Nuclear and Particle Physics
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Module - 02
Nuclear Force
Lecture - 03
Nucleon Scattering

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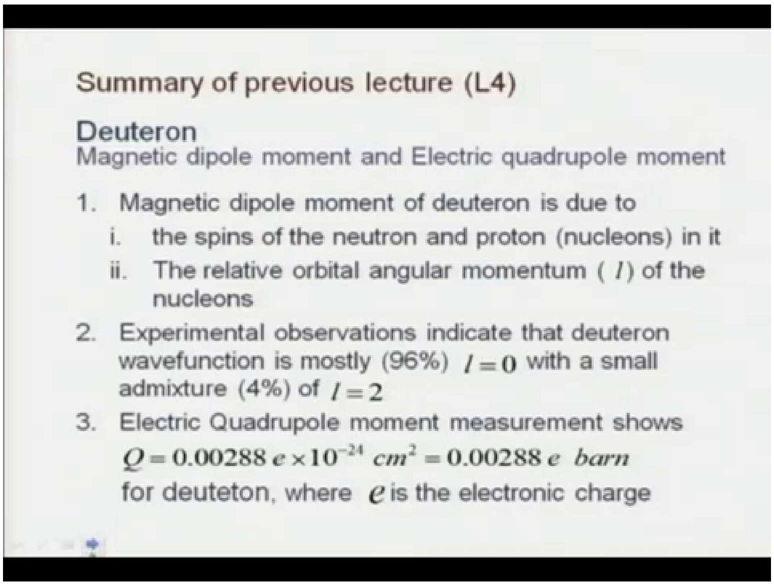

We will continue our discussion on the Nuclear Force today. So, let us recap what we did in the last lecture. We discussed the deuteron specifically its magnetic dipole moment and the electric quadrupole moment and we saw that the magnetic dipole moment of the neutron arises due to the nuclear, the spins of the nucleons and also due to the orbital angular momentum of the nucleons inside the neutron. And experimentally it is found that orbital angular momentum l equal to 0 state is predominant in deuteron or deuteron is predominantly in l equal to 0 state about 96 percent of the times in that state and 4 percent it is in the illegal to 2 state.

And the quadrupole moment, that is the electric quadrupole moment as measured to be 0.00288 e bond the bond is 10 power minus 24 centimeter square just to remind you, all right.

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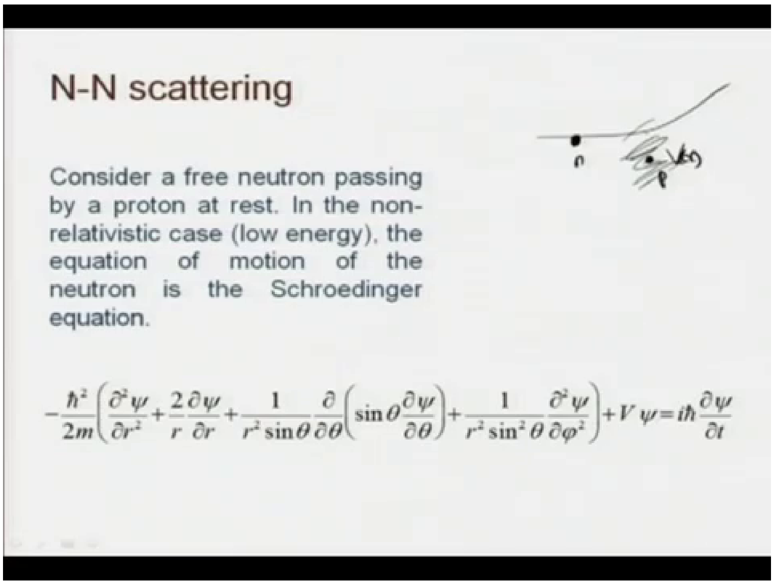

So, today we will discuss the nuclear nucleon scattering. We are basically interested in trying to understand the nuclear forces looking at the different systems, nuclear systems and the processes. So, deuteron which is a simple which is one of the simplest a bound states of the nucleon two nucleons bound state a neutron and a proton. And a study of the properties of this deuteron will give a lot of information about the nuclear force, but that will not be enough. For example, in we saw that in deuteron neutron and proton in parallel spin state and we do not have any bound state with anti parallel or spins aligned anti parallel to each other for example, and if you want to study the nuclear force spin dependence of the nuclear force with anti parallel spins a details about that etcetera we need to actually go into different systems.

Since, we do not have any other two nucleon bound states we rely on something else some other process which is basically scattering of the nucleon or nucleon. So, we basically we will bring this one nucleon near the other one, another one, say a proton is brought near neutron you can have actually a proton static also like you can haves hydrogen gas in a, as a target and neutrons with some kinetic energy can be sent to this target. And then study the system. Study what happens? What is the behaviour of the neutrons when they encounter or encounter the protons? So, such scattering studies reveal a lot about the proton neutron and proton proton and neutron neutron interactions and their forces first, then a strong nuclear force between them.

