Nuclear and Particle Physics
Prof. P. Poulose
Department of Physics
Indian Institute of Technology, Guwahati

Lecture - 33
Interacting charged fermions – 1

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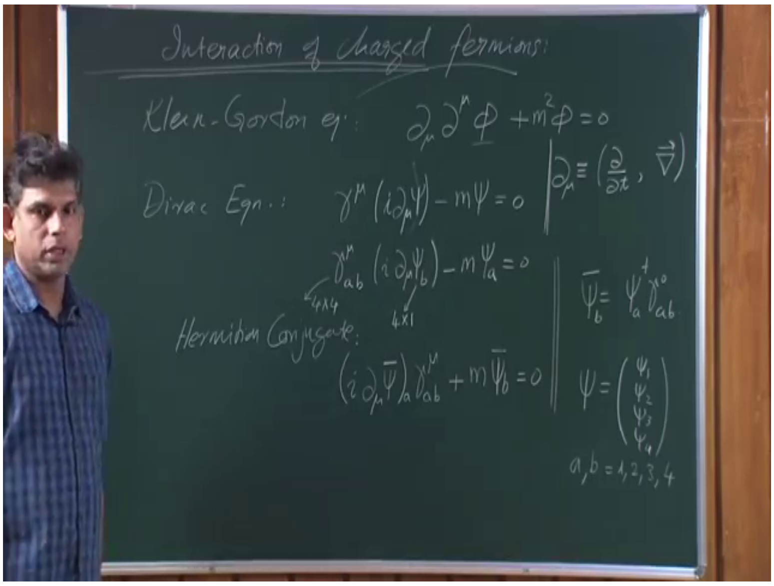

Today, we will look at how to deal with the interactions of electrically charged fermions. So, let us summarize first what we have learnt so, far in the particle physics part of this course.

So, we have said that in the relativistic case when we are dealing with particles, which are moving it or having a high energies and moving at relativistic speeches, then we have to consider the evolution equation for the equation of motion different from that of the Schrodinger equation. And, what we found as the Klein Gordon equation as one of the suitable equations to describe the dynamics of particles including their relativistic kinematics.

So, this in the notation that is now familiar to us, I will write this as dou mu dou mu phi plus m square phi equal to 0, where phi is the wave function that represents the charged particle. This is the equation which is quadratic, in the derivative both time and special derivative. Let me for completeness sake write what this means this is partial derivative with respect to time and the gradient operator. This we have not included the factor of C, which is usually the, to keep both the terms at the same dimension.

So, usually it is written as one over C this thing, but we are using a system of units which is in which C is taken to be 1 and the Planck constant h cross is equal to 1. Now, this is basically the Klein Gordon equation describing whatever particle that is represented by the wave function phi.

Now, Dirac equation is again relativistic description of elementary particle or the particles, this free particle the 1 that we have written down phi is also considered as a free particle there is no potential, there is no interaction there is only this mass term and the time derivative telling us how this for free particle will evolve with time? And the kinetic energy part described by given by the gradient of that, in Dirac equation we have a slightly different case where we have gamma mu an object, which we have defined clearly and described in detail in some detail and I dou mu which is essentially the momentum operator apart from the sign and apart from the constant h cross, which we are taking as 1 this acting on psi ok, this minus m psi is equal to 0.

So, that is the Dirac equation so, the differences between these 2 which we have already gone through, but let me just remind you gamma is a 4 by 4 matrix and let me denote the indices matrix indices explicitly, these I dou mu which is not a new matrix, which psi is a matrix, which is single index that is it is 4 by 1 matrix.

So, you have a 4 by 4 matrix multiplying 4 by 4 matrix let us, but we have minus m psi a. So, this is for each component of this a component of psi and a component of this product. Now there is a conjugate equation that I can describe, which is important in our case in the earlier case the conjugation was just a charge conjugation operator and psi phi would go to phi star that would do.

But in this case we have the Hermitian conjugate equation. So, that is basically I dou mu psi an object, which we will define in the minute psi bar and gamma mu this if it is a, then a p minus another plus m psi bar b is equal to 0.

So, the psi bar is defined as psi dagger with a gamma 0. So, psi bar b is a a b psi itself as we saw is psi 1 psi for component object psi 1 psi 2 psi 3 psi 4. So, this a and b runs from this is the summary of things that we have kind of discussed in the relativistic quantum mechanics or relativistic particle dynamics. We are dealing with free particles which mean particles like electron which is freely propagating in the space without an electric field in a region without an electric field or any other field.

