Nuclear and Particle Physics

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Module – 03
Nuclear Models
Lecture – 02
Shell Model, Collective Model

Today, we will continue our discussion on the nuclear models.

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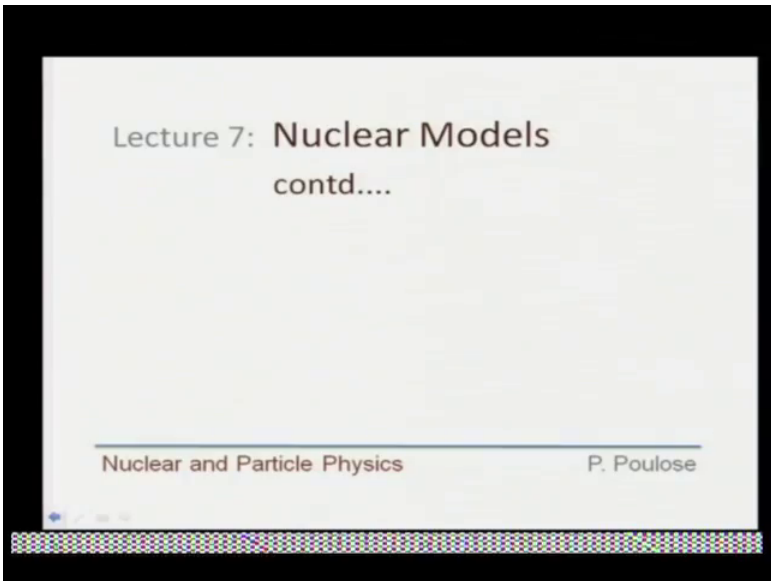

So, in last class we had studied some the shell model and we said shell model is a good description of the nucleons, especially when it comes to odd-A nucleons nucleons with odd atomic number which means either the proton or the neutron number is even, while the other is odd.

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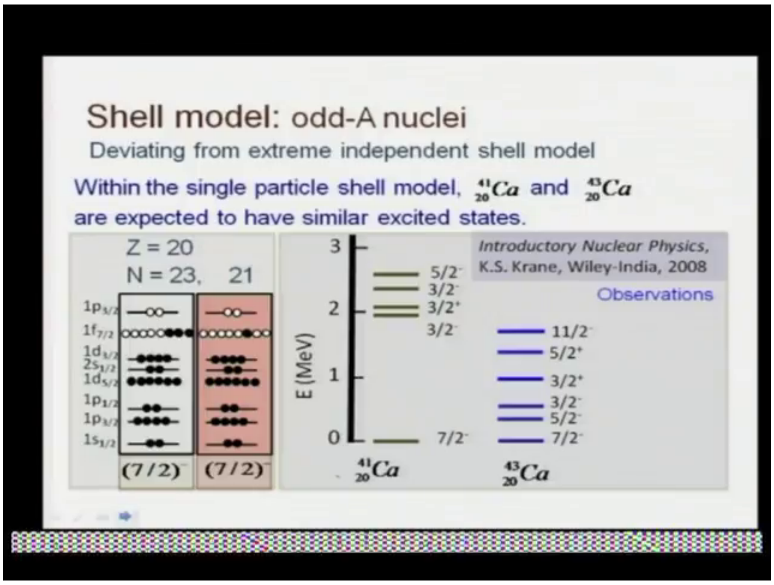

And in the extreme independent single particle shell model we said all the properties of the nucleon nucleus like this is described or given by the unpaid single neutron or proton in the particular nucleus under consideration. Mean, which also says that all the neutrons and protons try to pair with each other which means that if there are 2 neutrons in the nucleus they will pair with each other and there will not be any kind of they will try not to interact with the outside world or outside the system and become an kind of an inert this thing.

So, if there are 11 neutrons and then 10 neutrons are paired, and there is 1 neutron which is left without any pair and this neutron will dictate the properties of the nucleus concerned if the protons are unpaired already paired with each other. We saw that not only that the spin and unger momentum of the sorry, the spin and the parity of the nucleus in the ground state that can be explained very well in such cases with odd neutron odd atomic, so odd mass number.

And, but if we look at the excitation states or excited states of these nuclei that also can be very well accommodated within this shell model, in its extreme independent version. And towards the end of the discussion we had this last time we said in some cases some of the excited states cannot be accommodated just considering 1 neutron or proton, but we may actually think about say 2 or 3 neutrons placed in different shells and take the combined effect of these as we saw in the case of oxygen and fluorine in the last time.

Again this, we will continue with this discussion and then we see that there are some limitation certainly with these extreme independent version of the shell model because it is 2 can restrictive. It is not very realistic to think that only one particular nucleon which is unpaired is causing or the properties of the nucleus. So, we will take up an example for here which clearly illustrates that we will have to do slightly better than this extreme particle extreme independent shell model.

