Nuclear and Particle Physics
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Lecture – 44
SSB and Higgs Mechanism

Welcome everybody. Today, we will discuss an important aspect of a Gauge theory; that is the Spontaneous Braking of Gauge Symmetry.

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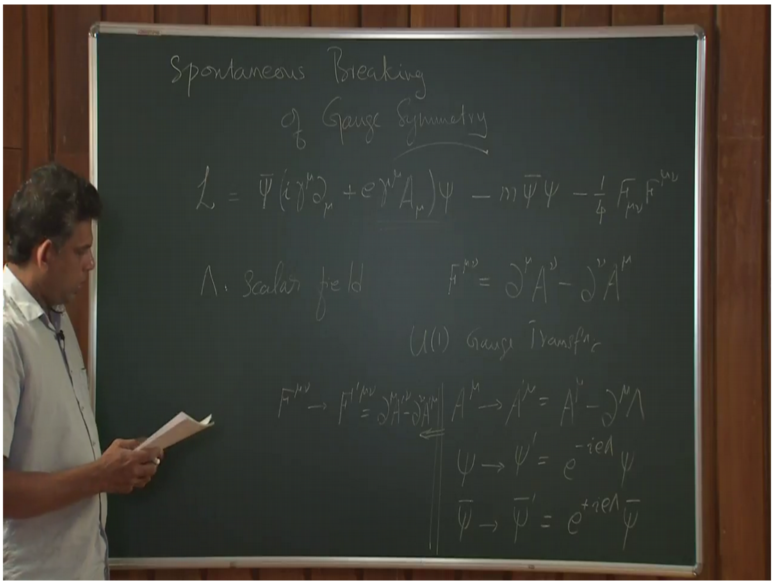

We learned that charged fermions like electron interacting with an electromagnetic field or photons can be understood with the Lagrangian like this; where, we have the kinetic energy term and the interaction term together here of the fermion particle represented by psi. Then, we can have the term which corresponds to the mass of the fermions.

Then, the purely electromagnetic part here the second term here, this along with the psi bar gives the information about the interaction of the current due to the electron or any charge fermion with the photon which represents the electromagnetic field in this fashion. Here F mu nu we had discussed earlier is a compact way of writing derivative of the photon field or the electromagnetic potential A mu.

And we saw that such a Lagrangian is invariant under the gauge transformation, where if we change A mu to some A prime mu which is equal to A mu minus derivative of some scalar function lambda; psi the fermionic field going to some psi prime which differs by the differs from the original fermionic field by a phase factor like this and this is not independent, but it is easy to write it or it is more convenient to just write it here.

So, 1 e e power i e lambda psi bar, a psi bar. We are now familiar with is the conjugate field. Lambda is some Scalar field. So, it is any differentiable scalar field that can give you this thing ok.

So, this is something which we know and here we look at this Lagrangian we won't go into the details, but we had done it earlier and you can check that explicitly putting all these things here, taking the read derivatives appropriately and then, see that this is invariant under the this first term is invariant under the transformation given here.

Psi bar psi is trivially invariant under this transformation because you have an exponential minus i e lambda here coming from this and then, plus i e lambda coming from psi bar and they will balance each other. This F mu nu is actually invariant in itself that is if you take this to F mu nu prime; then, dou mu A prime nu minus dou mu A prime mu.

As you can see, this will give dou mu A nu and dou mu dou nu lambda which will cancel from a similar term coming from the other term here. So, this is invariant under that. And we will get the equations of motion from here the Dirac equation with the interaction term and again, when you look at the equation of motion of the photon you will get the Maxwell's equation from here that is this.

Now, let me look at the electromagnetic part of this in a little detail.

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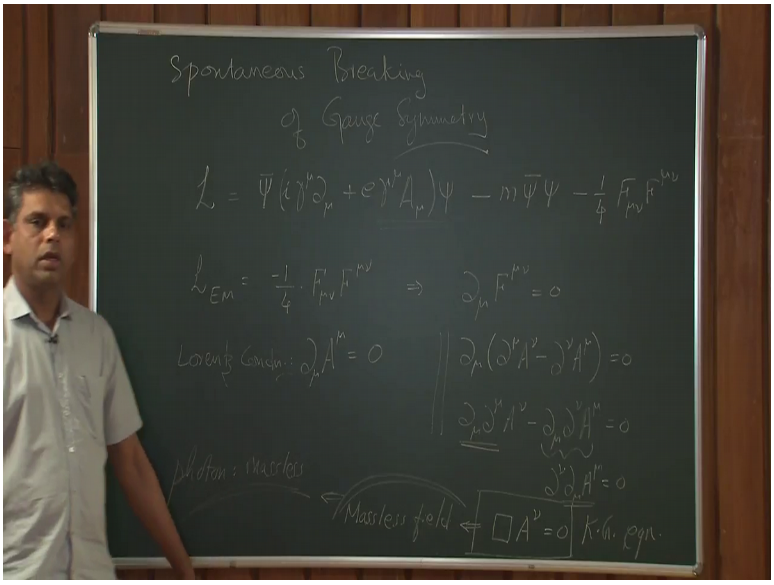

