Nuclear and Particle Physics
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Module – 03
Nuclear Models
Lecture – 01
Shell Model

Today we will discuss the Nuclear Models.

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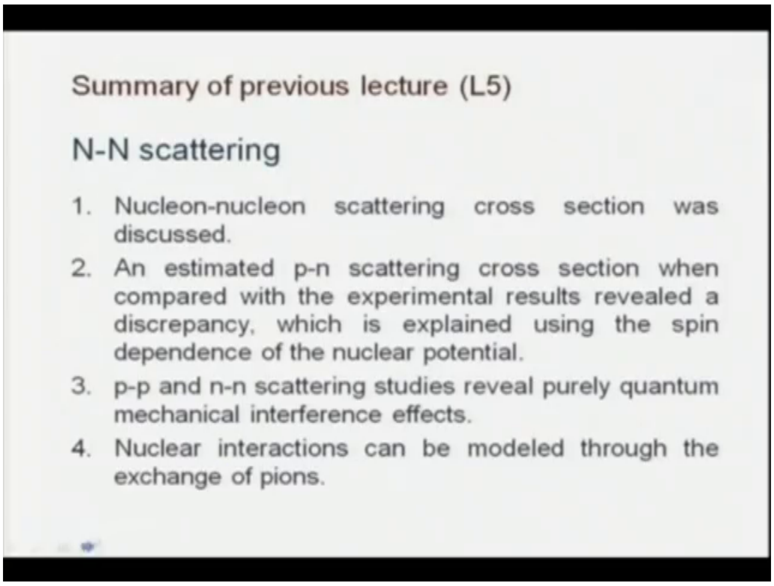

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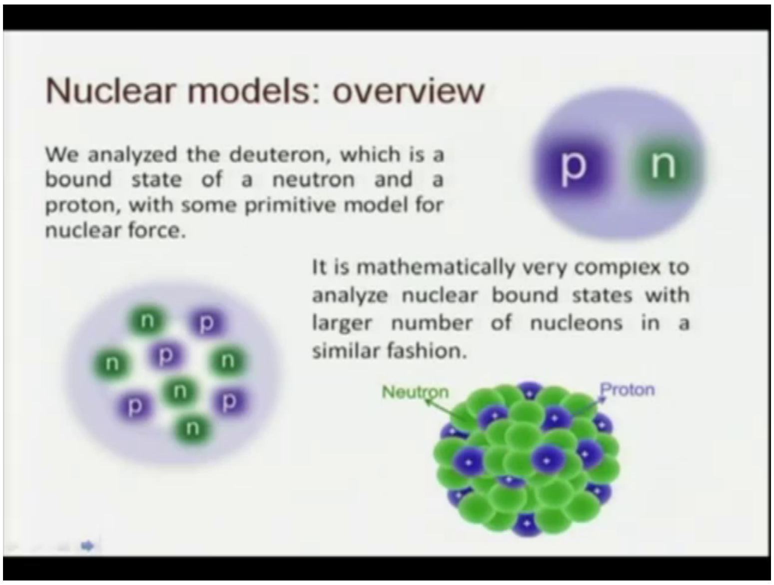

In our earlier discussions we had considered the bound state of neutron and proton called the deuteron; at least, understand the deuteron we considered square well kind of potential to represent the nuclear strong nuclear interaction between the neutron and proton. Solve the Schrodinger equation, and computed various quantities like the energy and various other observable, observables and found that we could explain the observed experimentally observed facts quite well.

And when we come to more complex nuclei with more number of neutrons and protons in it we immediately realize that this approach of trying to understand the nucleus from interaction of individual neutrons is not quite feasible. This is so, even for light nuclei like barium with 4 protons and 5, 4 or 5 neutrons in it. And it is unimaginably complex when it comes to iron or uranium or elements like that heavier elements like that where there are 90 to 100 neutrons protons and more than 100 neutrons etcetera in it. So, what we had to do is to take a different approach.

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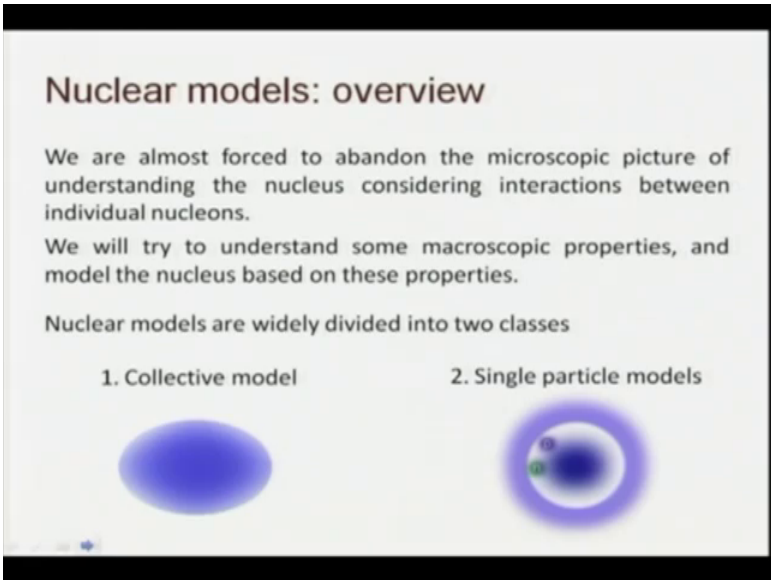

And this is not unknown to physics problems. For example, we know how to solve the quantum mechanically solve quantum mechanically the hydrogen atom understand it from the point of view of interaction between proton and electron. But when we consider hydrogen gas which has a lot of millions of hydrogen molecules in it or atoms in it we end up actually with not being able to understand this hydrogen gas from the fundamental interactions point of view, rather we take a different approach. We consider a statistical approach where we consider microscopic properties of the gas as a system.

