Advanced Condensed Matter Physics
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Lecture – 15
Phonon Green function and Hartree Fock approaximation

So, having discussed electrons greens function elaborately. let us just touch upon the phonon greens function and before that let us emphasize the importance of greens function for a moment.

(Refer Slide Time: 00:41)

Phonon Green's functions

Phonon Green's function:

$$D(q, \lambda, t-t') = -i \langle T A_{q,\lambda}(t) A_{-q,\lambda}(t') \rangle$$

q: momentum, λ: polarization.

$$A_{q\lambda} = a_{q\lambda} + a^{\dagger}_{-q\lambda} \Rightarrow A_q = a_q + a^{\dagger}_{-q}$$

Full phonon propagator in the interaction representation.

$$D(q, t-t') = -i \frac{\langle T A_q(t) A_{-q}(t') S(\infty, -\infty) \rangle}{\langle S(\infty, -\infty) \rangle}$$

So, the greens function the pool of the greens function rather, gives the excitation energy is of the fully interacting system.

We have started looking at different orders of the greens function that is weather the greens function contains one interaction term or 2 or 3, and so on and it turns out that they are quite complicated, because a third order greens function would necessitate calculation of over 5000 diagrams without taking into account the cemetery requirements. But never the less greens functions are very useful because the polls give the excitation energy. And will also see that even a fully interacting greens function that is taking into account all orders of interaction, it can be solved exactly, and the form is not to complicated, and this is done by using dysons equation which is what we will learned.

But before that let us review a little bit on the phonon greens, function ah; however, most of the most of our discussion will stick to the electrons greens function. So, the phonon greens function is written as, let us write it with a d and q is the momentum, will write a lambda and it is t minus t prime has the same form as we have seen for the fully interacting greens function and it is a time ordered product of A q lambda t and a minus q. Lambda t this capital is are combination of the bosonic operators of the operators for phonon will see in a moment.

Ah now q is of course, the momentum that we have seen earlier, and lambda is a polarization. Most of the cases the interactions that we considered in the for the phonon system they do not mix polarization. So, it is safe to get rid of this polarization index lambda. And the A q's without the polarization index are written as a q plus a minus q dagger and or equivalently one can write it as A q equal to a q so, this is say involving the lambda, and then we can sort of remove the lambda as well and can write it with.

So, will remove lambda in this step and write it without that, and these are the combinations. So, A q operators are combinations a q S and a minus q dagger. So, the full phonon propagator in the interaction representation is written as D q or we can write it inside has we are written about.

So, q a t minus t prime equal to a minus a time ordered of A q t a minus q t prime and the S matrix and this is S matrix in the denominator. So, this is the full phonon greens function, very similar to the electron greens function that we have seen including the S matrixes which were written earlier. So, if you open up the A q's the capital A q's in terms of the small A qs one can write down the greens function ah, or let us write down the non-interacting phonon propagator, without the S matrixes.

(Refer Slide Time: 06:03)

Non interacting Phonon propagator:

$$D^{(0)}(q, t-t') = -i\langle T\, A_q(t) A_{-q}(t')\rangle$$

$$= -i\langle T\left(a_q e^{-i\omega_q t} + a_{-q}^{\dagger} e^{i\omega_q t}\right)\left(a_{-q} e^{-i\omega_q t'} + a_q^{\dagger} e^{i\omega_q t'}\right)\rangle$$

At zero temperature,

$$\langle a_q a_q^{\dagger}\rangle = 1, \quad \langle a_q^{\dagger} a_q\rangle = 0.$$

$$D^{(0)}(q, t-t') = -i\left[\Theta(t-t') e^{-i\omega_q(t-t')} + \Theta(t'-t) e^{i\omega_q(t-t')}\right]$$

Fourier transform,

$$D^{(0)}(q,\omega) = \int_{-\infty}^{\infty} dt\, e^{i\omega t} D^{(0)}(q,t) = \frac{1}{\omega-\omega_q+i\eta} - \frac{1}{\omega+\omega_q-i\eta}$$

$\eta > 0$ and infinitesimal.

So, this is equal to D 0 q t minus t prime equal to minus i T A q t a minus q t prime it is equal to minus i t. So, there is a q exponential minus t omega q t. So, the that is the time dependence of these operators plus a of minus q dagger exponential i omega q t and A of minus q exponential minus omega q t prime and plus A q dagger exponential i omega q t prime.

And so, understanding that the angular brackets are for the non-interacting ground state and. So, at 0 temperature is known about phonons or bosons that that A q A q dagger is equal to 1, whereas, the a q dagger a q is equal to 0.

