Nuclear and Particle Physics
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Module - 02
Nuclear Force
Lecture - 02
Dueteron – properties

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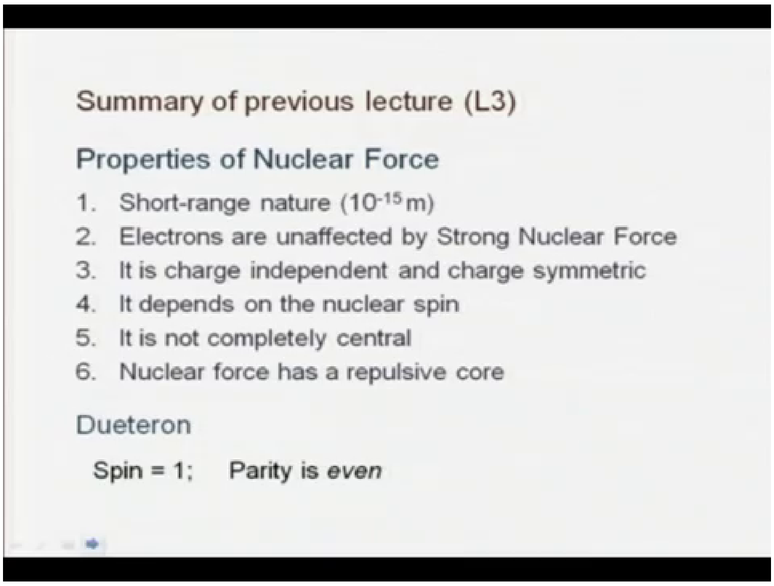

So, today we will continue our discussion on the nuclear forces. So, let us recap what we did in the last class. Basically we looked at the properties of the nuclear forces and essentially we said that we argued that they are short range in nature meaning they are they are, they do not exchange beer or their influence do not extend beyond something like 10 power minus 15 meters which is basically the typical dimension of nuclei.

And there are particles who do not take part in such a nuclear interaction for example, electrons do you know take what in nuclear interaction otherwise we would have seen the effect of these at the atomic level at the structure of the atom or spectrum atomic spectrum would have influence, would have been influenced by the nuclear forces which is not the case and therefore, we know that electrons are not influenced by the nuclear strong nuclear force.

And then we see that they are both protons and neutrons behave in exactly same way or very similar to each a similar way in under the influence of nuclear strong force and therefore, they we can say that they are independent of this fact that whether the nucleon is a proton or a neutron we call this is the charge independence, and the fact that the force is symmetric and then this as this change from proton to neutron is also called the charge symmetry of the nuclear force. And then the other thing that we considered in the last lecture is that it depends on the spin of the nuclear actually. So, the nuclear spin then spin of the nucleus is important when we consider the nuclear force.

And another important thing is unlike the columbic or electromagnetic interactions and even the gravitational interaction which are both central in nature which means that if you write down the potential, corresponding to the nuclear force or nuclear interaction then you will see that you cannot write this and rarely in as a central force potential. So, we will depend not only or not only on the distance between two points, but it also depends on the position relative angular if in spherical polar coordinates if you consider r theta and phi as the coordinates then it not only depends on r which actually gives the distance between two points, but it also depends on the polar angles theta and phi that also.

So, it is this deviation from the centrality is not large, but it is still not completely central that is another thing that we observed or discussed in the last class. And now a another thing is this repulsive core for the existence of the repulsive core in the nuclear force which actually says that as you go closer and closer the interaction actually becomes stronger and stronger, but it stops at some point. These interactions becoming more and more attractive becomes stop such some point and then further when you go closer than these distance there is actually a repulsive cores, core there you cannot go any further than that its basically the observation of the constant nuclear density that tells us essentially that, there is a repulsive cores core for this one a nuclear interaction.

And then further we discussed the deuteron which is one of the simplest bound states of two nucleons that we can think of. So, you take proton and neutron and then they can form bound states and this bound state is called a deuteron and this deuteron has a various different properties which again depends on what the nature of the nuclear spin nuclear force etcetera. And one of the properties of a deuteron which is a quantum mechanical objects now, is basically the spin of that system and it is observed that the spin of electron is equal to 1 you can experimentally determine this and it is also shown that or experimentally seen that the parity which we defined yesterday and discussed a little bit in detail that it is the even parity that the neutron has.