So, let us look at some simple basic ideas of the nuclear nucleons scattering here n, n, n stands for either proton or neutron. So, for a low energy systems if we are having a more kinetic energy compared to the mass of this say for example, a few a mevs up to a few mev kinetic energies because the mass of the neutron is about 1000 mev, a few mev can be considered as a nonrelativistic region. So, in such cases what will dictate the equation I mean the scattering process is the Schrodinger equation basically.

So, the Schrodinger equation in spherical polar coordinates is what we have written down here. So, it has the psi which is basically the wave function that represents the particle in the nucleon and it is in municipal in general function of all the 3 spherical polar coordinates r theta and phi. And there is a potential V which is basically the nuclear potential that we will cause the scattering to happen or the cause the interaction to happen. Scattering is basically like you have a proton somewhere and say for example, a neutron comes near that and then scatters off to some directions and this is the basically the a neutron proton for example. And when the neutron comes near the proton it sees the nuclear potential which we denote by V.

And to understand the nuclear scattering we need to actually have some model potential so we will take a simple model potential which is basically a square well potential, as 3 dimensional square well potential which is actually spherically symmetric.

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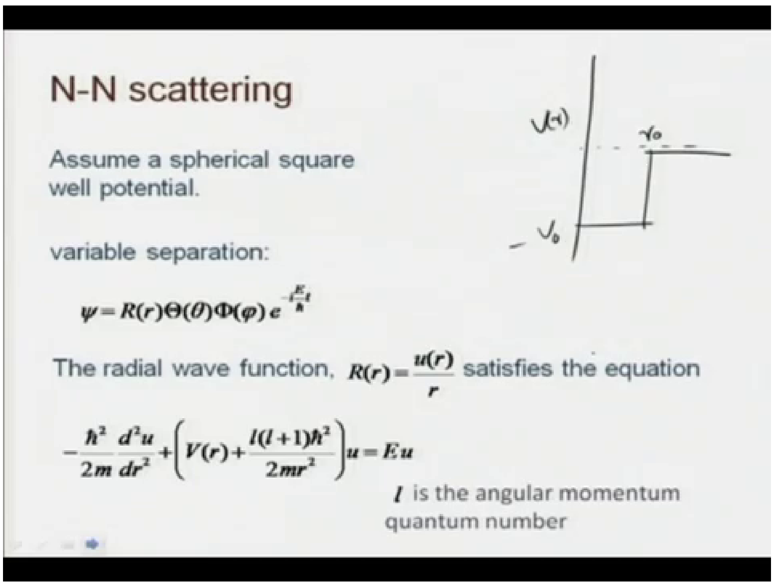

So, we will consider the potential V r to be some value V negative V 0 for small r up to some radius r 0 and beyond which it is equal to 0. So, with this potential we will actually consider the equation Schrodinger equation. And again we will separate the variables by defining psi the wave function as product of 3 functions which are of functions of variables r theta and phi separately and only functions r is a function capital R is a function only of the variable r, theta is a function of the variable theta, phi is a function of the azimuthal angle phi, and time similarly is separated from the special part of this one. And from the equation easily it is seen that the form of the time dependence is exponential minus i over h cross time t.

And now the radial part of the wave function the is we will rewrite as u over r another function u and this u will now satisfy the equation minus h cross over 2 m d square u over d r square plus V plus l into l plus 1 h cross square over 2 m r square u, e u where, this is the equation Schrodinger equation satisfied by the radial part wave function. Here l is the angular momentum quantum number.

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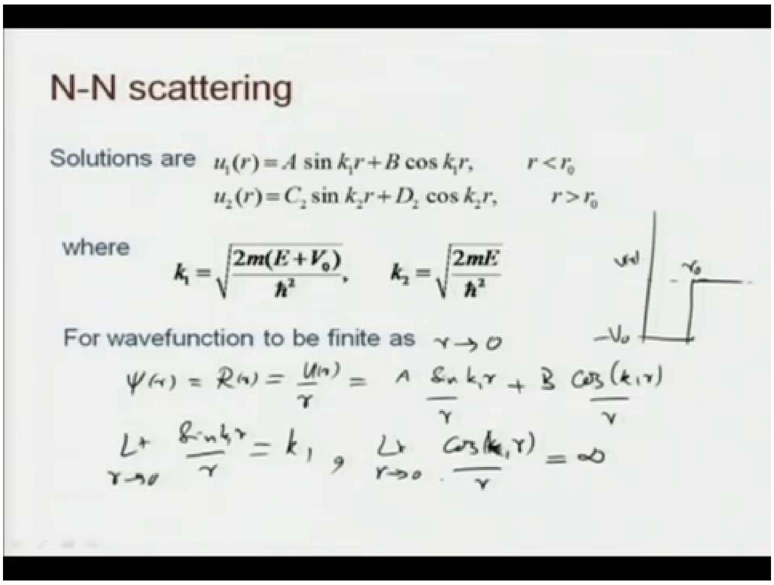

