Nuclear and Particle Physics

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Lecture – 37
Scattering Cross Section Revisited 2

Cross section can be written in terms of the transition rate, the flux and the transition rate W fi; the initial flux times the initial target density and the final state phase space available, in this fashion.

So, now let us look at it in a little more detail and then write it in a more useful form. First let us put in the expression that we had gotten for the transition rate W fi.

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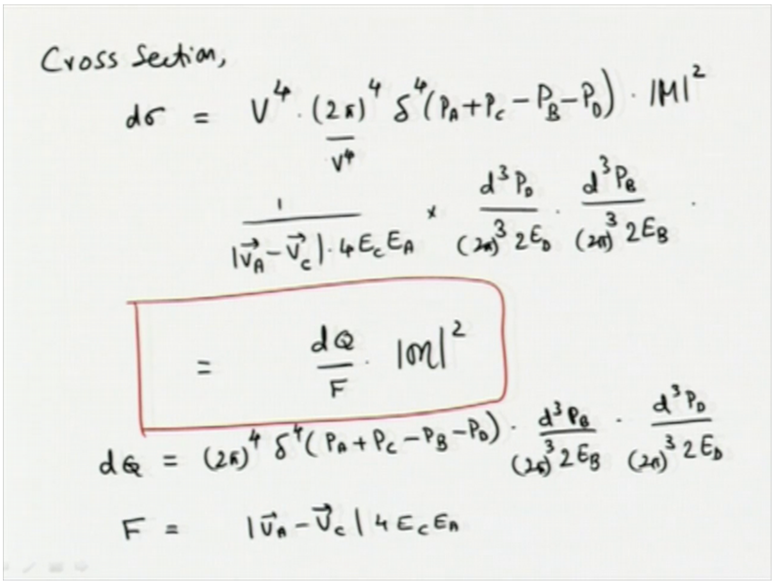

So, 1 V power 4 was already here in d sigma which is what the cross section that I will denote; let me write it here. So, cross section now because we have the phase space factor written as a differential phase space volume element or differential volume elements in the momentum space.

We write the cross section as a differential cross section d sigma and this times the W fi is essentially 2 pi power 4 over V power 4, delta P sum of the initial momenta minus sum of the final momenta; these are the 4 momenta. And then you have the invariant amplitude M square and the flux factor, which is V A minus V B the relative velocity the magnitude of that times; 4 times E C E A all right. So, this is E C V C.

Then the final stage available d 3 P D over 2 pi power 4 2 E D d 3 P B 2 pi power 2 pi power 3 2 E B. The volume factor coming with this two and also with the charge number densities initial flux coming with initial flux and target density are taken in the V power 4 in the numerator written as the first factor.

Let me write it in a book compact way as d Q over F times M square where d Q is the V power 4 factors cancel. So, there is a 2 pi power 4 and delta 4 P A plus P C minus P B minus P D and d 3 P B over 2 pi power 3 2 E B d 3 P D 2 pi power 3 2 E D. This essentially depends only on the momenta which mean and a energy in 3 momentum and the energy the or the 4 momenta.

So, therefore, actually this is in some sense it is independent of the dynamics, it only looks at what are the kinematic variables ok. And the phase space factors also included in this and the flux initial flux F is V A minus V C, the relative velocity between the two colliding particles the magnitude of that 4 times E C E A.

