Nuclear and Particle Physics
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Module - 04
Radioactive Decays
Lecture - 01
General Properties

Today we will discuss the radioactive decays.

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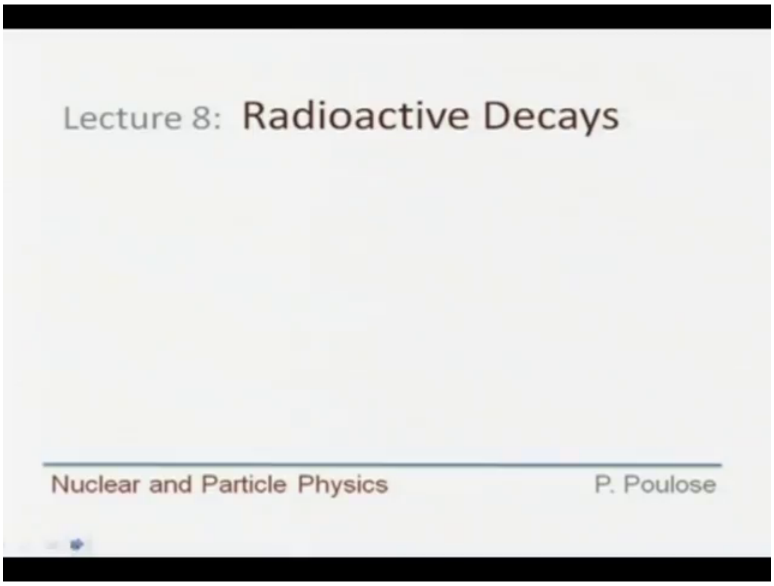

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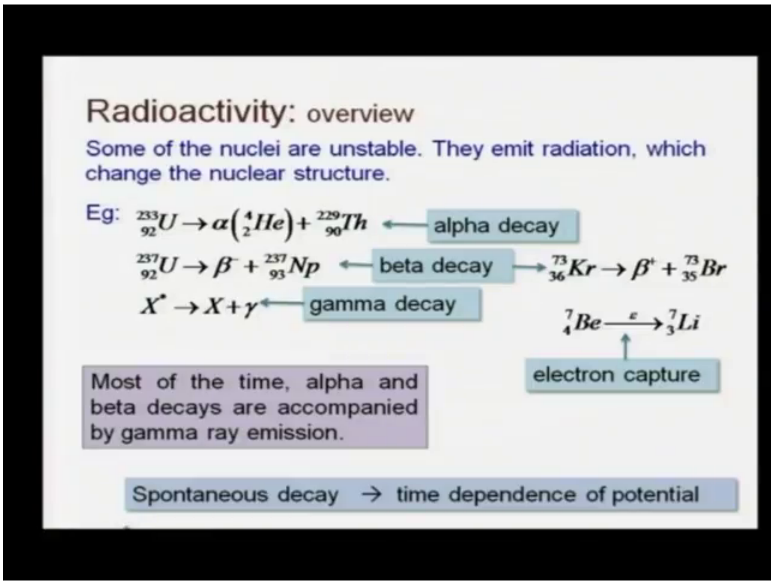

As you must already be familiar with radioactive decays of nuclei; there are many nuclei that is found in nature which decay over time converts itself into some other nuclei and artificially also we could make nuclei decay into other nuclei.

So, in this discussion we will consider some aspects of this radioactivity of the nuclei. So, examples of radioactive nuclei or uranium which has 92 protons, it can undergo what is called a alpha decay by emitting helium nucleus, and converting itself into thorium which was a 90protons in it.

Another example is uranium again 92 237 which emits a negatively charged particles. In fact, electrons and the z number in this case goes up by one unit, becomes 93 from 92 and that is another example, then there are many nuclei which when they are in excited state emits photons gamma rays and converts itself I mean comes down to d exits by omitting this photon. So, the first one is an example of alpha decay, since helium nucleus is called alpha particle and the second example is a example of a beta decay. So, the emitted particle is called beta particle with negatively charged particle, we will see that this negatively charged particles are nothing, but electrons coming from the nucleus.