And let us say we want to concentrate on a particular l value. So, we will stick to l equal to 0 case and in that case the solutions will look at the solutions, see looking at the equation Schrodinger equation we have V nonzero up to r is equal to r 0. So, for that part we and will consider again energy to be larger than or energy to be positive because we here we are talking about the scattering of electron or the sorry a scattering of neutron over a proton and in that case electron, so the neutron incoming neutron has kinetic energy and we will assume that the total energy is larger than the potential and, it is positive there.

This gives us the solutions of the equation as u 1 equal to some constant A time sine of k 1 r plus some another constant B cos k 1 r for the region r smaller than r 0 and a very similar thing for r larger than r 0, but now we have another wave vector k 2 and the coefficients C 2 and B 2 can be different or in general different from A and B respectively. So, k 1 and k 2 are given in terms of E and V 0. So, for region r less than r 0 we have the potential nonzero and we have an expression for the wave vector in this fashion which is equal to under root 2 m E plus V 0 over h cross square and k 2 we do not have any potential the potential is 0 in that region and therefore, we have this to be 2 mE over h cross square under root.

For the wave function to be finite as r goes to 0. So, we have the potential let me draw the potential again. So, we have the potential starting from 0 some nonzero potential minus V 0 and it is equal to 0 from r equal to r 0 onwards and we want for the region 1 where u 1 is the solution, r tending to infinity we have to include the point r equal to 0 as well. So, as r tends to 0 if we want the solution to be a finite solution we need to keep V equal to 0, why because the wave functions psi r now if we are actually considering l equal to 0 solution then the angular part we will only give some kind of a overall constant we will neglect that for the time being.

So, the total wave function is essentially r apart from the time dependent part which is again trivial and we will not write here and this in terms of u is u over r and this then because of u 1 it is for the region of interest near r equal to 0 it is A sin k 1 r over r plus B cos k 1 r over r. So, we know the limit of point 0 sin k r over r is equal to same k 1 r there is equal to k 1 which is finite no problem. But if we take limit the second term limit r going to 0 of cos k 1 r over r this is going to be infinity.

So, if we want the wave function to behave properly as r goes to 0 then we cannot take the second part or b cos k 1 r as a solution as part of the solution. So, we will like to take b equal to 0. So, we will consider that.

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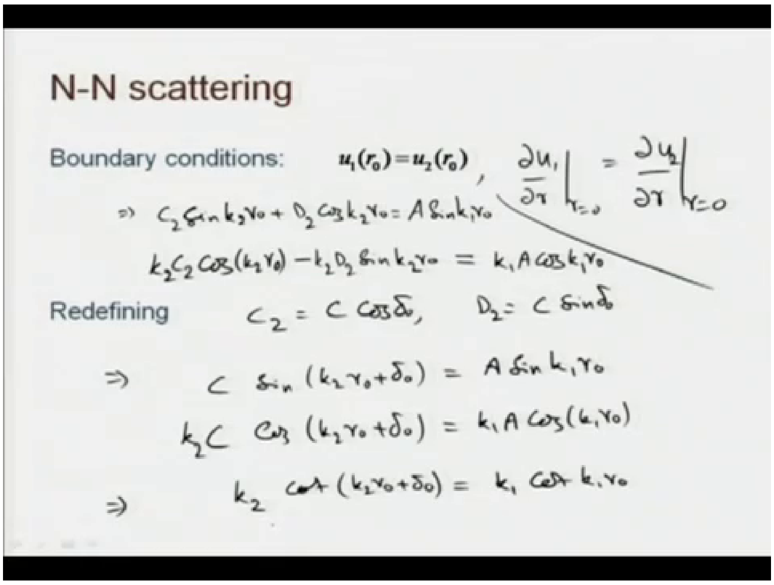

And there are other about the other boundary is r equal to 0 sorry r equal to r 0 where the potential actually drops down to 0. So, here we have the boundary condition u 1 r 0 equal to u 2 r 0, that is the boundary that is one of the boundary conditions. The other boundary condition is the derivative of u 1 with respect to r at r equal to 0 is equal to the derivative of u 2 at r equal to 0. So, they should be equal. So, these are the two boundary conditions.

Then this gives us 1, C 2 sin k 2 r 0 plus D 2 cos k 2 r 0 equal to now we have only one term which is A sin k 1 r 0. So, here and this is from equating u 1 and u 2 at r equal to 0 and when we take the derivative that will bring out k 2 here and then you have C 2 sin will give you cos after differentiating. So, cos k 2 r 0 plus now minus k 2 D 2 sin k 2 r 0 and this should be equal to k 1 A cos k 1 r 0. So, these are the two boundary conditions now.

