Nuclear and Particle Physics
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Module - 09
Electroweak Interactions

Lecture - 09
Scattering processes

[noise]

We will look at the scattering cross section between 2 particles [vocalized-noise] like electron [vocalized-noise] or electron and positron [vocalized-noise] or any [noise] charged particle, which could be described using Dirac equation.

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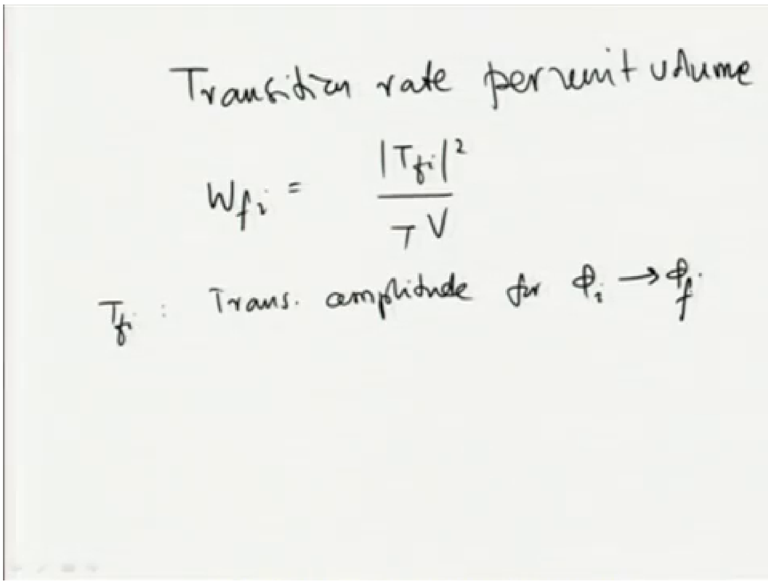

So, [noise] um let us go back to our earlier discussion on the scattering cross section, where [noise] we had ah said that the [noise] transition [noise] rate per unit volume [noise] can be [noise] ah written ah in terms of the transition amplitude [noise] like a this [vocalized-noise] [noise] W f i let me denote this transition rate [noise] [vocalized-noise] equal to [noise] T f i square over, whatever is the time taken for the same 2 per unit volume [noise] [vocalized-noise] ok.

So, [noise] T f i as we know is the transition [noise] amplitude for phi i to go to phi f, [noise] when it undergoes an interaction sorry [noise] 2 phi f [noise]. When in undergoes an [noise] interaction in time interval T and in a volume special volume V ok. So, usually we consider that there is no interaction before this with outside this time interval [noise] one way to see it is that [vocalized-noise] consider the Rutherford. Kind of scattering the particle, which is coming from in the beam coming towards this one is free actually until it actually [vocalized-noise] it comes close to the wall foil or whatever the nucleus, that it will interact with [vocalized-noise] and there is a short time interval where the interaction happens and then it flies off again asymptotically as a [noise] [vocalized-noise] free particle [vocalized-noise].

So, that S a kind of picture we have, [noise] now ah [noise] we had [noise] in the earlier lectures [noise] described [noise] T f i as 2 phi power 4 [noise] delta P A plus P C minus P B minus P D [noise].

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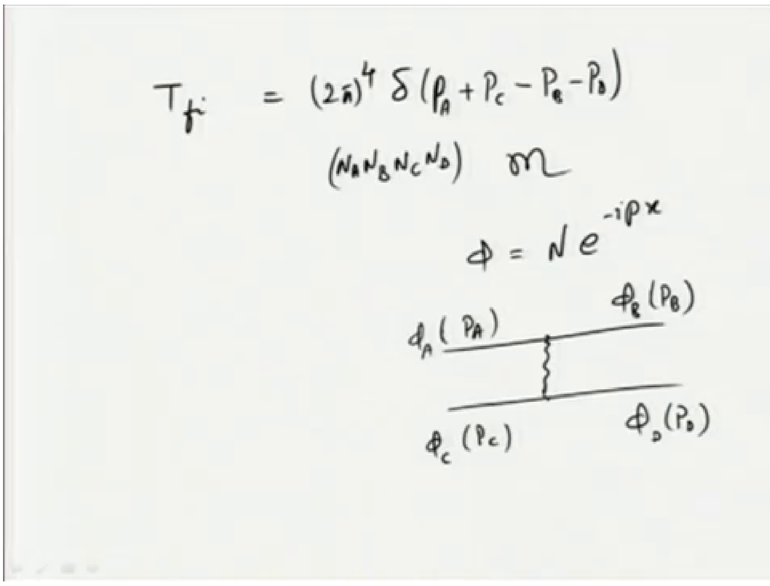

So, let me [noise] first consider the T f i itself is equal to this into whatever is the [noise] normalization constant N A N B N C N D [noise] [vocalized-noise] [noise] and [noise] m [noise] invariant amplitude [noise]. This is for wave functions like and [noise] e power i P x [vocalized-noise]. So, [noise] there is a particle um [noise] initial particle with momentum [noise] say phi A [noise] with momentum P A [noise] coming in after interaction goes out us [noise] phi B with momentum [noise] P B [noise] and it is interacting actually with another particle phi C [vocalized-noise] P C, which goes to phi D [noise] with momentum P D [noise].