Sometimes similar to the is a particle similar to electron comes out, but with positive charge. In fact, they are the anti particles of electrons, which are called positrons. So, this positron emission is called beta plus decay for example, krypton with 36 protons emits positron, and its z number now comes down by one unit to become 35 bromine. And the d excitation and emission of photons is called gamma decay, and another process that can happen is apart from the nucleus in the atom there are these atomic electrons. So, the nucleus sometimes captures absorbs one of the low lying electron, which are moving around the nucleus in an atom say for example, if you take beryllium with 4 protons and 7 neutrons in the 7 mass number which means 4 protons and 3 neutrons in the nucleus has 4 electrons moving around in the beryllium atom around the nucleus.

now this neutron, the nucleus can capture one of these electrons and thus reduce its charge by one unit. So, the z number is reduced by one unit and so, it becomes lithium nucleus with 3 protons and 4 neutrons. We will see later that which actually converts one proton in the nucleus by a absorbing an electron to neutron, we will be come to the details later. This particular kind of reaction process is called the electron capture. So, these are the basic spontaneous activity that can happen in an atom, and there are many such atoms which exhibit these decay properties.

The alpha and beta decays are followed by gamma ray emission which means that the nucleus produced by alpha or beta decay, themselves are actually in an excited state and they further decay down to the exciter ground state by emitting gamma rays. You can ask question like when we discussed the atomic models earlier, we had discussed some nuclear potential and discuss the energy levels different energy levels possibly occupied by the nucleons say for example, in the shell model we considered good saxon kind of potential and then we could solve the Schrodinger equation get the energy states of the nucleons.

These are stable states. So, such states cannot actually decay they are there their lifetime is infinite. So, they are there for infinite time. So, the in order to I mean how does this a decay happen then. So, for such spontaneous decay, the nucleon should be or some of the parts of some nucleons will should be experiencing some time varying potential with this. So, this time varying potential is needed to whose time varying part of the potential is needed to understand the quantum mechanics of spontaneous decay.

We will not go into the quantum mechanics of the spontaneous decay, but rather we will consider some common properties of the radioactivity itself, basically I mean mostly derived from experimental observations etcetera.

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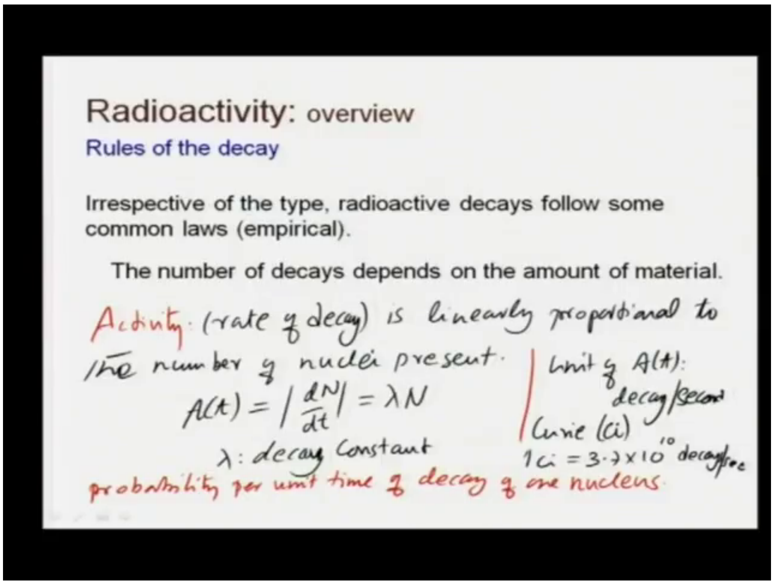

So, let us look at some of the common properties of radioactivity. Basically there are observations large number of observations which are made on radioactivity is radioactive nuclei, and then we understand a lot of lot about the common properties that such decays follow. These properties are mostly independent of what type of decay that follows for example, whether it is alpha decay or beta decay or gamma decay, the decay properties many some of the basic properties of the decay rules of the rules that the decay process follows are independent of what type of radiation happens.

One thing that we know from experiments is that the number of decays depends on the amount of material that is present what does that mean? It means that if you look at the decay rate say let me call this the activity. So, this is basically the rate of decay, it is observed that this activity is linearly proportional to the number of nuclei present in the sample.