Now, let us redefine to make it in a slightly a nicer form we redefine C 2 equal to C some constant cos delta and D 2 equal to C times sin delta some delta 0 let me take it as a delta 0. So, this will give the boundary conditions to be C sin k 2 r 0 plus delta 0 equal to A sin k 1 r 0 k and k 2 C cos k 2 r 0 plus delta 0 equal to k 1 A cos k 1 r 0. This will give if you divide one by the other that the second one by the first one then that will give you k 2 cause by sin is cos. So, k 2 r 0 plus delta 0 equal to k 1 cos k 1 r 0. So, in principle we can find out this delta 0 from non k 1 and k 2, k 1 and k 2 are given in terms of e and V

0. And suppose we know also the range of the potential r 0 from other data or other experiments or the other computations and then in principle we can invert this to get delta 0.

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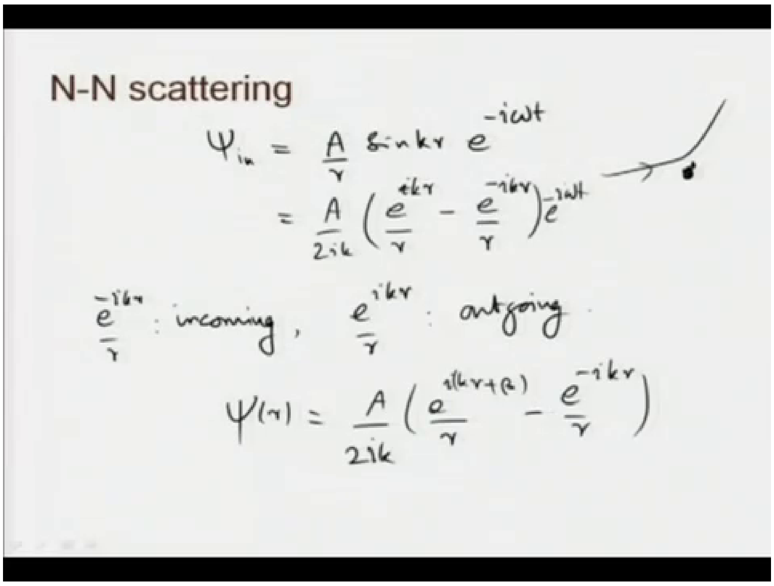

Now, let us look at it a little more detail like we have we had the incident wave function psi in as A sin k r exponential minus i omega t omega is the frequency which is e over h cross. So, if we consider some potential scattering some particle scattering over or a wave function quantum mechanical wave function scattering by a A potential at the scattering center then we can in principle think about analyzing the scattering in this fashion. We have incoming wave which let us write down us a sin kr e power minus I omega t, where now r is the radial distance from the scattering center. So, this actually I can write down A over 2 ik sign kr, I will write in terms of the exponential as ikr minus e power plus minus ikr over r over to i in the e power i omega t a over r, this over r.

You had a u with A sin k r and in a spherical part of that the wave function we have u by r which is A by r sign kr e power minus i omega t and then that can be rewritten in terms of exponentials as A over 2 i k e power ikr over r minus e power minus i k over r.

Now, you can ask the question what happens when a particle comes, wave function like this comes and encounters this scattering for a center. So, here actually in this way of writing this plane wave this has two parts one e power ikr over r which is actually called a spherical wave, which is in this case you know in general it is actually a an incoming, it corresponds to the incoming spherical wave and minus k ikr is the incoming one, e power ikr over r is the outgoing spherical wave. We will not go into how exactly we arrive at this conclusion, but essentially we will have to look at the e power i omega t the evolution of this and then we will see that for physical, from physical arguments we can come to the conclusion that e power minus ikr over r corresponds to the incoming wave incoming meaning coming towards the center and e power ikr corresponds to outgoing its going towards the wave from the scattering center, so two spiritual waves.

And general scattering theory argument I will say that a, well when the scattering happens and when we look at what is the resultant wave after the scattering then we have some psi r which is very similar to the in incident wave, but with a slightly changed outgoing spherical wave ok. The incoming spiracle wave part remains the same and outgoing spherical wave actually picks up a phase beta we denote that by beta here all right. So, this is the only thing that and that happens.

