So, now, this is written as the right hand side is written as curl of curl of dB dt.

So, this is a gradient and divergence of dB dt and minus del square dB dt now this term is equal to 0 because one can swap the order of space and time derivative and can write this as ddt of divergence of B which one knows by the second Maxwell's equation that this is equal to zero. So, this term becomes equal to 0 and hence my curl of curl of B or dB dt rather it is nothing, but equal to minus del square times dv dt. So, if you put this into the equation number let us call it as 5 putting 6 in 5 dB dt is equal to alpha and this and dB dt. So, doing a time integral this can be written as B equal to alpha del square B.

So, this is the equation that gives the space variation of B and now since the variation according to this diagram the variation can be in the x direction. So, let us convert this 3 dimensional equation here by to one dimension because we can safely assume that del B del y and del v del z to be equal to 0 and hence we can write it down as a being equation 7.

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$$B_x = \alpha \frac{\partial^2}{\partial x^2} B_x .$$

$$B(x) = A e^{-x/\lambda_L} + B \exp^{x/\lambda_L} \qquad \alpha = \lambda_L^2$$

$$B(x) = \underbrace{B_{ext}\, e^{-x/\lambda_L}} + B_{ext}\, e^{x/\lambda_L} .$$

At $x = \lambda_L$.

$$B(x) = \frac{B_{ext}}{e}$$

λ_L is called as the penetration depth.

$\lambda_L \sim 500\ A^0$ in ordinary Superconductors.

We can write it down as ah B x equal to alpha and d 2 d x 2 B x this is really ah partial differential if you like it is like del del and so on and this has a simple solution which is equal to B external well this is like let us not write be external. Now we can write it as the solution of this can be written as A exponential minus x over lambda l plus B exponential x over lambda l where lambda l is nothing, but.

So, alpha is lambda l square and one can easily see that that these because as x goes to 0. So, this superconductor is say from 0 to some d that is the width of the superconductor in that case at x equal to 0 B is equal to B external and so on. So, this can be written as B of x equal to B external x minus lambda l plus B exponential lambda l and of course, this will blow up as x becomes large. So, we can simply. So, this is again sorry B external. So, we can only talk about this term and this term says that the B which is inside which exists inside the super conducting sample falls off as exponential minus x over lambda l.

So, at x equal to lambda l B x equal to B external divided by E so, which becomes equal to E minus 1. So, this is called as lambda l is called as the penetration depth lambda l is of the order of about 500 angstrom in ordinary or conventional superconductors all right. So, we will come back to this discussion when we talk about the other energy scale for the superconductors namely the coherence length and distinguish between the type 1 and 2 superconductors; meanwhile let us do the thermodynamics.

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Thermodynamics of Superconductors

Consider Gibbs free energy of a superconductor. The magnetisation is M, magnetic induction is H. The work done in bringing a superconductor from infinite ($H_{ext}=0$) to a region where $H = H_{ext}$ exists is given by,

$$W = -\mu_0 \int_0^{H_{ext}} M\,dH = \frac{\mu_0 H_{ext}^2}{2}$$

We have $M = -H$

Gibbs free energy per unit volume, g

$$g_S(T, H_{ext}) = g_s(T, 0) + \frac{\mu_0 H_{ext}^2}{2}. \qquad B_{ext} = \mu_0 H_{ext}$$

$$\text{or, } g_s(T, B_{ext}) = g_s(T, 0) + \frac{B_{ext}^2}{2\mu_0} \qquad (1)$$

So far, we have been doing electrodynamics now let us do thermodynamics of. So, thermodynamic route is usually the simplest route to study phase transitions and so, let us consider Gibbs free energy of a superconductive.

So, if the magnetization is m and the magnetic induction is H then the work done in bringing the superconductor into a region where the magnetic induction H external from with the magnetic induction is H external; that exists from a region which is far away where the H external is equal to 0 is given by the work done expression which I am going to write let me write this because these are important points. So, the magnetization is M M and magnetic induction is H the work done in bringing a superconductor from infinity where H external equal to 0 to a region where H equal to H external exists is given by w equal to minus mu 0 0 to H external md H which is equal to mu 0 H external square over 2.

