Nuclear and Particle Physics
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Module – 11
Electroweak Symmetry Breaking
Lecture – 01
Spontaneous Symmetry Breaking

We will continue our discussion on the electroweak interactions understanding it through the gauge transformations so, we said that we had to consider gauge symmetry with the group s u 2 L cross u 1 y.

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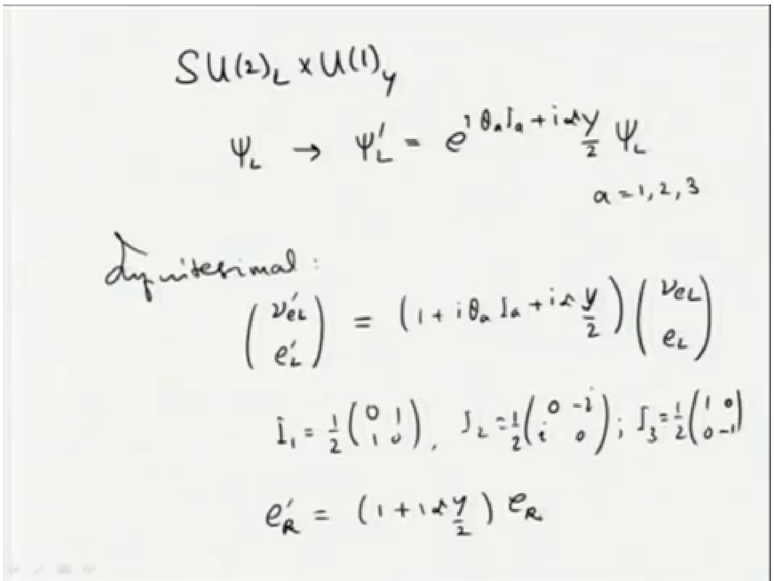

Where L denotes the that fact that only the left handed particles are charged under s u 2 and y denotes that there is a charge called hyper charge which dictates the interaction or transformation through u 1.

And psi L left handed particles transforms like psi L prime equal to e power i theta a I a plus i alpha y earlier we had denoted this hyper charge parameter along with the hyper charge as y for reasons that will be clear as we go on we will denote it by alpha now, reason is that later on we will consider the scalar field where we will usually denote that by the standard notation phi and we do not want to confuse you with this thing.

So, you will change the notation here compared to the earlier this one this L a is equal to 1, 2, 3 y is whatever the hyper charged is and essentially for infinitesimal transformation we write this is a doublet for doublet us I mean for usually the all the particles leptons and quarks that has a nonzero s u 2 charge is isospin it is taken in a doublet of su isospin equal to 1 by 2.

So, let us say nu e L, e L corresponding fields the primed version of this is equal to 1 plus i theta a I a plus i alpha y sorry here y by 2 is there y by 2 again there is 1 over 2 or y is a convention this and acting on nu e L here I a is 2 by 2 matrix the first term and the last term there is no 2 by 2 matrix, but we assume that there is a unit 2 by 2 matrix.

And in particular I can be written as I 1 as the Pauli matrix by 2 0 1 1 0, I 2 is equal to the second Pauli matrix sigma 2 minus i is i 0 again 1 over 2 and I 3 equal to 1 over 2 1 0 0 minus 1 and this factor of 1 over 2 on y is mainly due to this. So, this is the way it is and the right handed field transforms as 1 plus I alpha y by 2 e R no isospin, isospin is equal to 0 for this c.

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$$\mathcal{L}_e = i\left(\bar{\nu}_{eL}\ \ \bar{e}_L\right)\gamma^\mu D^L_\mu \begin{pmatrix}\nu_{eL}\\ e_L\end{pmatrix} + i\,\bar{e}_R\gamma^\mu D^R_\mu e_R$$

$$D^L_\mu = \partial_\mu + ig\, I_a W^a_\mu + ig'\frac{y}{2}B_\mu$$

$$D^R_\mu = \left(\partial_\mu + ig'\frac{y}{2}B_\mu\right)$$

Now, let us look at the Lagrangian, let us only look at the electron part. So, we have i nu e L e L bar gamma mu D mu acting on nu e L nu e L e L this is one and this is a D mu which act on the left handed doublet for the right handed field I have i e R bar gamma mu D mu R e R D mu L is dou mu plus i g I a W me mu a i g prime y by 2 B mu.