L electromagnetic is I will write only this part. This means that we are not considering the electron there. So, it is a free electromagnetic field only this one; no interactions. So, that gives us the Maxwell's equations F mu nu equal to 0; no current, it is in free space.

Now, this we had we could write as in terms of A mu and the vector potential as derivative of dou mu A nu minus dou nu A mu. This is dou mu dou mu A nu minus dou mu dou nu A mu which is equal to 0. Now, we had earlier mentioned that any electromagnetic field in general electromagnetic field can be represented by an A mu with and we can always choose there is a freedom to choose this. So, that this is satisfied that is the Lorentz condition is satisfied.

In that case here, this is nothing but we can swap this derivatives easily and you will get this equal to 0 because this is basically the divergence for divergence of A mu, Lorentz condition says that this is satisfied. So, (Refer Time: 09:50) we can always choose an a mu so that this is satisfied and then, in that case we have now. So, let me represent this dou mu dou mu as we had done earlier (Refer Time: 10:03) box A nu equal to 0.

Sometimes it is represented as box and then, you can also write it as box square; there are different convention notations, but we will I do not know what exactly we had chosen earlier. But whatever it is this is going to represent this particular thing here. Look at that and this we recognize as the Klein Gordon equation ok. So, this is the Klein Gordon equation of equation representing the dynamics of A mu and from here, we can note that it is a mass less field ok.

So, this mass term is absented that is all right; that is what we have in case of photon. Photon is mass less. So, all these are consistent as long as the as far as electromagnetic interactions are concerned. But then, we generalize a similar study or we generalize this to the case of weak interactions.

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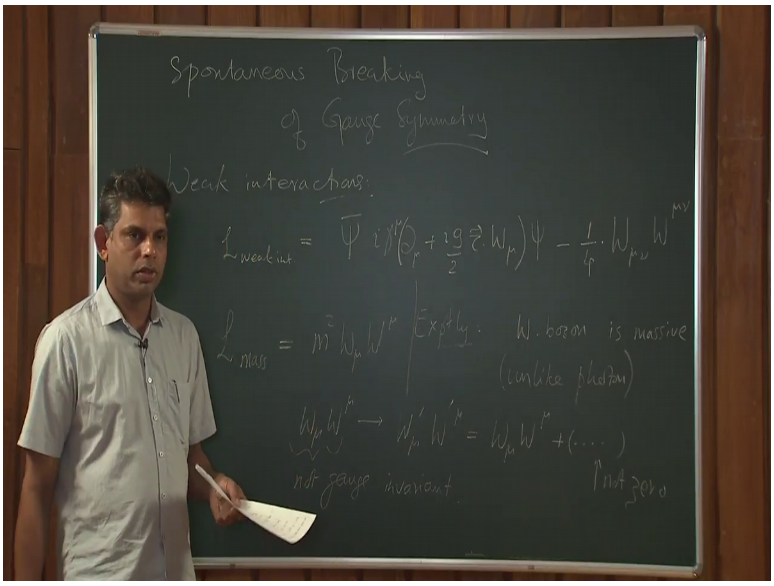

So, for weak interactions, we again can write the Lagrangian; where, electron is interacting with the weak field now, weaks nuclear interaction fail. So, we have this dou mu here and then, we have plus i g over 2 tau dot W. We will not go into the details of this thing we have mentioned this in the previous lectures. There is the field corresponding to the weak interaction like the photon here we have a W and to represent the weak interaction a weak interaction field and the field corresponding to the weak interaction.

This then, you also have similar to the pure electromagnetic field case. Here again, we have a field tensor W corresponding now to the weak field and the Lagrangian term; which talks about the weak interaction fields evolution in the absence of anything else. So, nothing is there and then, this is purely it is going to talk about the motion or evolution of the electromagnetic field. Experimentally it is seen that W boson corresponding to the quantum of W mu similar to the photon; this is a massive particle, unlike the photon.