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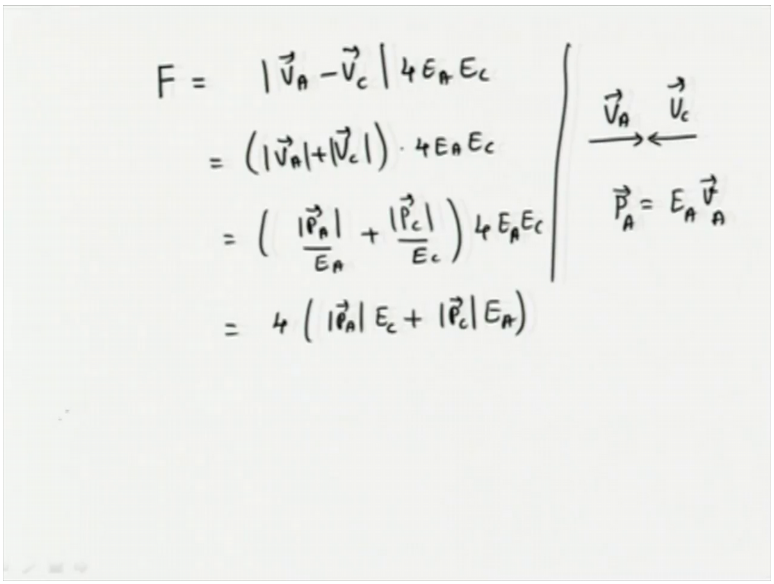

Let me first look at the flux factor F, which is equal to V A minus V C over 4 E A E C. Now, for a colliding beam V A and V C are opposite to each other, and in that case V A minus V C is essentially some of the their magnitudes or the magnitude of V A minus V

C is V A plus V C the magnitudes of individual the sense; times 4 times E A E C. This is true when their angles are 180 degrees with each other and then magnitudes can be just added the difference is this the magnitude some of the magnitude.

Now, other thing to notice is that I can write P momentum 3 momentum as E A V C A V A the to relativistic momentum is written in this fashion. So, in that case I will be able to write this as P A over E A plus P C over E C 4 E A E C, which is equal to 4 times P A E C plus P C E E A.

Now, in the now we will go to a specific reference frame.

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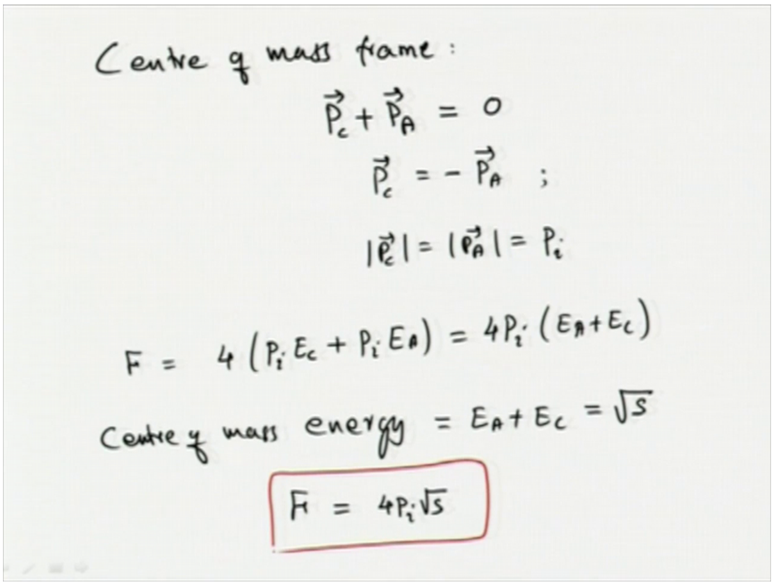

Let me called the centre of mass frame, define it as the reference frame in which some of the 3 momentum of the initial particles initial states is equal to 0. So, here we have P C plus P A as the sum of the 3 momentum of the two initial particles only two of them are there, which is equal to 0. This tells you that P C is equal to minus P A and if we take the magnitude I can write magnitude of P C is equal to magnitude of P A is equal to let me denote it by P i.

So, in that case F becomes 4 times magnitude of P A is P i E C plus magnitude of P C, which is again P i E A; I can take the P i out of the bracket then it becomes E A plus E C.

So, flux in the colliding case is basically 4 times the initial momentum a magnitude of the initial momentum times the some of the initial energies. So, if I denote centre of mass energy, which means the total energy of the initial particles in the centre of mass frame is equal to E A plus E C, which I denote as root S we will see later on why we call it root S. And in this notation, we have the flux initial flux as P 4 P i root square root of centre of mass energy ok.

So, we have sigma here the differential cross section as d Q over F M square and F is what is given here in the C.

