Nuclear and Particle Physics
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Module - 10
Gauge Symmetry
Lecture - 02
Weak Interaction-continued

Continuing our discussion on weak interactions, we discussed the fact that weak interactions violate parity they also violate charge conjugation symmetry and the combined symmetry of charge conjugation and parity is conserved in most of the in most of the weak interactions. Now let us look at it little in a little more details, let us look at the parity of quantum number how we can assign the parity in to the wave functions.

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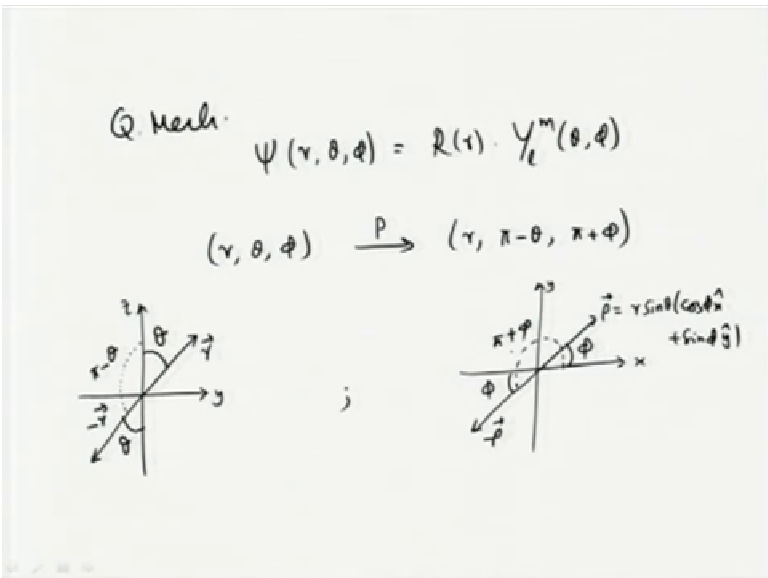

Quantum mechanically, wave function let us say r theta phi let us consider the case where this can be separated into the radial part and the angular parts the angular part is called the spherical harmonics like in the case of hydrogen atom, what does this what does parity do to this, r theta phi the polar coordinates under parity goes to r pi minus theta and pi plus phi, how do we see that.

Let us look at the theta r is just the magnitude of the position vector which remains the same whether it is in one direction or opposite to that so, for parity will not do anything on that that is scalar quantity. Now theta let us take the y z plane and consider the position vector lying in that in, that case this is your theta the polar angle and under parity it will go to minus r and this angle is now theta and what is the polar angle of minus r?

It is the angle measured from z axis which is pi minus theta it is clear. So, under parity theta goes to pi minus theta, for the case of azimuthal angle let us go to the x y plane. So, it is the x y plane, in the x y plane projection of the position vector let me call that rho which is equal to r sin theta cos phi x plus sin phi y as you know let us say it makes an angle in this particular situation a phi with the x axis, under parity it will go to the opposite direction with phi here and what is the angle that minus rho makes with x axis it is pi plus phi.

So, that is what you have here and what about Y l m theta phi, how does it go to what does it go into under parity.

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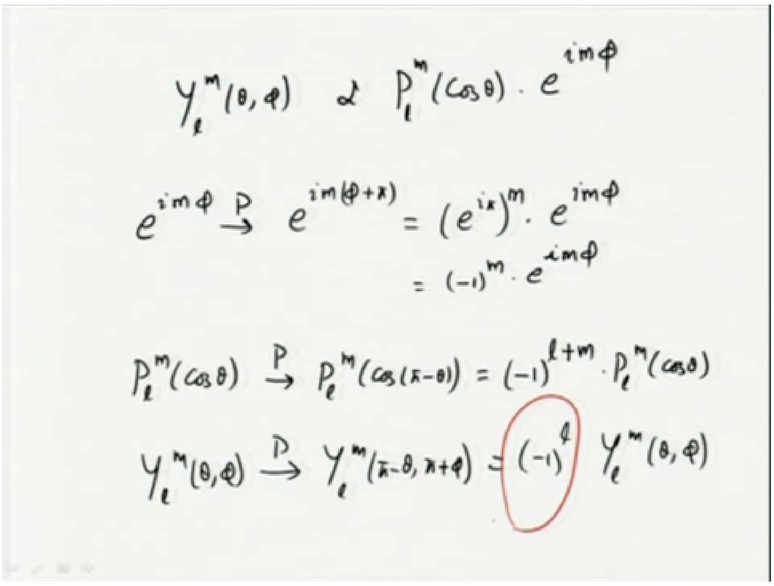