Now, what is a mass term we need in the Lagrangian? Some quantity like W mu W mu; if you go back to the Klein Gordon equation in one of the early lectures when we discussed the scalar fields, you will see that there was m square phi square there; instead of phi square we have a W prime W mu W nu. Why am I writing u prime? It is not there here yet.

So, there is W mu W nu here. Now, when we take the transformations similar to the field transformation without going into the details let me say this thing. We can when we look at a W mu prime W mu prime which is basically the transformation of W W or W W W mu W nu goes to W mu prime W nu prime which is now original term plus some non-zero coordination which means that such a term is not Gauge invariant under this.

Again, the gauge transformation corresponding to W W mu into W nu we had written earlier we can look at it, but that is not really the focus of today's discussion. So, we will not spend a lot of time on that.

You can ask the question that is fine. Gauge symmetry is broken or violated. This is the Lagrangian is no more gauge invariant, but we need mass therefore, let us add that. That is how physics is done; you understand the physical phenomena around you. If something is not explained according to our assumptions or something goes against as our assumptions, we will change it, change the assumptions.

So, that the new setup is in agreement with experimental results or the observations. Since, W boson is found to be massive, we need this. So, we can say that we don't want to insist on gauge symmetry.

Then, there is a little bit of uncomfortable situation because there are a lot of nice features of this gauge symmetry; one of that is it says when you do the Quantum theory of this interactions and the quantum theory of these particles evolution involved I mean including their interactions with different fields and they try to compute different observables like the probability computations in the scattering or the scattering cross section calculations.

We see that gauge symmetry helps there in terms of getting reliable computation, reliable quantities or I would I should say the other way round, in the absence of this case symmetry or in the presence of symmetry breaking terms like the mass square, mass term in the Lagrangian physical quantities like scattering cross section etcetera will become un computable; meaning that when you try to compute them there will be there are difficulties there.

Again, we will not go into the details of this and just mention the name that there is something called Renormalization which actually guarantees that there won't be any difficulties or any practical problems that comes up with simple the computations of these observables can be handled in a mathematically consistent way. Theories which respect gauge symmetry which are called the Gauge theories are renormalizable or are guaranteed to be well behaved; that is one reason where we think that we should somehow try to maintain the gauge symmetry.

Now, question is how do you get that? How do you get the mass at the same time keeping that good features of the gauge symmetry, the invariance of the Lagrangian under the gauge transformation? Now, this is the topic of today's discussion. So, there is a way of breaking this symmetry. Lagrangian will still be invariant under the symmetry; but then, we will also be able to generate mass.

So, this is called the Spontaneous Symmetry Breaking. I will towards the end mention why is it called Symmetry Breaking still where Lagrangian is invariant under the symmetry is something else which actually breaks the symmetry, we will come to that. And that will also explain why in the sense, why is it called Spontaneous Symmetry Breaking?

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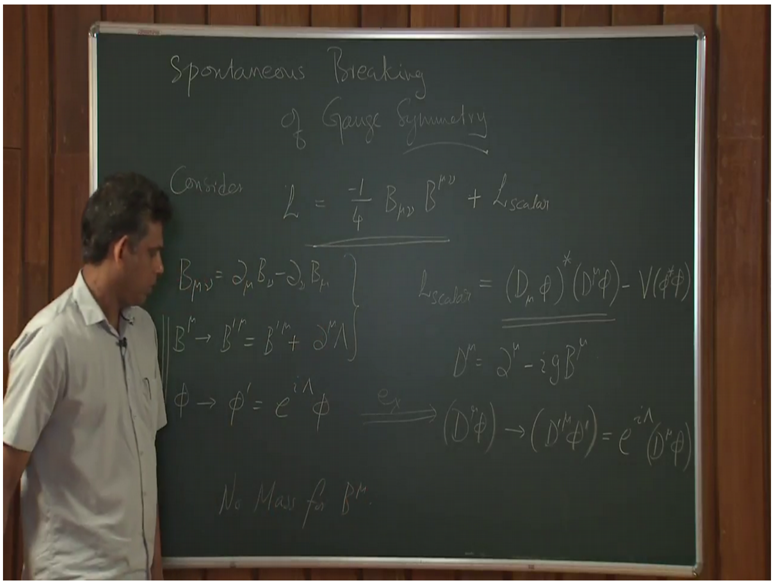

So, let us consider a Lagrangian of some field which we denote by B mu and the corresponding field tensor as B mu nu. We do not want to take the photon because we know photon is mass less and then we do not want to add confusion with the using the same symbol here. But this is a kind of a toy picture or a just in a tour model and just to understand what is spontaneous symmetry breaking.