Something similar to that is considered in the case of nucleus as well. So, we instead of trying to build the model for build a model for nucleus from individual interactions of the or interactions of the individual nucleons we consider some kind of macroscopic properties of this and then try to understand the nucleus from that angle. Here again there are widely two different classes one is actually not really going into the details of the structure of the nucleus, nucleons inside the nucleus, but collectively how they behave all of them together.

Like one of these liquid drops model so that we mentioned in the beginning of the course and other is to take the approach similar to the atomic cases where basically the properties of the atom are understood in terms of a few of the electrons.

So, in the case of nuclear models as well there are some models in which discusses the nuclear properties or tries to understand the nuclear properties from a few of the nucleons which can be considered separate from the rest of them. We will come to that there. In fact, we will actually discuss this single particle kind of model in detail to start with and then touch upon the collective behavior or collective models towards the end of this set of lectures, ok.

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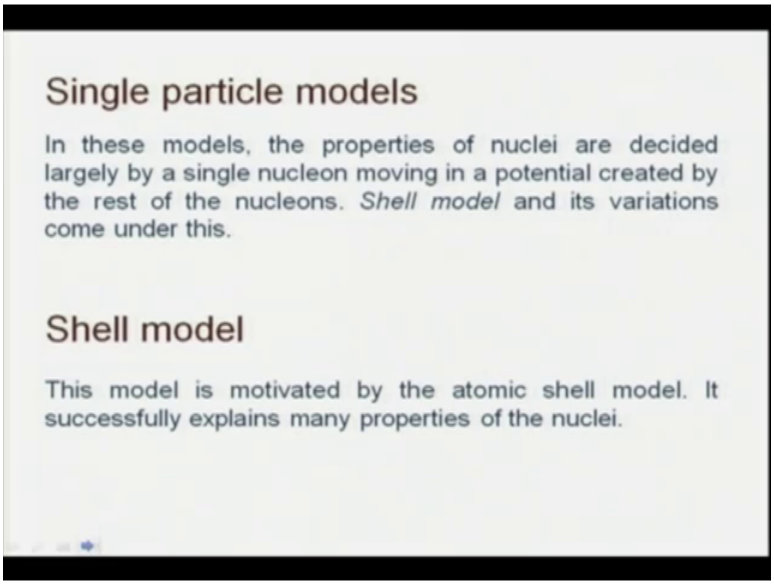

So, single particle model as I said which discusses the nucleons a few nucleons which dictates the properties of the nucleus and they will be moving in a kind of a potential generated by the rest of the nucleus. Shell model with different variations of this as it comes under this and remarkably this kind of simple approach actually simple model very well explains many of the properties of the observed properties of the nucleus.

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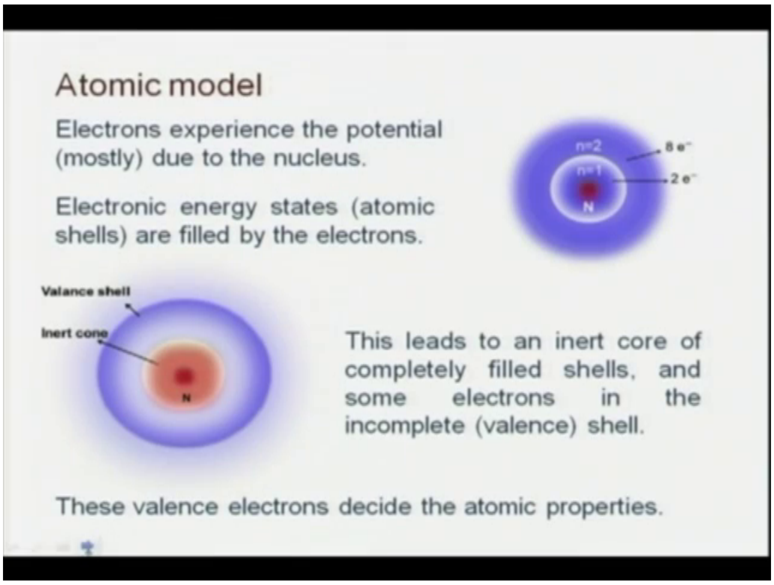

So, we will discuss this in a little detail. But before actually coming to the nuclear shell model let us consider look at what is the atomic shell model just recap that.