So, combining these information the 0th order greens function for phonons is written as minus i and a theta function t minus t prime exponential minus omega q t minus t prime plus a theta t prime minus t exponential i omega q t minus t prime. And this is the form for the 0eth order greens function for phonons. And now the Fourier transform is given by D 0 q omega which is equal to a minus infinity to plus infinity d t exponential i omega t D 0 q t.

And if we do that then this would be omega minus omega q plus I eta and. So, this integral will yield omega plus omega q minus i eta; where eta is a eta is greater than 0 and infinit symbol is small. So, what I did is that, I have just put this D 0 q t from the line above of the step over when then did that integration infinitism alright.

So, this is the form for the zeroeth order greens function.

(Refer Slide Time: 10:10)

$$\boxed{D^{(0)}(q,\omega) = \frac{2\omega_q}{\omega^2 - \omega_q^2 + i\eta}.}$$ Non-interacting Phonon Green's function.

At finite temperatures, $\langle a_q a_q^{+} \rangle = N_q + 1$

$$\langle a_q^{+} a_q \rangle = N_q = \frac{1}{e^{\beta\omega_q} - 1}$$

Putting it in $D^{(0)}$,

$$D^{(0)}(q, t-t') = -i\left[(N_q+1)\, e^{-i\omega_q|t-t'|} + N_q\, e^{i\omega_q|t-t'|}\right]$$

Electron-phonon interaction

$$V = \sum_{q,k,\sigma} M_q A_q c^{+}_{k+q\sigma} c_{k\sigma}$$

And if you simplify it looks like q omega it is equal to 2 omega q and omega square minus omega q square plus I eta. So, this is the non-interacting phonon greens function. So, at finite temperatures what happens is that these expectations have values that are that involve the number operator for a given q, which is this is equal to in q plus 1, and this is equal to the number operator in q which is given as the bose distribution function given by this.

So, the total I mean the greens function above is a given by at finite temperature. So, it is D 0 q t minus t prime equal to minus i putting it in D 0. So, this and then there is a n cube plus 1 exponential minus i omega q and the mode of t minus t prime plus n q exponential i omega q a t minus t prime.

And that is the 0eth order propagated for phonons, and then one can do a Fourier transform and write it in omega A q omega space, let us take a typical electron phonon interaction the we shall not considered phonon phonon interaction, because phonons are the quasi particles. So, it is assumed at the quasi particles are a non-interacting. So, electrons can interact with phonons and electrons of course, can interact with electrons, here will only consider electron phonon interaction.

So, interaction is of the form some over q k sigma there is a vertex or a strength of the interaction which is M q and it is A q and C k plus q sigma dagger C k sigma. So, there are 2 fermion operators, one creation one annihilation an along with that the phonon operator which is A q and M q is the strength of the interaction.

(Refer Slide Time: 13:42)

Notice that:

$$\langle T A_q \rangle = 0.$$

Because each $A_q \rightarrow a_q, a_q^{\dagger}$ operators.

$$\langle a_q \rangle = \langle a_q^{\dagger} \rangle = 0.$$

quadritic in V

$$G(k, t-t') = G^{(0)}(k, t-t') + \frac{(-i)^3}{2} \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \sum_{q_1 q_2} M_{q_1} M_{q_2} \langle T A_{q_1}(t_1) A_{q_2}(t_2) \rangle \sum_{k_1 k_2 \sigma \sigma'} \langle T C_{k\sigma}^{\dagger}(t) C_{k_1+q,\sigma}^{\dagger}(t_1) C_{k_1\sigma}(t_1) C_{k_2+q\,\sigma'}^{\dagger}(t_2) C_{k_2\sigma'}(t_2) C_{k\sigma}^{\dagger}(t') \rangle$$

Now, something interesting can be noticed here, that this term is equal to 0, because each A q contains A q and a minus q dagger kind of operators.

And the ground state expectation of each one of the terms a q or a q dagger is bound to be equal to 0. Because if you create a boson in the ground state, then you have one boson more which will have 0 overlap with the original ground state and same with the other term which is a q. Does the terms involving linear powers of A q capital A q would all be present? And not only that all the terms with odd powers of A q will all be absent. So, it is only even powers of A q which will be present, and the one can write down a linear in terms of I mean quadratic. So, quadratic in V is what survives, and A quadratic term in V is then the full greens function can be written as t minus t prime and it will be G 0 k t minus t prime, and plus minus i q divided by 2 minus infinity to plus infinity d t 1 minus infinity to plus infinity unity d t 2.

And now, I have a sum there is A q 1 and q 2, each one will have a vertex M q 1 and M q 2. And now I will have the phonon operators and A q 2 t 2 and sum over k 1 k 2 sigma prime is equal to t and C k sigma dagger t C k 1 q plus q dagger sigma k 1 plus q sigma t

1 and C k 1 sigma t 1 C k 2 plus q dagger sigma t 2 and C k 2 sigma star t 2 and. So, there are these so, one creation with sigma the other could be this is sigma prime, and then there is a C k sigma coming from the definition of the greens function for electrons is this.