That is described by this is the, this equation that is the kind of picture that we have. Now today as we promised we are going to look at how interaction can be understood? We had already seen how that is done understood in the case of particle, which obey the Klein Gordon equation.

So, just to summarize or to give in a very short way what is the difference between these 2 that without going into any details, let me introduce another word field and wave function I will use interchangeably somewhat loosely. So, in relativistic quantum mechanics we usually call this wave function, whereas a proper mathematical treatment is done in the framework of quantum field theory, where we will actually have to promote these wave functions to what is called the fields.

So, there are some certain differences, but I do not think we should worry about that at the moment. And that will actually take us a little deeper into the aspects, which are beyond of course, description this is a good representation of any scalar particle a complex field ok, phi which obeys the Klein Gordon equation is a good representation of the scalar particle. Were firm unique field like electron is represented by the Dirac equation, it is not there are certain features the spins of this is not going to be adequately addressed in Klein Gordon equation ok. Therefore, one has to actually consider the Dirac equation to treat the electron with including it is spin part use in a mu.

So, let me do not go into any more details of that we will assume that electrons and particles like electrons, the fermionique particle half integer spin particles obey the Dirac equation and usually they are represented by this psi, with 4 components like that in the Dirac notation. Whereas phi is just 2 degrees of freedom here, it is a complex number 1 real part and 1 in the other part.

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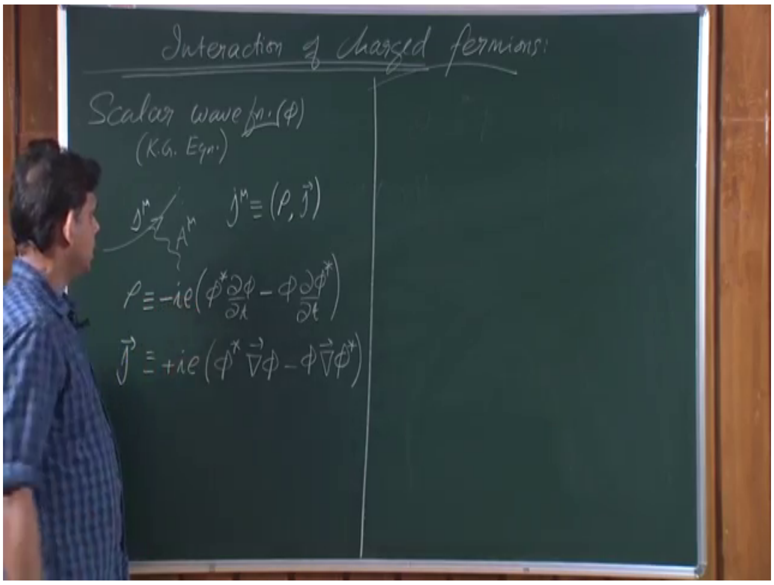

So, let me consider the K G equation or the scalar particle, when I say scalar wave function that is a wave function, which represent which a obeys in the Klein Gordon equation that is. So, let me write in bracket Klein Gordon equation and represented by phi.

Now, where a current of this phi f I particle represented by phi moves that can be thought about as current by the way complex scalar particle represents the charged scalar particle ok. So, this current let me denote by letter j mu and supposing there are photons or the electric field, in this region the, this notation is also familiar to us.

So, j mu interacts with this in a graphical way we can represent it in this equation there is a current of phi and then there is the electric field electromagnetic field we are represented interaction by this. Where j mu is equal to there is the 0th component which is rho and represented by rho and j and rho itself is defined as minus phi sorry rho itself is defined as i phi star time derivative of phi minus phi time derivative of phi star, i e minus i e.

This can be interpreted as the charge density electric charge density and j is represented by i e here it is a plus minus i e, and phi star gradient of phi minus phi gradient of phi star interpretation here is the current that is the electric current itself that is represented by this j.

So, in sense if you look at the electro dynamics the Maxwell's equations and all the electrode and classical electrodynamics this notation is clear there. We usually represent charge density by a rho and we represent the current density by j ok. So, that is what we have. So, this part we had discussed earlier.

So, in case you have forward and maybe you can check the previous lectures. Now coming to so, for electron it is I think I made a sign earlier correct sign and this will be for electron with minus e charge it is minus I e sorry for that.