In the case of atomic model we have a nucleus and lot of electrons moving in the potential generated by the nucleus. And when we look at the energy states of the electrons we see that we can actually consider different energy states and then consider these energy states as shells different shells and labeled them first shell, second shell, etcetera we will say n equal to 1, n equal to 2, n equal to 3 etcetera and we can work out how many electrons can be put in each of these shells. And it turns out that in the case of n equal to 1 2 electrons can be put in and n equal to 2 can accommodate 8 electrons. So, together n equal to 1 and 2 can accommodate 10 electrons, and further like that n equal to 3 can accommodate 18 electrons etcetera.

So, if we are considering an atom with say 2 electrons helium then we will actually consider to put 2 electrons in n equal to 1 ok, and no other shells are filled. If we are considering hydrogen then it has only one electron again that electron will be in n equal to 1, and that n equal to 1 shell can accommodate 1 more electron. If it is a lithium n equal to 3 then 2 electrons are there in n equal to 1 and the third electron is in an equal to 2 and n equal to 2 can accommodate another 7 electrons, but lithium has no more electron, so it is vacant.

So, we have some of these shells in any atom some of the shells are filled completely and some of the shells are I mean there are other shell which is not filled completely. So, this shells which are completely filled are called inert core, and the shell which is in completely filled we say valence shell. And properties of the chemical properties by enlarge of the atom is dictated by enlarge dictated by the valence electrons. So, the electrons in the violence shell. So, the core does not really or the electrons in the core does not really bother about chemical interactions, chemical reactions that is the kind of atomic feature.

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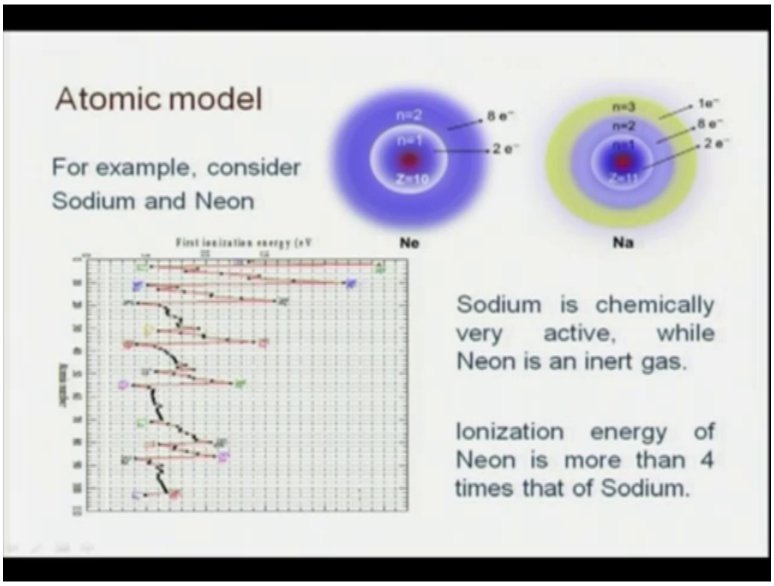

For example, if you consider say neon and sodium. Neon has 10 elect 10 protons and 10 electrons. So, n equal to 1 and n equal to shells are completely filled n equal to 2 can take 8 electrons and it is completely filled together n equal to 1 and 2 as 10 electrons it is completely filled.

So, there are no and in completely filled shells. So, there is no valence shell whereas, in the case of sodium when you actually have 1 more electron that extra that at 11th electron goes to n equal to 3 n equal to 3 can otherwise accommodate 18 electrons. So, it is not completely filled and its sodium has a valence shell in that. What does this tell us in terms of its chemical properties? Actually when you look at the chemical properties sodium is very active chemically while neon is an inert gas, neon has completed its n equal to 2 and then it is inert it does not really actively very eager to participate in interaction with other atoms ok. Whereas, the sodium has an incomplete shell and then the electrons in that is loosely held to the sodium atom and they actively participate in interactions with other elements whenever there is a possibility.

So, although a neon and sodium differ only by 1 electron and 1 proton of course, they are chemically very distinct different, and another thing is the ionization energy. Suppose you want to take away one of the electrons from these atoms a sodium will easily give one electron away whereas, neon it is difficult to take one electron from the energy needed to release one electron from the atom is called ionization energy, and the energy to take away one electron from this thing is called the first ionization energy and for the sodium first ionization energy is much small very small compared to that of neon.

In fact, when we look at look at the ionization energies of other elements as well basically you see that say for exam in the case of helium you have the completely filled shell n equal to 1 and it is not interested in giving up an electron easily, so it has a large ionization energy.

Then lithium has very small ionization energy about 5 electron volts, it has only one extra electron compared to helium and then it changes differs a little, but not very drastically and then it comes to be when it comes to neon which has the n equal to 1 as well as n equal to 2 completely filled it is again more than 20 electron volts. And then the next one sodium next element sodium has very small ionization energy 5 electron ones, and further on argon completely fills n equal to 1 n equal to 2 and n equal to 3 and there are no more electrons.

So, it is also an inert elements, its ionization energy is also large and you can see in this graph that we have plotted. There are such spikes in the spectrum and these spikes helium neon corresponding to helium, neon, argon, krypton, xenon etcetera correspond to completely filled shells.