So, this case each of the normalization we consider as N A N B and C N D in variant amplitude, we had written [noise] down earlier [vocalized-noise]. In the case of [noise] ah wave functions which obey the Klein Gordon equation as well as the case of wave functions which obey the Dirac equations [vocalized-noise].

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$$|T_{fi}|^2 = (2\pi)^4\, \delta^4(P_A + P_C - P_B - P_D)$$
$$\times (2\pi)^4\, \delta^4(P_A + P_C - P_B - P_D)$$
$$(N_A N_B N_C N_D)^2 \cdot \; |M|^2$$

$$(2\pi)^4\, \delta^4(P_A + P_C - P_B - P_D) = (2\pi)\, \delta(E_A + E_C - E_B - E_D)$$
$$\times (2\pi)^3\, \delta^3(\vec{P}_A + \vec{P}_C - \vec{P}_B - \vec{P}_D)$$

So, what we want in the transition amplitude transition from a rate is [noise] T f i square ah probability square ah amplitudes probability amplitude square gives the probability [vocalized-noise] [noise]. So, this gives me [noise] 2 phi power 4 and let us write [vocalized-noise] the delta function 2 of them other.

So, let me write it first us [noise] D P A [noise] plus P C [noise] the initial momentum P B minus P D final momentum [vocalized-noise]. And duplicate it [noise] that is another [noise] P A [noise] plus P C minus P B minus P D when is square it and you have um [noise] N A N B N C N D square [noise] in variant amplitude square, [noise] [vocalized-noise] N A and B [vocalized-noise] if ah we will come to the normalization later. First let us look at [noise] 2 pi [noise] power 4 [vocalized-noise] [noise] delta 4 [noise] P A [noise] plus P C minus P B minus P D square [noise] ok. This I can [vocalized-noise] split into [noise] 2 pi into delta E A plus E C minus E B minus E D [noise] into [noise] 2 pi delta 3 [noise] 3 momentum P A plus P C minus [noise] P B minus P D [noise] 2 pi power 3 [noise].

(Refer Slide Time: 07:35)

$$(2\pi)\,\delta(E_A+E_C-E_B-E_D)\cdot(2\pi)\,\delta(E_A+E_C-E_B-E_D)$$

$$=(2\pi)\,\delta(E_A+E_C-E_B-E_D)\int_{-T/2}^{T/2} e^{-i(E_A+E_C-E_B-E_D)t}\,dt$$

$$=(2\pi)\,\delta(E_A+E_C-E_B-E_D)\int_{-T/2}^{T/2} dt$$

$$=(2\pi)\,\delta(E_A+E_C-E_B-E_D)\cdot T$$

$$\left[(2\pi)^3\,\delta(\vec{P}_A+\vec{P}_C-\vec{P}_B-\vec{P}_D)\right]^2=(2\pi)^3\,\delta^3(\vec{P}_A+\vec{P}_C-\vec{P}_B-\vec{P}_D)\cdot V$$

So, when I [noise] have 2 such delta force [vocalized-noise] I have [noise] 1 term or 1 part 2 pi [noise] and delta E A [noise] plus E C minus [noise] minus E B minus [noise] E D into the same thing 2 pi delta [noise] E A plus E C [noise] minus [noise] E P minus E d. So, let me write [noise]. So, the first one as it is [noise] E A plus E C minus E B minus E D, [noise] second one in the integral form minus [noise] e power minus i [noise] E A plus E C minus [noise] E B minus E D t integrate over t d t. And let me take the interval as minus T by 2 2 [noise] T by 2 were essentially the integration in interaction happens [noise].

So, this now look at this ah the integral inside this has an exponential E A plus E C minus E B minus E D [vocalized-noise] T [noise] owing to the [noise] other delta function this can be said to equal to 0 [noise]. So, essentially we have [noise] 2 pi delta E A plus E C minus E B minus E D [noise] [vocalized-noise] integral d t minus d by 2 plus [noise] d by 2 [vocalized-noise].