So, for example, if you take a calcium 41, the calcium has 420 protons in the nucleus and depending on its isotope it will have 40, 41 or 43 or some other number on that total number of nucleons in it. So, if you particularly take calcium 21, it has 21 calcium 41 sorry, there are 21 neutrons in it apart from 20 protons and if you take calcium 43 it has 23 neutrons in it. This proton distribution if you look at the shell model the 20 is a magic number for us we know that from experiment as well as our model also can predict that, corresponding to this 20 there is a closed shell and therefore, the protons are completely filled in that closed shell and then we do not expect them to be actively participate in any other any interactions or produce or give rise to any particular property of the nucleus in that fact.

So, now this distribution of the neutrons will be considered calcium 20, 43 has 23 neutrons which are distributed in these shells and 20 of them are in the inert core with the magic number filling the magic number 20 and extra 3 neutrons are placed in 1 of 7 by 2 when we consider the woods action potential with spin orbit coupling in nature. And if you look at the calcium 41 it has a similar kind of distribution proton exactly the same kind of thing as the calcium 43. For neutron which will distribution there are 21 neutrons and 20 neutrons are again placed in the inner core filling the magic the shells with magic number 20 and one extra neutron is placed in 1 f 7 by 2.

So, if you look at calcium 43 and 41 then you see that in each case there is one unpaired neutral in the 1 f 7 by 2 and we will imagine that this is reflected in its spin parity assignments of its ground state as well as some of the at least low lying at least the first few excited states, right. The prediction for the ground state IP the spin and parity is 7 by 2 minus in both case.

And let us see what is the kind of observation that we have regarding the excited states. When we look at the excited state we will see a little different picture then what we actually would expect compared to within the shell model single particle model. In the case of calcium 41 we see that there is one extra nucleon which is their outside this magic number closing the inner inert core and it is a, and the excited states like there is first excited state 3 by 2 minus which can be very well explained by considering the extra nucleon, the neutron absorbing the energy or taking the energy and jumping to excited to or going to the stage p 3 by 2 p 3 by 2 has l equal to 1 therefore, odd parity state and j is 3 by 2 therefore, the spin of the nucleus corresponding to this is expected to be 3 by 2 and then we see that there is a 3 by 2 minus state.

And then there are other states like 3 by 2 plus etcetera which maybe you again accommodated by saying that the 1 d 3 by 2 breaks the pair and then take it to 7 by 2. So, that the neutron in the 7 by 2 f 7 by 2 is paired and you can half you half now an unpaired neutron in the 1 d 3 by 2 and therefore, it is in the nucleus is in the i equal to 3 by 2 and p positive corresponding to d shell which was illegal to 2 state 2 which is ok. But when we look at calcium 43 case should have been similar if all is well or if we agree with the single particle shell model there is one unpaired electron neutron in the 1 f 7 by 2 which could be taken to 1 p 3 by 2 as the first excited state.

But and the energies should also be the similar to excite 1 neutron from 1 7 by 2, 1 f 7 by 2 to 1 p 3 by 2 there is no breaking of pair in either case as per the model was it is just taking that particular neutron in 1 f 7 by 2 into 1 p 3 by 2 in both cases. But contrary to this expectation we see a very low lying less than half of the excitation energy corresponding to calcium 41, calcium 43 has very low lying 3 by 2 minus, but that is not the first excitation excited state either there is 5 by 2 minus even before that how do you accommodate that and there are not only these, but many other low lying excited states 3 by 2 plus 5 by 2 plus 11 by 2 minus etcetera even before you reach to MeV which is needed to break a pair of the nucleons inside the nucleus as expected.

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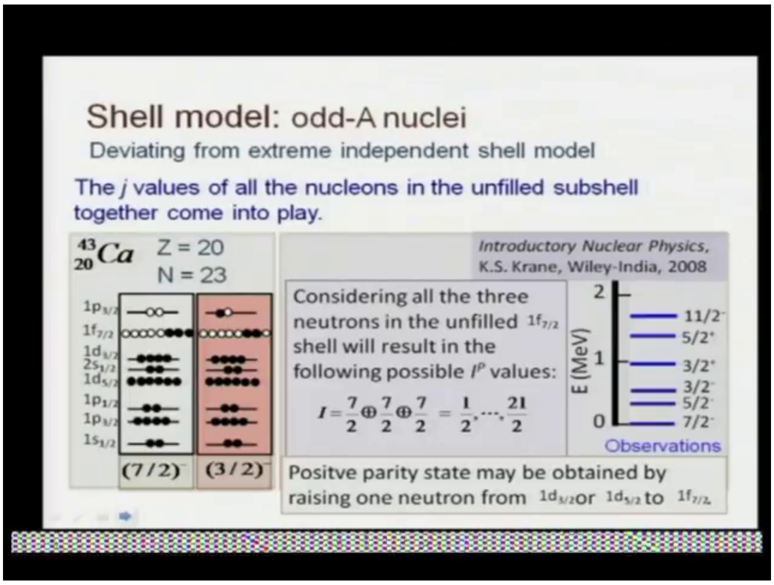

So, we need to actually modify it a little our model. And let us say we want to actually consider the case not just of the single neutral which is unpaired in that outer shell unfilled complete not completely filled or partially filled shell. But we will take let us take all the elect, all the neutrons in that partially filled 1 f 7 by 2. In this particular case of calcium 43 there are 3 of them and we will consider all the 3 of these things ok.

