Nuclear and Particle Physics
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Module - 09
Electroweak Interactions
Lecture - 08
Dirac Equation – continued

We are discussing the Dirac equation. Now for interacting particles, what should be do interacting particles.

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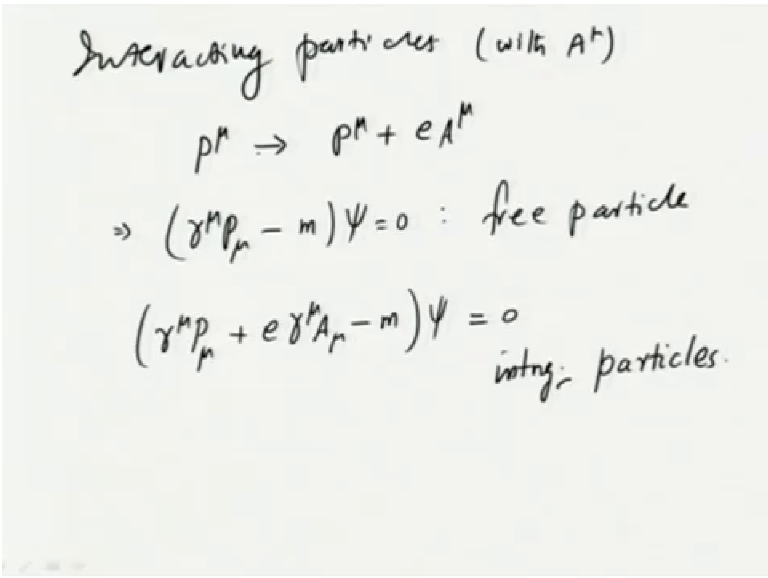

So, the interaction with an electromagnetic field represented by A mu. So, this we had already discuss will change P to P mu plus charge of the particle times e A A mu, but charge of the particle is minus e electron like particle.

And this gives gamma mu P mu minus m psi for free particle, that is changed to gamma mu P mu plus e gamma mu A mu minus m psi oh sorry is equal to 0 equal to 0 for interacting particles. This involves the kinetic energy term, the total energy the free particle term plus the interacting potential term in the operator.

So, we had to first will first isolate the potential exactly what it is and then use the perturbation method to understand the transition amplitude and eventually the cross section. What did we do just to remind us we are interested in and isolating the potential of interaction.

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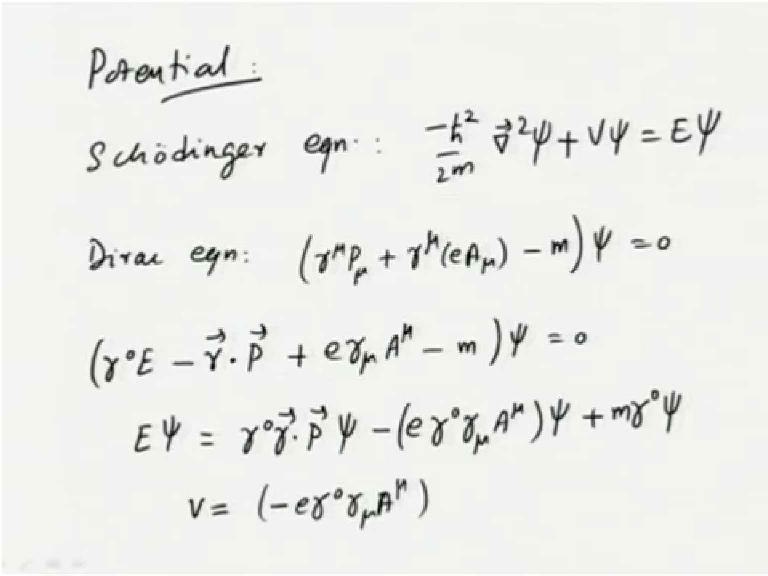

For that let us first look at what we did in the case of Schrodinger equation ok. What we have in the case of Schrodinger equation? It is minus h cross square over 2 m del square psi plus v psi equal to e psi i not put h cross equal to 1 in this for clarity, but in the other equations I have not put this h cross square I will just take them to be equal to 1.

So, Dirac equation with interaction term is gamma mu P mu plus gamma mu e A mu minus m psi equal to 0. Will expand the first term gamma 0 e plus minus gamma dot P, then we have plus e gamma mu A mu minus m psi equal to 0. So, I will take e to one side just to compare it with the Schrodinger equation multiply it by gamma 0 that will give you e psi equal to take the rest of it to the right hand side I will have A gamma 0 gamma dot P psi, minus e gamma 0 gamma mu A mu psi plus m gamma 0 psi, this is the Dirac equation with interaction term.

Comparing with the Schrodinger equation, first term as the kinetic energy term, second term is the potential. So, we it should be positive the potential. So, V therefore, V is equal to minus e gamma 0 gamma mu A mu. So, that is the potential coming in the Dirac equation.