So, these are the terms that are that will be present. And now in order to write the on the electronic part, as the non-interacting means functions, we have we have studied how it looks like.

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Worry about the phonon propagator at the moment.

$$\langle T A_{q_1}(t_1) A_{q_2}(t_2)\rangle = i\,\delta_{q_1+q_2} D^{(0)}(q_1, t_1-t_2)$$

You just worry about the phonon propagator at the moment.

So, that gives me easily T A q 1 t 1 A q 2 t 2 i delta q 1 plus q 2 D 0 q 1 t 1 minus t 2. So, that is the; write it neatly so that is so, delta q 1 plus q 2 will be equal to 0, which means q 1 equal to minus q 2 and that will be with a i and then we have to in a compute the non-interacting phonon greens function which is what we have learnt how to calculate it here. So, that will be the energies square.

So, the electron greens function of course, as a many contributions, and there will be a since there are 5 rather there are 3 greens a 3 electron 3 creation operators, and 3 annihilation operators for the electrons, there will be 3 factorial combinations which would allow us to write down them down in terms of the non-interacting prince function for the electrons so, which we have learned. Now let us discuss more on the so, will stop with the phonon greens function here, and discuss with continue discussing with the electron greens function. And so, there are some things about the electron greens function which we have seen shortly or rather briefly. But we haven't discussed them in great detail.

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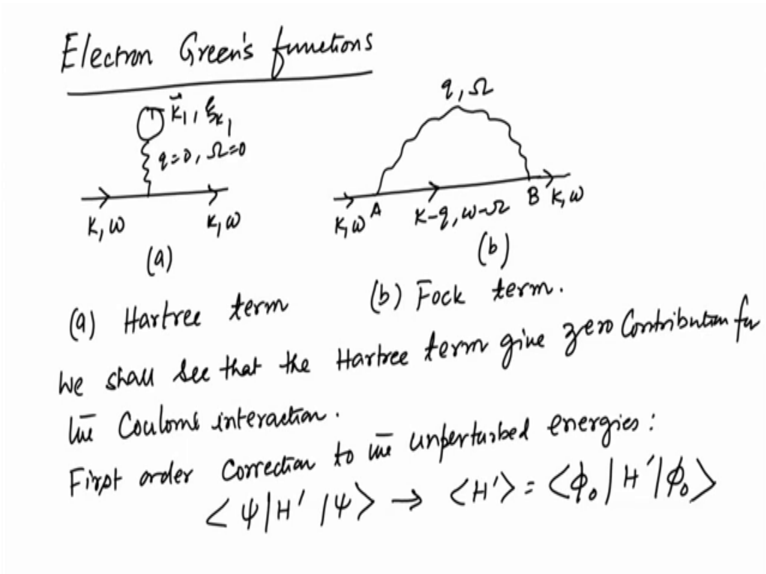


So, let us discuss them the ones that we have we have started coming back to the electron greens function yesterday or the in the previous discussion, you might have seen that that we had diagrams of this kind. So, this is a q equal to 0 and there is a k, and then we are written it with time, but if we Fourier transform will get an omega here, will get a omega here, and this is some k 1 or k 2 depends on which of the diagrams you are calculating, and this will correspond to energy k 1 and so on.

And this is q equal to 0 and omega equal to 0, it is an instantaneous interaction. And similarly, there is another term that we had considered earlier, that is this one, in which there is a k omega there is a k minus q omega minus omega. And there is a q omega that is being carried by the vertex and then finally, the free electron greens function is given by this. So, what happens is that, at this point let us call this as A, at this point there is a vertex or there is a weekly line which has a strength a V q, and has an energy dependence which is goes has omega so that takes away some momentum and energy.

So, the electron is left with k k minus q and omega minus omega finally, they reunite here at the point b and continues as k and omega. So, these 2 classes of diagrams is what we had seen. So, this is let us call this as a and this as b. And c that what can this they tell us, and how we actually use them to calculate something that is of important to us. So, a is known as the Hartree term. And b is known as the fock term. So, we shall see that, the Hartree term gives 0 contribution for the coulomb interaction.

And since were discussing the first order perturbation, there is no change in the wave function, we only get renormalised energies. And the first order correction is computed as the expectation value of the interaction hamiltonian between the ground state. So, correction to the unperturbed energies is obtained by psy H prime and a psy. And in this case, we know that this H prime would have to be between the non-interacting states.

So, the underlying assumption is that in presence of the perturbation, the system hasn't gotten too far away from the ground state. So, the ground status are still good states for the problem, and we simply want to calculate the change in energy and for for the interaction term to be squeezed between the non-interacting downstairs.