And then again excited state will be could be 3 by 2 minus as I said earlier and the absurd ones are like observations are like this, but then the expectation with this new arrangement or new interpretation that for the relaxed session that is made to the shell model. That, it is not that the 2 of them 2 of the nuclei in the outer shell one of as 7 by 2 is paired, but all the 3 of them are treated at the same kind of footing and we can consider the effect of all these 3 together.

So, basically when we compute the angular momentum total angular momentum of the nucleus then you will have to consider all the 3 neutrons which means there is a 7 by 2 plus 7 by 2 plus 7 by 2 that you had to add according to the angular momentum addition rules. And then that will give you let us see what it will give you basically you are adding 7 by 2 and 7 by 2 that will give you 0 1 2 3 up to 7 because 7 by 2 plus 7 by 2 the maximum is 7 and 7 by 2 minus 7 by 2 is the minimum when you add this vectorially or as per the angular momentum addition rules that will give you a 0 and with each of these you have to add now I have 7 by 2.

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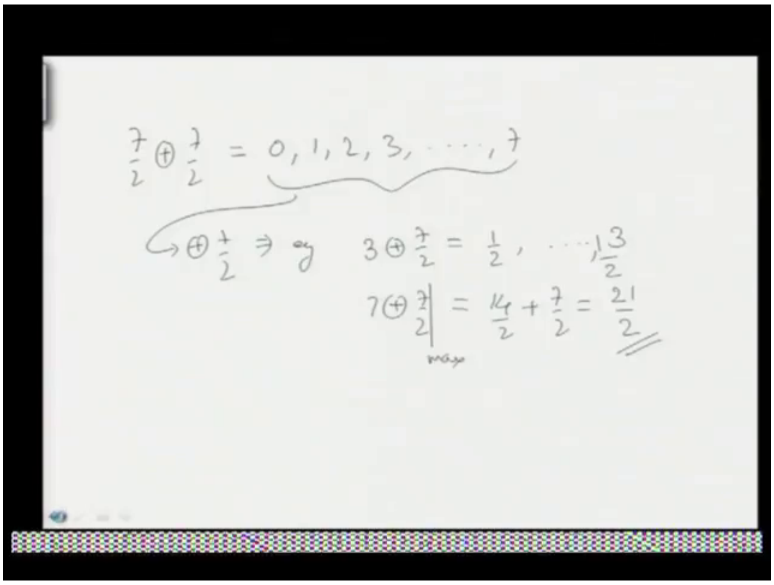

So, that will give you say for example, you take 0 and then consider this that will give you 7 by 2, 1 and 7 by 2 that will give you 6 by 2 sorry that will give you 5 by 2 and 7 by 2, 2 that will give you 2 minus that is equal to 5 3 by 2, 5 by 2, 7 by 2 and like that.

So, when you consider 3 for example, you will get 3 plus say for example, 3 by 2 plus 7 by 2 will give you sorry not 3 by 2 it is 3 into 3 plus 7 by 2. So, that will give you either minimum value is half and the maximum is 3 plus 3 by 2 is 10 by 2 10 and half it is 21 by 2 right and there is a 7. So, 7 is basically, so this is, this will give you actually the minimum and you have now 7 plus 7 by 2 that will give you 14 by 2 plus maximum is 14 by 2 plus 7 by 2 is equal to ok.

So, this is the absolute maximum that you can get from this 7 by 2 plus 7 by 2 plus 7 by 2 and in the case of minimum it is 1 by 2. So, you can say that with this combination you will have basically minimum of 1 by 2 and a maximum of 21 by 2. And obviously, all of this what excited states can be accommodated within any of this. So, the excitation energy is not really taking the electron from somewhere to somewhere some higher state, but within that it is actually getting a higher energies all right and so this is kind of slightly a different approach that you have to consider compared to the extreme independent shell model.

Now, the positive parity other of these positive parity system. So, maybe accommodated by considering a 1 d 3 by 2 or 1 d 5 by 2 breaking a pair and then going into that, but the catch there is that it these are very low lying and the energy needed to break one pair of such a nucleon pair neutron pair is about 2 and then you would expect an excited state with 3 by 2 plus or 5 by 2 plus have with energy around to maybe a minimum at least 2 by about 2 or maybe. Whereas here it is much less than that for 5 by it is about half of that for 3 by 2 and 5 by 2 has little more than that. So, we need certainly to go beyond this shell model kind of a description to accommodate everything even within the excited states, I mean even within the observations of excited states.