Just note that this is proportional to legendre polynomial azimuth associated legendre polynomial as a function of cos theta and exponential i m phi. These are the phi and theta dependence of the spherical polar coordinates spherical harmonics Y l m and the azimuthal angle under parity goes to pi plus phi and e power i m phi will go to e power i m phi plus pi under parity of course, which is equal to e power i m. Let me write it as i pi m e power i m phi which is equal to minus 1 e power i pi is minus 1 power m e power i m phi. There is a factor of minus 1 power m that the e power i m factor i m phi factor picks up for the azimuthal part of the wave function picks up.

What about the polar angle part under parity, it will go to P l m with cos theta cosine of pi minus theta now and without going into analyzing it we will just give you the result it is equal to minus 1 over l plus m, you have to just look at how the legendre are associated legendre polynomials behave under theta are going to pi minus theta P l m cos theta and together Y l m theta phi under parity goes to Y l m pi minus theta pi plus phi is equal to minus 1 power l plus m plus m l plus 2 m minus 1 power 2 is always positive. So, we do not have to worry about it, so it is minus 1 power l Y l m theta phi.

So, this factor decides whether the total wave function is even under parity or odd under parity if it is even under parity maybe if l is odd then it is an odd function of parity immersion under parity if it is even if l is even then it is even under parity.

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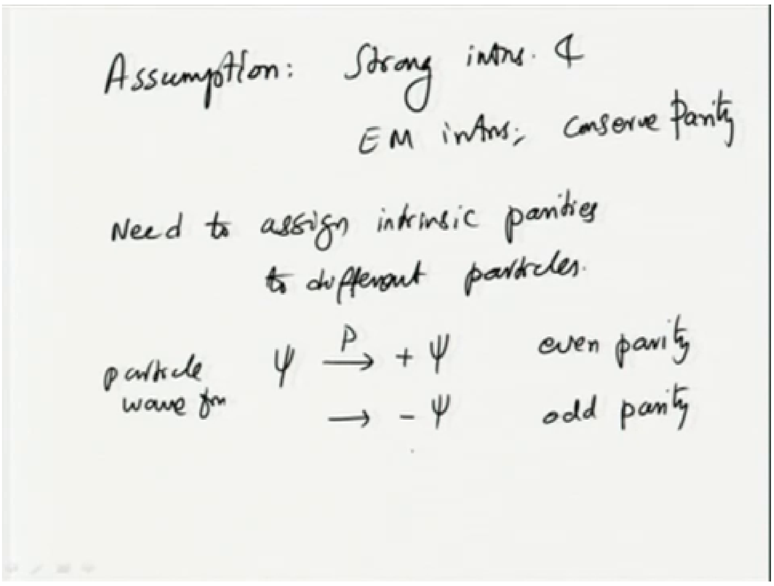

Now, assumption to start with that strong interactions and electromagnetic interactions conserve parity, this need to be checked experimentally and experimentally we have not seen any violation parity violation in strong interaction or electromagnetic interaction, but for this we need to assign intrinsic parities to various different particles, when you talk about intrinsic parity we do not have any wave function like this that we can and orbital angular momentum that we can think about the particle is a particle which has maximally it can have a spin for this thing.

So, without going into how exactly that comes about there is the question that will ask is looking at various different reactions is it possible to assign intrinsic parities to particles different particles consistently. So, that the strong interaction and electromagnetic interactions conserve parity answer is yes this can be done consistently. So, what we had to do is, to say psi the particle wave function how does it undergo, how does it change under parity, it can either pick up a plus sign or a minus sign. So, this is called even parity and this is the odd parity.

So, even an odd parity for these particles can be assigned consistently so, how do we do that convention.