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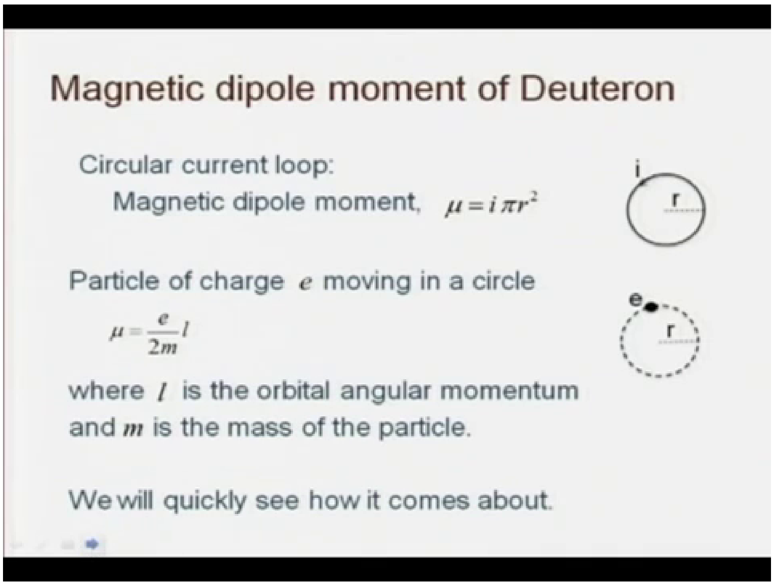

Now, let us look at other properties of the deuteron. One important another property is the magnetic dipole moment of neutron. To start with let us consider what is magnetic dipole moment itself. So, you classically if you consider a current loop let us take the simple case simple loop. a circular loop carrying some uniform current i and radius let us take the radius to be r then we can a define the magnetic dipole moment to be equal to the current times the area of the loop which is i times pi r square where r is the radius of the circle of circular loop.

Now, in the case of particles where particles and their motion again a have the magnetic dipole entering in this, for example, consider an electron or any particle with charge any charged particle having charge say e which is moving in circles this charge moving in the circle is can be equated to some current flow in circle. So, if l is the orbital angular momentum and m is the mass of the particle then the magnetic dipole moment which we denote here by mu equal to e power 2 m times the l the orbital angular momentum of this thing. How do we actually see this? It is straightforward actually to do the simple calculation to arrive at this expression.

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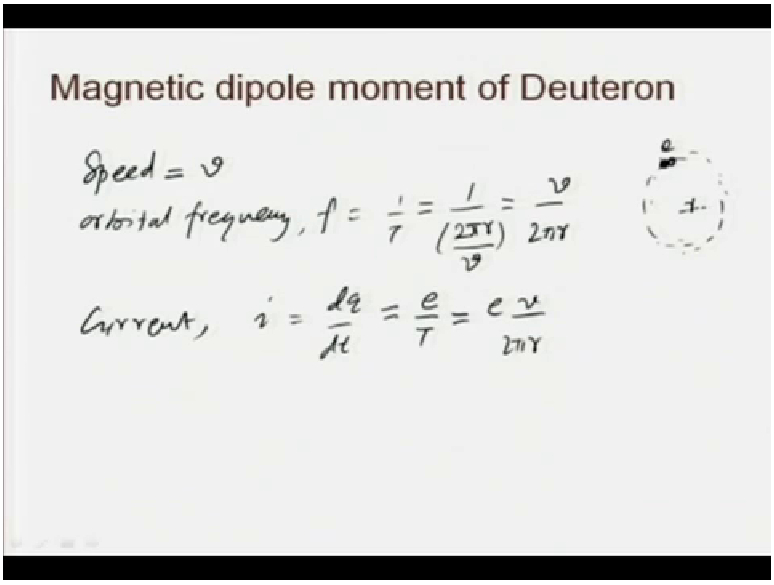

So, we are considering an electron moving in a circular orbit or in charged particle with electric charge e moving in a circular orbit of radius r, let me take the radius to be r similar to the earlier expression, earlier diagram. And then we know if we consider the speed angular speed sorry not the angular speed the speed with which this one the particle is moving in the circle is equal to v then orbital frequency is equal to 1 over T which is equal to 1 over T. The time period is it is 2 pi r distance would be covered with the speed v in one second v is the distance travelled and for 2 pi r distance what is the time taken is 2 pi r by v. So, the frequency turns out to be v over 2 pi r.