So, we have introduced 2 not 2 3 W mu and one B mu. So, total 4 bosons vector particles vector fields here and they are the ones which help us have the gauge invariants for the Lagrangians for the right handed particles and the transformation is with the covariant derivative dou mu plus i g prime y by 2 B mu and this is because I is equal to 0 for this particular.

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$$W^a_\mu \to W'^a_\mu = W^a_\mu - \frac{1}{g}\partial_\mu\theta^a - \epsilon^{abc}\theta^b W^c_\mu$$

$$B_\mu \to B'_\mu = B_\mu - \frac{1}{g'}\partial_\mu\alpha$$

Now, we said how the size transform here then W transform like W mu a goes to W mu a prime equal to W mu a 1 over g derivative of theta a epsilon a b c theta b W mu c.

B mu goes to B mu prime equal to B mu minus 1 over g prime dou mu alpha. So, this is familiar to us s u 2 W the gauge bosons that correspond to the s u 2 transformation W mu is transformed in a similar way compared to the g mu we introduced in the case of s u 3 a the blue on fields whereas, B mu which corresponds to a u and gauge transformation transform similar to the a mu which we had in the electromagnetic transformation so, will not spend too much time dwelling on this because we are familiar with these things.

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$$\mathcal{L} = i\bar{\psi}_L \gamma^\mu D^L_\mu \psi_L + i\bar{\psi}_R \gamma^\mu D^R_\mu \psi_R$$

$$- \frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g\,\epsilon_{abc} W^b_\mu W^c_\nu$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

Now let me write down the Lagrangian i psi L bar gamma mu D mu L psi L plus i psi bar gamma mu D mu R psi R, this is what we had written down earlier as well this is gauge invariant now, but now to make the new part nearly introduced vertical a physical particles we need to give add the kinetic energy term.

So, it is the usual notation that we consider one over minus 1 over 4 W mu nu a W mu and nu a minus 1 over 4 B mu nu B mu nu, where W mu nu a is defending an exactly similar way as g mu nu dou mu W nu dou nu W mu and a non abelian term g epsilon a b c W mu b W nu c. Whereas, B mu nu is correspond to the u 1 and it will be exactly similar to f mu nu dou mu B nu minus dou nu B mu.

There is nothing no third term and we have only one B. So, we cannot make an anti symmetric combination with one particular component object. So, this is the Lagrangian which is invariant under s u 2 cross u 1 and everything seems to be fine what gives what is missing, there is something missing here. The kinetic energy term is there no problem and there are interactions terms through this D mu D mu L has both B mu and W mu in it.

So, there are interactions of the left handed particles with both Ws and B right hand side right handed particles in the second term of the Lagrangian has D mu R which was only B mu in it apart from the normal derivative. So, right have a particles interact with B not with the W's, but what is missing is, the mass term where are the mass terms we had written down the mass term was in the case of electromagnetic interaction and QCD

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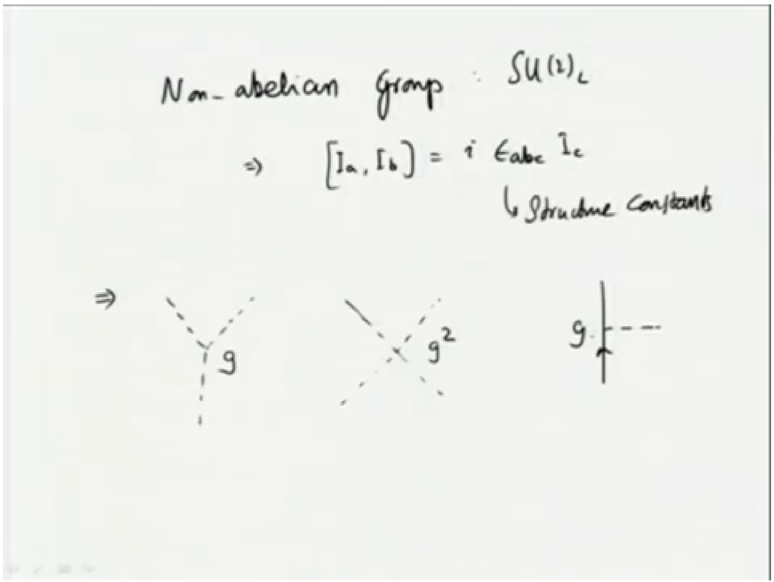

Nature of s u 2 group leads to or cannot be answered either way I a I b equal to i epsilon a b c I c. So, these are again called the structure constants that that is what we had used and looking at the W mu nu very similar to the g mu nu a, we have here again self interaction of the Ws. So, this leads to self interaction of the W gauge bosons both triple interaction and quadratic interactions this is proportional to do g the coupling constant and this is proportional to g square in addition to the interaction with fermions Ws or half of these interactions whereas, the B do not have any self interactions it is similar to the photon.