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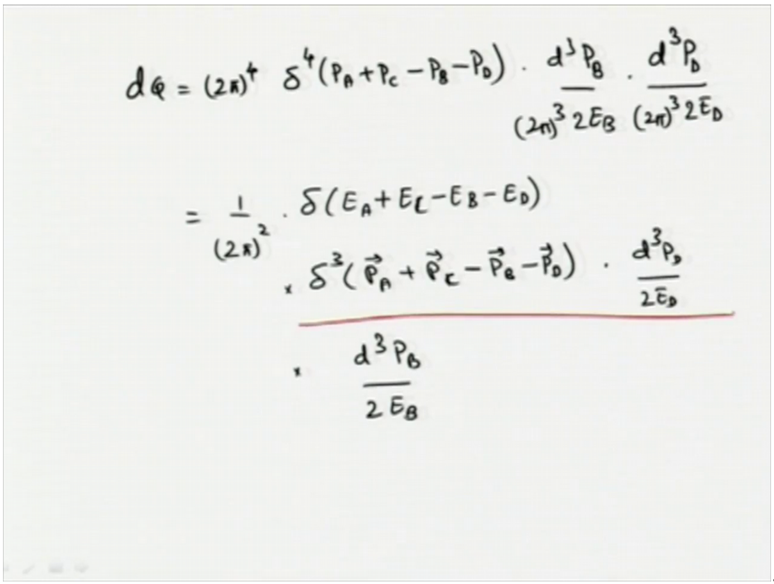

We will come to the factor d Q this is equal to 2 pi power 4 delta 4 P A plus P C minus P B minus P D ok.

So, now let us look at d Q, which is 2 pi 4 delta 4 P A plus P C minus P B minus P D, into d 3 P B over 2 pi 3 2 E B d 3 P d 2 pi power 3 2 E D. Firstly, there is A 2 pi power 4 factor in the numerator and then 2 pi power 6 factor in the denominator.

So, altogether we have a 2 pi power 2 in the denominator and let me split this delta function into delta energy E A plus E C minus minus E B minus E D ok. Into the other delta function rest of it, which is a 3 dimensional d cube P A 3 vector P C minus P B minus P D, all 3 vectors now. Then you have d 3 P D over 2 E D and you have d 3 P B over 2 E B. Now, let us look at the this line here second line here.

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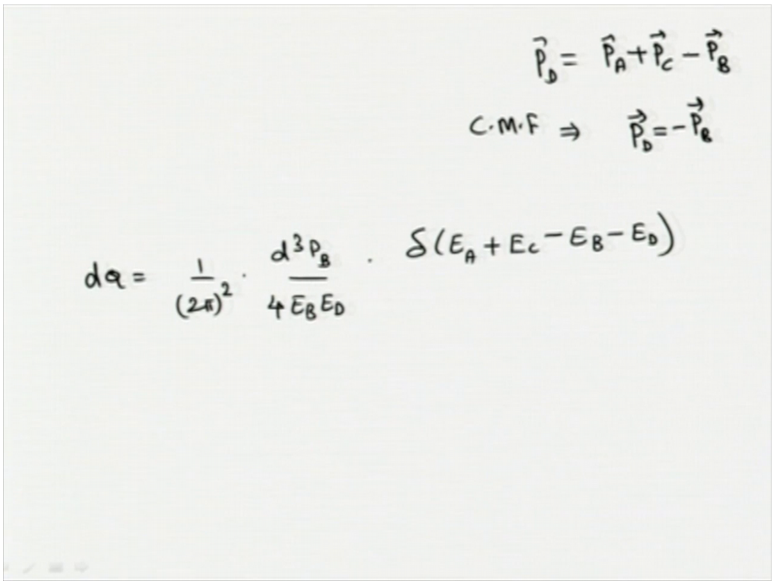

So, that will give you P D equal to P A plus P C minus P B. This in the center of mass frame center of mass frame which means P A plus P C is equal to 0 P D is equal to minus P B ok. And d Q will be 1 over 2 pi power 4 2 pi power 2 d 3 P B over 4 E B E D, I clubbed 1 over 2 E D with this.