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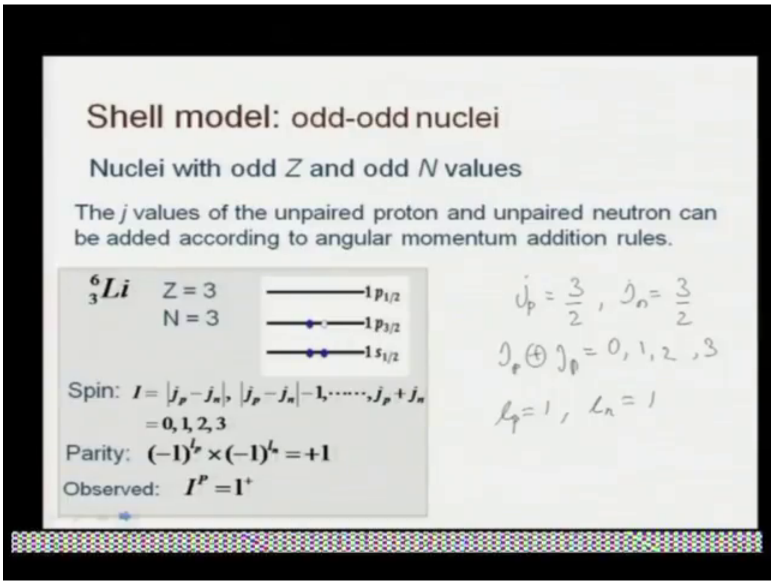

Now, what about the odd a nuclei with the z number of proton odd z number and odd n values? In this case we had to come to some kind of a conjecture then it could be that a proposal could be that the j values of both the unpaired you know each case when we consider the proton and neutron there as one unpaired proton there is one unpaired neutron. So, considered the total j value of both of these and then consider that as the total energy I mean angular momentum of the nucleus.

Say for example, in the case of lithium 6 you have 3 proton, 3 neutrons both protons and neutrons are in the same I; 1 p 3 by 2 states the unpaired single neutron and you can ask the question what is the spinner value. Spin as we said is a combination of, so j p here is equal to 3 by 2, 3 by 2 and j n is also equal to.

Now, 3 by 2 and 3 by 2 has to be added. So, j p plus j n will give you 3 by 2 minus 3 by 2 is 0 which is the minimum and 3 by 2 plus 3 by 2 is 3 which is the maximum and in between there are 1 and 2. So, the possible lye values are 0, 1, 2 or 3. And parity, parity has to be is a multiplicative quadrant number. So, what you has to consider is, what is the angular momentum of this single unpaired proton which is equal to 1 because it is in the p state and l n is equal to 1 again because that is also the neutron is also in the p states.

So, when you take minus 1 power l p in that give will give you 1 and minus 1 power l n that will also give you minus 1 and together when you take the product of these 2 that will give you a positive these one all right. And what is observed you could ask the ground state is all right, it is observed with spin 1 and parity plus which is accommodated within this. We are one of the possibilities.

Another example that you can take in the same similar system situation is aluminum 26. So, it has 13 protons and 13 neutrons in it. So, we can ask the question again what are the spin values let us see.

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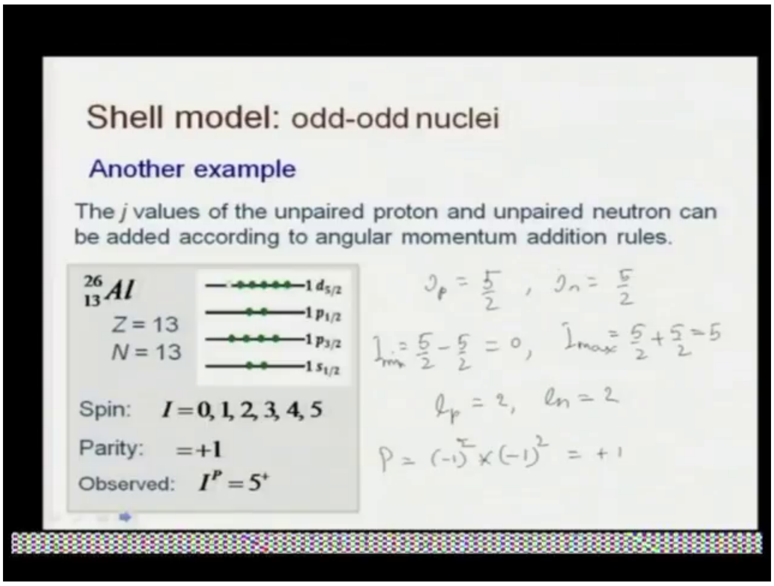

So, it is j p equal to 5 by 2 it is in the 1 d 5 by 2 state and j n is equal to 5 by 2 because you have again 13 neutrons and 5 by 2 minus 5 by 2 is the minimum i, which is equal to 0 I max is going to be equal to 5 by 2 plus 5 by 2 and that is equal to 5. So, I the possible I values are 0 1 2 3 4 and 5, and parity l p is equal to 2 and l n is also equal to 2 that will give you parity equal to minus 1 power 2 into minus 1 power 2 which is equal to plus one right. So, parity is plus 1 observed. I p is 5 plus well 5 is are only there in that that is simulated.

How does one prefer 5 instead of 4 3 or 2 or 1 or 0? We need to look at further details about this thing the wave function etcetera, and then it is slightly beyond the discussion that we have here. And I think we will not go into the details of any of this. Actually for those of you are interested certainly there are enough books in loading the text books that we are considering giving a lot of details of calculations and various other examples are also discussed.