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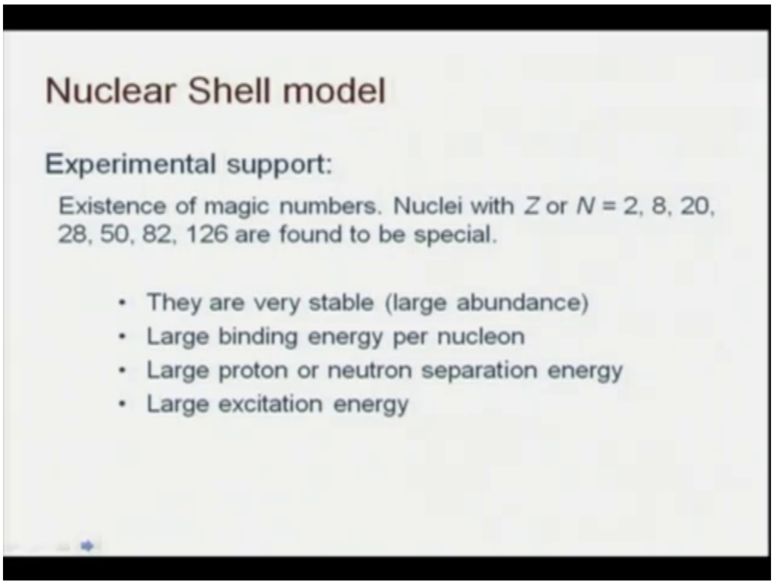

Now, you can ask the question supposing that we imagine something similar happening in the case of nucleus with nucleons filled in this kind of shells, other kind of closed shells that we can think about we can have and then do we have observations corresponding to this like in the case of atomic picture. And answer is yes, when we consider different isotopes we see that nuclei with proton number Z or the neutron number N 2, 8, 20, 28, 50, 82 etcetera though these nuclei are found to be special in the sense that they are very stable compared to the nucleons the nuclei when the nearby with the near with the similar Z added numbers.

And they have a large binding energy they nucleons since in them are tightly bound large proton and neutron separation energy the separation energy in the case of nucleons is similar to the ionization energy this is the energy needed to take a proton or a neutron away from the nucleus.

And then it also has large excitation energy if you want to in give some energy and then excite the nucleus then also to take an electron or one nucleus from ground state to excited state you need to give larger energy. So, there is a big gap there it looks like.

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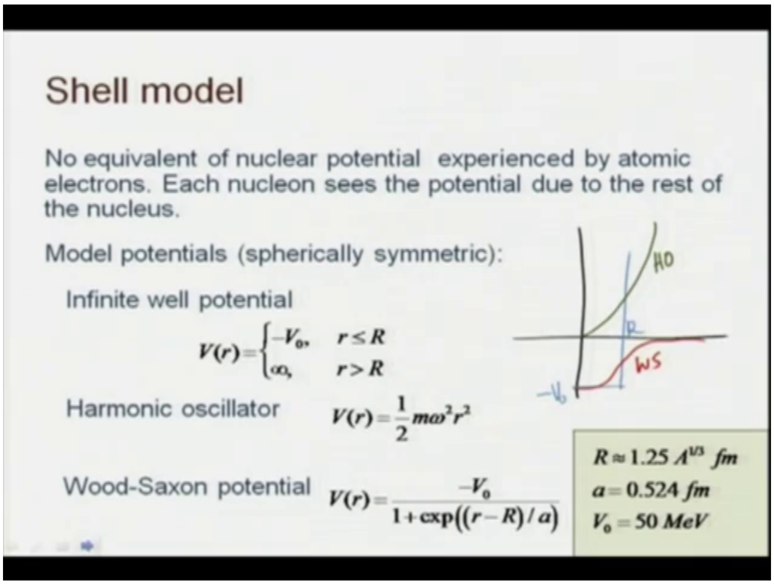

So, there is some support for this kind of picture, but when we try to work out the details and ask the question what is the potential that I should put in the Schrodinger equation corresponding to a particular nucleus. We immediately face a problem a difficulty compared to the atomic case. In the atomic case we had the electrons experiencing potential due to the proton the nucleus an external agency whereas, in the case of a nucleus we have lot of neutrons and protons in it and then we do not have any other external agency an extra thing that there that can that provides the potential.

So, what people thought is all right, when we consider a particular neutron or a particular proton then we can think about some kind of a potential experience by this particular nucleon and the potential can be due to the rest of the nucleus. So, the rest of the nucleons come together ok. So, there is strong interaction potential and this potential can, I mean in the this kind of an effective potential that you can thought of a particular nucleon experience due to the nearest of it. Now, you had to model this the potential is taken to be spherically symmetric in the case of shell model.

So, one immediate thought is to actually considers an attractive potential which is confined to or which is which has a short range ok, it is the size of the range of this potential is almost the same as the size of the nucleus ok. So, you can consider similar to the deuteron case again the potential which is minus V 0 attractive constant potential up to the radial distance r equal to capital R which is the size of the nucleus. And beyond that it is infinite it could be I mean it is a infinite well kind of a potential and positive. So, it is not attractive beyond that ok. So, this is certainly a toy potential that we can consider and you can work out the energy spectrum etcetera.