So, that is the work done in order to bring it from infinity to a place where H external magnetic field or magnetic induction exists. So, we have m equal to minus H for a superconductor. So, let us write down the Gibbs free energy ah. So, Gibbs free energy per unit volume is given by g let us write it with a g small g. So, g s s stands for superconductor in T and at a temperature T and a magnetic induction external which is equal to g s T 0 and then plus this extra energy that it acquires because of because of the H external which is what we have shown here.

So, or g s the same statement if you want to put it in terms of the magnetic field it is just there will be a this B external square divided by 2 mu 0 because of the relationship between H so, your B external equal to mu H mu 0 H external.

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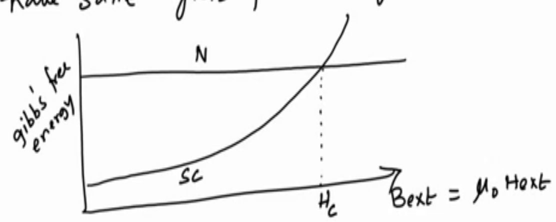
In the normal state, the magnetisation is very small, thus the magnetic work is also vanishingly small. Hence

$$g_n(T, H_{ext}) = g_n(T, 0) \qquad (2)$$

$g_n(T, H_{ext})$ increases quadratically with H_{ext}.

For some H_{ext}, the normal and the superconducting states have same gibbs free energy.



So, in the normal state the magnetization is very small thus the magnetic work done is also vanishingly small.

Hence g n th external equal to g n t 0. So, in the superconducting state g s t and at a given magnetic field or magnetic induction there is a extra term B external square by 2 mu 0 or mu 0 H external square by two; however, in the normal state the magnetization is known to be very small and we do not have this magnetic energy contribution as it is seen here. So, now, this tells that equation one tells you that there is a B dependence for g s whereas, there is no B dependence for or B or H dependence for g n. So, g n t H external it increases quadratically with H external. So, it increases quadratically we take H external. So, for some H external equivalently B external the normal and the superconducting states have same energy have same Gibbs free energy.

So, what it says is that if you plot the Gibbs free energy. So, this is Gibbs free energy as a function of either B external or H external does not matter they just get scaled by this and. So, for the superconducting state it is like this and for the normal state it is like this. So, this crossing point is called as H C or bc depending on which language you want to use ah. So, below H C this is of superconductor SC and this is normal.

So, the sc state the superconducting state has lower energy for all values of external fields or induction below H C or bc and as the external field crosses this H C, then the superconducting state has higher energy and the normal state has lower energy as you can see it is here.

So, then beyond that the normal state stabilizes. So, what one can look at is that we can call this as equation one or rather and this as equation 2 and if we equate one and 2 at H equal to H C.

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Equating (1) and (2) at $H_{ext} = H_c$.

$$g_n(T,0) - g_s(T,0) = \underbrace{\frac{H_c^2}{2\mu_0}}_{\text{positive}}.$$

Below H_c superconducting state is more stable.

<u>Example</u> For Pb at $T=0$, $H_c = 0.08$ Tesla.

Thus at $T=0$, the superconducting state is stabilized by <u>4.25×10^{-25}</u> Joule/mole. This is really a small energy!!

H external equal to HC, then g n t 0 minus g s t 0 equal to H C square over 2 mu 0 ah. So, this is positive and because this is positive and because this is positive the superconducting state is more stable than the normal state below H equal to H C. So, let us give an example for lead at t equal to 0 H C equal to 0.08 Tesla, thus at t equal to 0 the superconducting state is stabilized by 4.25 into 10 to the power minus 25 joule per mole and this is really a small energy. So, it is quite amazing that such a small energy actually stabilizes a superconducting state, but that is true of the order of 10 to the power minus 25 joule per mole, let us now talk about the critical.