Let us go on and then consider the next of the other kind of nuclei with again even a, but this time even Z and even N, both N and Z are even.

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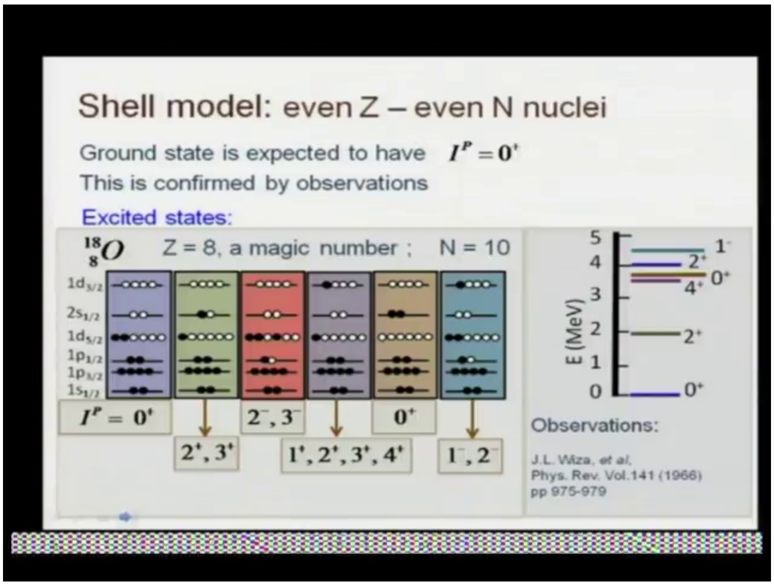

What about that? As per the extreme independent shell model with single particle unpaired neutron dictating the properties of the nucleus we expect I p to be 0 and positive because there is no one paired neutron no unpaired electron. Well, and if you look at more the even even nuclei most of them agree with this I think almost all of them which are observed agree with this and that is its confirmed.

And let us look at the excited states say consider the example of oxygen 18 Z is 8 which is a magic number and therefore, we think that will not be coming into play when the properties of the nucleus is concerned especially in the case of excited states, but neutron number is 10 this case. So, the configuration is 2 neutrons in the lowest one is half, 4 neutrons in the next 1 p 3 by 2 2 in the 1 p half completing the inert core with magic number 8 in a core.

And then you have 1 d 5 by 2 which has 2 electron, 2 neutrons in it, I p is anyway 0 there are no unpaired this once. How do we excite this? One way to excite it is 2 split the pair in the outer shell in complete shell 1 d 5 by 2 and take that to the next take one of those neutrons to the next level which is 2 s half. So, you have here you have here 1 neutron in s half and 1 neutron in d 5 by 2. Similar to the additions that we had done in the previous slides right, we will see that a combination of 1 by 2 and 5 by 2 will give you 2 or 3. So, that will give you a 2 or 3. And what about parity? Both are in the l equal to 1 is in the l equal to 2 state, the other is l equal to 0 state minus 1 power 2 will give you 1 plus 1 minus 1 power 0 will again give you plus 1 and product of these 2 will give you plus 1. So, the parity is positive.

So, we expect first excited state to be either in 2 plus or in 3 plus or all right. What other possibilities are there? Well, one could actually break to protons in the 1 p half although that combines the magic number shell 8 we can break one there giving lot of energy and then take that to the 1 d 5 by 2. So, that will give you 2 minus 3 minus 2 because it is again j equal to 5 by 2 and j equal to half states similar to the previous case, but in this we have one of that is a odd parity level. So, that combination will give you an odd parity or we could actually take the 5 by 2 break the 5 by 2 neutron again and then take it to not s by s half where do you have enough energy to go up to d 3 by 2 say that case it will be in 5 by 2 5 by and 3 by 2 combination will give you 1 2 3 or 4 all right, and parity is positive because both are even parity states.

Another thing could be that both of these 2 neutrons in 1 d 5 by 2 goes to 1 s half and then that will have a excited state with 0 plus perhaps. Well, you could also take the 1 p half break that take it to d 3 by 2 instead of d 5 by 2 or you could take it to actually s half you could take it to any of those things. So, the combination will here be either 1 or 2 minus. So, this is these are some possibilities. There are other possibilities as well that you can think about say for example, in this case we have in the last case we have actually jumped from one way, 1 1 s half we could actually go to s half instead of d 3 by 2 and there it will be 0 minus and 1 minus, all right.

Let us see what we have observed. This observation result is taken from this particular reference given here and the observation is that indeed the ground state is 0 plus there is around 2 MeV there is a 2 plus excited state and then there is a 4 plus and 0 plus 1 minus 2 plus etcetera. So, one thing is that we do not see any 2 minus 3 minus in the low in the first few excited state all right and 2 plus is; obviously, could be accommodated very well within this. But then and it also corresponds to breaking energy is 2 MeV which is all right, but then the other states are all around 4 or 3 and half 4 MeV what is special about this 2 plus I mean we do not see anything special that we could accord it to this here ok.