So, I can actually write down equation relating the activity and which is basically the rate of decay or the rate at which the number of particles come down d N by dt, actually the modulus of that just the magnitude is equal to some constant times N, where N is the number of particles present in that number of nuclei present. In that and the lambda is called the decay constant, if you take one particular atom there is considered in this fashion, there is one nucleus radioactive nucleus it has some probability to decay. If lambda is the probability to decay, if you take N such atoms such a nucleus then lambda times N is the number of particles that will decay in one second.

So, if lambda is the probability per unit time per minute per unit time probability per unit time of decay of one nucleus. So, if you have 1000 nucleus nuclei and lambda times 1000 was the number of nuclei that will decay in one second, if lambda is expressed in temper or decay per second. So, that will tell you the unit of radiate activity. So, the activity unit of A is decay per second. There is another commonly used unit called curie in honour of Marie Curie who was done a lot of work in radioactivity related science and she has won 2 Nobel prizes also for her work in radioactivity.

So, curie which is denoted as C i is basically defined as one curie equal to 3.7 times 10 power 10 decays per second. So, this also tells us that radioactivity is a kind of a statistical phenomena, which means if you take a particular nucleus you cannot say when it will decay or how long will it live there without decay. What you can say, but yes if you have 1000 such nuclei how many of them will decay in one second or one minute or some interval of time all right.

So, basically this kind of probabilistic nature is a common property of quantum mechanical processes. So, there is no surprise that radioactivity also has stat also is statistical in nature.

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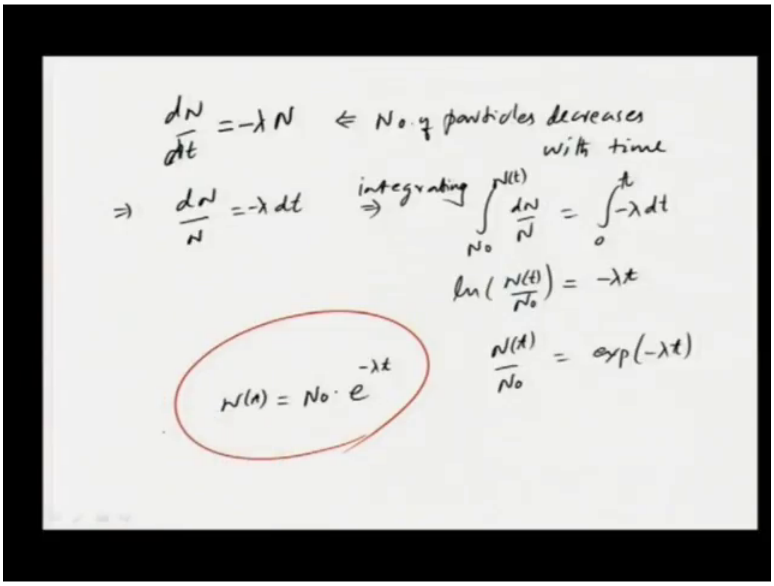

So, we said d N by dt the rate of decay is proportional to N and in fact, we said it is equal to lambda N, the absolute value was lambda times N. So, if you take lambda is positive like probabilities should be positive. So, it is a positive quantity N is also a positive quantity, but we know that d N by dt the rate actually decreases as time increases for some sort time interval dt, the number of particles we will actually come down. So, d N by dt is a decreasing quantity, it should be minus lambda N.

So, this negative sign actually says that the number of particles decreases with time all right. So, now we can try to get an expression for this, I will rewrite this as take the end to the left hand side, and write it as d N over N which is now equal to minus lambda times dt which goes to the other side. Integrating we will get say t equal to 0 we will switch on at time t equal to 0, and then ask the question after some time t what happens. So, integrate this from 0 to t corresponding N value and let us consider denote it as N 0 which is the number of particle present at t equal to 0 and N t is the number of particle at time t.

So, left hand side limits of integration are N 0 to N t and right hand side it is from 0 to t; and left hand side when you integrate one over and d N you will get logarithmic log N within the limits N t and N 0 it will come out to be, log of N t over N 0 equal to the other integration will give you minus lambda which is a constant which is taken out integral dt is t within the limit 0 to t will give you t simply t . So, now, you take the exponential on both sides; exponentiate this exponent of log is whatever that quantity N t by N 0 is equal to exponential minus lambda t or you can write N t equal to N 0 times e power minus lambda t.