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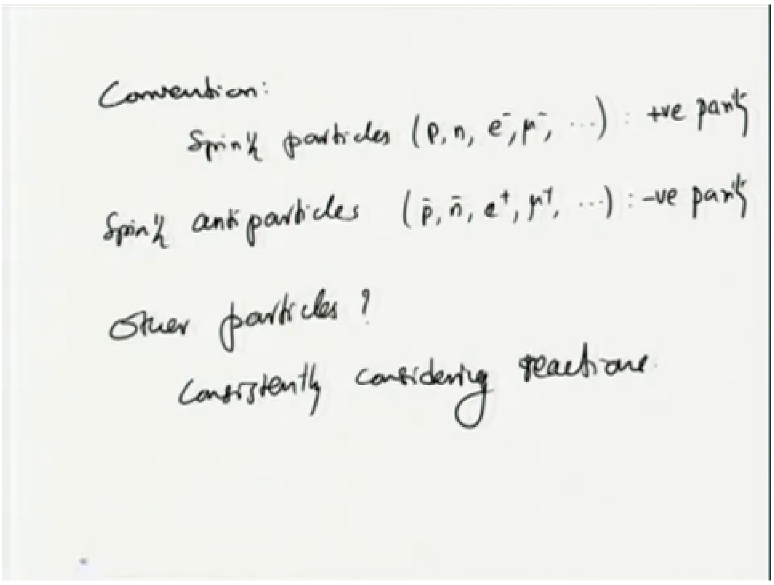

To start with spin half particles like proton, neutron, electron, muon, etcetera neutrinos etcetera or half positive parity anti particles spin half like anti proton, anti neutron, positron, mu plus etcetera half negative parity or I should I can write as even parity and odd parity other particles so, consistently considering reactions.

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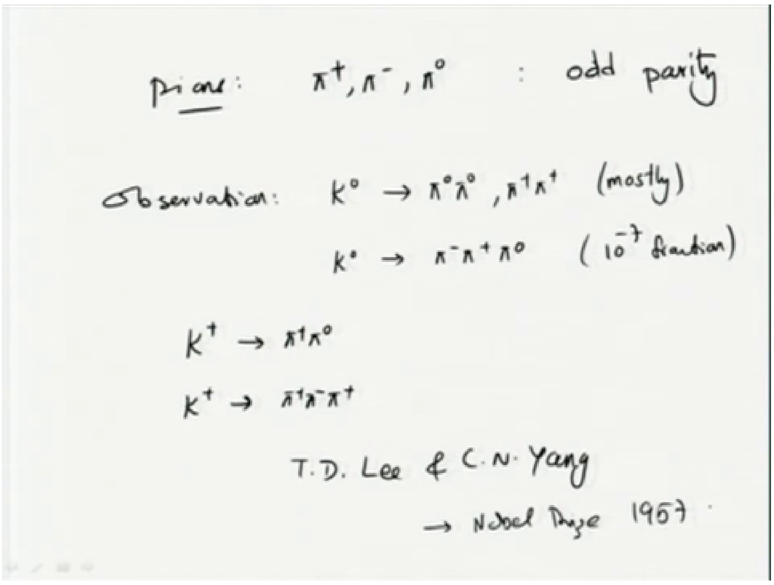

For example, pions pi plus pi minus pi 0 or half odds parity we will not go into how one beta means this or how one actually assigned this there is a consistent way of doing it this has been done.

Now, let us look at one particular reaction observation of K 0 decay it was observed that it could decay K into 2 pi ons pi zeroes or pi plus plus it actually it is mostly decays into this, but there was a small part of this decaying into pi plus pi minus pi 0. Something like 1 in 10 power minus 7 decays to 3 pions, now if pions has odd parity, parity minus 1 the final state of the first reaction decay gives together parity plus minus into minus the second reaction it is minus into minus into minus 3 times and that for tog together it is minus 1.

So, the K 0 decays is to final stage with positive parity mostly, but also there is a small decay probability for it to decay into negative parity final stage, which in other ways says that K 0 is a particle which cannot be assigned a proper parity to the one or the other way to interpret it is that, parity is not conserved in such decays. It can be gained do positive or negative verities there was also observation of similar of the particle K plus going into pi plus pi 0 or going into pi plus pi minus pi plus 3 pi 3 pions and 2 pions.

Again one final state is, even parity the other one is odd parity. In fact, historically speaking initially these particles were caught off to be 2 different particles because parity was violated in the final 0 they decay into 2 different final states with 2 different final parities, but then it was suggested by T D Lee and C N Yang that parity is actually parity could be thought of violated in weekly case like this and then further on it was experimentally found as we saw earlier that parity is indeed violated in C P variation and this fetched Nobel prize for Lee and Yang in 1957 combined C P variation is also observed in the weekly case weak interactions.