So, that will give me [noise] 2 pi 1 delta function as it is [noise] and ah factor T whatever is the time interval in a similar way we get [noise] 2 pi power 4 not to pi power 4, we have taken the time for [vocalized-noise] energy part away only the 3 momentum now. [noise] P A plus P C minus P B minus P D square of it let me denote it as the square of it equal to one of the delta functions as it is P A minus sorry P m plus P C minus P B minus P D [noise]. And then the conjugate variable which is basically the space coordinate.

And, now we have A 3 dimensional thing therefore, it is the spatial volume relevant to that that is coming into picture, [vocalized-noise] when we are integrating over the other or when we consider the other delta function as an in the integral form and then integrate it over that this V [vocalized-noise] [noise].

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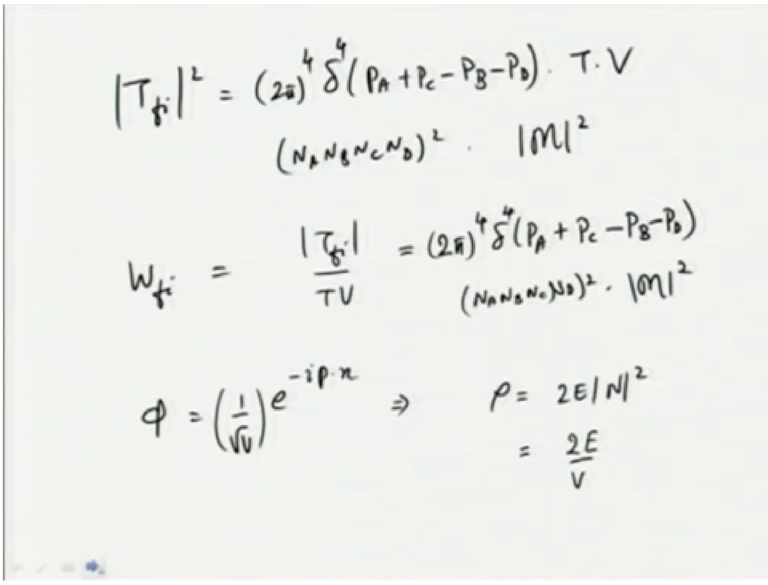

So, [noise] T f i square is 2 pi now I can write it as 2 pi 4 [noise] P A plus P C minus P B minus P D [noise] in 4 momentum and it is delta 4, [noise] and you have A T and V [noise] the time interval and the volume [noise] involved N. And um you have an N A [noise] N B N C N D square [noise] and invariant mass square [noise] in variant amplitude squares. Now W f i the rate is [noise] T f i [noise] over T V per unit volume rate per unit volume is now 2 pi the T V cancels with this T V. So, what you have is [noise] 2 pi [noise] power 4 delta [noise] P A plus P C minus P B minus P D, the 4 momentum delta 4 T V cancels with this and you have an [noise] N A N B N C N D square [noise] invariant amplitude square [noise] [noise].

Now, when we take phi to be [noise] 1 over root V in the case of 3 dimensional box normalization, we take this [noise] minus i P x [noise]. And that will give you as per our earlier this thing rho is equal to [noise] 2 E N square, if you remember or please look back in the case of Klein Gordon equation we can actually obtain this rho equal to 2 E N square as our [noise] ah density. Probability density or if you multiply it by E, then it will be charge density. So, number of particles essentially that it will correspond to um. So, this is 2 E over V for this [vocalized-noise]. So, this actually says that there are 2 E particles in V volume. So, the density is 2 E over whatever the volume B [vocalized-noise] ok.

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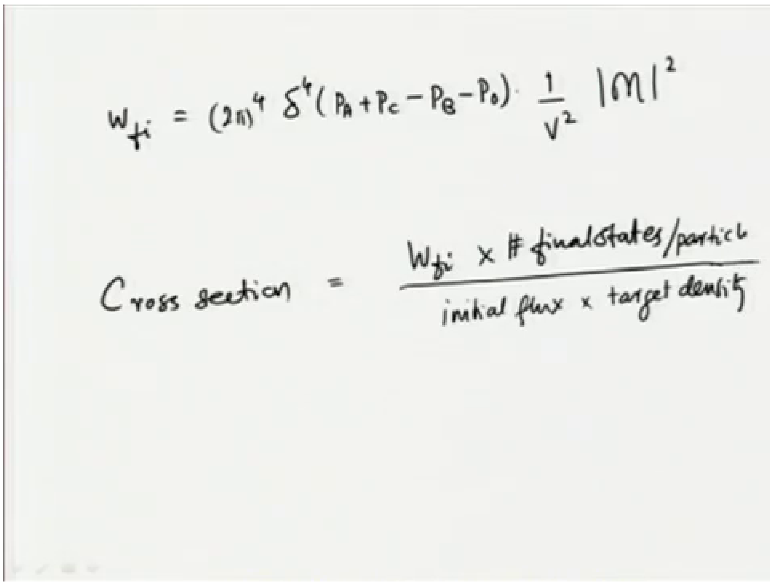