Then so, here this is with a field B mu, we can have B mu nu is equal to similar to the electromagnetic case. It can define the field tensor and then you will see you see immediately like exactly like in the earlier case that B mu going to B mu prime B mu minus minus or plus it does not matter. So, let us take plus dou mu lambda, one can check the gauge invariance of this explicitly. We will not go into that here.

But, we will add part to the Lagrangian; another part of the Lagrangian, L scalar. Where, L scalar is equal to D mu phi star D mu phi ok; that is the kinetic part and photon shell V which depends on phi star phi the product of phi star phi.

And D mu is dou mu that the partial derivative plus rather minus i g B mu. Now, in the gauge transformation here, we will consider transformation of phi as well with phi going to a phi prime which is exponential i lambda phi, the same lambda. So, these two together represents the gauge transformation here. Now, you can see I will leave that as an exercise that D mu phi along with this B transformation and phi transformation. So, you have a D prime mu and phi prime which is equal to exponential i lambda D mu phi. That part I will leave as an exercise for you to complete.

Now, you can see that this Lagrangian, this part of a Lagrangian is (Refer Time: 25:45) invariant because this dou mu phi star will pick up a minus sign here, in the same phase and you will get cancellation of the face phase these two together. V again is taken as the potential part is taken as function of the product of these two. So, from psi and psi star these phases will cancel. And therefore, the whole thing is invariant under this and then this part has nothing to do with phi, but it is invariant under this transformation as we know.

So, this is invariant. So, the Lagrangian is invariant under such a Scalar gauge transformations and we do not have a mass term which is purely B mu B mu (Refer Time: 26:38). So, notice that no mass for B mu. Now, let me do certain re parameterizations.

We need the scalar part of the Lagrangian.

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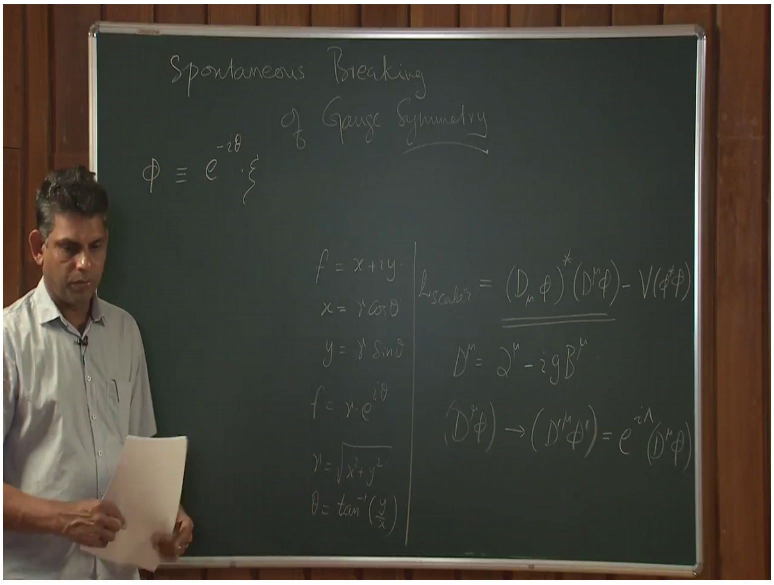

So, we will keep it here or rather yeah, let me keep it is it as it is will take it. Let me take phi which is a complex field as we see here; phi star is already denoted along with this and then, we also said in the case transformation it can break up a phase etcetera. So, we are talking about a scalar field. So, let me define this or parameterize it as a phase and another real a length scale real part and the magnitude and the phase.

So, that you can always do for example, you can consider a function x plus i y and write x as r cos theta and y as r sin theta that will give you r e power i theta and r is nothing but square root of x square plus y square and theta is when you divide this by this is tan inverse of y by x. This is very familiar to you.

So, you can always write i complex number or a complex quantity, complex field phi in terms of the magnitude and then, the phase factor, all right. Why are we doing it?