The other thing that you can do is to consider the harmonic oscillator potential which is also a well understood potential in quantum mechanics it is a confining potential it is 0 at r equal to 0 and grows to infinity at infinity. So, it is going up. So, let me draw this for you. So, if we consider the square well potential. So, it will look like negative say minus V 0 at some for radius less than R ok, and is infinity otherwise outside and the harmonic potential is, let me use a different color l; let say green it is it grows like this. So, this is the harmonic oscillator potential, this is the harmonic oscillator potential.

And the difference between these 2 is that the harmonic oscillator is smooth it gradually goes to gradually rather is rather fast a square to infinity, but there is no abrupt changes where as in the case of infinite well potential it changes drastically just at r equal to capital R, it changes from minus V 0 to infinity.

Now, both of these are not really a realistic thing to happen the harmonic oscillator potential is too shallow and not constant that are equal to are inside the nucleus whereas, we know very well that inside the nucleus all the observations indicate that the potential is constant, ok. And therefore, a different kind of I mean a an additional sorry some kind of a compromise between these 2 are made and then there was just proposed the Wood and Saxon that we can look at a potential which is constant more or less for r less than capital R and changes over smoothly, but rather fast to 0 at r is equal to R. So, it will look something like this it is goes r equal to R it is where it should have been.

So, it goes to, so this is the Wood-Saxon potential WS. It is smooth transition not abrupt like that. So, that is a like slightly better than the square infinite square potential square well potential and there are 2 parameters here one is V 0 the other is a and of course, the radius itself we know can be considered to be 1.25 times third power one-third power of 4 third root of mass number in Fermies occurs, 1.25 Fermi a power 1 over 3 and other parameters V 0 and a are adjust address.

So that a to reproduces what we want to have in the energy spectrum etcetera are calculated and then compared with the observations and then some kind of an agreement between the experimental absolute spectrum and observed and theoretical prediction;

gives what the values of a and V 0 should be and then it turns out that a equal to 0.524 Fermi and V 0 the depth of the potential equal to 550 MeV a gives a good approximation is the, approximate sent to the observables.

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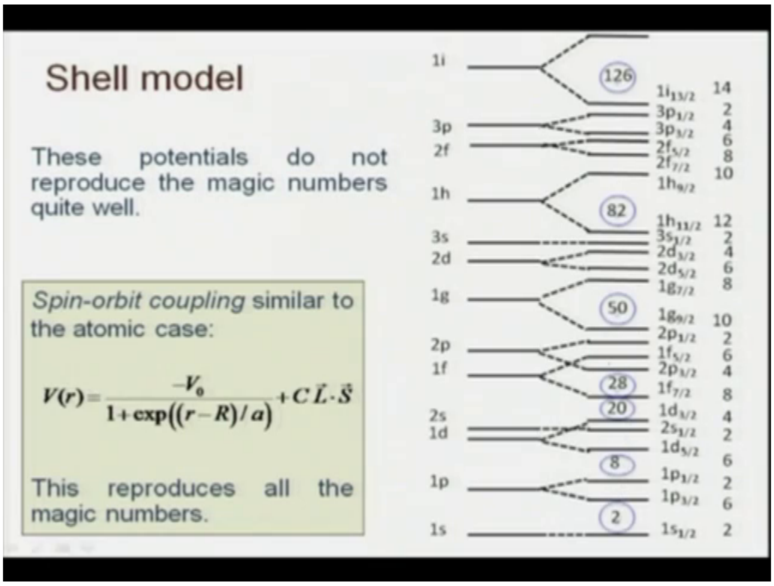

How about the magic numbers? Well, there are these magic numbers there are gaps in the spectrum large gaps in the spectrum or shells different shells in the spectrum and then you can think about these gap, large gaps positions of these magic numbers and, but none of these even the Wood-Saxon potential. Quite well reproduce the magic numbers not all of the magic numbers are reproduced and some other number is also come in etcetera.

So, some correction to this was thought then similar to again the atomic case we consider the spin orbit coupling in the case of nucleons right and this correction potential correction term is added to the potential. So, the new potential looks like minus V 0 over 1 plus exponential smaller minus capital R over a, which is the Wood-Saxon tau plus some factor C times L dot S which is the spin orbit coupling them and now this remarkably reproduces all the magic numbers.

So, the spectrum with the V 0 parameter value is given in the previous slide ok. Gives this kind of a spectrum energy spectrum with relative, roughly relative weightages like where are there are large jumps there are large gaps between these energy levels we can think of the magic numbers corresponding to those ok. And in we again label these energies levels by similar to the atomic case as 1 as N S j like one, in case of 1 s half the one correspond to the principal quantum number, s corresponds to l equal to 0 the angular momentum quantum number and half corresponds to the total angular momentum value j. So, 1p 3 by 2 corresponds to principal quantum number 1, l equal to 1 and j equal to 3 by 2. So, s is l equal to 1, p is l equal to sorry s is l equal to 0, p is l equal to 1, d is l equal to 2, f is l equal to 3, g is l equal to 4 etcetera ok, all right.

So, remarkably this and without the spin orbit coupling the spectrum does not really reproduce the magic numbers well, all right.