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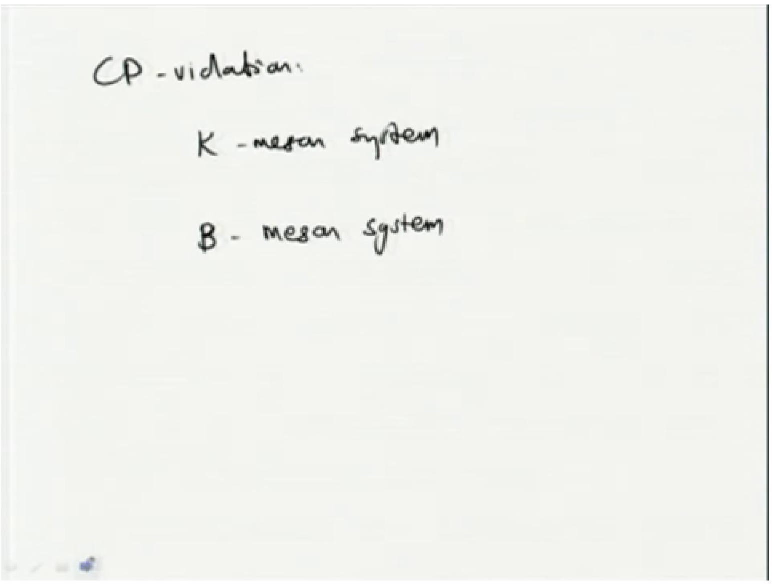

So, for example, if you consider again K meson system and more recently B meson system both of these so, C P violation, there are dedicated experiments there had been dedicated experiments running for a long time many decades investigating the C P violation in the K meson system, experiments accelerators were tuned to electron positron colliders for example, could be tuned to energies which will abundantly produce K mesons different excited states of K mesons also and similarly one could tune this collider energies to B meson masses and you can produce these masons abundantly.

They will decay in different channels looking at their decay properties decay channels one can study the C P properties and then it has been found that C P violation is a feature of K meson system and B meson system. C P violation unlike the parity variation is somewhat smaller when I say smaller what I mean is the following consider the case of beta decay in beta decay electrons coming out are always observed to be left handed there is no right handed electron which is coming which is observed in the beta decay whereas, parity conservation or parity transformation would take left handed electron to right handed electron. So, if parity was a good symmetry or if things were symmetric under parity then equal number of these 2 left handed and right handed electrons would have been still in beta decay.

So, a non observation of right handed electrons or left handed positrons in beta decay says that parity is violated and maximally that is there is no (Refer Time:18:56) right handed electron at all observed, but in the case of C P variation when we consider 2 processes which one of which can be the C P transformed state of case of the other one experimentally we see both of this, but the probability for both of these to occur are not the same that is the kind of situation that we see in experiments.

For example, if something happens at hundred percent some decay particular decay happens at the rate of say hundred in a particular time interval for a particular sample and a sample which can be obtained by C P transformation decays to the C P conjugate of that final stage not 100 times, but something like say a 90.

So, there is an asymmetry between the C P conjugated state and the original state or the interaction C P a particular interaction and the C P conjugated interaction. So, this asymmetry if it is not equal to 0 then we can say that C P is violated and it is seen that it is not it is never like 50 percent or 100 percent it is always something like a few percent or less than a few percent that way.

We will see now the theoretical developments and then eventually we will conclude that whatever we have observed the C P violation as well as parity violation as well as C P violation, whatever we have observed so, far can be accommodated in our theoretical framework. So, let us look at the theory in a theoretical development again.

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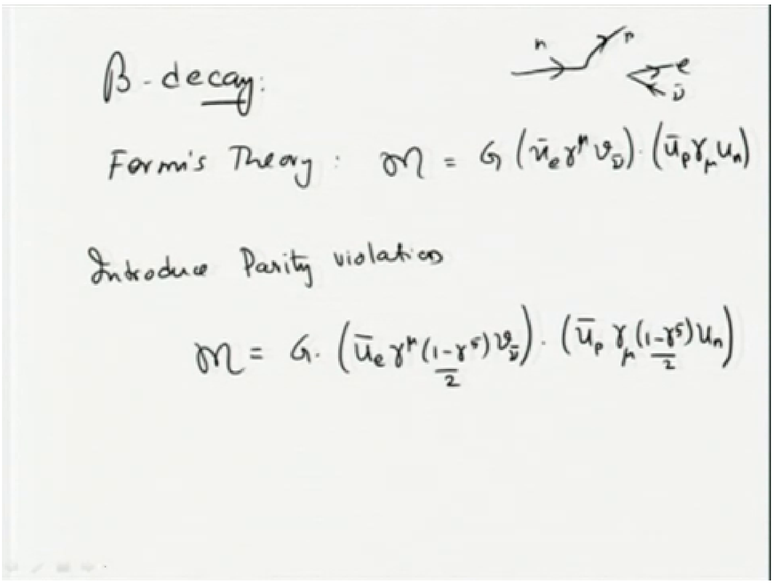

We did talk about the theory of beta decay which is which was proposed by Fermi. So, Fermi's theory was that you consider the invariant amplitude like cun 2 current interacting with each other mu e a u e bar gamma mu v nu bar and. So, this is one current which is the electron and the neutrino current and the other one is neutron, proton, neutron going into proton so, that is u p bar gamma mu u n.