So, now, the current associated with this the current associated with it. What is current? Current is change rate of change of charge. So, if you stay at one point on the circle and then ask what is the charge passing through that point in unit time. So, that is what you call the current. So, that is equal to actually charge e will be passing through that T is the time period, then e by T amount of charge will be passing through a particular point on the circle in unit time. So, that is how you know that is what basically the meaning of this time period. In fact, the particular particle will be going the time taken for a particular particle to arrive at that point for the second time is T. So, e by T is the charge per unit time passing through that point. So, now, because we already know 1 over T is equal to v over 2 pi r we can write this I to be equal to e times v by 2 pi r, all right.

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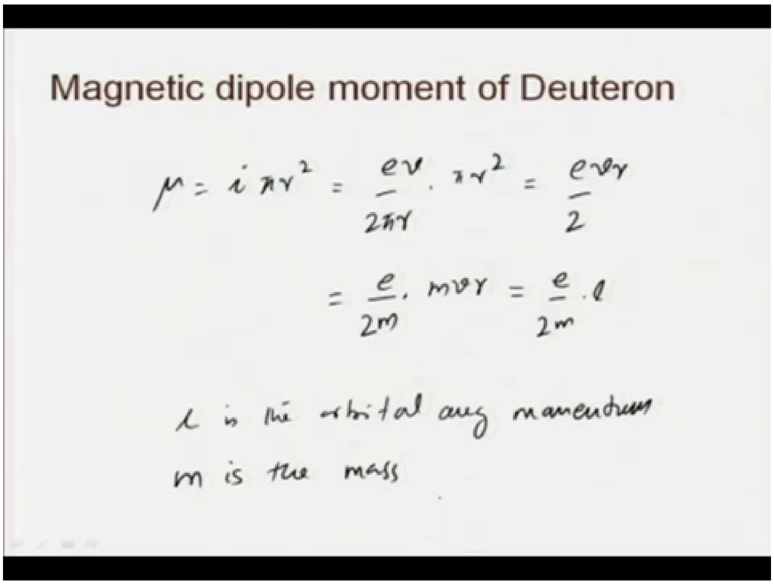

So, this then we will tell us that now I have say the mu is equal to from our earlier expression for the magnetic dipole moment of a current loop with current i and area pi r square here we have i is set is equal to ev over 2 pi r times pi r square and this is equal to e v over 2 r evr over 2 well I can actually write.

So, this is fine, but I can actually write it in a slightly different way I will write it as e by 2m, where m is the mass of the particle I have divided it by m and therefore, I had to multiply by m to be equal to the earlier expression. So, m times whatever is left is in vr. You know mv is the momentum if v is the speed and momentum linear momentum the linear momentum times radius for a circular motion is basically the orbital angular momentum let us denote that by l. So, I can write the magnetic dipole moment of a particle which is moving in a circle with orbital, orbital angular momentum l and mass m electric charge e is equal to e over 2 m times l. So, l is the orbital angular momentum and m is the mass of the particle which is moving ok.

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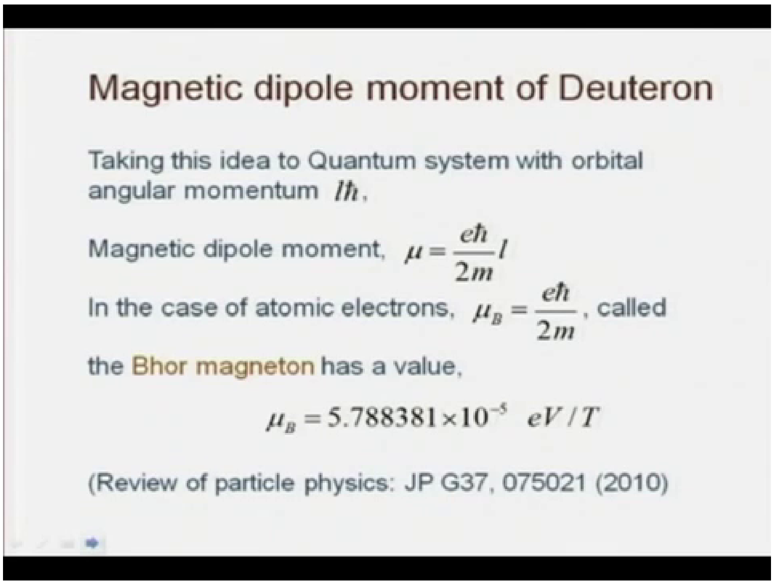