And you have a delta E energy function E A plus E C minus E B minus E D ok. The rest of the things are or taken care of the other delta function and the other integration goes away giving us this condition all right.

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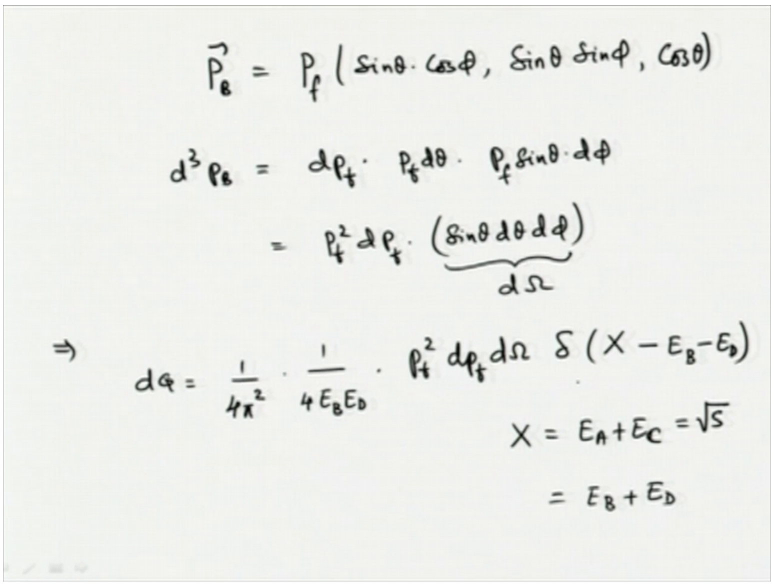

Now, let us look at this other integration d 3 P B d P B the 3 dimensional P B, let me write as P f the magnitude sin theta cos phi in polar coordinate system, sin theta sin phi where sin theta theta is the polar angle and phi is the azimuthal angle of their P B and cos theta.

In this case d 3 P B is essentially oleum element here when you vary the are combined the radial part or P f, then you have a d P f and when you change theta to d theta by d theta then you have then you have P f d theta, and for the azimuthal case you have P sin theta d phi. This is basically the similar to the normal volume element in R theta phi coordinates, here we have P f square d P f sin theta d theta d phi and this is what you recognize as the solid angle or differential solid angle.

So, this will give you then d Q equal to 1 over 4 pi square let me write it in this fashion, 1 over 4 E B E D P f square d P f d omega, delta E A plus E B let me write that as X minus E B minus E D, X is equal to E A plus E C the initial energy, which is equal to root S. And it is by this delta function it is also equal to E B plus E D let me do a do it a little bit of manipulation here.

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$$X = E_B + E_D$$

$$= (m_B^2 + |\vec{P}_B|^2)^{1/2} + (m_D^2 + |\vec{P}_D|^2)^{1/2}$$

$$|\vec{P}_B| = P_f = |\vec{P}_D|$$

$$= (m_B^2 + P_f^2)^{1/2} + (m_D^2 + P_f^2)^{1/2}$$

$$\frac{dX}{dP_f} = \frac{P_f}{(m_B^2 + P_f^2)^{1/2}} + \frac{P_f}{(m_D^2 + P_f^2)^{1/2}}$$

$$= P_f \left(\frac{1}{E_B} + \frac{1}{E_D} \right) = P_f \left(\frac{E_B + E_D}{E_B E_D} \right)$$

I write X equal to E B plus E D, E B is M B square plus P magnitude of b P B magnitude square under root plus similarly m d square plus P D square under root.

But, we have P B equal to P f which is also equal to P D as we saw. So, this is equal to let me write it as m B square plus P f square power 1 over 2 plus m D square plus P f square power 1 over 2. Let me differentiate this as a with respect to P f, that will give you P f over m B square plus P f square 1 over 2 plus P f m D square plus P f square 1 over 2 P f is common.

So, this is essentially P f 1 over m B square plus P f square that is equal to 1 over E b plus 1 over E D, which is equal to P f into E B plus E D divided by E B E D ok.