So, this is basically the familiar exponential law or rule of exponential law of radioactivity all right some consequences of this.

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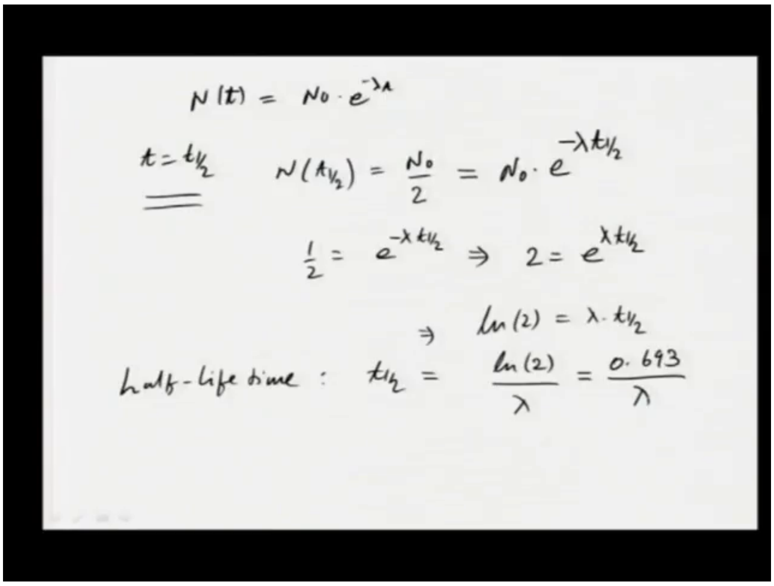

So, we have N t equal to N time t is equal to N 0 exponential minus and if in particular we take t to be equal to say some t half, let me denote t equal to t half or some particular time and N t half. So, t half is such that N t half is equal to say N 0 by 2. So, we started at time t equal to 0 we had N 0 partial a nuclei, and after t half time we have N 0 by 2particles or nuclei in the sample, but according to the exponential law this is related to N 0 as N 0 times exponential minus lambda t 1 by 2.

Right both sides we have N 0. So, we can actually divide by N 0 on both sides, and you will get 1 over 2 equal to exponential minus lambda t by 2 or invert this take 1 over on both sides you will get 2 is equal to exponential plus lambda t half, now we take the logarithm on both sides. So, that will give you log 2 is equal to lambda t half. This time needed to reduce the number of nuclei from N 0 to N 0 by 2 is called half life of that particular nucleus.

So, half life t half is equal to logarithm of 2 divided by the decay constant lambda, which logger 2 log 2 can be written 9 6 3 truncating it up to third decimal place divided by lambda. So, half life is a measure. So, this is a probabilistic this thing as we said we cannot talk about the time that one particular nucleus will stay alive, but in the sample we can say that the number of the time taken by the sample to reduce its number or the amount by half, this is called the half life of the sample. So, that is equal to logarithm of 2 natural logarithm if 2 divided by the decay constant lambda all right.

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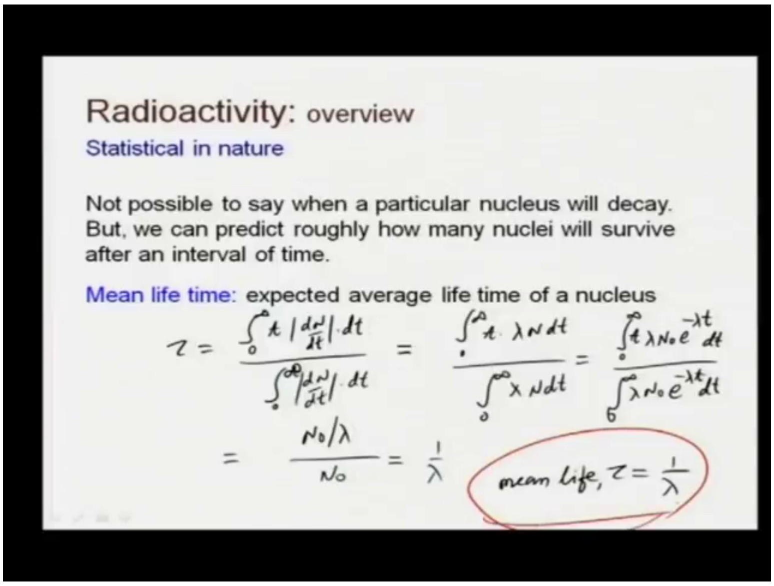