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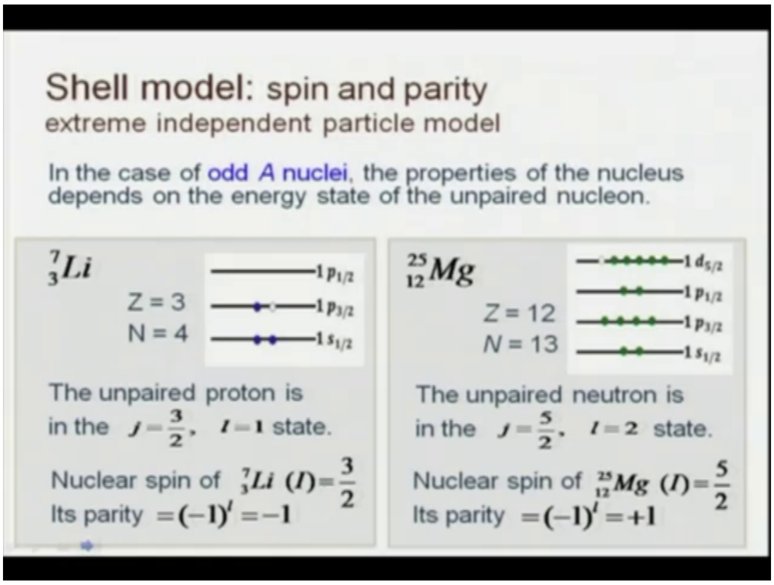

So, now, that is one very good thing and there are other things is that the shell model is actually very successfully explains. Especially this model is successful with odd mass numbers, either the proton or neutron number is even and the other number is odd ok. So, the especially the spin and parity of this thing is extremely well reproduced by the shell model all right. But what is the assumption? Assumption is that, so there are different now versions of this shell model one is that called the extreme independent particle model which actually says that all the properties of the nucleus or the properties of the nucleus like the spin disparity etcetera are decided by the neutron which is unpaid ok, before that I should say the neutrons and protons paid neutrons are paired with each other.

So, if you have 2 neutrons in the system then they are compared with each other due to some spin arrangement or spin arrangement say possibly. And if there are 3 neutrons 2 of them are paired and then there is an unpaired early, if there are 4 then there are 2 pairs, if there are 5 then there are 2 pairs and one unpaired etcetera.

Another thing is that the neutrons and protons are taken separately. So, the neutrons see a kind of a potential that we saw in the previous slide, emerging from whatever the background nucleolus and similarly a single proton also sees this one. And then so these 2 energy spectra that we discussed in the discussed here are actually separately for neutron and separately for proton. So, incidentally the magic numbers you for proton you know that we do not have any boundary or any elements, with proton number 126 of a, so this magic number is yet to be observed. Up to 8 2 it is fine, and what is the next magic number in the case of proton we do not know.

Whereas in the case of neutrons up to 126 is observed and then in the case of shell model you can go up it can actually to predict other magic numbers and in fact, the next one is 184. So, we do not have any observation yet there in the case of neutrons even. So, neutrons and protons are separately considered. And then say let us say consider the extreme independent particle model and in this case lithium as an example has 3 protons and 4 neutrons, all the neutrons are paired and in the case of neutron 2 of them are paired and one and paid neutron.

And the extreme independent version of the shell model says that properties of the nucleus is decided by this single unpaired nucleon in the case of lithium 7 it is the unpaired proton and according to the shell model with Wood-Saxon plus the spin orbit coupling.

The first 2 the 2 electrons are in the one s half state and sorry not the electron who the 2 paid protons are in the 1 s half state and the unpaired proton is in the 1p j equal to 3 by 2 state. So, this spin of the unpaired proton sorry is the spin of the lithium 7 nucleus is the J value of the unpaired nucleon here the proton which is 3 by 2 that is the prediction of the extreme independent particle shell model; And what about the parity? Parity in quantum mechanics goes as minus 1 power orbital quantum number orbital angular momentum, so minus here it is p states which is l equal to 1 states and minus 1 power 1 is minus 1.

So, we expect lithium 7 to be odd parity nucleus with nuclear spin 3 by 2 and in the experimental observation says that this is correct and you can take some other example say magnesium 12 protons and 13 neutrons in it.

So, since proton number is even all of them are paired and then they do not really bother or they do not gives rise to give rise to contribute anything to the spin and parity ok. And when we consider the neutrons, so they are 13 in number 2 in the 1 is half, 4 in the 1p 3 by 2, 2 in the 1p half and remaining 5 in the 1d 5 by 2 ok. So, the unpaired this 5 in the 1d 5 by 2 can be thought in terms of 2 pairs and one unpaired neutron, yes neutron. So, the unpaired neutron in this case is in the state J equal to 5 by 2 and l equal to 2, and correspondingly nuclear spin of magnesium 25 is I equal to 5 by 2 and it is parity is minus one power l here l equal to 2 which is the d state and so therefore, that is equal to 1.

So, indeed magnesium ground state of magnesium, we are talking about the ground states the minimum energy configuration and the magnesium 25 is found to be in I 5 by 2 p plus.