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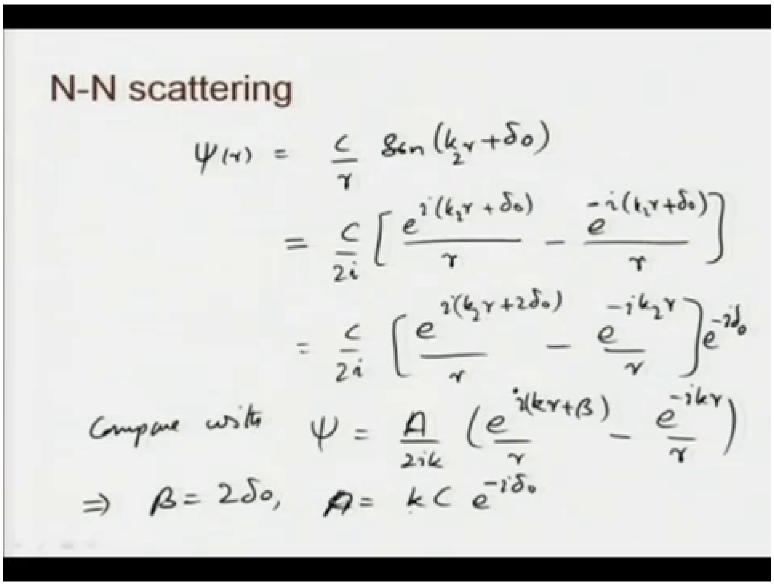

$$\psi(r) = \frac{C}{r}\sin(k_2 r + \delta_0)$$

$$= \frac{C}{2i}\left[\frac{e^{i(k_2 r + \delta_0)}}{r} - \frac{e^{-i(k_2 r + \delta_0)}}{r}\right]$$

$$= \frac{C}{2i}\left[\frac{e^{i(k_2 r + 2\delta_0)}}{r} - \frac{e^{-ik_2 r}}{r}\right]e^{-i\delta_0}$$

Compare with $\psi = \frac{A}{2ik}\left(\frac{e^{i(kr+\beta)}}{r} - \frac{e^{-ikr}}{r}\right)$

$\Rightarrow \beta = 2\delta_0, \quad A = kC\,e^{-i\delta_0}$

When we have scattering of a quantum mechanical wave function like this for then this particular thing from this angle when we look at the wave function that we had written down which we had obtained using the square well potential and the scattering of the square well on the scattering of the square well potential we had written down this as C over r sine k 2 r plus delta 0.

This is essentially equal or you can write now this one as e power ik 2 r plus delta 0 divided by r. So, r I am taking inside. So, it is C minus e power minus ik 2 r plus delta 0 divided by r by 2 i of course, and that time dependence is also taken away there is exponential minus i omega t which is always there we are just not writing that here.

Now, this can again be written C over 2 y e power ik 2 r plus 2 delta 0 over r minus e power minus ik 2 r over r, but what is left out is exponential minus i delta 0. So, I have taken exponential minus i delta 0 out of this two terms and now this is in the form like this compare this which psi equal to C over, sorry earlier we had written it as a over 2 ik e power minus ikr plus beta plus ik 2 a minus divided by r, e power minus ikr over. Now, this tells us that we have if we identify beta equal to 2 delta 0 and C or A as k C e power minus i delta 0 then we have the earlier expression in the same form as the general expression.

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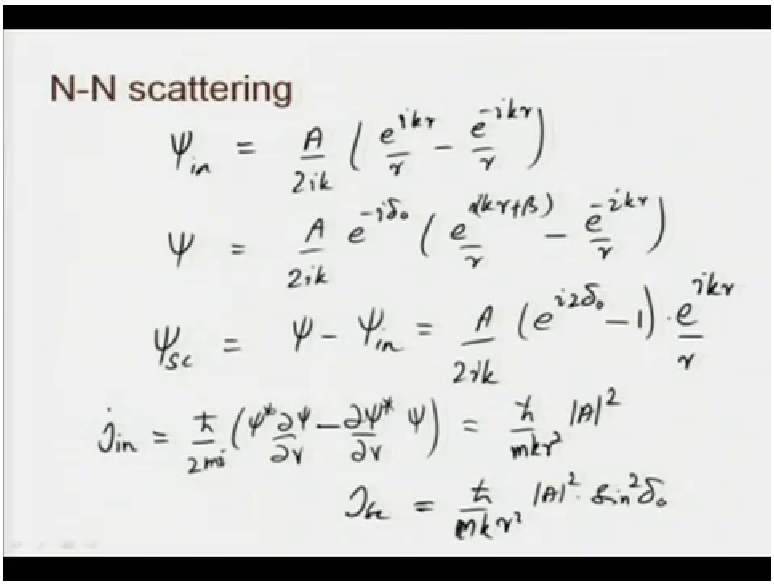

Now, once you have the wave function I like this in this fashion. So, we have let us say let me write it again psi incident is equal to A over 2 ik e power ikr over r minus e power minus ikr over r and after scattering we have A over 2 ik e power k over 2 ik e power minus i delta 0 e power ikr plus beta minus e power minus ikr over r. And if I now consider the scattered wave function as psi minus take away the incident wave function from this that will amount to be A over 2 ik e power 2 i, should I wrote a later i 2 delta 0 minus 1 e power ikr over r.