So, as I said it is statistical in nature not possible to say when a particular nucleus will decay, but can predict how many nuclei will survive or decay after a particular interval of time. We can also define what is is called the mean lifetime, which is essentially the average time of the lifetime of the nucleus which is defined as tau which is equal to integral 0 to infinity t d N by dt, the rates which is it will decay times the time interval around the time t divided by integral 0 to t, d N by dt dt. If you wait up to infinity, this can be written also as 0 to infinity t d N by dt is nothing, but lambda N dt divided by 0 to infinity lambda N dt is equal to integral 0 to infinity t lambda N at time t is nothing, but N 0 times exponential minus lambda t exponential law into dt divided by 0 to infinity.

Lambda N 0 e power minus lambda t do. So, if you do the integration we will leave it as an exercise for you the numerator will turn out to be N 0 over lambda simple integration you can use integration by parts and the denotinator is; obviously, lambda N 0 are constant and then integral e power minus lambda t dt from 0 to infinity will give you one over lambda. So, this will simply give you N 0 and therefore, the whole thing is just one over lambda. So, we can say the mean lifetime tau is equal to 1 over lambda right. So, this is this inverse of the dk constant.

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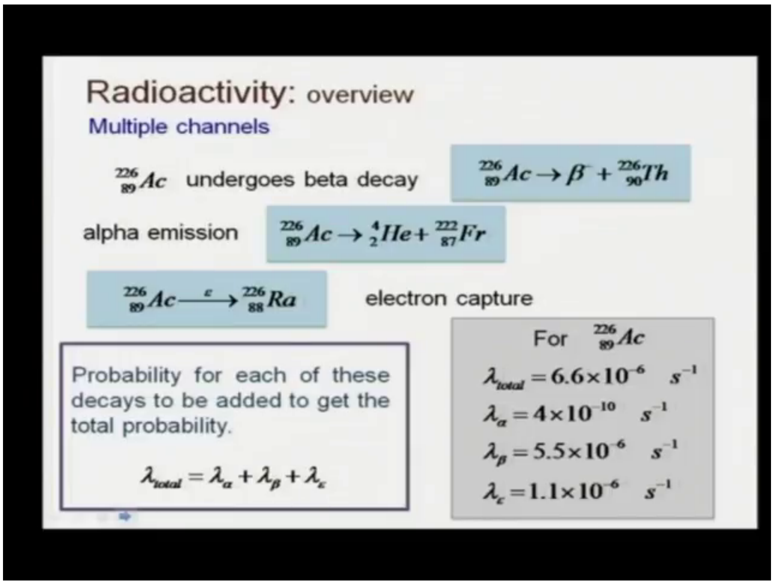

The now let us look at some other thing a particular nucleus we said decays radioactivity, but it can actually decays either through beta decay it can also possibly decay to other channel other type of radioactive decays can undergo. Say for example, if you take actinium which has 89 protons and with atomic. So, the mass number 226 if you take this isotope of actinium, it is observed that it beta decays to thorium 90. And it also undergoes alpha emission to francium with 87 protons in it, not only there it can also transform to radium by electron capture. So, all these 3 types of decays are possible by actinium.

Now for such cases where there are multiple channels multiple possibilities of decays is possible. So, the to get the total probability, you add all these probabilities which means you have to add lambda alpha which is the probability per unit time of alpha decay of actinium, add probability of beta decay of actinium lambda beta and add the probability of electron capture lambda epsilon of the actinium, and the total of these will give you the total probability of the decay of radioactive I mean radioactive decay of actinium. Again remember that we are talking about probabilities for these decays. So, when we concentrate or focus on a particular actinium nucleus it will decay either through beta emission or through alpha emission or through electron capture or it may not decay for a long time also. But if you take one thousand or some amount of large number of this actinium nuclei then you will see that the ratio the number of decays, alpha decays and the number of beta decays and number of epsilon capture the electron capture are in these proportions though the probability for these to happen are in this proportions. Actually these are these probabilities are experimentally measured by looking at the intensity of each of this decay process that alpha ray beta ray and waste word. So, look at the amount of the number of electron capture.