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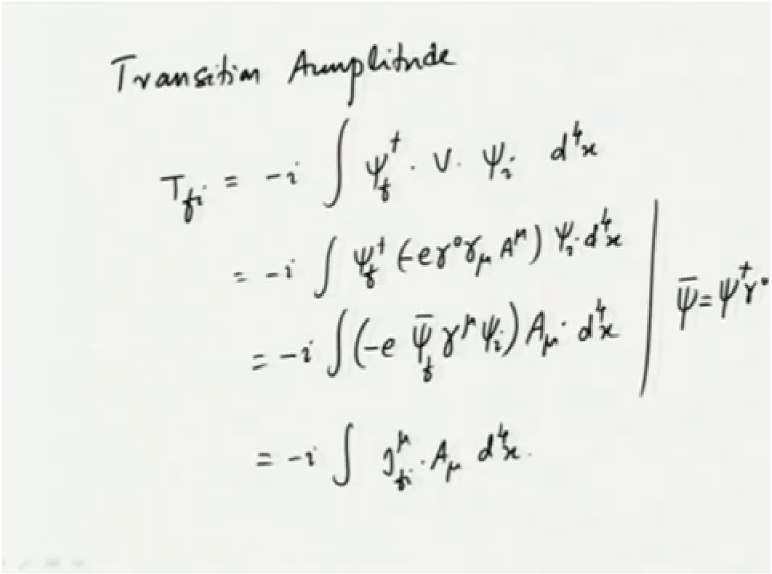

Now, using perturbation theory we have the transition amplitude T f i equal to minus i integral psi f dagger, v psi i d 4 x that is equal to minus i integral psi f dagger, minus e gamma 0 gamma mu A mu psi i d 4 x.

I will take the gamma 0 with psi f dagger write it as psi bar. So, I have minus i integral minus e psi bar gamma mu psi A mu. So, psi f psi i d 4 x this is nothing, but the j f i mu A mu d 4 x.

So, that the transition amplitude is exactly in the same form as we had earlier with this j mu. Now defined in discussion again like in the earlier case we consider interaction between 2 charge particles ok.

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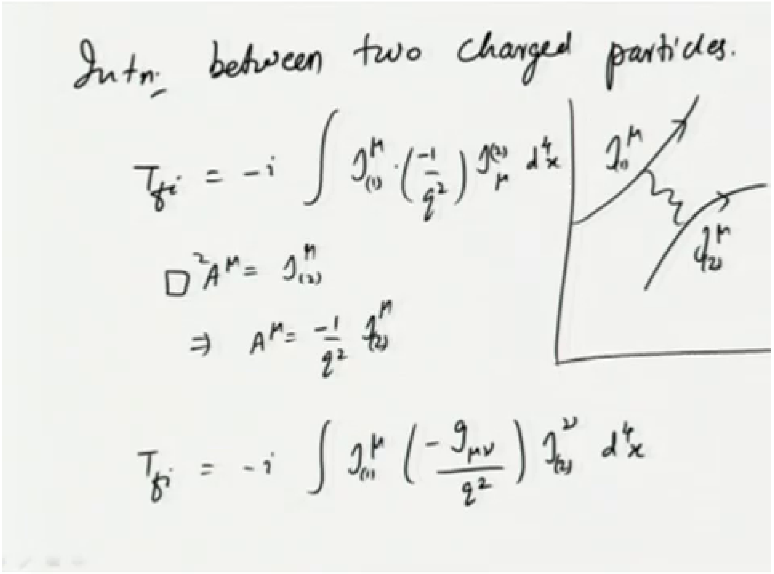

So, the A mu is caused by some other charged particle in that case I have T f i is equal to minus i integral j mu of 1 particle interacting with the A mu, which is cause by another particle, but again we can take j 1 mu j 2 mu and del square A mu is equal to j 2 mu this is just to remind you and that gives you A mu equal to minus one over q square j 2 mu this is what we had earlier.

So, we have here minus one over q square j 2 mu or I could write T f i as minus i 2 contravariant currents j 1 mu minus I, there is no i minus g mu nu over q square j 2 nu d 4 x d 4 x.

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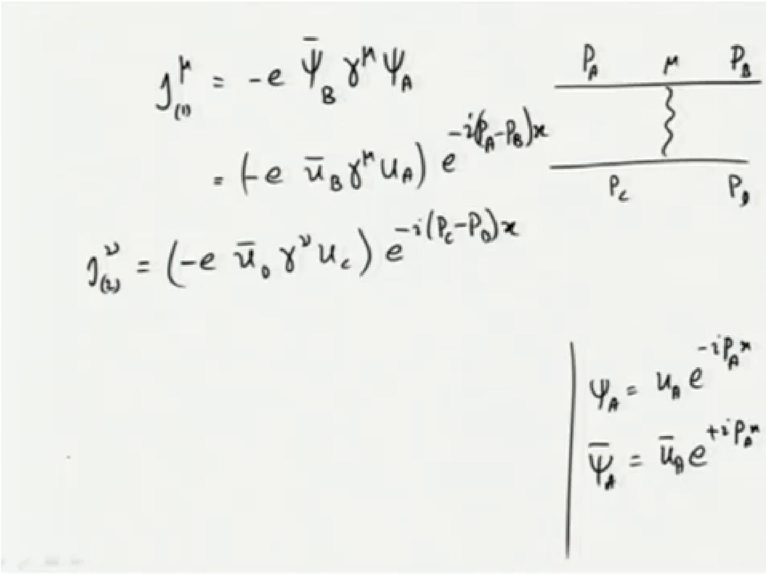

Now, let us look at each currents j 1 mu I can think about A particular situation I have A incoming particle with momentum P A outgoing, but after interaction causes P B. In that case minus e psi A sorry this is the final particle. So, psi B bar gamma mu psi A is what I have. So, when I have psi A written as u A e power minus i P A x similarly for psi b and the bar conjugate will actually take u bar and exponential minus i plus i P A x.