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Mass term:

$$\mathcal{L}_m = \bar{\psi}_L M_L \psi_L + \bar{\psi}_R M_R \psi_R + \bar{\psi}_L M \psi_R + h.c.$$

3rd term: $\bar{\psi}_L M \psi_R$ is not $SU(2)_L \times U(1)_Y$ invariant.

$$m_R \bar{\psi}_R \psi_R = m_R \cdot \bar{\psi} \frac{(1-\gamma^5)}{2} \frac{(1+\gamma^5)}{2} \psi$$

$$\psi_R = \frac{(1+\gamma^5)}{2}\psi \qquad = \frac{m_R}{4} \bar{\psi}\left[1-\gamma^5+\gamma^5-(\gamma^5)^2\right]\psi$$

$$\psi_L = \frac{(1-\gamma^5)}{2}\psi \qquad (\gamma^5)^2 = 1 \Rightarrow \bar{\psi}_R \psi_R = 0$$

Let us look at the Mass term, we know that electron has mass so, are the quartz and muons and tau articles neutrino do know do (Refer Time: 14:31) fine within the standard model and then it has a tiny mass experimental determined which we will come to in a different way later on (Refer Time: 14:40) in a completely different way. Let us focus on the masses of the other particles the electrons, all the charged leptons electron, muon and tau electron and all the quarks. They are found to be massive and then how do we generate. Firstly, why could not we write a mass term in the earlier Lagrangian?

Let us see how to write a mass term l mass what do I do, I have to have a term psi L m psi L why should I club this with this psi L with psi L it is because a that is the only way to do it with a gauge invariant with gauge easier to gauge invariance because psi L transform in a way which will be canceled by psi L bar. So, this is fine, but the other let me write down the other the thing we can have let me call this m L psi bar R m R psi R ms are some coefficients it could be metric it could never be a metric in the first it could be a 2 by 2 matrix.

In the second it need not to be it cannot be a matrix anyway there is only one term plus you can also have if you want psi bar L m psi R and the Hermitian conjugate of this thing plus Hermitian conjugate if you want. Now problem with this terms are the following let me start with the last term. So, the third term psi L bar m psi R is not or cannot be made s u 2 L cross u 1 y invariant actually problem is faced with s u 2 L ingredients.

That is because the doublet transform in a particular way including the s u 2 part and right handed wave function you know field transform without including any isospin particle at this thing. So, this will not be concerned there is no way to cancel that from other phase factors from of psi R and psi L bar. What about the other term? Let us look at first psi R bar m R. So, let me write it as m R psi R, this is equal to whatever m s psi bar 1 minus gamma 5 by 2 1 plus gamma 5 by 2 psi for psi R is equal to 1 plus gamma 5 by 2 psi and psi L is equal to 1 minus gamma 5 by 2 psi.

So, this is equal to m R by 4 psi bar 1 minus gamma 5 plus gamma 5 minus, gamma 5 square psi gamma 5 square is equal to 1 that makes this psi R bar psi R equal to 0 for any psi this is equal to 0 so, we cannot have an interaction like this a term like this possible.

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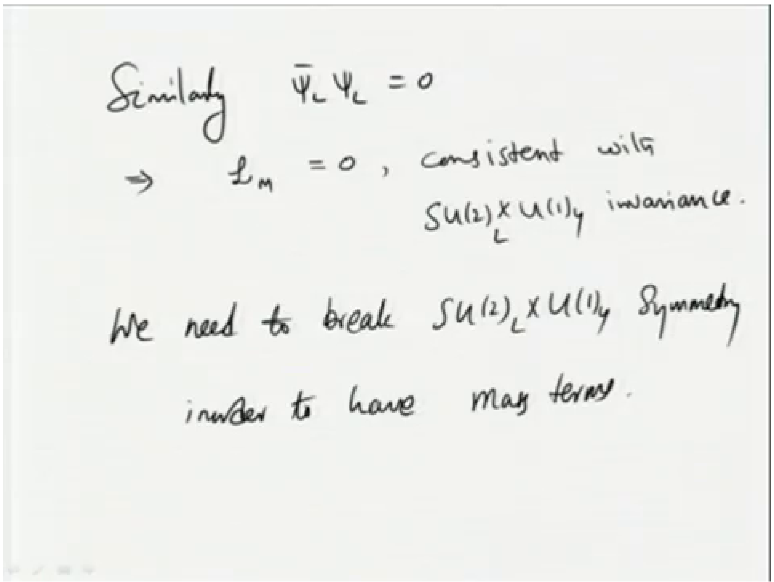