That can happen all right now in this particular case of actinium experimentally that is found that the total decay probability is 6.6 times 10 power minus 6 second inverse or decays per second. So, in one million actinium atoms and nuclei then in one second 6.6 of this will decay or in 2 seconds 13 of these will decay. Beta decay probability is about 5.5 times 10 power minus 6, which means out of this 6.6 which decay in one second from a sample of 6 million sorry one million actinium.

5.5 of this is basically beta decay and 1.1 decays through electron capture or in 2 seconds 2 of these will decay through electron capture eleven of this will decay through beta decay and the total of 13 will decay, and its expected that none of these will decay in 2 seconds through alpha decay. Because alpha decay probability is 4 orders of magnitude smaller compared to this other 2 beta and electron capture probabilities. So, if you want to observe the alpha decay you need to actually have take a much bigger sample or wait for a longer time.

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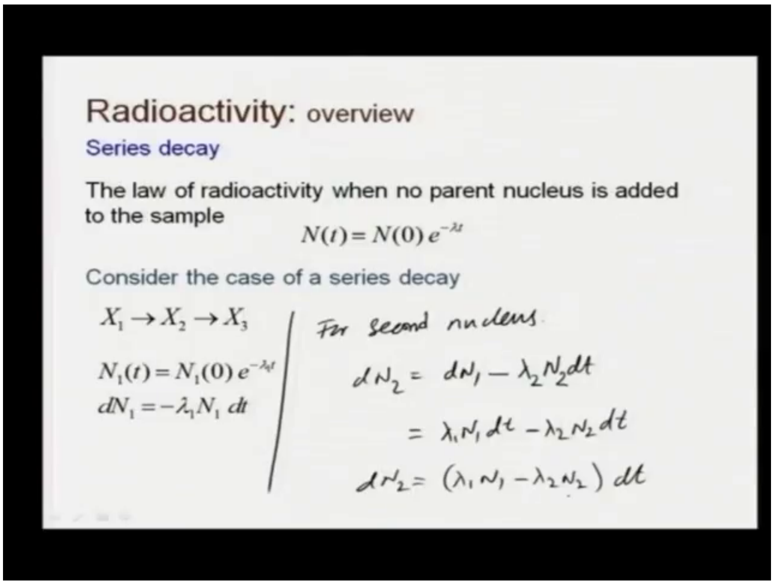

Now, we have another possibility, the law of radioactivity that we considered that when one particle decays.

To another particle we said the number of particles present in that sample at time t is equal to number of particle presented in that sample at time t equal to 0, times exponential minus lambda the time elapsed t. This is true as long as no parent nuclei are added to this, if you add more particles to it then this decay there is a number of particles present at a time t will not be this. So, let us look at such situation say consider in case of a series decay, which means you take the nucleus of type one denoted it by X 1 which decays to X 2 and X 2 is not stable it decays further to X 3 and for a given sample of X 1 you start some sample of X 1 at t equal to 0 pure X 1 no X 2 or X 3 in it and you do not add any more X 1 type of the X 1 nuclei into it. So, you just leave it there what happens X 1 will decay into X 2. This follows the radioactive exponential rule that we spelt out earlier N 1 at some time t is equal to N 1 at t equal to 0 times exponential minus lambda 1 t correct. And the number of decays that happens in dt time interval is d N 1 equal to minus lambda 1 N 1 dt. Now for the second nucleus let us see what happens; d N 2 is equal to correct d N 1.

This would have been the case if X 2 is stable whatever N 1 X 1 that decays he is converted into X 2. So, d N, N 1 which is the number of X 1 type of nuclei decayed N dt seconds dt time, is also equal to the number of X 2 type of nucleus present in that sample if X 2 is stable. If X 2 is not stable you have to subtract the number of N 2, X 2 which will decay to X 3 which follows the radioactive law which is lambda 2, N 2 dt. Let me repeat that since su is not stable you have to subtract lambda 2 N 2 dt which is the number of X 2 which is decay to X 3 all right which is equal to d N 1 is lambda 1 N 1 right in time dt minus lambda 2 N 2 the time dt or I can write d N 2 as lambda 1 N 1 minus lambda 2, N 2 dt. Note that I have not added minus sign, when I wrote d N 1 because here we are considering not the number of d N which are decay it is true that numerically it is equal to that, but we are actually considering the other way around saying that what is the number of d N 2 which is the number of X 2 type of nuclei which are added to the sample which are produced. So, which is a positive quantity. So, lambda 1 N 1 dt number of X 2 type of atoms nuclei added to the sample in dt type.