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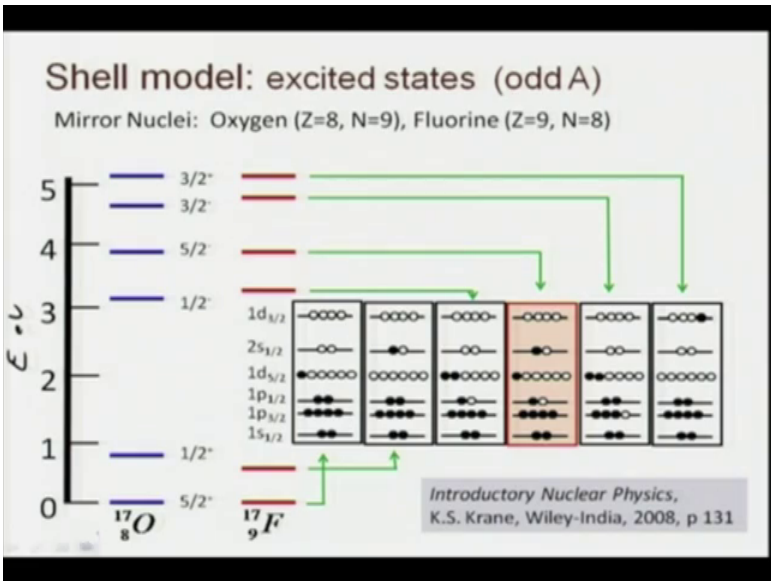

And you can go on, go on and then ask what about other nuclei that has odd a value and then remarkably the predictions of the shell model agrees with all of the observed cases. There is another thing where the shell model is very successful in predicting the absorption practically agreeing, in agreeing with the observation variable which is the excited states of odd in nuclei, odd a nuclei.

As an example let us take oxygen and fluorine which are mirror nuclei in the sense that oxygen has Z equal to 8, N equal to 9, 8 protons and 9 neutrons. Fluorine has 9 protons and 8 neutrons. So, Z and N are interchange in the case of oxygen and fluorine since all of these are blind to whether it is a proton or a or a neutron a strong interaction is blind to this therefore, we expect exactly similar kind of situation in the case of oxygen as well as flow rate, as far as the nucleus of these are concerned.

The ground state configuration is that there are for oxygen, we will not worry about the proton because all the protons are paid. So, the 9 neutrons are filled in this fashion, 1 is half 2, 1p 3 by 2, 4 1p half 2 and the unpaid single neutron is in 1d 5 by 2 indeed ground state I value the spin is equal to 5 by 2 and parity it is l equal to 2 state therefore, parity is equal to plus 1. It is x, it is agrees with the observation.

Same is the case with fluorine, but instead of neutron now we are talking about the protons, the neutrons are 18 number they are all pair and they do not come into picture when we discuss the extreme independence shell model. And let us ask the question what about the excited states. When you give some energy to these what could happen? One possibility is that this neutron let us consider the oxygen now a similar thing follows for the fluorine just that proton is neutron replace by neutron is replaced by proton. So, the unpaired neutron in the 1d 5 by 2 of oxygen 17 can jump to 2 s half when you give some energy into this. And what would be the I value there? The spin it is 1 by 2. And what is the parity? It is an s state l equal to 0 state therefore, its parity is plus.

So, we expect an excited state with I equal to 1 by 2 and p plus 1 plus indeed vc. In fact, the lowest the first excited state is 1 Ip 1 over 2 plus, same is the case with fluorine. There is a slight difference in actual magnitude of the excitation energy because of other factors. Configuration is also I mean one possible configuration that we can think about is the one which we discussed just now.

Well, if you give more energy what can happen? It cannot further jump to 3 by 2 it can go from s half 2 s half to 1d 3 by 2 which is the next energy state, right. So, indeed that will have I 3 by 2 and positive parity even parity will call positive parity as the even parity negative is or parity. Indeed there is an excited states 3 by 2 plus and for both oxygen and fluorine, is that what do you have or nothing in between this first excited state and excited state with 5 electron volt, this is the exit session the y axis the perpendicular axis is actually excitation energy which is in E V, excitation energy in E V ok.

So, E in electron volt, 5 electron volt is there anything in between; experimental observation actually says that ok. So, this configuration is this. Experimental observation says that there is something which is in between, which has excitation energy about 3 electron volt and I is equal to 1 over 2 parity or both fluorine and oxygen we have this observation. Now, you ask the question how do we accommodate this in this thing and we would imagine that this next there is we do not see any possibility of the odd nucleon to jump from 1d 5 by 2 to a state with j equal to half, but odd parity we do not see that I mean have I equal to 1 by 2, but even parity that is already observe ok, but we do not have any other I equal to 1 by 2 with odd parity. So, what is the explanation that the shell model provides.

Well, we can actually think about a situation where we break the pair one of the pairs ok. Let us say the bare pairing also requires some kind of a there nucleon proton proton or neutron neutron pairing also has require some energy to release that this thing experimentally it is found that this pairing energy is something like of the order of 2 electron volt and then you need something of that order to release that is see. If you give something of that order 2 electron volt then it will break them pair.