So, now there is, this suggests that again we have to slightly go beyond the single particle shell model and accommodate other aspects of this.

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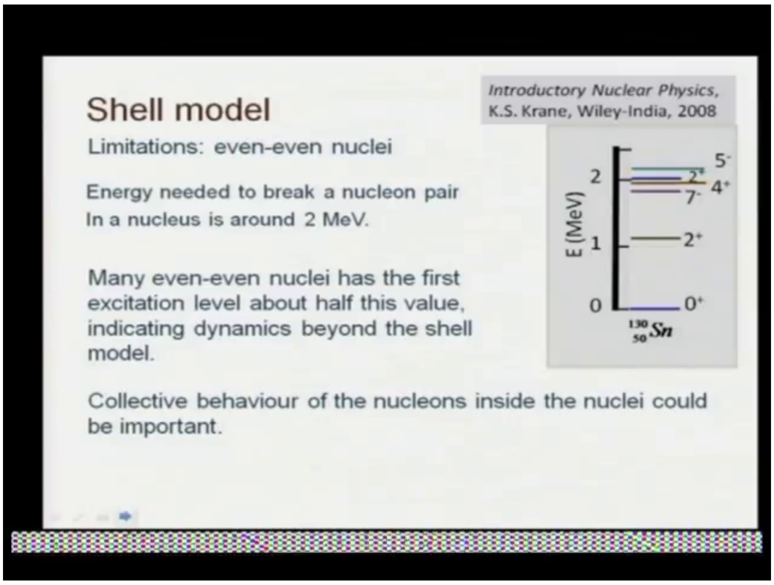

When we actually consider other even even nuclei, many of these nuclei even even nuclei, nuclear with even broad number of protons and even number of neutrons in it they all have the same kind of feature, they have all one single 2 plus state at low energies. So, in the case of tin for example, even at one day maybe which is half the energy needed to break a nuclear nuclear pair and there are lot of many many states around 2 MeV, but this is a single isolated state say first excited state very low lying excited state and this is kind of a common feature of many such even even nuclei.

So, certainly when has to actually to accommodate this thing one has to go beyond using and one may people think to accommodate all this is to consider some apart from the either along with the single particle in the shell model description you consider also effect of other nuclei some kind of a collective behavior of these nucleus. Or for the even even nuclei in fact, we will work without this shell model considered only kind of collective models. So, in such cases what happens is that the excited state will have a other modes to be considered collective modes to be considered we will come to that in a little moment.

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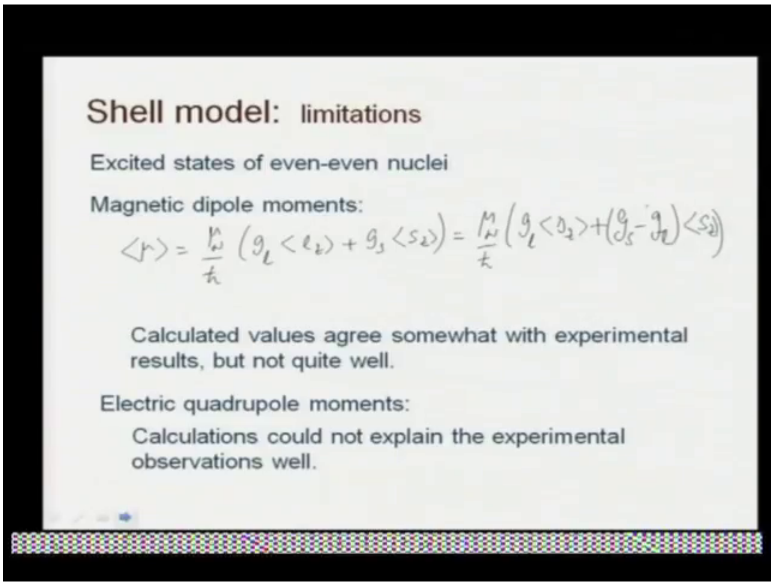

And then there is there are other properties like the magnetic dipole moment, quadrupole momentum, electric quadrupole momentum etcetera what are the predictions of these there? There again when as limited predictability or limited explanation that the shell model can give for example, the magnetic dipole moment mu is equal to the nuclear magnetron divided by h cross and g l a times expectation value of l z plus g s times expectation value of e s z this. But remember we have included the l s coupling in our model.

So, l z and s z are not really a good really the good quadrant numbers them in the system will or have any definite l z physical states will not have any particular l z or l z values rather it be it will be having the total angular momentum j and j z values. So, we have to rewrite this in terms of j and j z. So, basically you can write l z as j z minus l z. So, this will turn out to be, so there now finally, this will be j s into l z minus s z when you write it in this fashion and sorry j z missed; and this j z is l z plus s z, so l z is j z minus s z. So, there should be g s minus g l term there and s z.