So, total change in the number of X 2 type of nucleus in time dt is therefore, lambda 1 N 1 the number added minus lambda 2 N to the number decade all right.

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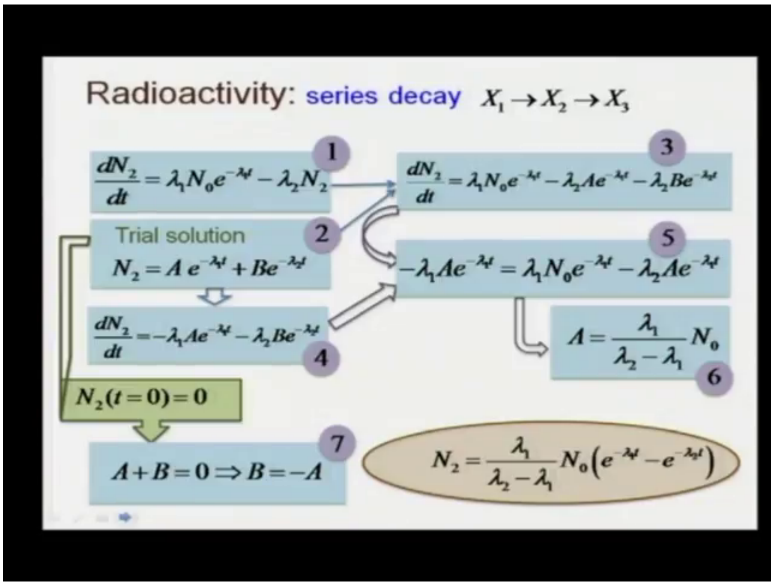

So, we can actually write this as d N 2 by dt is equal to lambda 1 N 1 now N 1 I can write as N 0 e power minus lambda 1 t single exponential now minus lambda d N 2 correct. So, to get N 2 solve for N 2, we will actually put in a trial solution let us assume let us consider N 2 equal to A some constant times exponential minus lambda 1 t it looks like from equation 1 that it has some term which is like exponential minus lambda 1 t.

Then plus B exponential minus lambda 2 t, this is again expected because there is N 2 in equation 1 and N 2 itself is expected to also have a decay part right it decays to X 3, the X 2 type of nuclei decays to X 3 and therefore, exponential minus lambda 2 t kind of term is also expected in this. So, we will take this trial solution we do not know how much of this the weight factor A and B are, but we will find that now putting these 2 together. So, take out a trial solution equation 2 put this back in equation one the second term, that will give you d N 2 by dt the first equation, d N 0 by dt is equal to first term intact lambda 1 N 0 exponential minus lambda 1 t.

Second term is minus lambda 2 N 2, but N 0 is given by equation 2 therefore, second part of equation 1 has now 2 terms lambda 2 a exponential minus lambda 1 t and lambda 2 B exponential minus lambda 2 t is called this equation 3. We can also get d N 2 by dt directly from equation 2 our trial solution, that will give differentiating the equation 2 will give you d N 2 by dt now our left hand side and right hand side is minus lambda 1 a constant times exponential minus lambda 1 t minus lambda 2 B exponential minus lambda 2 t. Now equation 3 and 4 the right hand side left hand side is the same we are talking about the rate of change of N 2 that is the number of X 2 type nuclei.

At some time t in an interval dt how many of them will change the rate of change at some time t. So, the right hand side should be the same right and right hand side in equation 3 has 3 terms equation 4 has 2 terms. So, and the second term in equation 4 and the third term in equation 3 are the same lambda 2 B exponential minus lambda 2 t. So, we when we equate the right hand sides of 3 and 4 we drop this because it is exactly the same and equate the rest that is minus lambda 1 A e power minus lambda 1 t is coming from equation 4, the right hand side of equation 4 that should be equal to lambda 1 N 0 exponential minus lambda 1 t, minus lambda 2 a exponential minus lambda 1 t coming from equation 3. So, from these 2 we can we get.