Here we can say that the outermost completed subject is 1p half with 2 neutrons in the case of oxygen and if we break that and put that one of the neutron from there to 1d 5 by 2 ok, like this. The neutron in up in the pair in 1p half breaks away and goes to 1d 5 by 2 and joins that 1d 5 by 2 unpaired neutron becomes a pair. Again we have one single neutron unpaired, but now that is in 1p half states and this 1p half state has j equal to 1 by 2 and l equal to 1 therefore, it is an odd parity I equal to 1 by 2 nucleus state. So, we give some kind of an explanation for this observation and it is, it is satisfactory. We ask other, any other, yes, there is a 3 by 2 minus odd parity I equal to 3 by 2.

How about this? Well, we can break a pair now not from 1p half, but from 1p 3 by 2, well that is another possibility. So, 1p 3 by 2 one pair is breaks and joins 1d 5 by 2. This should certainly be energy required for this should be larger compared to the previous case one half odd parity case because the energy required to shift it from 1p 3 by 2 to 1d 5 by 2 is larger. So, it is it all these faults in well gets the explanation procure. We asked further is that all experimentally observe this thing, but it does not stop there we have something called 5 by 2 minus, now we have a big problem even if we break one of the pairs available and try to put it in somewhere one of the put it in pair it with the 1d 5 by 2 we do not have any possibility of getting 5 by 2 minus. This is a problem with the which cannot be explained with extreme independent shell model.

So, one possible way is actually to consider this kind of a possibility. Instead of one nucleon s not paired we actually consider say for example, the neutron in 1p half breaks away from its pair goes to 2 s half. This energy is more than that is needed to break away and go to 1d 5 by 2 indeed, and this goes and then stays in 2 s half.

Now, we have not one, but 3 unpaired neurons and you say that it is not just a single particle extreme independent model that we can consider we consider a combination of this as these 3 N paired neutrons and work out the find out what is the effective or resultant angular momentum states available and see if that agrees with observation ok. Incidentally this particular case of oxygen and fluorine is very beautifully discussed in introductory nuclear physics by Kenneth Krane. So, you can read that for more details. In fact, he discusses it in much further details giving other examples as well.

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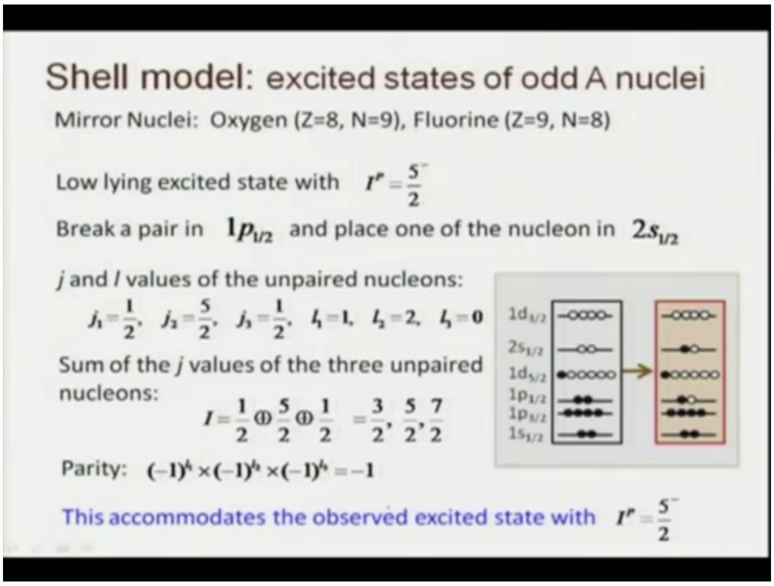

But, now let us focus on this particular problem of single particle state, yeah a single particle shell model. So, the low lying excited state with breaking a parity a pair in the, so there is a low lying 5 by 2 minus which is observed how do we explain that all right. So, as I said it is possible that you break 1p half and then take that neutron to 2 s half, so you have 3 unpaired neutrons and then j and N values of this j values of these 3 let me call this j 1, j 2, j 3, these 3 unpaired neutrons as half 5 by 2 and j equal j 3 equal to 1 by

2; l values are l equal to 1, l 1 equal to 1, l 2 equal to 2 d state and l 3 is equal to 0, 2 s state.

So, according to the quantum mechanics angular momentum addition rules the total angular momentums possible are 3 by 2, 5 by 2 and 7 by 2. And the parity possible are minus 1 power l 1 times minus 1 power l 2 times minus 1 power l 3; l 2 and l 3 are even l 2 is equal to 2 and l 3 equal to 0. So, that gives plus 1 plus 1 and l 1 equal to 1 gives you minus 1 and resultant a parity is expected to be minus. So, we have a possibility of 5 by 2 minus parity, without parity in this 3 case.

So, this is a possible explanation certainly, and we I mean this is one of the possibilities you may be able to find other possibilities. Like for example, I do not know maybe you can break instead of 1p half and you can break the 1p 3 by 2 and take that to 2 s half and then instead of half you will have a 3 by 2 then there is still you will have a 5 by 2 existing possibility and parity is also conserved. So, this is another possibility ok.

So, you think about different possibilities and then see if that can be accommodated. It can be accommodated, this kind of possibilities are there only thing is that we have to deviate from the extreme independent shell model where there is only one unpaired nucleon. So, here we have 3 nucleons and then this is what it comes out to be all right. There are more to it shell model.

We will discuss other possible other successes, successful explanations of other observations and as well as we will discuss some of the limitations of the shell model that we will do in the next lecture.