And now, there is we can write down the probability current corresponding to these two psi j the psi in and psi out and I will leave it as an exercise which I will spend out a explicitly at the end of the lecture. So, you can work it out. It corresponds to this h cross over 2 mi psi star dou by dou r psi minus dou psi star dou r psi and for the psi in this is equal to h cross over mk r square mod A square and corresponding to the scattering or scattered wave function it is h cross over m k r square, j square sin square delta 0. So, this psi square delta square part is basically the difference between the initial incoming current and outgoing or the scattered current.

Now, what does this mean actually? How do we actually physically understand this?

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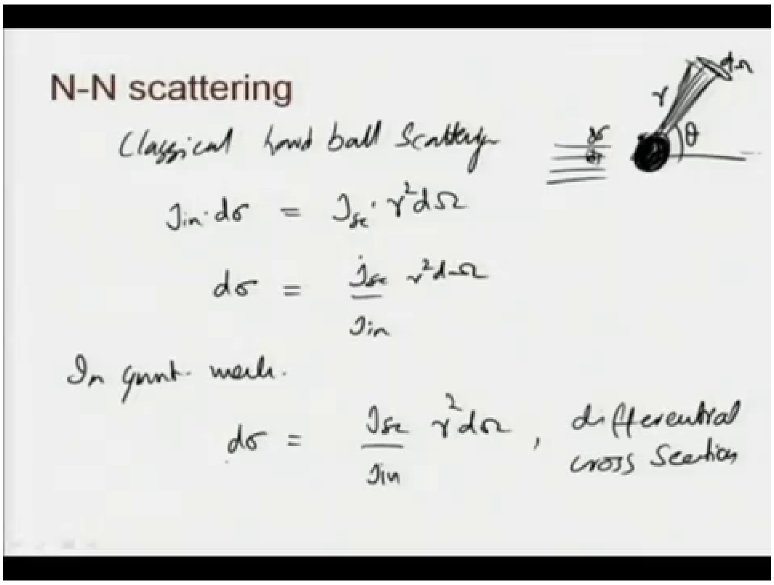

In the scattering we have, let us say we want to understand these in terms of some physical quantities. So, let us say what is happening, let us consider some kind of a classical hard ball scattering. So, if we have a beam of incident particle and then encountering some kind of a hard ball, part of it hits this surface of the world and then some of it will go into as scattered into some region and something else other things will cater to other regions. So, let us focus on some particular region.

Say let us consider some small area of cross section cross section denoted by d sigma. So, if J incident is the incoming current that is the flux of the particle coming in number of particles crossing unit cross section in unit time. So, this if I multiply by some cross section d sigma that will give me the number of particle crossing that area in unit time.

So, this many number of particle let us say scatters off into the region here into a solid angle d omega. I presume that you are all familiar with what is mean by solid angle. So, the area corresponding to d omega is if it is r distance away from that the scattering center or the scattering the ball then area corresponding to this is r square d omega. So, if J scattered is the number of the scattered the current or the flux of the particle which is scattered into the scattering solid angle d omega then the number of particle in the area r square d omega will be J sc..

So, suppose we assume that the particle which are crossing d sigma cross section, scatters into solid angle d omega at some angle theta and phi, in that case, so this is good we have writing it. So, let me do that again. So, this is the distance r, this is at an angle theta and I do not want to complicate the diagram by writing also the azimuthal angle, but there is that azimuthal angle as well.

Now, we can define or rewrite this if we know J sc and J in and if we know what is the solid angle the detector for example covers, then we can get the d sigma from here or look we can from an experiment from a scattering experiment like this we will be able to say, for a detector of d omega solid angle kept at a distance r from the scattering center if J sc is the current or the flux of the particle that it receives and J in is the incident flies then those particles would have go through this d sigma cross section correct. So, this is the kind of physical interpretation that we can give from here considering or comparing the two currents.

And in quantum mechanics we can actually define again d sigma as J sc of the probability current and J in is the probability of the incident particle times r square d omega as the cross section or differential cross section. And for a particular particle or a wave function corresponding to a particular particle this will be interpreted as the probability for a particle to scatter into solid angle d omega when it encounters the potential which we have considered all right. So, this is the interpretation that it that we will give here.