(Refer Slide Time: 25:06)

$$\langle \phi_0 | H' | \phi_0 \rangle = \frac{1}{2} \sum_{\substack{k_1 k_2 q \neq 0 \\ \sigma, \sigma'}} V_q \langle \phi_0 | c^{\dagger}_{k_1\sigma} c^{\dagger}_{k_2\sigma'} c_{k_2+q\,\sigma'} c_{k_1-q\,\sigma} | \phi_0 \rangle$$

$q=0$ Hartree

Fock

$$V_q = \frac{4\pi e^2}{q^2} \qquad \delta_{k_1, k_2+q}\, \delta_{\sigma\sigma'}$$

$\boxed{\text{Hartree terms don't contribute for Coulomb interaction.}}$

Look at the Fock term: $k_1 = k_2 + q$, $\sigma = \sigma'$

Also a fermion loop brings a $-$ive sign.

$$\langle H' \rangle = -\sum_{k\sigma} \langle c^{\dagger}_{k\sigma} c_{k\sigma} \rangle \underbrace{\frac{1}{2} \sum_{q \neq 0} V_q \langle c^{\dagger}_{k+q,\sigma} c_{k+q,\sigma} \rangle}_{-\epsilon_{HF}}$$

$$\langle H' \rangle = \sum_{k,\sigma} \epsilon_{HF} \langle c^{\dagger}_{k\sigma} c_{k\sigma} \rangle$$

And this can be written as, it is equal to half ah, and will be k 1 k 2 q, now should not be equal to 0, and sigma and sigma prime with a V q will just come in a moment y q is not equal to 0, and then I have a 5 0, and then I have I need some space here. So, I will just write it C k 1 sigma dagger C k 2 sigma prime dagger, it is right it a little lower than that because I need some space upstairs. And then we have to write the 5s also.

So, 5 0 I mean there in the same. So, let me so, there is a V q, and then there is a C k 1 sigma dagger C k 2 sigma prime dagger C k 2 plus q sigma prime and C k 1 minus q sigma. And this has to be taken with the 5 0. If you look at the coulomb term that we had written earlier this is exactly the form that we are taken. And what is V q? V q is equal to 4 pi e square by q square and this blows up at equal to 0.

So, q equal to 0 term from the summation will have to be dropped. And now look at the pairings, will write down the pairing so, if we can take a pairing between this and this, and this and this, both correspond to q equal to 0. And we can also take pairing between the first and the third one, and the second and the 4th one, both will correspond to delta k 1 is k 2 plus q. And of course, also delta sigma prime. Now you see that the Hartree term corresponds to q equal to 0; which is not included here at all, any of the 2 terms I mean any of that the pairing comes from these pairing these lined terms.

The terms that are being connected by the horizontal lines they are not included. So, Hartree terms Hartree diagrams, they do not contribute for the coulomb interaction term. So, this is one nice information which see every information in this regard is chooseful, because we had to calculate a large number of diagram. So, were trying to reduce by using symmetries as much as possible. So, Hartree terma, now there are 2 search terms, if you remember one with the k 1 and the other with the k 2 and both will not contribute. So, Hartree terms do not contribute for coulomb interaction.

So, this is something that is interesting and important. So, will just put a line here now let us look at the fock term. And the fock term is a is simply. So, this lower one is the fock. So, this is the Hartree, and this is the fock. So, this term looks like so, this demands that k 1 is equal to k 2 plus q, remember all these keys are vectors, they are a wave vectors ah, but we are writing it without the vector notation you can put them, but it becomes someone Messi to write every time with vector, but understand their vectors.

And this also demand that sigma equal to sigma prime. So, there is so, there is also a fermion loop that brings a negative sign. So, my H prime simply becomes equal to k sigma C k sigma dagger C k sigma, and half then, there is a V q q naught equal to 0, and then there is a C k plus q sigma dagger C k plus 2 sigma.

And that is the combination, now if we call this as a minus Hartree fock energy. So, including the Hartree because Hartree does not give any contribution, then we can write down this term as a k sigma and there is a Hartree fock, and energy of the Hartree fock, and then C k sigma C k sigma. So, the expectation value of H prime between the non-interacting ground states is equal to sum epsilon H f which is the Hartree fock energy given by this term which is the sum over there is a half factors well. Sum over q naught equal to 0 of V q and then this expectation value.

So, that is the total Hamiltonian including this energy correction, at the Hartree fock level is equal to.