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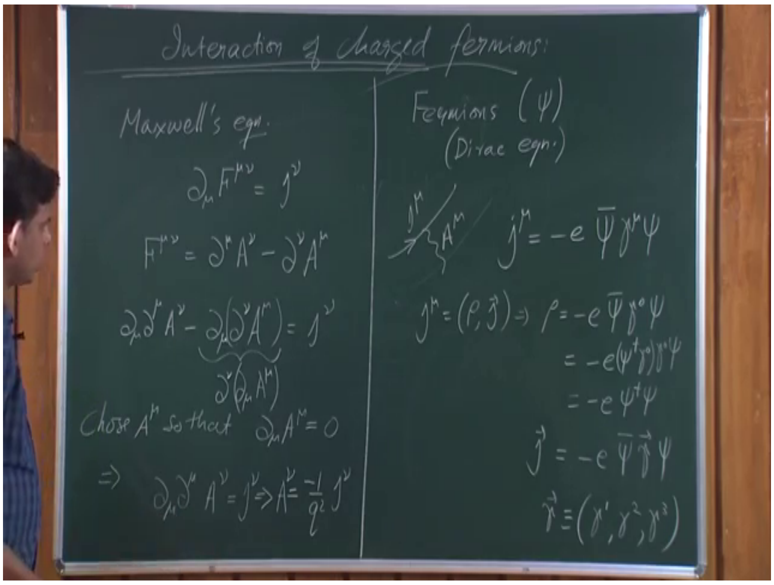

Now what about the fermionic fields so, fermion represented by psi they obey Dirac equation again picture is the same, we have a current and the current is interacting with a mu so, this is like an electron and on the way here encounters electromagnetic fields then how does it interact and that is the way we represent it.

So, this is again we can do in the same way only thing is what is j mu here j mu is for electron minus I the charge of that psi bar gamma mu psi. So, psi bar is known gamma mu here is the Dirac matrix and psi is the wave function. Now this again when we write it in terms of the charge density and current density we have rho equal to minus e psi bar gamma 0 psi ok. This is nothing, but minus e psi bar itself a psi dagger gamma 0 ok, then there is another gamma 0 psi which is equal to minus e psi dagger psi since gamma 0 square is equal to 1, j is equal to minus e psi bar gamma vector psi.

When, I say gamma vector is basically gamma vector is defined in a sense as gamma 1 gamma 2 and gamma 3. We will go back and remind ourselves about one thing that is the electromagnetic field itself the electromagnetic field in itself as the Maxwell's equations Maxwell's equations were written as dou mu f mu nu equal to j nu. Well j nu is the current density in the source of electric field current density of the source of the electric field. And we can write it in themselves rho and j and the, that is what we have in the classical electrodynamics case help me nu are the field tensors, which we are again familiar with we had discussed it earlier.

So, this is what it is this can be written in terms of f mu nu is essentially the mu in terms of the potential or the photon field it is rho mu rho nu A mu. And when you put this back we have dou mu dou mu A nu ok, minus dou mu A mu dou mu dou nu A mu equal to j nu , but this if you focus on this then this is essentially there is no preference preferred order in which this differentiation need to be done and this this can be interchange I can follow or do this differentiation first and this the second rather. I can actually simply change this order in which these 2 act on A and nothing changes now I have something like this we can choose this A so, that dou mu a mu is 0 always ok.

So, this is something which we will assume now or rather it is something which we had discussed earlier. So, we will say that how a we have chosen an A, which has this particular condition satisfied that choose A so, that dou mu a mu that is basically the dot product between these 2 are the for divergence of a vanished. This can be done always you can choose A mu (Refer Time: 21:35) and that will not change any of the physics aspect the meaning, that it is the same electric any given you know photon electron electromagnetic field can be represented by some a, which can always be written in terms of some divergence less in that will give you dou mu dou mu a nu equal to j nu or this will give us a solution ok. Without going into the details we can find the solution this remember what do you mean by solution, this for given j source the electric field electromagnetic field everywhere represented by A mu satisfy this equation that is what you have?

Supposing you have an electron or some distribution of charge, moving distribution of charge, something like that or a nucleus moving that will create an electromagnetic field surrounding that is, now what is that electric field that is represented by a how does it behave that is dictated by this equation?