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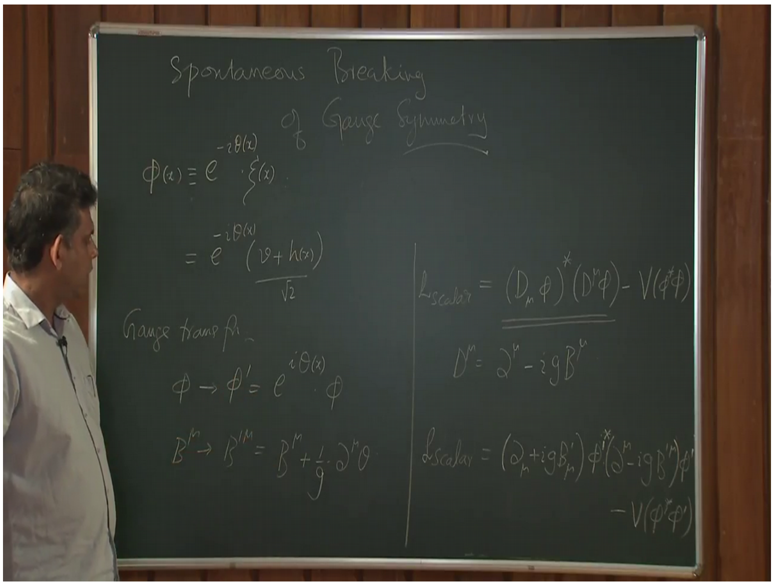

And then, I will write this psi as in a particular fashion. So, that is equal to e power i theta or minus i theta some v plus h x over square root of 2.

So, here I will introduce the position dependence explicitly. All of these parameters depend on x and here, we cannot have too many functions independent. So, we have theta which still depends on the x and the psi we have written in a particular faction fashion a constant plus the function h x. Why have bit than this, it will be clear in a moment.

Once you do this, now we can actually consider the some Gauge transformations. I will take a specific Gauge transformation; where, phi goes to phi prime which is equal to e power i lambda now, but lambda is taken to be theta and B mu will go to e prime mu which is B mu plus 1 over e 1 over say g, we have a g here. So, we will take that g, derivative of theta.

So, this is invariant under such a transformation the Lagrangian L scalar is invariant under such a transformation which means that I can very well write we do not need this. So, (Refer Time: 31:40) we can write the same a scalar L scalar as D mu prime. So, I will write a B in the explicit form. So, we have a dou mu plus i g B mu prime plus because I am taking here this star acting on phi star. Then, we have the dou mu minus i g B prime mu I saw I wanted it to be written in terms of the phi prime and phi.

So, here a phi prime plus or minus V phi prime star phi prime; Since, V is a function of the product of the complex conjugate phi star and phi that is automatically invariant under such a transformation and there is no B there. And this term is guaranteed to be invariant under such a simultaneous transformation of phi and B. We have this right.

Let me take a look at this,that is I just rub this off.

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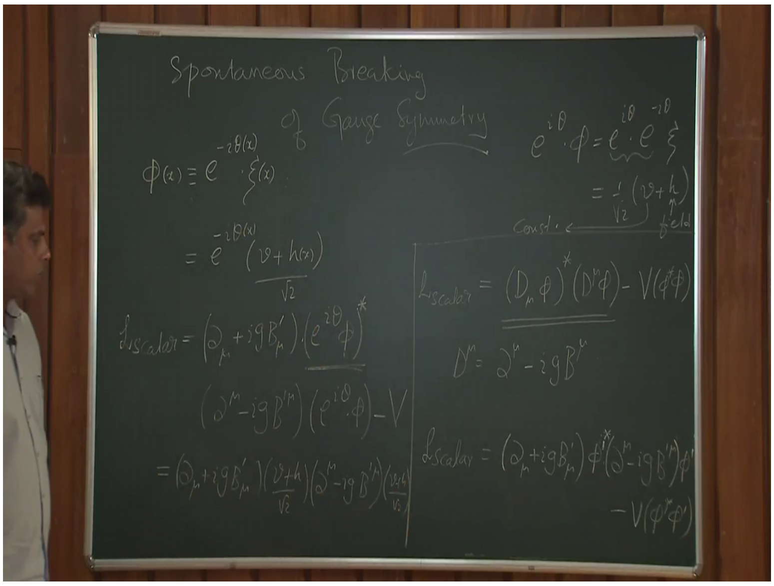

But it is basically L scalar dou mu plus i g B mu prime, I will keep it as it is; phi star is e power i theta phi and star of this. Then, you have dou mu minus i g B prime mu e power i theta phi minus V. V now, I am not considering any changes in V. I will just keep it is as it keep it as it is. This is equal to dou mu plus i g B prime mu.

What do you have here is phi e power i theta and phi which is e power i theta; e power here minus i theta into psi that is equal to 1 over root 2 v plus h. Remember that h is a field which is a function of space time co-ordinates; whereas, v is a constant ok. So, we have that.

So, I can write this. This is nothing but v plus h by square root of 2 and you have dou mu minus i g B prime mu same thing v plus h over root. That is a real quantity now. Therefore, star will not affected. So, this is what we have; I will rub these things.