Now, s z itself is not a good quantum number, but you can write s z in terms of j l and s values. So, we will not go into the details of that, but you can do that yourself simple quantum mechanics. Now, one can actually for a each each of these cases each of these nuclei you can try to compute this mu expectation value of mu and compare it with the experiments. Then when we do that we see that they agree roughly, but not quite well. So, there has to be some kind of some kind of corrections that we have to think about additional inputs needed for needed to explain everything. So, it is roughly the same roughly the expected value, but not completely.

Similarly for the quadrupole moment when you go to the quadrupole moment what happens is a electric quadrupole moment you again you can compute this things we will not go into the details, but you will see that here it fails even, the predictions are even worse than the magnetic dipole moments predictions.

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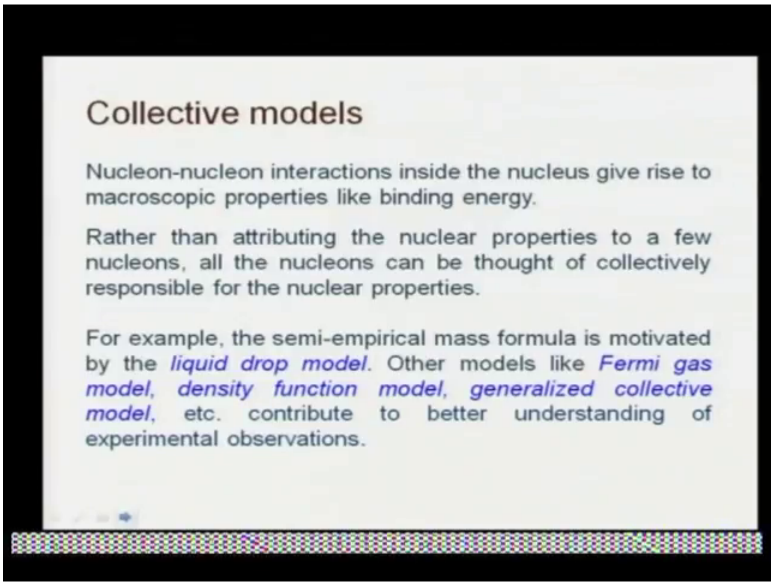

So, clearly one has to go beyond the shell model; obviously, I mean this was a simple very simple model. So, we are not surprised that in fact, we should be very surprised that lot of these properties considering the spindle parity of the excited states as well unexplained very well in this within the model and suddenly one has to accommodate other aspects of the nucleus into the model ok.

So, the collective models are should I you take a different philosophy, it says that it is not any particular nucleus nucleon is not special, soeverything is collectively giving rise to these properties. And say for example, the liquid drop model. In fact, when we discussed the semi empirical mass formula we had somewhat considered this kind of a nucleus, the nucleus which is a kind of a uniformly distributed uniform distribution of this. We did not really talk about nucleons inside that the granule the finer details of the these one we said that take it as a kind of a sphere or a liquid drop and then think about what is the density of that; what is the volume term that will enter in the mass or mining energy and what is the surface term that will come in there etcetera. We did not talk anything about anything about the nucleons inside, all right.

So, and the constant density which is experimentally observed can also be explained very well within the liquid drop model for example, but liquid drop model cannot explain all the excited states again and there are various problems with that. So, there are there are variations of this there are other models like Fermi gas model considering all the nucleons us independent objects like gas molecules non interacting gas molecules and confined in ammonium and you can work out the details of the this model and then see how it works; something which confines them to the model the particular volume is what the Fermi gas model will consider. Then there are other models generalized to collect to give model density function model etcetera I am just giving you the name. So, that you can go up and look up these models and then understand them yourself. We will not go into the details because that is beyond the scope of this course.

But we will say certainly that all these models will contribute better understanding of the experimental observables like observations. Like say the excitation states of even even nuclei, especially for the even even nuclei these collective behavior is more prominent compared to odd a nuclei or odd odd a sorry nucleons the nucleus.

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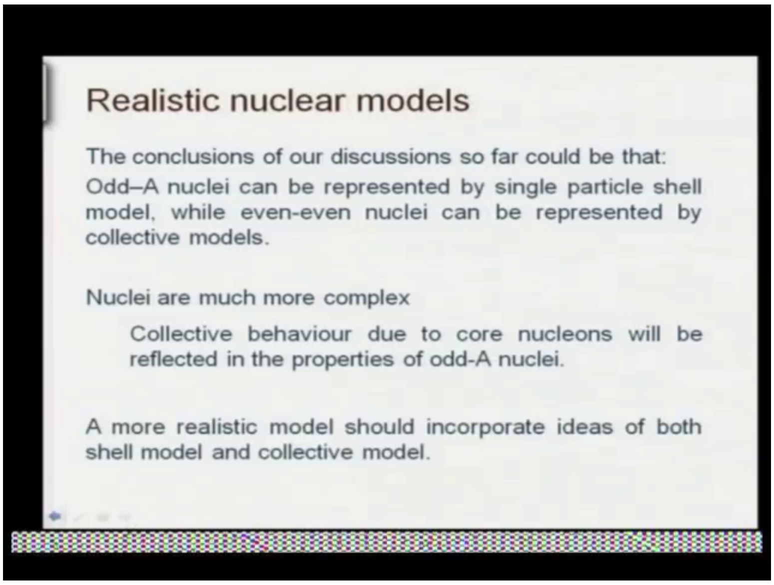

Then actually in a case of a realistic model one need to perhaps take combination of both of this thing some kind of a uniform approach need to be considered. One has to, I mean one cannot actually say that well odd nucleons there are I mean say let us say roughly we can categorize these two, one type of nuclei where we can consider the properties the model corresponding to that nucleus as kind of a shell model and the other kind of nuclei say even even nuclei considered with model like collective models.