This cannot have parity violation it is because it is a current vector current vector current interaction to introduce parity violation one can look at the interaction in a similar fashion with a slight change the current is now u e bar gamma mu 1 minus gamma 5 by 2 v nu bar u p bar gamma mu 1 minus gamma 5 by 2 u n. So, famous theory is not I mean it is modified, but it is not modified drastically. So, instead of a vector, vector current vector current interaction now we have a mixture of a vector and an axial vector current.

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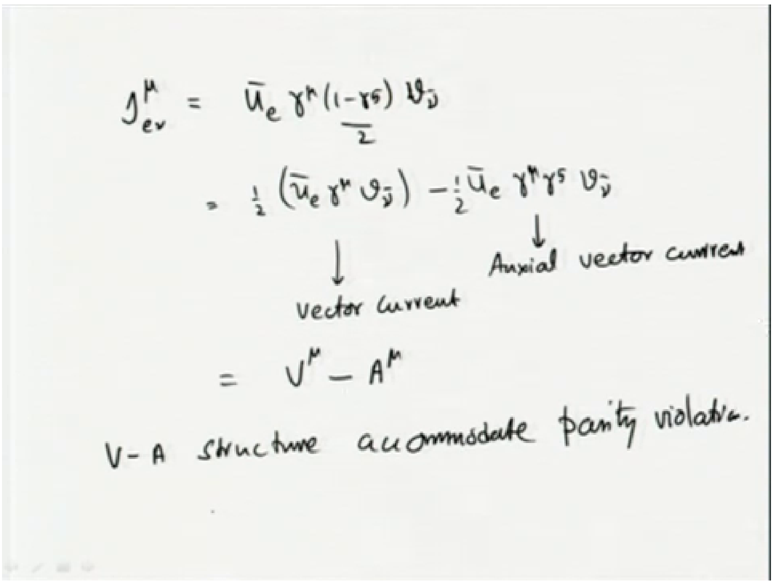

So, let me write down one of these currents say the J e nu electron neutrino current as u e bar gamma mu 1 minus gamma 5 by 2 v nu bar, this can be written as 1 over 2 u e bar gamma mu v nu bar minus u e bar or 1 over 2 into u e bar gamma mu gamma 5 v nu bar, first one is a vector current, second one is an axial vector current. So, there is a vector minus axial vector we call it V mu A mu.

So, this basically the V minus A structure, the V minus A structure of the current accommodates parity violation how because under parity we will transform in a slightly different way compared to A mu. So, there would not be any particular way to particular parity transformation property for V minus A. So, that indirectly that this theoretical structure will when you compute the cross section or the decay prob probabilities you will see that this will lead to violation of parity that this particular structure and the parity conjugated structure will not give the same decay probability or transition amplitude. C P variation on the other hand requires slightly more involved analysis, we will come to that somewhat later.

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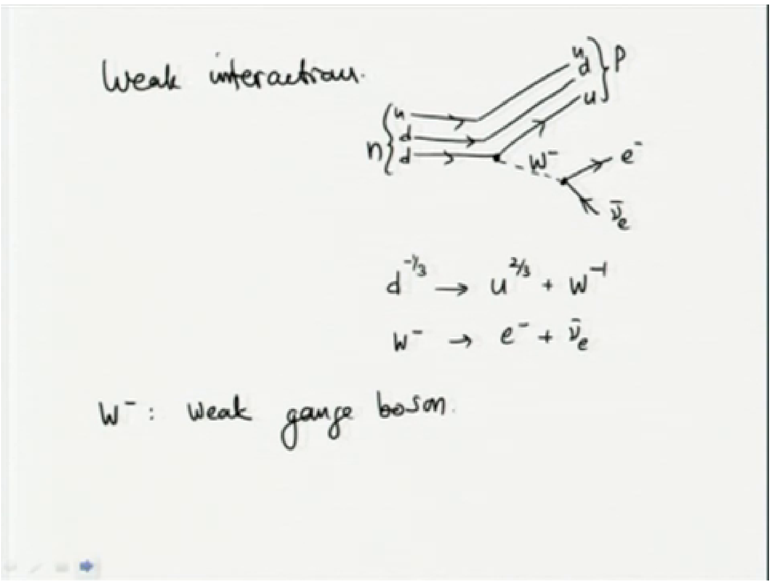