So, that is way one way to interpret this [vocalized-noise] with all this um that put in [noise] W f i is equal to 2 pi power 4 delta 4 P A plus P C minus P B minus P D [noise] one over V [vocalized-noise] ah [noise] V square, because each of this. So, this says that N is equal to one over root V [noise].

So, we have not one over [noise] and this [noise] in to yes 1 over [noise] V square, which is N A N B N C N D and you have the [noise] invariant amplitude square [noise]. We need want to relate this to the number of a the cross section [noise] ok. So, the cross section [noise] of this scattering process [noise] is equal to or cross section of the interaction is rate per unit volume [noise] into number of [noise] final states [noise] ok, per particle [noise] divided by [noise] um so, the cross section into the cross section into initial flux into target density [noise] ok [noise].

So, the cross section times initial flux times the target density is essentially going to give you the W f i the rate into the number of final states, whenever we just going into available final state into ah per particle [noise]. So, this is what basically the cross section is ah we had discussed this earlier.

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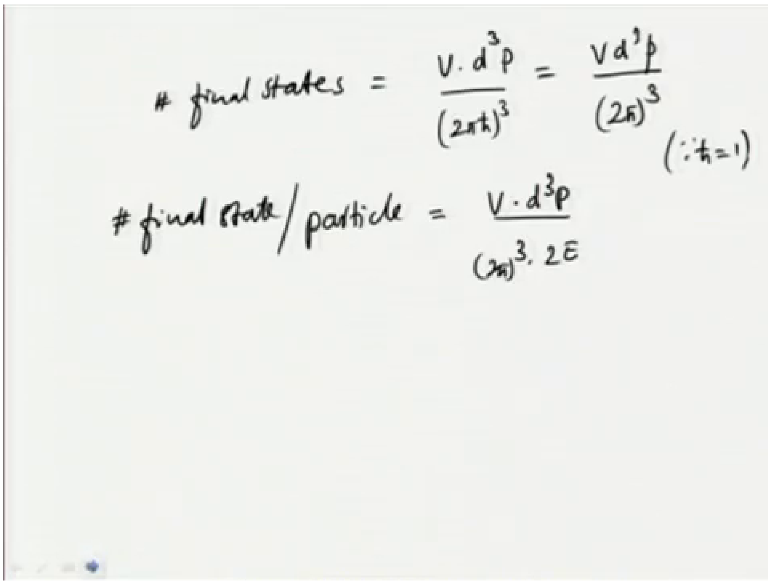

Now [noise] [vocalized-noise] um [noise] number of [noise] final states is equal to as we said earlier the phase space volume V plus d 3 V [noise] when we consider a volume element between P and P plus d P then it is V d 3 P [noise] over h cube [noise] 2 pi h cross q [noise] and since we are taking h cross to be equal to 1 this is 2 pi 3 [noise] for h cross is equal to one [noise] in our units [vocalized-noise].

So, number of final set we said in volume V we have essentially 2 E number of articles that, but that is not the normalization told us. So, 2 e particles in V volume will give you final state [noise] available for 1 particle [noise] is equal to V D 3 P over 2 pi 3 2 E [noise] for 2 E it is this much and then for one particle it is one over 2 pi of this [noise].

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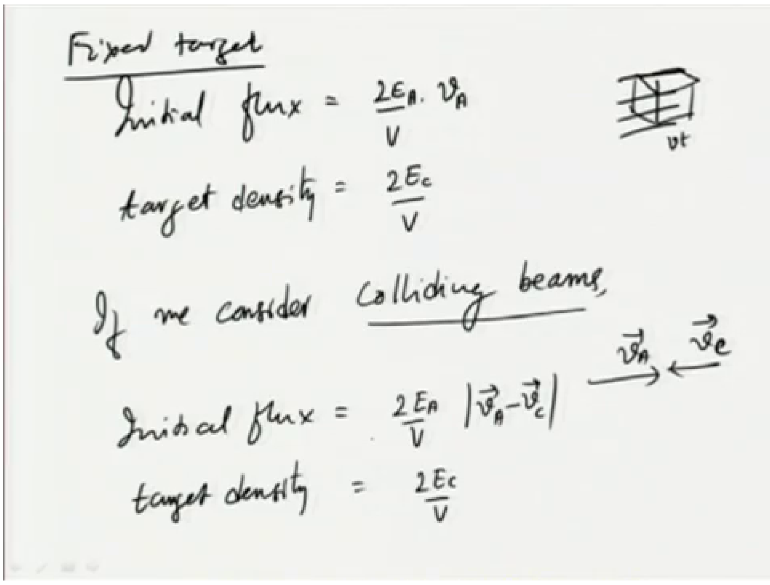