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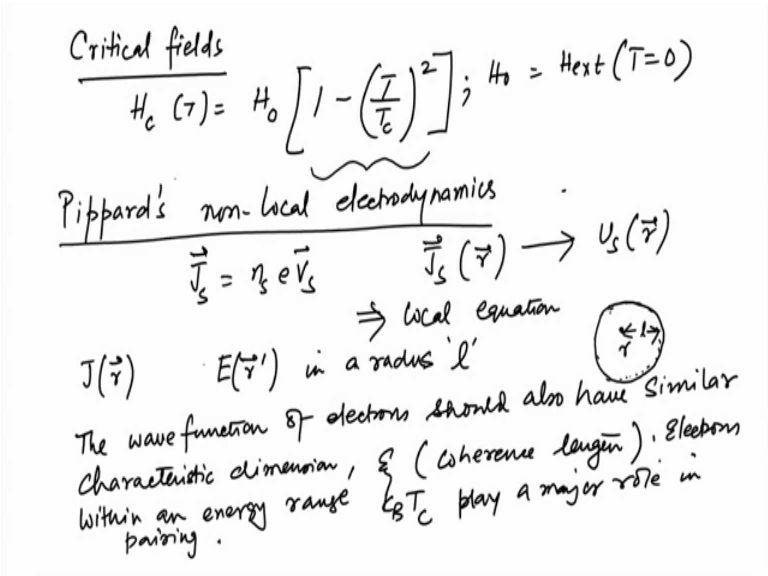

Fields which is what we are just introduced and the variation of the critical fields with temperature is given by we give this without truth is H C of t it is equal to H 0 will tell you what H 0 is it is 1 t 1 minus T by T C whole square where H 0 equal to H external at T equal to 0.

So, this is the external lea applied magnetic induction at T equal to 0. So, that is H zero. So, it is clear that at t equal to T C this term vanishes. So, H C of t goes to 0. So, there are many things that are important in this particular context and we have talked about electrodynamics and we have talked about thermodynamics and we found that some important information is encoded in both of them, now let us talk about.

So, another interesting comment from Pippard which is encoded in Pippards local electrodynamics or rather non-local electrodynamics and let us see what it says if you look at this equation j s equal to ns e v s this is what we have written this looks like that J s at r is related to v s at r.

So, it is a local equation. So, J s at r is simply determined by the v s of at that point r what Pippard say that the current density at a point r depends on. So, J s at r it depends on E the electric field at r prime. So, which is centered around r in a radius say l ok. So, at a given point r the electric field the current density at this point will depend upon electric field in the region which is spread all over r prime where r prime is centered around r with a radius which is l.

So, every point inside this circle is r prime and all of that those r primes will contribute to the current density at r. So, this is the non-locality which Pippard thought i s more relevant and realistic and this can be you know thought of to be a spread over a region of radius l and. So, this l is actually related to the characteristic dimension of the electron wave function and so, this l is related or rather let us write it.

The wave function of electrons should also have similar characteristic dimension that is extent which is called as let us call that at psi which is called as a coherence length. So, electrons within an energy range K T C where T C is the transition temperature play a major role in pairing. So, this is pairing of electrons.

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The momenta of these electrons have an uncertainty

$\sim \Delta p \sim \frac{\Delta E}{v_F} \sim \frac{k_B T_c}{v_F}$.

Thus the position uncertainty $\Delta x \sim \frac{\hbar}{\Delta p} \sim \frac{\hbar v_F}{k_B T_c}$.

$\xi = a \frac{\hbar v_F}{k_B T_c}$ $\quad a \sim 1$

$= 0.8$ in BCS theory.

$\Downarrow$

Coherence length. Pippard suggested that:

$$J_s(\vec{r}) = (\text{Const}) \int \frac{\vec{R} \left[\vec{R} \cdot \vec{A}(\vec{r}') \right]}{R^4} e^{-R/\xi} \, d\vec{r}'$$

Introduce a dimensionless parameter.

$\kappa = \lambda / \xi$

So, the moment of these electrons have an uncertainty of the order of delta p which is equal to delta E by H this is v f which is nothing, but K T C over v f thus the position uncertainty. So, we are just using a Heisenberg's uncertainty relation the position uncertainty is a delta x is equal to H cross by delta p H cross v f by K T C thus psi equal to a H cross v f by K T C.