Now, let us look at the weak interaction further n going to p, but we know n has one d quark another d quark and another u quark and proton has one u quark another d quark a d quark and another u quark . So, one can add the quark level think about a d quark converting into a u quark and then electron and antineutrino are created. We want to understand it in a way similar to the case of electromagnetic interaction that we had considered meaning we want to actually connect these 2 coins by some propagator something taking this information from this conversion of the d quark to u quark that information goes to the pair clear and then that is connecting the electron, neutrino current.

This is again thought of as if happening through the exchange of something propagation of something similar to the photon propagation in the case of electromagnetic interaction, we denote this by w minus w for some weak particle or weak boson why boson because it turns out that this is a vector particle which has spin one. So, spin one integer spin boson, it is a similar to the interpretation of photon as the quantum or the representation of the electromagnetic field here we consider w particle as the representation as the as the quantum h picture of the weak field.

So, w is a particle when you get I mean which is the quantized form of the weak field and we call this w this need to have a charge because electric charge is changed when you convert a d quark to a u quark. So, to start with d quark has minus 1 by 3 electric charge this converted into and this first vertex here converted into a u quark with 2 by 3 electric charge plus w goes on with minus 1 electric charge sum it up we will see that right hand side has minus 1 1 plus 2 by 3. So, this is one bit which is equal to minus 1 by 3 charge.

Similarly, when w goes to electron and antineutrino anti neutrino does not have any charge w has minus charge coming bringing into that vertex which is taken away by the electron (Refer Time: 29:50) ok. So, that is w minus going into e minus plus antineutrino. So, this is roughly the kind of split of this process there are 2 currents associated with these, one associated with the conversion of d to u and production of w and this w is then decays to e minus and nu bar so, w minus is the weak gauge boson.

We can look at other processes it is possible to have a Beta plus decay.

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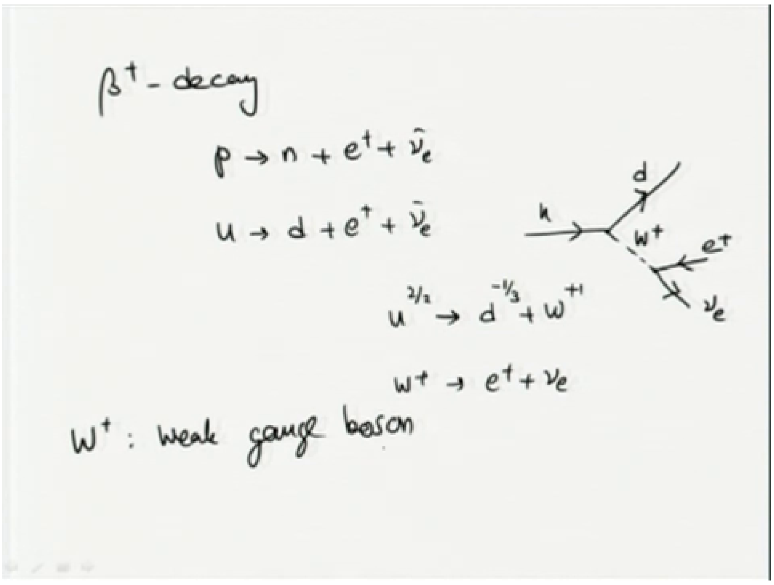

Also which is essentially p going to n plus positron plus neutrino and this can be thought of as conversion of a u quark into a d quark plus electron plus a positron plus anti neutrino. So, this picturized as u quark into d quark with the w plus now because u has a charged 2 by 3 go into d with charge minus 1 by 3 plus w plus 1. So, that charges are added up properly correctly this will take go into positron when a positron goes out we put the arrow in the opposite direction so, it is e plus and nu e.

So, positron takes away the charge which is the w u plus brings to that second vertex. So, w plus goes to e plus plus nu e. So, we have another gauge boson w plus. So, weak there are 2 weakly was on snow w plus and w minus it turns out that one is the antiparticle of the other.