So, now this is just what we have discussed in the last two slides is basically the classical idea. So, you have some electric some current and then what is the magnetic dipole moment associated with that. Now, in quantum mechanics the orbital angular momentum of systems particles actually quantized. So, basically usually you write these angular momentum in terms of in units of h cross.

So, the angular momentum can be written as l h cross, where l is basically some individual and muriatic dipole moment in this case can be written as mu is equal to e h cross over 2m l. So, we take that h cross from the angular momentum for the particle or the whatever the system that you are considering for the particle say and we write that as eh cross over 2m times l. Now, this quantities, so for any particle now you can say that if the angular momentum of the particle is l what is the magnetic dipole moment associated with that for a particle of mass m and charge e. So, this is called actually magnetron whatever is the factor that is multiplying the l to get mu is called the magnetron and in particular if we are talking about electrons. So, electron basically we can write the magnetron as mu b which is a standard notation equal to eh cross over to m e. So, e is the, m e is the mass of the electron now.

So, this for the electron is called Bohr magnetron you must have already come across with this one in quantum mechanics class. So, this is called Bohr magnetron and you can actually experimentally determine the value of this it is found to be equal to 5.788381 times 10 power minus 5 electron volt for Tesla. So, it is the units of Bohr magnetron is this or any magnetron is electron world energy unit per Tesla.

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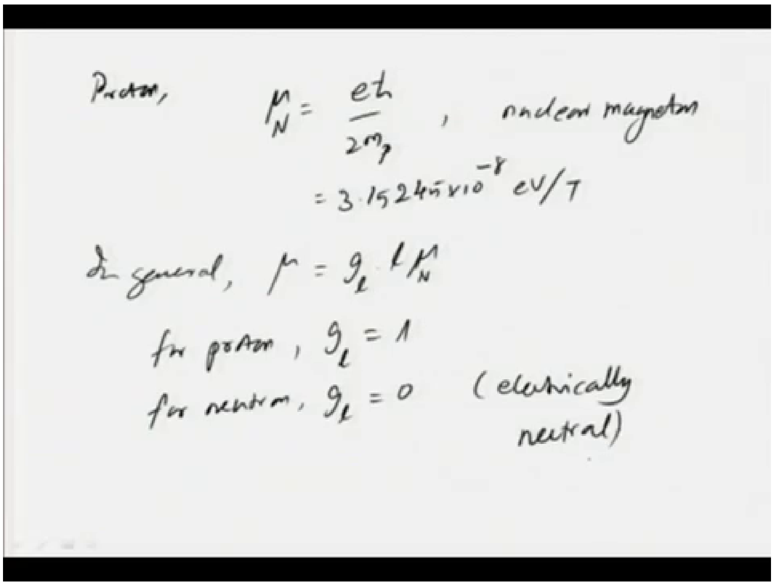

Now, for proton for proton we have similar expression mu is equal to eh cross over 2 mp. So, if you are talking about the magnetic orbital angular momentum with the some proton which is moving even with angular momentum l, then this is the kind of thing that you will have. You can again write down the proton the magnetron corresponding to proton and write the magnetic dipole moment in terms of this, this magnetron. So, this is actually called the nuclear magnetron.

And if you calculate the value of this is equal to 3.15245 times 10 power minus 8 electron volt for Tesla and this is again taken from the same reference as we said earlier. We can write in general, we can write for a particle for nuclear particles the magnetic dipole moment mu due to its orbital motion is equal to g l some factor times the angular momentum and nuclear magnetron. So, know for proton this is found to be actually equal to this g l is equal to 1, so for (Refer Time: 16:21). So, for other particles we can think in a similar way and then get what ask what is the value of g l, that is.