Where a is a number which is of the order of 1 and a equal to 0.8 in BCS theory. So, this introduces and others in length scale for the problem which is known as the coherence length. So, a coherence length is the second length scale in addition to the penetration depth that we have introduced earlier. So, Pippard suggested that. So, basically that j s at R should be written as an integral rather than writing it as n s into E into v s.

So, this is equal to some constant which is not so, important. So, this is R; R dot a R prime this is that R prime that we were talking about an R and this is the R by psi d R prime. So, this is how the non-locality in the super current density comes that the super current density at a given position R depends on the electric field in this particular fashion that there is a region which is centered around R prime that every point in that region contributes to the electric field and ah characteristic length scale comes out of it and we have introduced this earlier.

Now, we can talk about dimensionless introduce a dimensionless parameter kappa which is equal to lambda by psi and these are. So, for typical superconductors lambda equal to 500 angstrom as we have already said psi equal to about 5000 angstrom.

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For typical superconductors,

$\lambda = 500 A^{0}, \quad \xi \simeq 5000 A^{0}$.

$\kappa < 1$.

In 1957, Abrikosov found that for some class of superconductors

$\kappa > 1$. $\Rightarrow$ used as a distinguishing feature of type-I and type-II superconductors.

A threshold value of κ for which such flux penetration starts to occur is $\kappa = \kappa_c = \frac{1}{\sqrt{2}}$

At this value of κ_c, the flux penetration starting at a lower critical field H_{c_1} and reaching an upper critical field H_{c_2}.

So, kappa is typically less than 1, but; however, in 1957 Abrikosov found that for some class of superconductors this kappa can be greater than 1 and this is used as a distinguishing feature of type 1 and type 2 superconductors. So, to remind you in type 2 superconductors the flux lines penetrate the sample till some threshold value of the magnetic field and if we increase it beyond a certain threshold then superconductivity disappears.

So, a threshold value of value of kappa for which such flux penetration occurs or starts to occur is kappa its say that value of kappa is called kappa c which is equal to 1 over root 2. So, I mean this is in terms of that so, ok. So, this says that at this value of at this value of kappa the flux penetration starting at a lower critical field H C one H C one and reaching an upper critical field.

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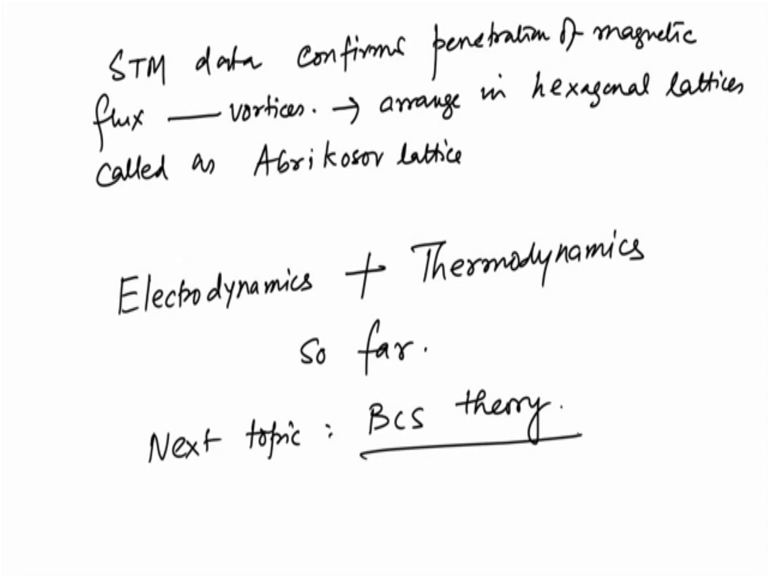

So, the scanning tunneling microscope data STM data confirms penetration of magnetic flux which are called as vortices and these vortices actually line up in the form of a hexagonal lattice in a regular lattice in the form of an hexagonal lattice and they arrange in hexagonal lattices and which are called as called as Abrikosov lattice.

So, we have studied mainly the electrodynamics and thermodynamics of superconductors. So far, and have gotten quite a few information useful information about superconductors and the next thing would be doing BCS theory which is a microscopic theory of superconductors.