And, initial flux [noise] equal to [noise] number of particles per unit volume [noise] into [noise] the velocity of a particle right right essentially look at what is happening there is A B in which is going in [noise]. So, what is the initial [noise] flux in a cross sectional area number density and in A unit are A cross section if you take the number density [noise] for unit are A cross section the volume per unit time is going to be this unit area into v t responding to that this thing and when t is equal to 1 it is just b.

So, it is cross section times weak the [vocalized-noise] area [vocalized-noise] times v t is the volume. So, these times for unit area it is equal to 1 into V into 1 [noise]. So, that is the volume. So, 1 into V into 1 is V essentially [vocalized-noise]. Since we are considering initial beam as the A particle beam and let me denote it by v A.

So, this is the magnitude of the velocity [noise] v A [noise] and [noise] if the other particle is A target particle, which is trust in A [vocalized-noise] in the lab frame target density is to be considered [noise] that is going 2 E C ah other particles.

So, essentially what we have is initially you have particles of type E A and [vocalized-noise] E A and C [vocalized-noise]. So, same way you have V um [noise]. Now um if [noise] we consider [noise] colliding beams [noise] then this V is actually the relative v. So, there are 2 particles then one is going with A velocity [noise] v A the other is going with A velocity [noise] v C [noise].

So, in the rest frame of C type of particle one beam the other we will have ah velocity v minus v A minus v C right [noise]. So, the in this case this is for fixed [noise] target [noise]. So, for colliding beams we have [noise] initial flux equal to 2 E A over V is the number of particles, but then this is going to be [vocalized-noise] v the magnitude of this relative velocity v A minus v C. Target density then [noise] [noise] gives 2 E C over V, because it is in the rest frame of that particle that we are considering the initial flux that is why the velocity is the relative velocity [vocalized-noise].

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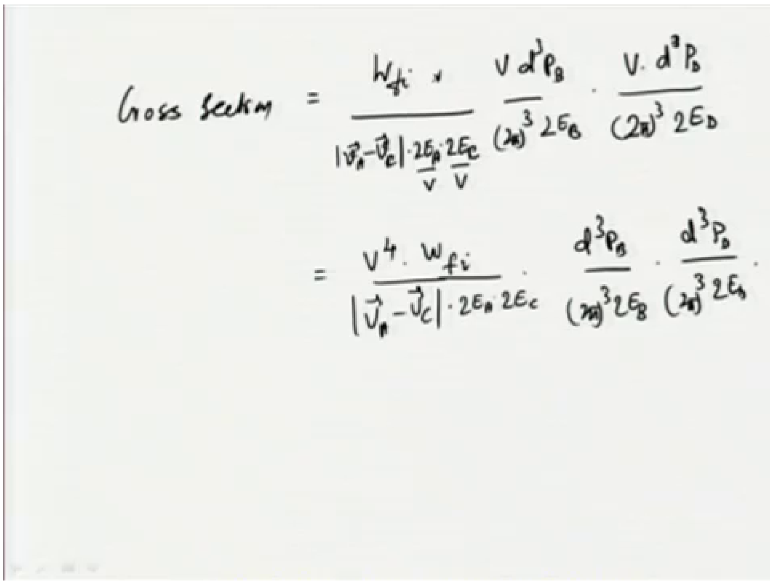

So, um [noise] that will give you the cross section [noise] equal to W f i [noise] into V D 3 P B over 2 pi 3 2 e B [noise] B D 3 P D over 2 pi [noise] 3 2 E D, [noise] initial flux times target density is [noise] v A [noise] minus v C [noise] 2 E A times 2 E C divided by V [noise] (Refer Time: 23:17) [noise]. So, we have a [noise] V power 4 [noise] W f i over V A minus V C magnitude of that, [noise] 2 E A [noise] 2 E C [noise] and the number of a space V V is not [noise] D 3 P B over 2 pi [noise] 3 2 E B [noise] D 3 P D over 2 pi 3 [noise] [noise] 2 E D [noise].