So, can we say what are that, what is the different type of q a mu yes. So, that without going into the details let me give you the solution that A nu equal to minus 1 over q square j nu, where q square is essentially the hmm momentum transfer under this one. So, what do you mean by q square I will come to let me just rewrite this. So, what we have us a mu is equal to minus 1 over q square j nu.

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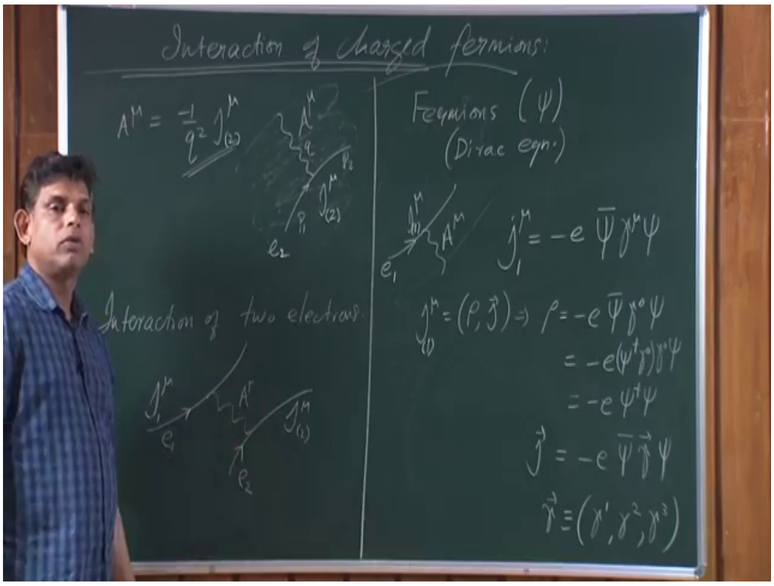

So, what does that mean this means in a region we have some electromagnetic represented by A mu ok. So, it is the same index A mu, but we know that is coming from say some electron or a nucleus presence of sitting electron or sitting in case for moving electron or moving in or some complex charge density , but that is represented by this thing. So, there is j corresponding to this how do we actually feel the presence of electromagnetic interactions? From the charge particles forget about the Newtonian way of action at a distance we demand light in special theory of relativity and same demander, that we have to have something some cause for effects the effect is that we feel the electromagnetic field their causes is that there is some charged particles moving or not moving.

Now this changes in that or whatever the presence of that has to be conveyed to us, that is done by the emission for the propagation of the photons or the presence of the photons photon field in that field. So, when for that when a photon is emitted ok. We can think about a emitting and reabsorbing etcetera or just a emitting, then it is emitted then you will have some momentum p 1 here and p 2 here and A q here.

So, this essentially is the momentum of the q let me do not going to further details, but essentially to complete description of the electromagnetic field the surrounding this in a space configuration coordinate picture, you will have to actually take all such a possible q and integrate over a for take the Fourier transform of this a to get this spatial part of this thing, but this is basically a particular photon is represented in that way.

So, this is what you have perfect q, whenever such a momentum transfer happens you can actually think about us or such a current happens, you can think about a photon which is represented by minus 1 over q square and this is the q. Now, that and this let us look at together for simplest for identification, let me denote this as electron 1 and the current due to that as j 1.

And this as some another charged particle or electron 2 and a current due to that is j 2. This 2 has nothing to do with these 2 for these components of j this is another some just an index to represent this particle and then identify that that is caused by this electron and this is talking about another electron that is all.

And, now let me look at these together. Here, we were saying that an electron encountering an electromagnetic field will interact with the photon in this manner, with the current representing the charged particle motion the current it is electric charge itself and then this is the see a mu (Refer Time: 28:08). Here, we have the source of the current itself. So, we have say we are saying that that a mu is kind of coming from this second another electron or this mu.

So, in a sense we can actually talk about interaction of 2 electrons yes someone current j 1 mu interacting with another current j mu 2 ok. So, this is a good picture, which we have to keep in mind.

So, 2 electrons say electron 1 and electron 2 interacting with each other can be represented by such a picture I mean in actuality it is much more complex than the, but main features of this interaction can be captured and in fact, calculated also in a probability for such interaction etcetera can be calculated or the scattering of an electron on an electron to get another electron on another electron can be captured very easily in by this thing. I will stop here then we will discuss further in the next lecture.