Similarly psi L bar psi L is also equal to 0. So, that tells you that Lagrangian mass term v is equal to 0 we cannot have 4 if you want consistent with s u 2 symmetric. What does this mean, this means that only possibility so, in this Lagrangian here only possibility is to have third term, first 2 terms are not possible technically I mean it is just a question of I mean they are identically when it where is nothing to do with gauge invariance or anything they are gauge invariant of course, but reason for that would be 0 is to do with the way the projections are multiplied.

Now the third term is the only hope for us, but we do not have gauge invariance for that, actually that is something which we have to live with we need to break s u 2 symmetry or s u 2 plus c 1 symmetry. In order to have mass term Lagrangian; this is for the fermions even for the gauge bosons.

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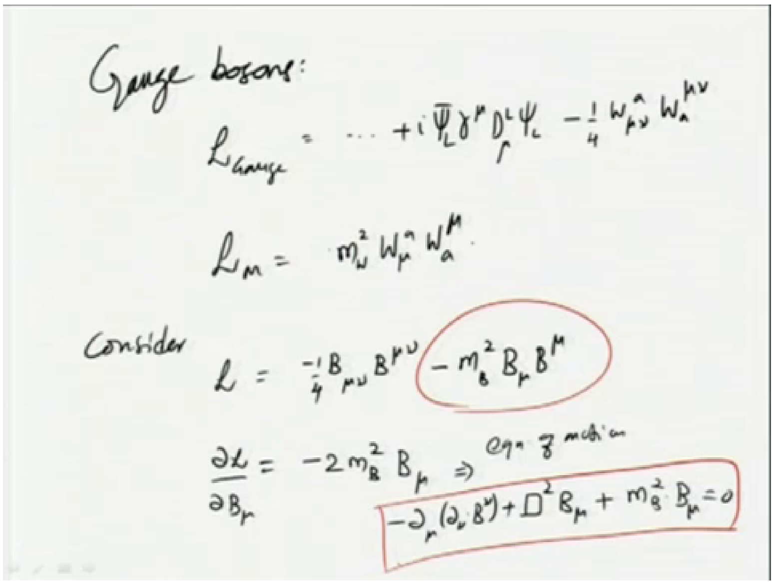

We have added interaction term for the various things including psi bar gamma mu D mu L psi L interaction is added through this D and we have added 1 over 4 W mu nu a W mu nu. Kinetic energy term for this and self interaction term, but the self interaction term always had 3 W's or for W's in it and wherever you we had 2 W's let us go back to that here, when you have W mu nu W mu nu 2 either you have 3 terms 3 W's with 1 derivative acting on one of those or we have 4 W's, where we had 2 W's we had 2 derivatives I think which is actually the kinetic energy term there is no mass term.

The mass term for gauge boson is essentially one which is equal to something some constant into W mu W mu. This is the kind of term that will give you the mass square how do I justify this all right. So, let us look at the Lagrangian dou mu W minus 1 over 4 F mu nu F mu nu for simplicity we will take a non abelian kind of this thing minus or let us take a B mu which is immediately familiar to this m square m B square B mu B mu what is the equation of motion. So, what this part is purely W derivative term the first term has only derivative terms.

So, that will give you dou L over dou B mu is equal to because of the symmetry minus 2 m B square B mu and dou L by you write down the equation of motion for this that will give you the equation of motion del square the B mu box square mu B mu minus, minus of minus plus m B square B mu equal to 0. So, the mass term in the Lagrangian is similar to the mass term for the phi the Klein Gordon equation part, these are Klein Gordon. So, this is for there is part of this thing is there then there is also term minus dou mu dou nu B nu.

So, this gives you this tells you that the mass term is similar to this, but now if you focus on that when the mass term you see that this is not invariant.