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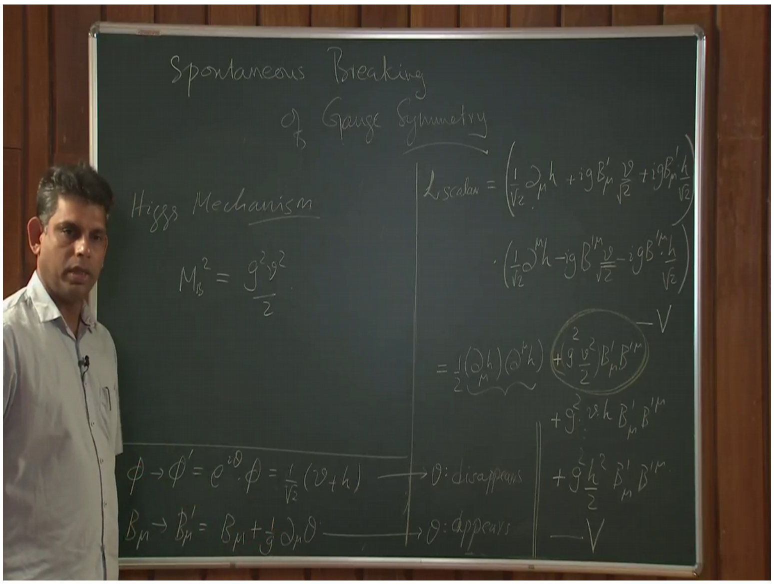

So, this gives us L scalar is equal to you have a 1 over root 2 dou mu h.

So, that is from here, dou mu dou mu acting on v is 0. It is a constant dou mu acting on h, we have there, there is a square root 2. So, this is there. Then, you have i g B mu prime. So, that is a plus i g B mu acting on v; then, you have i g B mu prime acting on interacting with h byte; that is coming from this here; Similarly, from that; now therefore, this.

So, there is a minus V here that potential. So, here we have 1 over root 2 dou mu h minus i g B prime mu v by 2 minus i g B prime mu h by root root; it is all square root of 2 here. And then you have a V which is not relevant at the moment it is important, but we will come that.

This is equal to now let me expand this. So, I have a 1 over 2 dou mu h dou mu h together this; then, you have dou mu into this. So, what is that? Dou mu h minus i g B prime mu v by 2, but then, notice that you also have 1 over root 2 dou mu h plus i g B prime mu v by root. So, they cancel each other. So, this is square root, that cancels. Similarly, this and this cancels; so, you do not have that cross term. So, the only term here is this.

Then, you have base times, it is taken care of this into this is plus g square v square by 2 B mu prime B mu prime, coming from the product of these two. This times, this is again you have plus g square h v by 2 B mu prime B mu prime.

So, there is one coming from here this times this and another coming from here. So, actually there is no 2. Then, you have this times this which is plus g square h square by 2 B mu prime. Now, this term, this Lagrangian along with the potential there minus potential V. Look at this term, v is constant; g is some constant like the charge 2 is constant and then this is the (Refer Time: 41:03).

So, this is like m square, if we denote this by m square; we have a mass term there in terms of B prime mu. So, let me look at this once again. Initially, we had a scalar field which is dou mu minus dou mu plus i g B mu phi and heavy potential B. This is an interaction term with in terms of phi when you look at this phi here.

So, this Lagrangian here written in terms of phi has no term which is purely quadratic in B or there is no mass term. Whereas, once we make the Gauge transformation that phi goes to phi prime which is equal to e power i theta phi which is equal to e power.

So, finally, we had e power i theta cancelling with e power minus i theta and a 1 over square root of 2 D plus h and simultaneously, we had B going to B mu prime which is equal to the original B mu plus 1 over g derivative of theta such a B prime when we write in terms of this phi prime; now, phi prime is without any theta here is essentially this whole thing we have a mass term here.

There are few things a couple of things we can notice. We may notice that theta disappears sorry from phi. The original phi had a theta that disappears in phi prime. So, this says that theta disappears. Original B did not obviously, have any theta, but after the gauge transformation, theta appears here.

So, this theta is now part of B and not part of phi that is interesting. What has happened is in this re parameterization, we have observed theta part of phi into B to become B prime and that B prime, then gets a mass ok. So, that is interesting. Now, we have mass term for B and this particular mechanism to generate mass to the scalar field is the celebrated Higgs Mechanism.

So, that tells you that mass of B is for the square of that this and B prime is now considered as the actual physical field. B mu alone will not suffice, B mu as to be added with theta part theta phantom added with dou mu theta. Now, let us look at, the potential part that we have not looked at so far.