We have also another nucleon which is neutron and what do you think is the value of g l there? It is actually a neutral particle, so a neutral particle moving in circles we will note generate a charge or current and therefore, we cannot actually think about magnetic dipole moment due to the motion orbital motion of the neutral particle the neutron say, this is equal to 0 right, because it is electrically neutral. Now, in quantum mechanics unlike in the classical cases we also have the concept of spin all right.

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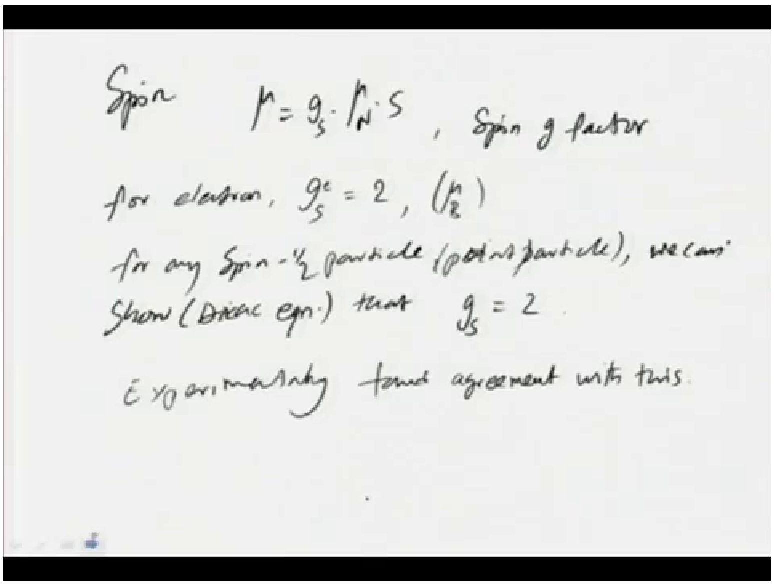

It is a spin, well it is sometimes given an analogy of top spinning a classical analogy of that. Well, whatever picture that you want to consider this quantum number spin is similar to angular momentum orbital angular momentum. So, the spin we will also therefore, have generate similar kind of physical effects say if there is a charged particle which is spinning you can think about the charge sphere or take a dielectric ball and then let us charge it. So, it surface has some charge electric charge let us say uniformly chargeable and take one diameter as its axis and then spin it then you can actually think about this motion of their charge for you the charge is moving and therefore, there is current now and then that current can actually thought about as developing magnetic dipole moment.

Similar to that although this classical analogy is not going to be exactly the same kind of a physical picture motion physical motion for the case of quantum mechanical spin, but I am for or for all practical purposes these charged particle spinning or the spin itself is actually for any particle charged or otherwise will actually give you kind of a magnetic dipole moment for them.

So, this again we can write down a similar expression say mu n whatever is the spin of the particle and the some factor g yes g factor. So, this g is called the g spin g factor. So, this spin g factor of the particle. So, you can ask the question like since protons have spin half neutrons also spin have particles what is the new dipole moment magnetic dipole moment of neutron and what is the magnetic dipole moment of proton due to its spin. Supposing it is not moving, but it is just having a spin on then what is the magnetic dipole moment that will be actually equal to whatever is the g s of the proton times the Bohr sorry the nuclear magnetron times the value of its spin, which is basically equal to 1 over 2 in units of h cross. So, all these things we are talking in terms of units of h cross because then h cross is already taken into account in the magnetron.

So, g s is called the spin factor and then say for example, even for electron you can actually ask what is the electron. What is the magnetic dipole moment due to its spin or in particular? What is the g s for electron? So, this is equal to 2. In fact, for any, here for example, we are talking about mu b as the magnetron which is the Bohr magnetron, not nu mu n, nu n is when you are considering proton and neutron with their masses of proton mass of proton considered in this and mu b is for the case of electron mass and we call that Bohr magnetron. In fact, for any spin half particle you can prove. So, that point particles actually which means the particle without any structure within it we can show that using what is called Dirac equation g s is equal to 2, the spin g factor is equal to 2.