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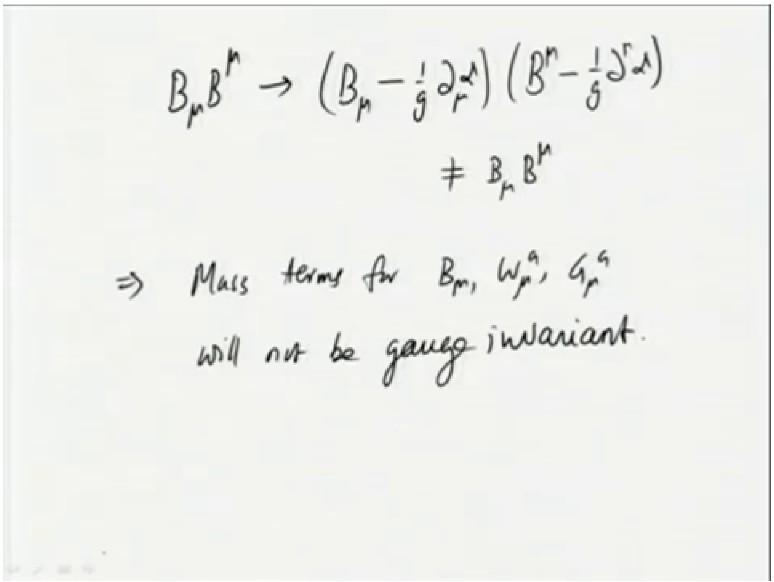

Under gauge transformation where B mu goes to B mu minus 1 over g dou mu alpha and B mu 1 over g dou mu alpha, this is not equal to B mu B mu it is not possible to have these things. This is the reason why we cannot have mass term for the gauge boson suspect thus therefore, that leads to mass terms for B mu W mu or g mu will not be gauge invariant.

So, there are 2 issues one is to generate the gauge boson the Fermion on particle, particle masses, other is to introduce masses for the gauge bosons. We will see later on that the gauge why should we have the gauge boson mass. We will see that the photon is mass less anyway and then the gluons are also mass less, but we will see that weak gauge bosons are massive wind gauge bosons are massive we have determined the mass of this experimentally we have discovered these particles.

And then we have seen that they have mass that is one thing, but the other thing is that it was expected that they are massive because they belong they correspond to the weak nuclear interactions which are short range and for short range particle interactions the exchange particle is expected to have mass, otherwise the range of the interaction will be infinite like the electromagnetic interaction. It is a different story why strong interactions are short range even with gluon as the exchange particle without any mass that will not consider for the time being.

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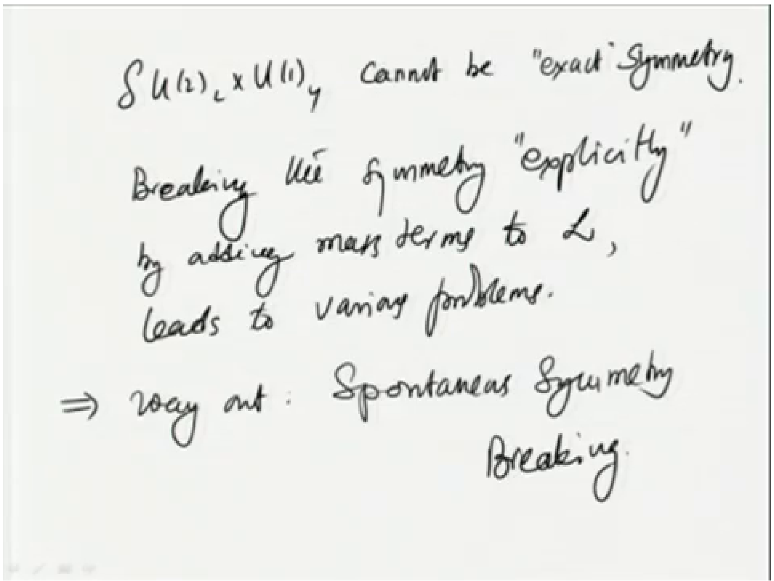

And we saw also that s u 2 L cross u 1 y cannot be an exact symmetry, because there are masses for all these particles and then to generate the mass for this particle we need to break the theory break the gauge invariance. So, we cannot add any mass term explicitly to the Lagrangian and break the symmetry breaking the symmetry explicitly, why explicit we mean by adding mass terms to the Lagrangian this is not good idea not a good idea because this leads to various problems. So, in order to avoid all this problem there is there a way out is what is called spontaneous symmetry breaking.