We may need this or let me leave it there.

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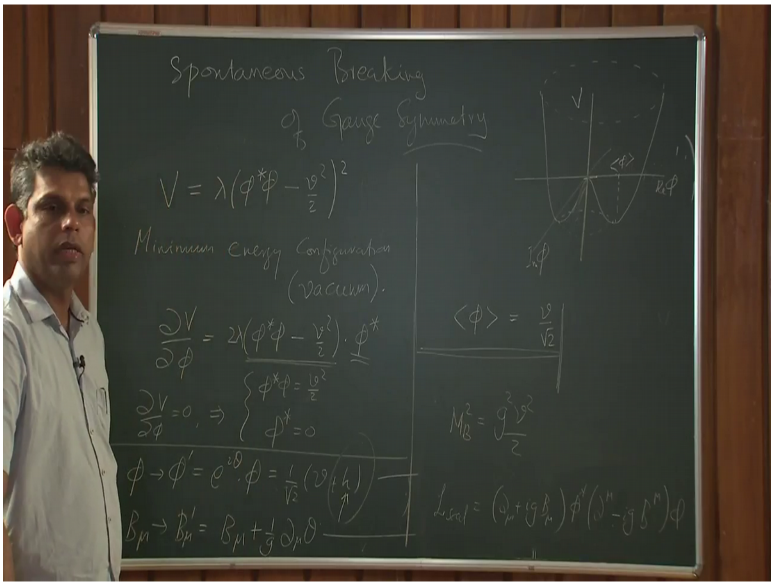

Potential V is (Refer Time: 47:12) and saw that we have a lambda phi star phi minus v square by 2 whole square. Now, this when we ask question about what is the Minimum energy Configuration, any physical system tries to have a Minimum energy Configuration.

So, that is one of the things and the lowest energy configuration is basically called the Vacuum which in particle physics ok.

So, we will say that the Minimum energy Configuration or the vacuum. So, the minimum energy configuration corresponds to the minimum of the potential because any system can have the potential energy and kinetic energy, the other part.

So, potential energy can be positive or negative; whereas, the kinetic energy cannot be negative. This is the always positive. So, the kinetic energy minimum of the kinetic energy is 0 and therefore, minimum of the potential will basically correspond to the minimum of the minimum energy configuration.

So, here we can ask the question, what would be the minimum of this which means we can go by the usual way we can get this by differentiating this taking phi and phi phi and phi stars 2 independently degrees freedom. So, this we have with the phi star here. Setting it to 0 ok, we get the extrema minimum or maximum. So, that will give you two ways; one is this is 0. So, phi star phi is equal to v square by 2 or this is 0. So, there are two configurations which corresponds to the extrema, derivative vanished.

If we take the second derivative, we will see that it is basically the first one which corresponds to the minimum or we can actually draw this potential in phi and so, here. either I write it as real and imaginary parts of phi and you will see that ok, you can actually think about a kind of circle in here. So, basically you rotate this about this axis, axis potential here and you will get all variations of real and imaginary part.

But, the point to note is that here there are two extremas to the turning points; one is when phi equal to 0 both imaginary and real parts of this equal to 0, the other is here which we can consider as the minimum of the potential. So, that part mathematically you can check what that this corresponds to the minimum of the potential and that means, that for minimum energy the expectation value is v by root 2 of phi.

So, phi I am just saying this has the vacuum expectation value or whatever is the value of phi corresponding to the vacuum is v by root 2. So, that phi star phi is v square by 2.

What does this mean? This means that when we say vacuum, in fact, we call the minimum energy configuration as vacuum assuming that in such vacuum or minimum energy configuration, we do not have presence of physical particles or physical fields. Then, quantum fluctuations or particles can be created giving energy or with energy larger than the minimum energy configuration, we can think of having potential having particle signature or the presence of here.

But here, what it says is that the Higgs field cannot be set to 0 even in vacuum. So, there is a presence of the Higgs field in the background when we look at the scalar field, the interaction of this field with other particles in the vacuum say the gauge fields in the vacuum gives its mass. So, we had say for example, we had written the mass of B equal to g square v square over 2 plus square of this thing. So, when be equal to 0, B is mass less.

So, if B was 0. So, we had if we had started with a potential just phi star phi whole square without p square v; then, this would have been 0. The presence of this gives the mass through the interaction that we saw. It is the scalar Lagrangian heard the term dou mu plus i g B mu phi. So, it is basically the interaction of this B with phi that essentially gave rise to the mass here. But there is no phi the physical field corresponding to phi which is essentially h is presence its value in the vacuum that is what that is what matters as far as the mass is concerned.