(Refer Slide Time: 32:06)

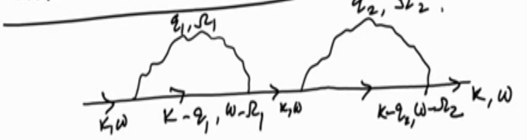
Interaction Hamiltonian at the Hartree fock level,

$$H' = \sum_{k,\sigma} \epsilon_{HF}\, C^{\dagger}_{k\sigma} C_{k\sigma} = \sum_{k} \epsilon_k\, C^{\dagger}_{k\sigma} C_{k\sigma}.$$

Look at the Fock term more carefully,

$$\underbrace{i\int \frac{d\Omega}{2\pi}\, \frac{1}{2} \sum_{q} V_q\, G^{(0)}(k-q, \omega-\Omega)}_{\Sigma(\omega).}\, G^{(0)^2}(k,\omega)$$

Look at $n = 2$ diagram



So, this is this is including so, this is just the ok, let us just write the Hamiltonian which will are only writing the interaction term. So, this is interaction we can put a interaction Hamiltonian. So, that was not the total Hamiltonian. So, this is equal to epsilon H S and a C k sigma dagger C k sigma. So, that is the, those are the 2 single particle operators, and there is sum over k and sigma simply we can write it as epsilon k C k sigma dagger C k sigma, that is the Hamiltonian.

Ah now if this Hamiltonian has some other terms; which are at the 0th level then that has to be added to this term and so on. So, let us look at the fock term more carefully. So, this is equal to i ah; now I will do the Fourier transform and go over to the omega space. This 2 py in the denominator is routinely put for the normalization. And then there is a half, and then there is A q V q, and there is a G 0, and a k minus q omega minus omega and then there are G 0 square and there is a k omega.

That is like saying if you go to this there are 2 G 0s with k omega on the left side and right side of the vertex, and then there is one G which is here and then the V there is a V q omega that we are written V q. But it is also function of omega will see that how that goes. Now let us call this thing entire thing here up to this to be something called because we are summing over q, and we are so, let us call this a sigma omega we are summing over q and we are also summing over capital omega.

So, this is equal to a sigma of omega. So, this is what we get from the Hartree fock term, where we writing one of the terms is equal to the part of the Hartree fock term to be equal to sigma of omega. Now look at n equal to 2 a typical n equal 2 diagram. So, there is a k omega, there is a k 1 minus q 1 omega minus omega one there is again k omega, there is now k minus.

So, this could be just k minus q 2 omega minus omega 2, and then there is a k omega and this carries this one carries A q 1 omega one and there is A q 2 omega 2.

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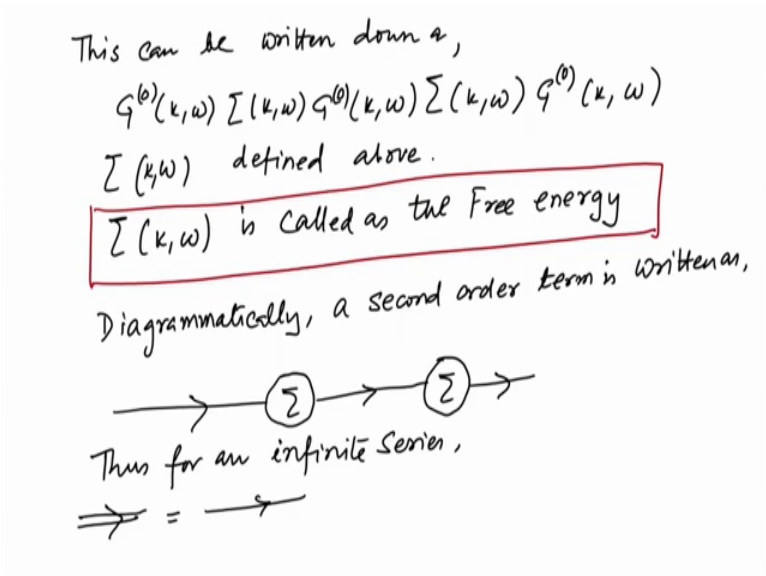

So, this one can be simply written down as G 0 write it k omega sigma k omega G 0 k omega sigma k omega and because, we are summing over that internal energy is which are capital omega one capital omega 2 etcetera, and this and so on. Sigma omega is defined above.

So, sigma can actually be a function of it is actually a function of both k n omega, because you are summing over q so, it is a k omega. This is called as sigma k omega is called as the free energy. And let me this is an important things so, let me box it in red colour.

So, this is called as a free energy. So, diagrammatically a second order term is written as this, and then there is a sigma, then it is this, and there is a sigma and then it is this and so on. So, at the second order, we have 2 such sigmas, and in the third order there will be 3 such sigmas.

And if we considered an infinite series, and then series we can write down of full greens function let us write it with a double arrow; which is equal to a single let us let us take it to the new page because this is going to be used.