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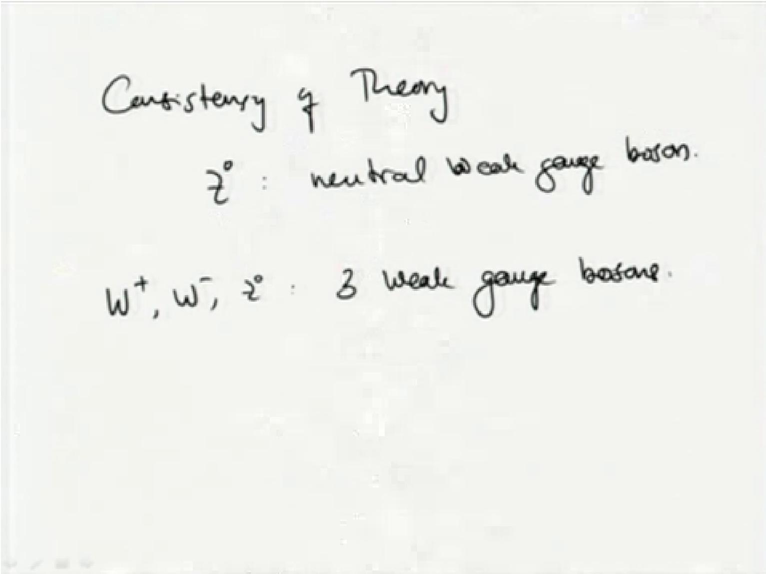

Then you will see that for the theory to be consistent when we discuss the theoretical model it will be clear we need another neutral weak gauge boson, this was in fact, prediction of the theoretical framework the standard model we will come to that in a little while when we actually discuss the formal framework. So, we have here w plus w minus and a z 0 3 weak gauge bosons right one is a neutral gauge boson the other 2 are charged.

Let us consider some other examples of Weak processes.

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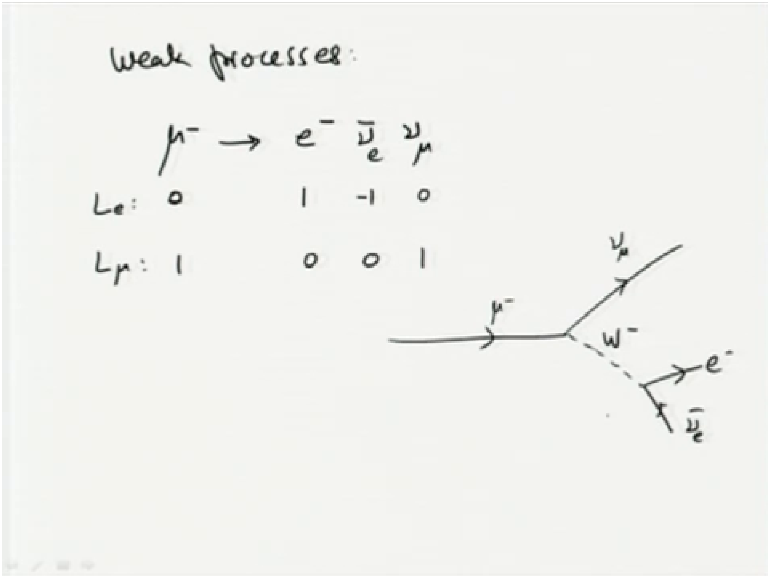

We have been talking about the muon which has which is like electron, but massive, more massive it can decay into electron and electron type of anti neutrino and nueon type of donor. In all these we will consider the electron number being conserved electron whenever when we say electron number muon type particles will have the electron number called muon electron number which is different from electron type electron number individually they are conserved.

So, let us see electron number electron type it is equal to 0 here, 1 here, minus 1 here, 0 here. So, total in the initial and final it is 0 l mu similarly is 1 here, 0 here, 0 here, 1 here. So, theoretically that is why we do not have nuon going to e minus nu e bar and nu mu bar and similarly in the beta decay it is the anti neutrino which is coming out not the neutrino.

Experimentally one can actually determine whether one particular thing is an antineutrino or an neutrino why for example, considering it is reaction with other particles for example, the antineutrino which is coming out from beta decay will not a collide with will not give a reaction like say for example so, this is one week process diagrammatically we can represent this as a muon going into nu mu and emitting a w mu which will go into electron and electron type anti neutrino.

So, as we said in the case of electromagnetic interactions we do not mix the type of the particles. So, with muon only muon type neutrino connect, similarly with electron only the electron type neutrino or antineutrino connected and there are no the fermion lines are continuous from one end to the other. So, this is one way to understand this and once you know the rules are with this associated with this like in the case of the Q E D we had written down the final rules for Q E D.