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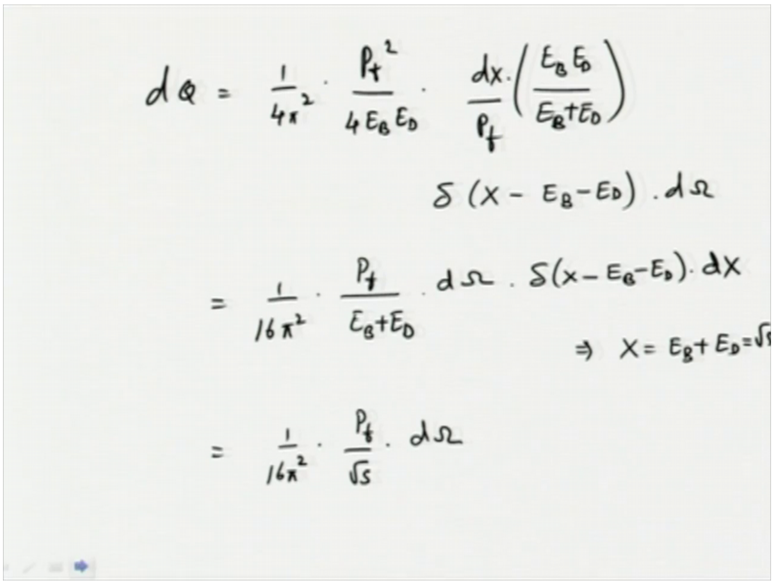

Now, d Q is 1 over 4 pi square P f square over 4 E B E D all right. And d P f d P f is now d x into E B E D over E B plus E D ok. This particular term is coming from d x by d P f is P f into x there is a P f factor. So, inverse of this is d P f by d x over P f.

Now, this is and you also have the delta function X minus E B minus E D and d omega of course. So, this is one over 16 pi square A P f cancels with the other P f. So, you have A P f there E B D E B E D cancels with the E B E D, there and then there is a and E B plus E D here and let me take the d omega along with this. And you have the delta function d E A and delta x minus E B minus E D and integration over d X.

Again, if you look at the last 2 factors delta function and d x integration you will see that you can integrate our d x now this new variable that we have defined.

So, this will give you again without bothering about the rest of the terms factors coming with this we can integrate this out because it is a delta function. And that will give you integration will give you X equal to E B plus E D since in fact, this is what we had used earlier or E B plus E D is equal to X we had taken it as root S.

So, that will give you 1 over 16 pi square P f over root S d omega as simple as that. And now let us go back to the expression we had for the cross section. It is d Q over F M square. And F is essentially 4 P i root S and d Q is 1 over 16 pi square P f over root S d omega.

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$$d\sigma = \left(\frac{1}{16\pi^2} \cdot \frac{P_f}{\sqrt{s}} \cdot d\Omega\right) \left(\frac{1}{4 P_i \sqrt{s}}\right) \cdot |M|^2$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \cdot \frac{P_f}{P_i} |M|^2.}$$

$$M = \left(-e\, \bar{u}_B \gamma^\mu u_A\right) \cdot \left(\frac{-g_{\mu\nu}}{q^2}\right) \cdot \left(-e\, \bar{u}_D \gamma^\nu u_C\right)$$

Putting these things together we have d sigma equal to 1 over 16 pi square P f over root S d omega is essentially what, the d Q is and you have 1 1 over 4 P i root S as the 1 over f factor and you have the square of the invariant amplitude. This is equal to 1 over 64 pi square S P f over P i M square d omega or I can write d sigma over d omega as 1 over 64 pi square S P f over P i M square.

So, this is the differential cross section and we will remember this; we will come back to this. But now we will give our attention to the M square where M we had written as minus e u B bar gamma mu u A for the first current minus g mu nu over q square. And which is the propagation propagator factor and then the second current u D bar gamma nu u C.

We will see what we can do with this M or taking the multiplying in the complex conjugate of this object, M square.

We will come to that in the next discussion.