Most of the time what you will have to do is to actually take combination of both in realistic models. Say for example, in the case of single particle shell model we do not really worry about what is the picture, what is the contribution of the nucleons in the completely shell completed shells. If you want to consider their role also then one way to think about is to consider some kind of an core inner core which collectively behaves like a collective body gives contributions as a collective body to the nuclear properties.

How does it give you some aids contribute to the energy levels? For example, you could think about vibrations of this collective body, liquid drop or a sphere which is not a solid sphere like a rigid sphere it can actually vibrate, it can also rotate. Well, a rotation if it is a completely spherical symmetry spherical symmetric body then rotation may not give you anything there is no axis of rotation. So, symmetry will tell you when you do not expect anything whereas, the most of the nuclei especially heavier nuclei are not completely symmetric as well as we know they deviate from sphericity and thus we can also think about rotational modes for this.

So, now, these, in this way we can accommodate some of the observations discrepancy that we had in the case of excited states of many nuclei especially in the case of even Z even N nuclei, even even nuclei may be contribution of these vibrational modes and rotational modes can give rise to this or can explain the disparities. Indeed these approaches are considered and it is found certainly that their vibrational and rotational modes certainly had to be taken seriously in the case of many modern many nuclei ok.

And they I mean people we can actually work out an explanation for many of the observations better than the purely the explanation purely based on shell model or collecting model that way. So, some kind of one realistic models what I like to actually consider kind of a combination of this thing.

We will not be able to go into the details of any of this thing ok. So, idea is basically to give you a flavor of the nuclear model and a starting point from where you could take off on your own. So, you can certainly go and then understand try to understand this in more details for.

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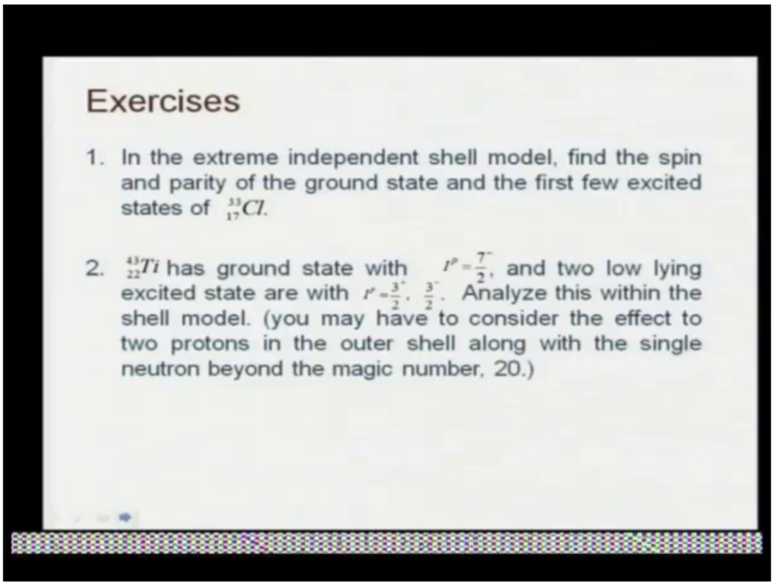

So, for the time being we will actually stop our discussion on the nuclear models here. We will enter with the 2 exercises that you could try on your own. There are many other exercises that you can think of also and also available in the textbooks that we have referred to.

So, let us consider these 2 questions one is that you take chlorine 13, it has 17 protons and 16 neutrons. So, find out the how in the an extreme independent shell model how do they does the electron and proton, so proton and neutron are distributed in different shells? Then what is your prediction corresponding to the in parity I, value the spin and the parity of the excited or the ground state the first few excited states etcetera you can easily work that out. And you look at Ti 43, 22 protons and 21 neutrons in it.

So, 21 is actually number one more than the magic number 20. So, you would I mean today's discussion is valid there saying that collective behavior is important there. So, you can check this out what are the values of these I values and p values the spin and the parity values for the ground state as well as exciter state that you expect. In fact, it is given that there is a low lying 3 by 2 plus and 3 by 2 minus how do you accommodate that within this that is what you can think about.

The hint is you can actually you also have 2 protons in the shell outside the just be just above the magic number closing, close shell with magic number 20. So, you may also consider these 2 protons going slightly beyond the single particle shell model approximation considering multi particle shell model. But, so, you can think about that in that direction. And then see if you can accommodate 3 plus and 3 by 2 plus and 3 by 2 minus in this thing.