Now, Dirac equation is for those of you who were not have, who have not heard of this is a equation similar to the Schrodinger equation, for a quantum mechanical system, quantum mechanical spin half system in particular for electrons say. They valid for any spin have (Refer Time: 22:04), but particularly for electron when you take into account the relativistic effects as well Schrodinger equation is a non relativistic equation. So, it talks about the non-linear motion of electrons. But if we want to consider relativistic electron then you have to actually have corrections to this Schrodinger equation. In fact, in order to have a different equation which is basically the called the Dirac equation and so that is the one which is the equation of motion of a relativistic electron or relativistic charged particle I mean this spin half particle.

Now, for similar to this we can actually this, this is therefore, an theoretical calculation using there Dirac equation you can show that spin half particles will have a g s is equal to 2 and experimentally it is we have found agreement with this. Now, you can ask what is the situation with proton right, protons.

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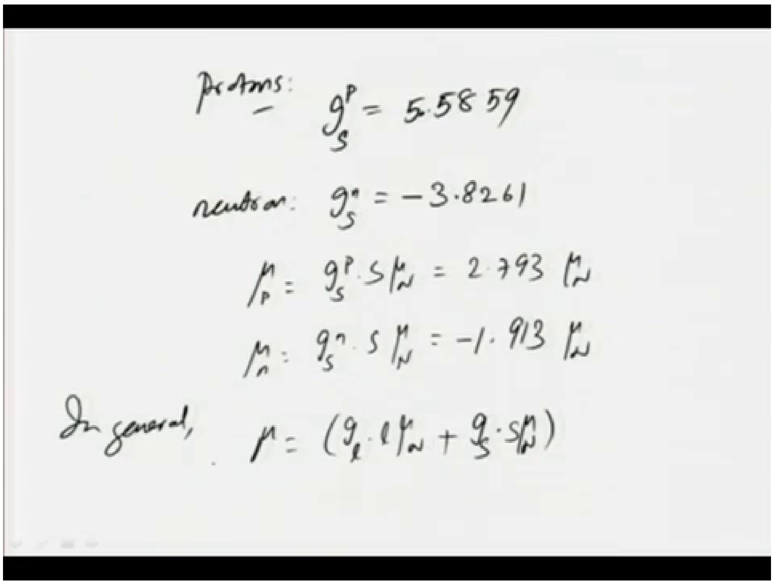

So, for proton you have a g s, let me denote it by g s of p which is experimentally found to be 5.5859 and I am not adding any I, this is a precisely measured quantity actually there are more than these decimal places that you can write down without any uncertainty and then and experimentally you also have some uncertainty at the end we are not doing all these things. So, up to 4 decimal places this is like this 5.5859 and this is not equal to 2 all those proton is a spin half particles.

So, how about the Dirac equation applied to proton? The Dirac equation applied to proton as a point particle would give you g s is equal to 2, g s of p deviates from 2 tells you that it is actually not a point particle it has a structure and indeed we have found that the proton is a composite object. It is made of what are called quarks. We will come to that at some other at a later stage in the same course in this course. And for a neutron we have a g s let me denote it was g s n, n for neuron there is found to be actually negative minus 3.86 yeah 8261, 8261. So, this minus sign actually says that when we consider the spin and the magnetic dipole moment as the vector quantities they opposite to each other spin itself is a vector and we have been talking about only the magnitude so far, but it is actually a vector quantity. So, when you write a mu equal to something times s that vectorially it holds good and if the coefficient is negative then it says that mu is anti parallel to s, for the case of neutron it is so, then for proton it is actually parallel to it.

Now, in general therefore, you can actually write down let me say. So, for example, in the case of mu proton therefore, mu p is equal to g s p times s times mu n which is equal to s is half, so half of g s p in units of 7 9 3 in units of nuclear magnetron. And for neutron it is g s of n s times mu n s is again half. So, this is minus 1.913 mu n.

In general if they are also moving with some orbital angular momentum then you can write the total magnetic dipole moment due to that say for example, a proton which is moving in circles then you will have a contribution from the orbital angular momentum and orbital and a contribution from spin. So, mu n is common to the worth of this, but anyway let me write it in this fashion and you know for we already discussed this for neutron g l is equal to 0, for proton g l is equal to 1.