So, it is not the Higgs fields interaction outside, I mean other ways in general that is giving this math, but its interaction which is there even in vacuum, even in the minimum energy configuration that gives this thing. We will not really be able to resolve this as the interaction of a particle like the phi field with B, but we will see its manifestation in terms of measurable mass of this particle.

And along with this we also have as a consequence a field. So, we did introduce this scalar field and then, there is a remnant scalar particle corresponding to that which is denoted here as h the field there. Now, this h is looked for such a scalar particle notice that we did not have any need of such a particle otherwise, when we study electrons nucleus, atoms or any other elementary particles. We do not really see this Higgs filed here and there was no reason to actually think about it, but for the generation of this mass.

And such a particle was discovered in 2012 at the LHC.

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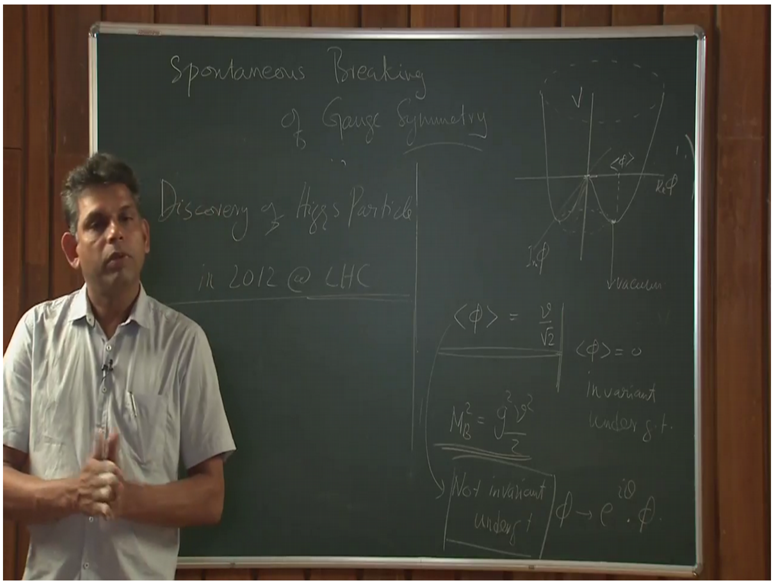

In 2012 at LHC and that was a great success of Higgs mechanism which therefore, was subsequently awarded the Nobel Prize. Now, one more point that I wanted to mention. So, just a final point in this discussion; Why are we calling it the Spontaneous Symmetry Breaking? Symmetry Breaking, where is the Symmetry Breaking? We saw that the Lagrangian is invariant.

So, we said this Scalar Lagrangian goes to itself when we do this transformation. So, there under the Gauge trans transformation, the Lagrangian is symmetry or even if you add the gauge field part that B mu nu B mu nu, lagrangian is invariant under this thing.

But, when we look at the vacuum configuration, if it is here phi equal to 0 or the vacuum expectation value of 0 is invariant under the gauge transformations we can say it.

As if we say vacuum expectation value is not 0 which is some value, it cannot keep changing it is the vacuum we have to choose a particular energy level, energy configuration and then, that is what it is. If we say it is V, whatever will be is that is the value of the field in that. Incidentally this is the measurable quantity; So, the L experimental because they interact with particles, we can actually measure this not from this. Ok sorry um.

So, here we have a toy model, but in actuality we have the gauge bosons where we can actually measure these things. So, in the standard model here I do not want to really write the value of this because this is not the realistic model that we discussed. But in the real standard model realistic Higgs mechanism explained in the standard model, this v is measured to be 246 g v. So, it is a measurable quantity. So, it is a constant, it is a fixed value; then, when we actually say we transform is this is not invariant under the gauge transformation because gauge transformation takes phi 2 e power i theta phi.

So, we would have gone picked up a phase. So, that is actually trying to move around. So, you are picking this thing here and then, it is like moving around these things. So, now, if you pick something there, it is a particular value that we are fixing and this particular choice that you are making here for the vacuum a particular configuration, particular value of the choice of v that we are considering field (Refer Time: 61:49) if I that you are considering the Vacuum is not invariant under the gauge transformation; whereas, the Lagrangian is invariant under the gauge transformation.

So, this is what is called named Higgs Mechanism which rely on this Spontaneous Symmetry Breaking. Ok.

We will stop here and we will continue with our discussion or other topics; we would not continue with this discussion here, but we will discuss other things in the next class.