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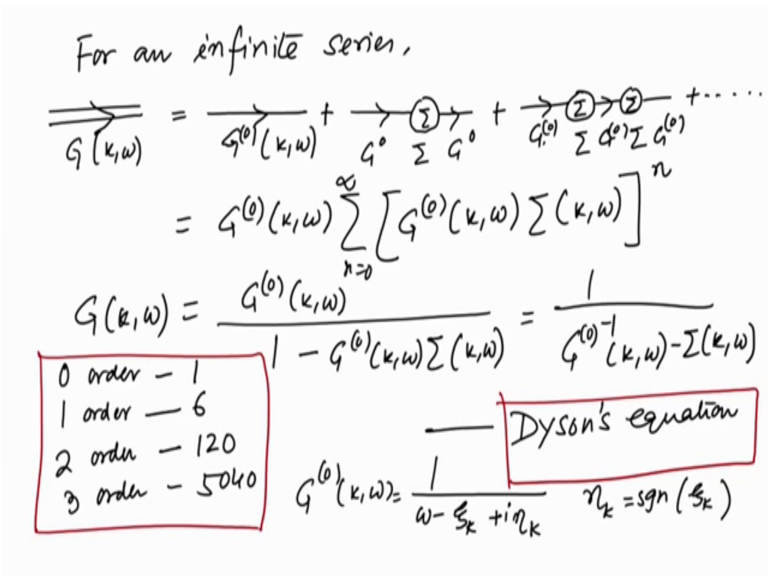

So, full greens function is written with a double arrow, double line with an arrow, and the non-interacting or the unperturbed one is written with the so, this is the G k omega. This is G 0 k omega. Now I will have a line and a sigma and a G so, this is a G 0 sigma G 0. I am dropping all these dependencies.

So, this plus, and then there is a plus sigma sigma. So, there is a G 0, G 0 sigma G 0 and so on. And going up to infinite order, one can write down a G p series, with a G 0 k omega, and then there is a sum over n equal to 0 to infinity, and then there is a G 0 k omega sigma k omega, and n, and this is equal to G 0 k omega 1 minus G 0 k omega sigma k omega is equal to 1 divided by G 0 inverse k omega minus sigma k omega.

So, this is called as dysons equation.

This is what we were saying earlier, that we could do a perturbation theory with up to first order or up to second order or up to third order. Now in some given problems, a perturbation theory can fall insufficient up to a particular order or given order, in which case one has to actually sum over infinite number of terms containing infinite number of H primes. Now formally that may look somewhat difficult because if we know that after so, we go from first order, we have first order we had 6 terms, second order we had 20 terms, or something like that, if I remember it correctly there are.

So, so, second order so, so, the from the 0eth order it goes up to so, 0th order there is one term. first order there are 6 terms, second order there are one 20 terms and third order there are 5040 terms. So, this is the magnitude of the problem that we otherwise need to find out. But here up to infinite order which is taking into account repeated interactions, infinite number of times one arrives at an expression for the greens function; which looks very simple with a G 0 and a self-energy to be computed and we have given the form of the self-energy. And this one is with a G is 0 as we already know, that this is equal to a non-interacting greens function which is psy k plus I eta k eta k being the sign of psy k.

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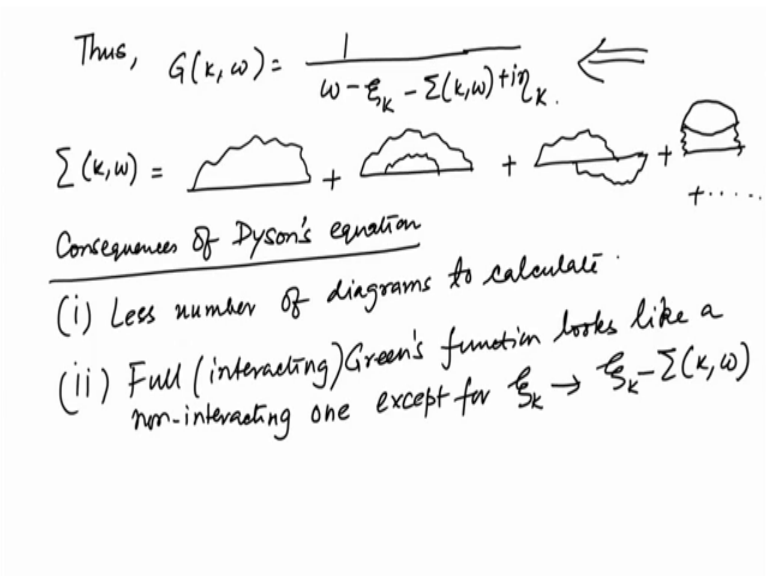

And in which case our full greens function looks like, minus sigma k omega plus i eta and i eta k. And so, sigma is equal to a term like this, flush a term like all these and so on and so on.