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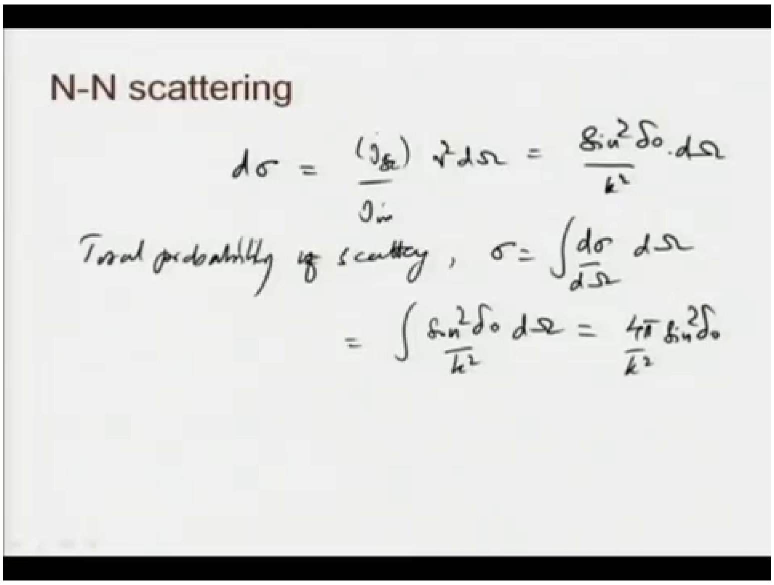

And now in our case we have d sigma equal to J sc is known J in is known therefore, putting it together we have say J sc over J in r square d omega corresponds to some or equal to sin square delta 0 over k square. And if we want the total probability of scattering we have to a integrate d sigma over d omega all right, this is basically d omega as well we know into d omega. So, that is essentially equal to integral sin square delta 0 over k square d omega. Since, this quantity integrand and does not depend on theta or phi we have 4 pi over k square sine square delta 0 as our total cross section.

Remember we have considered l equal to 0 case which is actually a spherically symmetric situation. So, that is why we have this result. If we take some other l value then we will not get such spherically symmetric situation and we will have a d sigma over d omega as a depending as a function of theta and phi in general all right. That we have some idea of what is the scattering, a nuclear nucleon scattering in particular.

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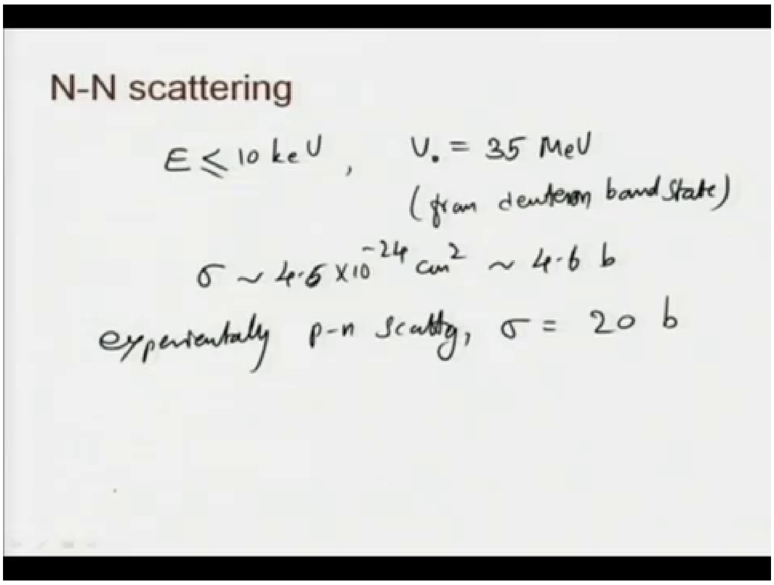

And let us consider in a specific cases by putting in some values. Let us take situation with energy something around 10 keV which is very small compared to the nucleon mass. And we know from neutron sorry from studies of the deuteron state we come to a number 35 mev for V 0. So, this is from deuteron bound states. Now, we can put this back in our expression and then let us say we can extract the delta 0 from there, earlier expression that we had written down and we can put that back in this cross section and it corresponds to something like 4.5 or 4.6 into 10 power minus 24 centimeter square or equal to 4.6 bar.

But experimentally proton neutron scattering cross section is given to be something like 21 for low e values. So, now, energy scattering values experimentally gives sigma is equal to about 20. So, what is happening? To understand this, what we do; we have to consider the spin dependence of the nuclear force.