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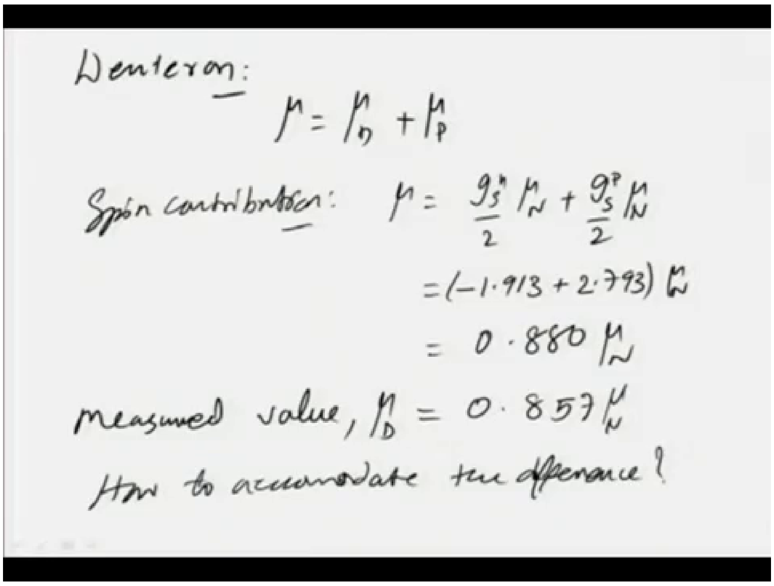

So, that comes to our main discussion which is basically deuteron. So, for deuteron what is the magnetic dipole moment of deuteron? As I said earlier the deuteron is made of neutron and proton. So, you can say that well there is a contribution to the magnetic dipole movement of deuteron from the magnetic from neutron and from proton.

So, let us first take the spin contributions, which means the contributions due to the spins of proton and neutron. So, that will be equal to mu is equal to g s of n times s times s is equal to 2, so half. So, this is divided by 2 times mu n plus g s of p again spin is 1. So, it is equal to mu times n which is equal to we said earlier in the previous slide that this equal to 1.913 plus 2.793 mu n which is somewhat equal to plus 8 point sorry it is

0.88033 goes etcetera and you can ask the question what about the measured or observed value which is basically the magnetic dipole moment of deuteron, let me denote it by mu d it is found that it is equal to, it is equal to 0.857 neutron, magnetic 7 nuclear magnetron. So, it is almost the same, but so not exactly.

Since, as I said earlier that we can actually measure this magnetic dipole moment precisely it is not very satisfactory to agree at the first digit decimal, but do not agree at the second decimal third decimal etcetera. So, we had to ask the question since we can actually experimentally measure this very precisely, then what about the discrepancy here. How will we accommodate the difference? How will we accommodate the difference? So, one of the things that we can think about is the orbital version of proton inside the deuteron right, that the orbital motion of the proton with respect to the center of mass of n-p system, now considering n-p system as the deuteron.

So, now let us go back to the last lecture where we had actually discussed the spin of the deuteron and when we discussed the spin of the deuteron and the parity of the neutron we said there is a relative what is the relative angular momentum of the n-p system which constitutes this deuteron. And from various arguments and from the observed experimental facts of parity even spin is equal to 1 etcetera we come to the conclusion that deuteron has an n-p relative n-p system with relative angular momentum l equal to either 0 or l equal to 2 or could be some kind of a combination.

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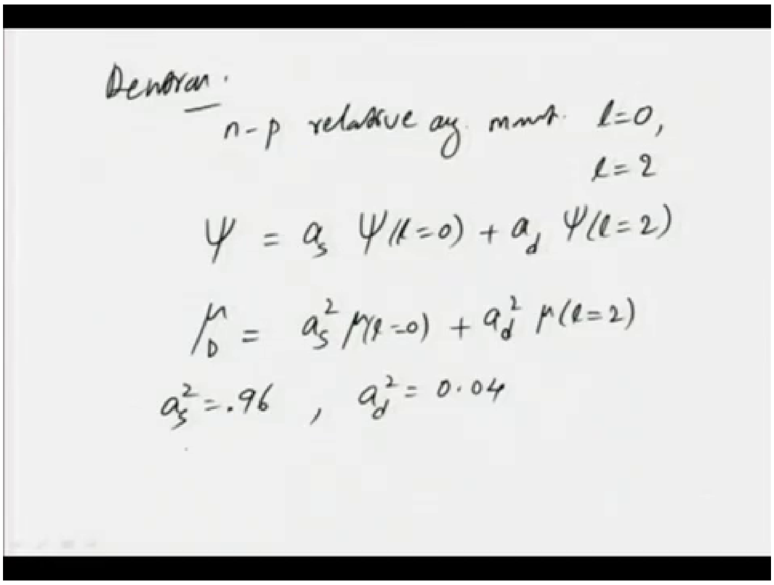