So, consequences of dysons equation so, this is the first one of them is that less, far less actually a number of diagrams to calculate. and second is of course, the interacting greens function, a full means interacting greens function, function looks like a non-interacting one, non-interacting one except for the energies are now replaced by well they have to resemble same, minus sigma k omega at this stage.

This is exactly the form of the G 0, now the energies are replaced by this psy k minus sigma k omega.

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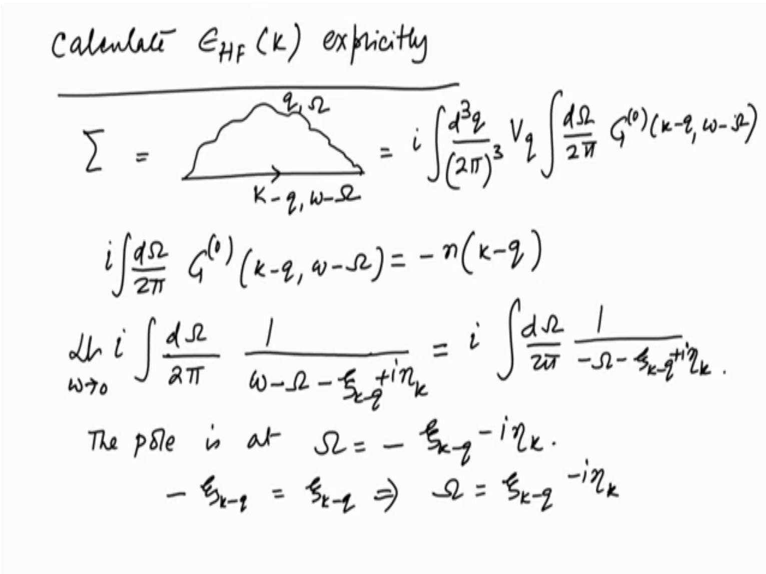

Now, let us calculate, calculate H of which is a function of k explicitly.

So, a sigma is written as, we are taking into account that same fock term. So, this is a k minus q, and omega minus omega and this is the q omega. So, this is written as we have said earlier, when we are discretize the or rather use integral for q ah, then for a 3-dimensional integral d will try it at d q, and the normalization factor used over the first is to pi whole cube. And there is a V q, and then there is a d omega over to pi, and then there is a G 0 and k minus q omega minus omega.

So, it is looks like that if we can calculate this integral, then we would be done in a doing an infinite order perturbation theory, or rather we can get the dyson equation on calculate the full greens function. And know what the self-energy is. So, this is the expression for the self-energy. And so, this requires us to calculate integral such as now there is one thing that one should look at is that we are talking about really equal time greens function, that is these 2 are have the same time; which means that the frequency associated with it is infinite is small or going to 0, in which case we have the omega small omega going to 0 then we have d omega by 2 pi and a G 0 k minus q omega minus omega, look like a simple minus n k minus q.

This was once introduced while writing it, and then we had again written it as a green function. So, so, this is that so, as omega goes to 0 will take the form of a number operator n k minus q. the reason is that this is d omega over 2 pi ah, and now this will be written as one divided by minus omega minus omega minus psy k minus q plus i eta k, and this is equal to i d omega by 2 pi we put omega going to 0.

So, limit omega going to 0 because these are equal time it happens instantaneously this interaction. And now we have a minus omega minus phy k minus q plus i eta k, and the pool is at is at omega equal to minus k minus q i eta k. Now it is always true that a corresponding to a minus energy, there is also a plus energy.

So, this if it is true, then we have the pool at k minus q minus i eta k.

(Refer Slide Time: 52:18)

Consider $\xi_{k-q} > 0$, $i\eta_k = i\eta$

Pole at $\Omega = \xi_{k-q} - i\eta \Rightarrow$ Pole is below the real axis.

So if the loop in enclosed in the upper half, then integral is zero.

$\xi_{k-q} < 0$, $i\eta_k = -i\eta$

$\Omega = \xi_{k-q} + i\eta \rightarrow$ one simple pole in the upper half plane.

$$\int \frac{d\Omega}{2\pi} G^{(0)}(k-q, \omega-\Omega) = 2\pi i \times (\text{sum of residues})$$

<u>residue</u> $\dfrac{\Omega - \xi_{k-q} - i\eta}{\Omega - \xi_{k-q} - i\eta} = 1 \Rightarrow$ integral $= 2\pi i$

Now, consider k minus q is greater than 0, and i eta k equal to i eta then of course, then it is taking the sign of the psy k. So, the pool at omega equal to psy k minus q minus 2 i eta. So now, the pool is in the lower half. So, pool is not in the upper half. So, the integral is equal to 0. So, if you want to a close it from top.