Similarly in the case of weak interactions also we will write down the final rules once you get the final rules then from the diagram one will be able to compute the write down their invariant amplitude for this process and subsequently compute the probability for it to happen or the decay with.

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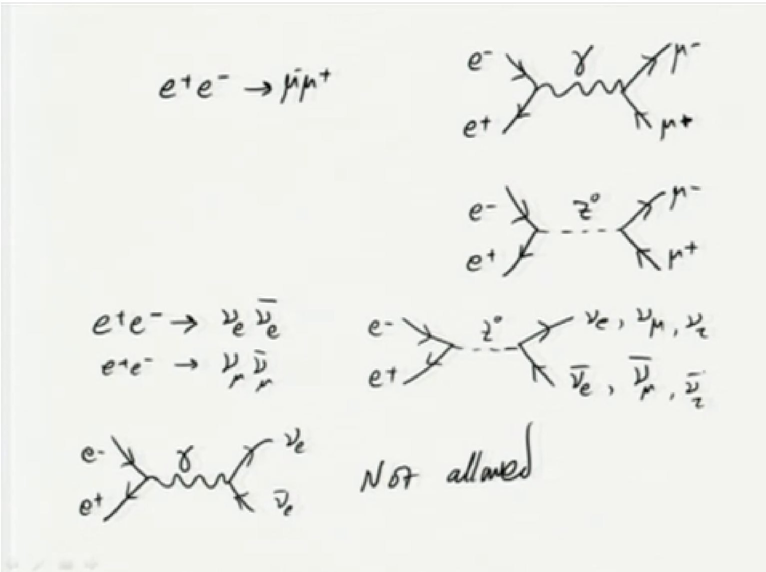

We can think about other processes which also involve weak interaction we had seen this process we had seen this in electromagnetic interaction there we had written down or drawn the diagram e plus e minus goes to mu plus mu minus through the exchange of photon represented by gamma it can also happen through the exchange of weak gauge bosons same final states mu minus and mu plus in the final shape same initial state, but the intermediate propagator is now different it is the z 0 that can carry this information. This is the weak neutral current interaction the neutral the propagator is neutral here and there is no current and the charge change along the current charge remains the same all along the current.

Now, so, when we consider e plus e minus go into mu plus mu minus we should be considering both of this, indeed the electromagnetic interaction here dominates compared to the weak interactions, but this statement is dipper statement depends on at what energies the electron and positron are colliding each other. So, many times this becomes very important for example, the neutral gauge bosons z 0 has a mass of 91, around 91 g e v.

So, if you tune the energy of the electron and positron center of mass energy to 91 g e v or it turns out that most of this reaction will go through the exchange of z 0 that there will be a production of z 0 which subsequently decays is to mu plus and mu minus. We will try to look at some of these processes later, but here we will just try to indicate that along with the electromagnetic interaction in many of these reactions we will have to also consider the weak interactions.

In fact, the neutral gauge boson and the neutral currents like what we have just now written down currents were discovered by the framework of standard model which we will come to later and it was subsequently discovered and that was a big triumph of the model and gave us a lot of confidence in our analysis. There are also other processes like e plus e minus going to 2 neutrinos a neutrino and an antineutrino of this type.

There is no way that this can go through using electromagnetic interaction in electromagnetic interactions, but in weak interaction it is possible to have production of z boson or exchange of z boson between nu e and n u e bar. Well rather than exchange here one can think about production of z 0 and decaying into which decays to nu nu similarly e plus e minus go into nu plus nu minus mu plus mu minus through photon can also be thought of as annihilation of electron and positron and creation of a photon which then pair creates a mu and mu mine mu minus and mu plus.

So, this is possible, but as I said it is not possible to how e minus e plus annihilating to a photon pair creating a neutrino and antineutrino. This is because photons do not interact with neutral fermions neutral particles photrons only photons only in interact only with charged particles at least directly like this. We will see that when we consider the higher order perturbation there are corrections to this statement photons can interact with neutral particles through higher order processes higher order perturbation process.

This is not allowed, incidentally this can also go to the same initial state electron and positron can also go to mu type neutrino and mu type antineutrino without being in conflict with any of those electron number conservation. So, because it is f 0 which is coupling to any type of neutrinos, but again you cannot have nu e and nu mu bar together it is either nu e nu nu e bar or nu mu nu mu bar or even nu tau nu tau bar.

We will stop our discussion here. And then, further go on with our discussion on the weak interactions in the next class.