So, all of this as exponential minus lambda 1 t as common, take that away and then you have A equal to lambda 1 over lambda 2 minus lambda 1 times N 0 right. Now this is one thing another thing is also we got A, but what about B, to get B we will look at another property which is basically that, we took X 1 only that is we took a pure sample of X 1 type of nucleus to start with. So, at time t equal to 0, N 2 type of X 2 type of nuclei was not present which means N 2 t equal to 0 is equal to 0. So, from trial solution we get A at t equal to 0 N 2 is A plus B which is equal to 0 means a plus b equal to 0 or B should be equal to minus A.

So, we get both A and B in fact, A and B are simply related with each other by a minus N and N 2 can be written therefore, as lambda 1 over lambda 2 minus lambda 1 times N 0 into exponential minus lambda 1 t minus exponential minus lambda 2 t. So, this is the case of a series decay of X 1 to X 2 to X 3. You can actually consider the exactly similar way other series expansion or the any in number of such series decays like X 1 to X 2 X 2 to X 3, X 3 to X 4, X 4 to X 5 up to wait for N and simply follow this algebra.

And then you will be able to get the number of an expression for the number of each type of this nuclei at some time t, which can be related with the N 0 which is the number of nuclei of first type pure sample of N 1, X 1 and with the decay constants of other the decay constant of the nuclei involved.

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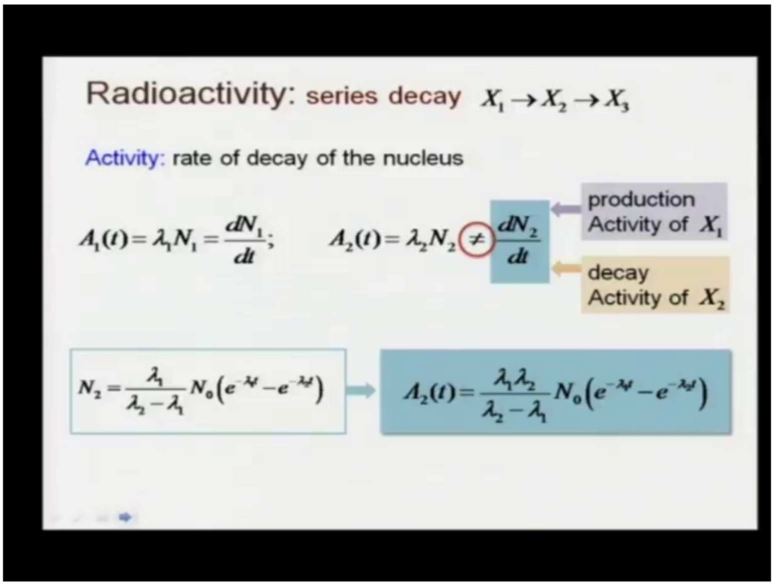

Rate of decay in this case is slightly different I mean you cannot write it simply as lambda 2 N 2. Say in the case of X 1 we can write the activity or the rate of decay as A 1 equal to lambda 1 N 1 because it only decays nothing is added to it whereas, in the case of X 2 type of nucleus, we have A 2 equal to lambda 2, N 2 which is correct that is the activity that everyone, but that is not equal to d N 2 by dt because d N 2 by dt has 2 parts in it, one is due to the production of X 2 type of nucleus coming from decay of X 1 and the other is reduction because of the activity of X 2.

So, but A 2 can be written as lambda 2 N 2, because the radioactive law always says that activity the number of particle which is dk is proportional to N 2 number of particles present that is the kind of observation that we have. So, N 2 is equal to lambda 1 over lambda 2 minus lambda 1 times N 0, times exponential minus lambda 1 t minus e minus lambda A exponential minus lambda 2 t and from this activity is lambda 1 lambda 2 by lambda 2 minus lambda 1 times N 0 into exponential minus lambda 1 t, minus exponential minus lambda 2 t.

With this will stop today is discussion we will go on with some other properties of the radioactivity in the next lecture.