Ah now also so, pole is above the pole is in the below the real axis, real axis so, if the loop is enclosed in the upper half, then the integral is 0. Now suppose a psy k minus q is negative, in which case my i eta k equal to minus theta. So, pole is equal to omega equal to psy k minus q plus i eta. One simple pole, one simple pole in the upper half plane and doing contour integral as you have d omega by 2 pi G 0 k minus q omega minus omega; it is equal to 2 pi i into some of residues pi i into some of residues. And the residue here is equal to 1. Because it is equal to omega minus k minus q minus i eta divided by omega minus k minus q minus i eta, which is equal to 1.

So, that gives the integral to be equal to the above integral to be equal to 2, pie I now you multiply it with I over 2 pi.

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Multiply with $\frac{i}{2\pi}$ $\Rightarrow$ $\frac{i}{2\pi} \times 2\pi i = -1$

$$\Sigma(k,\omega) = -\int \frac{d^3q}{(2\pi)^3}\, V_q\, n(k-q)$$

$$= -\int \frac{d^3q}{(2\pi)^3}\, V_{k-q}\, n_q = -\int_{|q|<k_F} \frac{d^3q}{(2\pi)^3}\, \frac{4\pi e^2}{|k-q|^2}$$

$$= -\frac{e^2 k_F}{\pi}\left[1 + \frac{1-(k/k_F)^2}{2(k/k_F)} \ln\left|\frac{k_F+k}{k_F-k}\right|\right].$$

Independent of Ω.

At this level it simply renormalizes the bandwidth.

So, the required expression multiply with i over 2 pi which was there here i over 2 pi. So, I over 2 pi, and then the whole thing becomes it gives you i by 2 pi into 2 pi I which is equal to minus 1, and one gets a minus 1 into k minus q ok.

So, then you recognize that a minus so, the sigma k omega becomes equal to a minus d q by 2 pi whole cube a V q, and n k minus q. I can q is being integrated over, I can use q as a dummy variable, and redefine k minus q A q. So, q becomes k minus q 2 that becomes equal to d cube q by 2 pi whole cube. And I have a V k minus q n q n q I'm writing it. So, I can simply write this as, because n q is the number operator which will have a value equal to 1 at t equal to 0, for q to be less than k firmly, where n q will have a value equal to 1.

So, I simply get at d q by 2 pi whole cube, and a 4 pi a e square over k minus q square. And this integral if you perform it becomes equal to e square k f over pi, and a one plus 1 1 plus 1 minus k k over k f whole square divided by 2 k over k f ah, and logoff k f plus k and k f minus k. And so, this is at this level it is independent of omega of omega.

So, omega means I should write it with the capital omega. So, it is it is simply so, the at this level at this level. It is simply renormalizes the bandwidth. There is an interesting cartoon on the perturbation theory that one can present this is a book by Matuk; in which he gives an analogy between a propagator or a full bring function and the drunken man.

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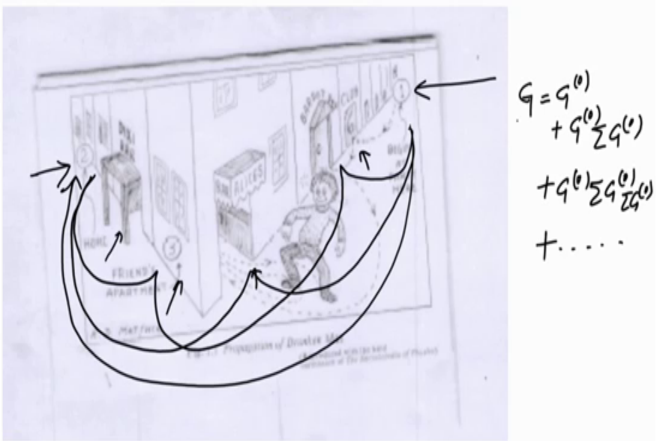

Propagator and the drunken man

So, the drunken man if you look at it carefully the though the resolution is not good. He started from one, which is his original location. And then while going back home, home is at 2 here.

He is undergoing repeated sort of deviations into various bars. So, these are alices bar, and then there are other bars which are written here. So, he goes and gets scattered by each one of them, and then he multiply time goes and he comes out by they are shown in the path, and then he goes here and then he before he goes home he goes to an Dixie bar and all that.

So, basically the he is full propagation is given by the full brings function G, and he is outside the bar the travel is given by the G 0, and then he could have gone. So, one option is that he could have gone directly from here to hear, or he could have just visited one bar; which is like one sigma. And so, it could have been like this bar that he visits and then goes back home, but, so, one is G 0 and then his scatters at the bar and then goes back home. I could do at 2 bars and then finally, goes back home.

So, which means that he goes hear, and then he goes here, and then he goes home and so on. So, his entire trajectory will be actually formed by taking the combinations of all these parts that he takes and this full propagation would be described by a perturbation theory that we have presented here.