So, l equal to 0 or it should be even that is what the parity argument said. So, it is either 0 or 2 it cannot be larger than 2 because of the spin arguments spin measurements.

And now let us say suppose we want to accommodate both of these l equal to 0 possibility and n equal to 2 possibility how do we actually consider this in quantum mechanics. So, the wave function of the deuteron then is mixture of l equal to 0 state plus a mixture of plus l equal to 2 states. These are the two possible l value, so the combination of this. So, when we consider a combination like this linear nation of this that is what the superposition linear superposition principle says that you can actually superpose two different states to get a new state and the actual physical state of the deuteron could be such a superposition.

So, let me denote it by a s, s corresponds to n equal to 0 say and a d let me denote d by l equal to 2 state usually in atomic systems that is what we denote l equal to d l equal to 2 state is d state. So, a d and a s are the coefficients issue. And ask the question whether this is a possibility and whether that agrees with everything including the magnetic dipole momentum measurements. So, what about the magnetic dipole moment here? Then the mu you will be then a s square. So, the psi actually gives the kind of amplitude for this one. So, if you want to find the magnetic dipole moment then you had to take the magnetic dipole moment operator and multiply it by psi and psi star integrate over d s that is what do you get it and I do not go into that. So, mu d is a s square times mu which is coming only from l equal to 0 kind of a state plus j d square from l times mu coming from l equal to 2 states.

We will not go into the details of this calculation any further we will simply say that experimentally whatever we have measured will agree he we say that a s square is equal to something like 0.96 and ad square is equal to 0.04 total into equal to 1. So, this says that there is this takes care of that discrepancy of mu d which we found experimentally and from the theoretical calculation of considering only l equal to 0 state. It is almost l equal to 0, but there is a small difference coming from l equal to 2 and taking together taken together that will now agree with the theoretical or the experimental value. So, this is basically what we want to discuss on the magnetic dipole moment of the deuteron.

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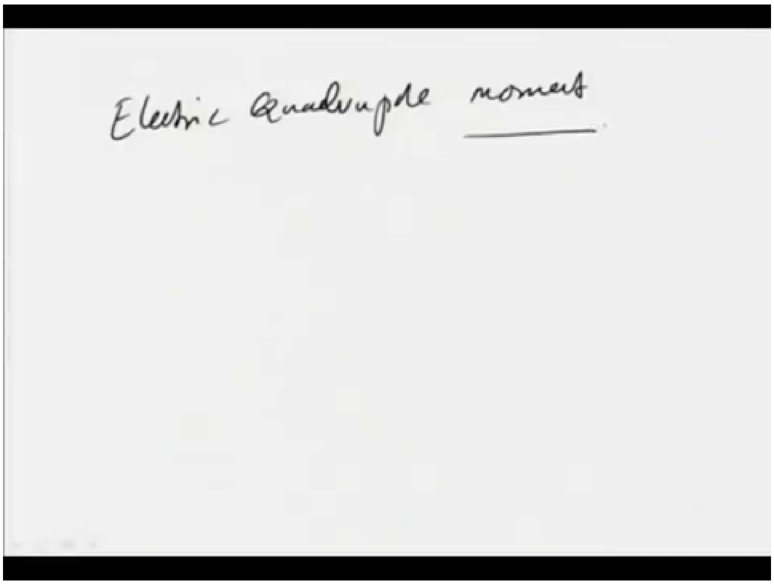

And there are some other properties of the deuteron which is basically another important property called the electric quadrupole moment. And we will come to that in the next session.