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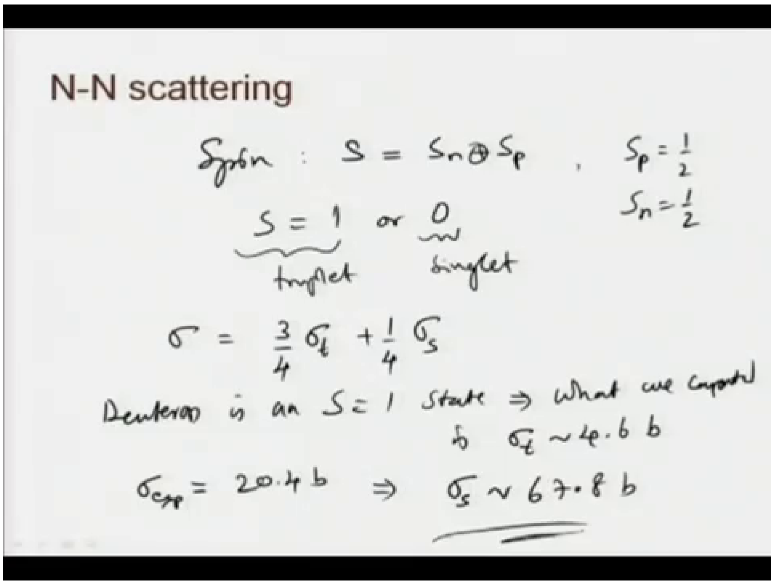

So, we have the spin states, then the nuclear say the neutron comes near the proton. The total spin of the system depends on the combination the angular momentum combination of the nuclear the spin of the neutron and the spin of the proton and both of these s p and s n hard spin particles and therefore, total spin s can be either 1 or 0 right.

So, we have either a triplet combination or a singlet combination. Singlet, spin 0 is called singlet because that has only one projection allowed 0 and triplet has 3 projections plus minus 1 and 0 allowed. So, it has 3 different as that states possible. That is why it is called a triplet.

Now, such cases supposing we have the total sigma and depending on the triplet us and singlet let me denote it by sigma t and sigma s respectively. And since the triplet has 3 states and singlet has one state we had to also have a multiplicity factor of 3 over 4 an averaging factor there and 1 over 4 here to find the total cross section. Now, what about the, whatever the value that we computed? Remember when we computed this we used actually the information from obtained from the deuterium about states and deuterium is sorry deuteron is S is equal to 1 state. So, this gives what be computed is actually sigma t which is equal to about 4.6 bar. Now, we know experimentally sigma is equal to 20 sailors to be more accurate 20.4 band and these two values put together will also tell us that sigma s is about 67.8 bond.

So, we can somewhat understand what is the discrepancy, why is this discrepancy coming and then there can be other nucleon the scattering processes which can actually verify this indeed we will not go into the details of all these things. But lot of experiments are consistent with the understanding and that we have from such elementary studies. And you should also remember that when we solve the scattering problem we are using the potential which is we used a potential which is very croud approximation of the nuclear potential actual nuclear potential. So, there will be corrections to that. And considering was as aspect, so we can actually try to understand the nuclear force in a better way.

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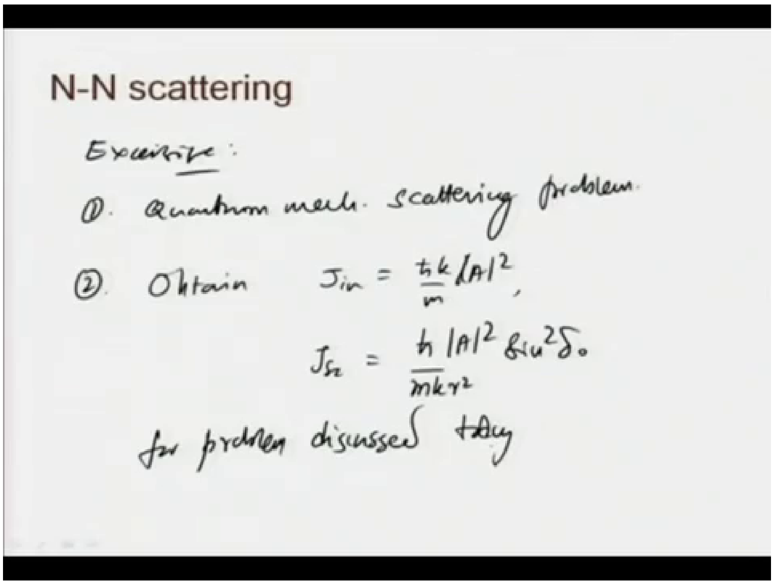

Now, before we conclude let me just suggest some of the questions, some of the as I mean some of the things that you can work out exercises, that you can work out. One thing is I would urge everybody to go through the quantum mechanics is a take it take your favourite quantum mechanics textbook and then go through the scattering theory section on scattering theory.

So, quantum mechanics can scattering problem right. So, almost all the books will be dealing with this some in detail more detail than the other, so you take the book that you think is best and read through that and understand the cross section problem daily is, scattering problem, how to analyze a scattering process. So, that will give you a better understanding of today's discussion.

And another thing that we did not really work out in detail is the current that we can state.

So, work out obtain J in equal to h cross k over m absolute value of A square and J sc is equal to h cross A square over m k r square sin square delta 0 for problem discussed today. So, this is easy to work out, but some I am leaving it to you to do that, all right. With that we will conclude today's session.