So, psi A bar is going to be u A bar e power plus i P A x keeping that in minds we have minus e u B bar gamma mu u A and exponential minus i P A minus P B x. Similarly for the second current mu the second current let us consider P C and P D as the initial and final moment. That case I have minus e u d bar gamma nu u B not u B u C e power minus i P C minus P D x putting together I have T f i.

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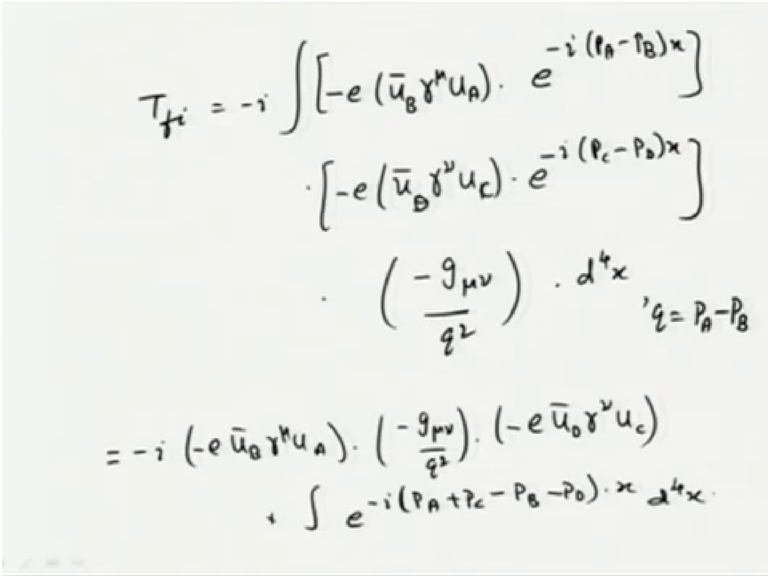

T f i equal to minus i integral minus e u B u B bar gamma mu u A e power minus i P A minus P B x this is j 1 and j 2 is minus e u D bar gamma nu u C e power minus i P C minus P D x. And you have the propagator g mu nu over q square where q square is now the momentum transfer, which is P A minus P B, which is also equal to P C minus P D.

This is equal to minus i minus e u B bar gamma mu u A minus g mu nu over q square. So, let me write it slightly up q is equal to P A minus P B, then again the that from the other current u D bar gamma nu u C, into integral e power minus i P A plus P C 2 of the initial momentum minus P B minus P D the final momentum x before x and this integral is nothing, but A delta function.

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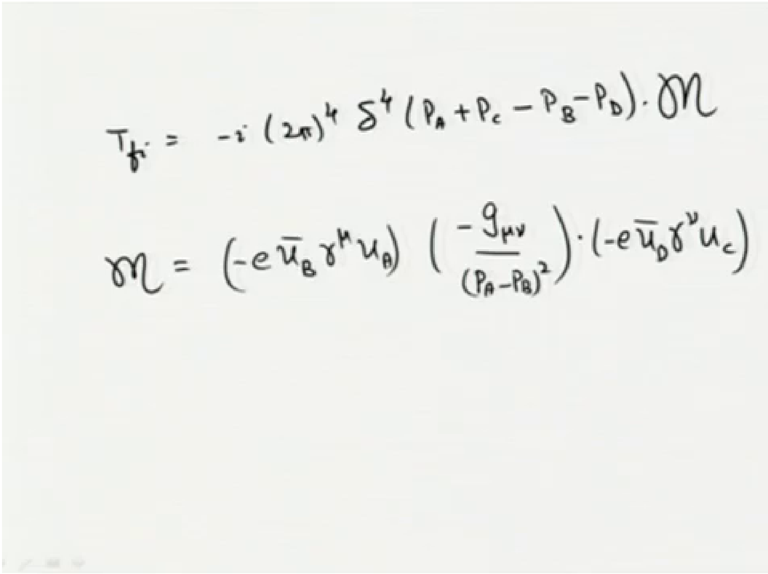

So, we can write T f i s minus i 2 pi power 4 delta 4 P A plus P C some of the initial momentum minus P B minus P D minus some of the final momentum, into an invariant amplitude m is equal to minus e u B bar gamma mu u A propagator factor g mu nu over P A minus P B square ok. And minus e u D bar gamma nu u C we will look at the cross section how why we can write the cross section in the next class.