Nuclear and Particle Physics
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Module – 09
Electroweak Interactions
Lecture – 07
Dirac Equations

So, today we will learn we will continue our discussion on the relativistic quantum mechanics, how do we discover describe particles and their interactions in that.

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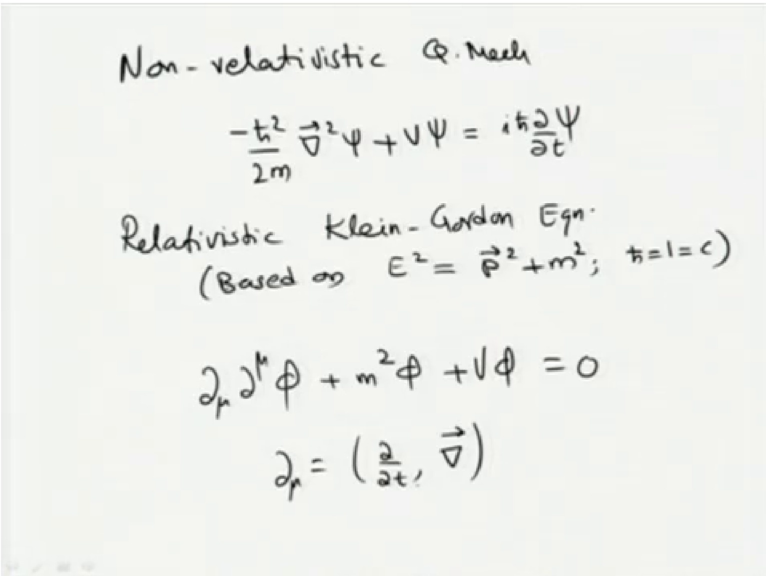

So, in the case of a non-relativistic quantum mechanics, we had the Schrodinger equation minus h cross square over 2 m grad square psi plus V the potential interacting potential psi equal to i h cross dou by dou t the energy operator acting on the wave virtual site, and as we said this is not adequate to describe the relativistic dynamics. So, for the relativistic case we had discussed what is called the Klein Gordon equation. This is basically based on the energy momentum relation E square is equal to P square plus m square where we are now working in units of h cross is equal to 1 is also equal to c.

In that case we said we could write down for a wave function particle described by a wave function phi dou mu dou mu phi, plus m square phi plus V phi equal to 0 where dou mu is now, dou by dou t, and we discussed some the discuss the interaction of charged particles, and to some extent found out be the expression for transition amplitude in this case.

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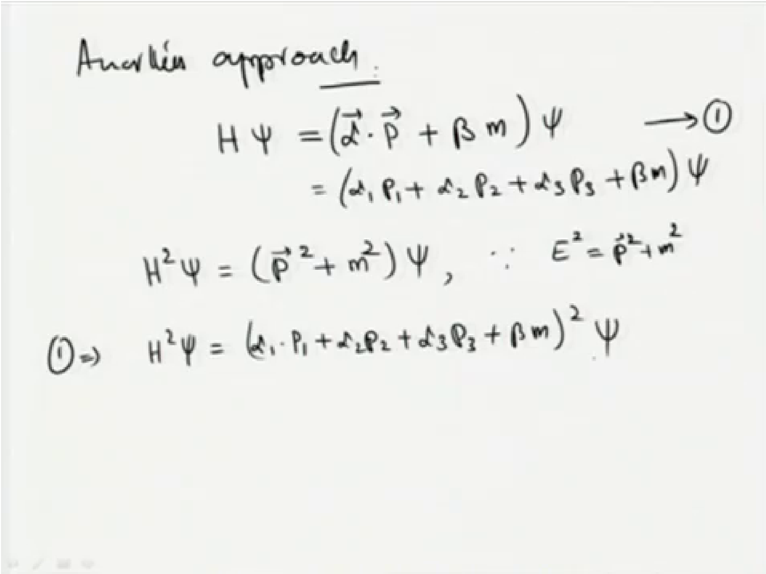

Another approach to get to the relativistic equations is due to Dirac, and we will see that we can get some other dynamical equation, describing the wave functions evolution which is known now as the Dirac equation. In the case of Klein Gordon equation, we used a square is equal to P square plus m square, we in the case of Dirac equation what we will do is to linearize this thing.

Remember the problem in Schrodinger equation was that it is quadratic in the coordinate in the position coordinate, and or the second order derivative in the position coordinate, but first order derivative of time parameter. And then in Klein Gordon equation we took both of them to second order.

And in Dirac equation we go the other way, and then try to get the first order kind of s. So, it is now written as some coefficient alpha dot P alpha is taken given a symbol of a vector, and dotted with P the meaning is essentially that alpha 1 some coefficient multiplying the first coordinate of momentum another coefficient alpha 2, P 2 plus alpha 3, P 3 plus beta m psi. So, this is actual meaning of this.

But in a compact way we can write it as alpha vector dot P, whether alpha is actually a vector or not we will see later. Now what are this alphas and betas for that we will actually gain rely on the energy momentum relation. We know that H square psi should be P square plus m square psi. Since energy operator they have a Hamiltonian operator square of that when it acts on psi, it will give you the square of energy as the value if psi j s the Eigen state of energy, and initially we take psi as the Eigen state of energy, which is a physical state.

So, E square is equal to P square plus m square is what V half, but from 1 if you take H square, and if you take the square of the operator on the right hand side, you will have alpha 1 dot P 1 dot P 1 plus alpha 2 dot node or actually in this ordinary multiplication, alpha 3 P 3 plus beta m square psi that will give you.

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$$
\begin{aligned}
\Rightarrow \quad H^2\psi = {} & \left(\alpha_1^2 P_1^2 + \alpha_2^2 P_2^2 + \alpha_3^2 P_3^2 + \beta^2 m^2\right)\psi \\
& + \left(\alpha_1\alpha_2 P_1 P_2 + \alpha_1\alpha_3 P_1 P_3 + \alpha_1 \beta m P_1\right)\psi \\
& + \left(\alpha_2\alpha_1 P_2 P_1 + \alpha_2\alpha_3 P_2 P_3 + \alpha_2 \beta m P_2\right)\psi \\
& + \left(\alpha_3\alpha_1 P_3 P_1 + \alpha_3\alpha_2 P_3 P_2 + \alpha_3 \beta m P_3\right)\psi \\
& + \left(\beta\alpha_1 m P_1 + \beta\alpha_2 m P_2 + \beta\alpha_3 m P_3\right)\psi
\end{aligned}
$$

Let me call this equation then energy equation as 2. H square psi then a is equal to so, I have alpha 1 square P 1 square, plus alpha 2 squared P 2 square, plus alpha 3 square P 3 square, plus beta square m square psi plus, other terms alpha 1, alpha 2, P 1, P 2 plus alpha 1, alpha 3, P 1, P 3 plus, alpha 1 beta m P 1 acting on psi plus similarly, alpha 2 alpha 1, P 2, P 1 plus alpha 2, alpha 3, P 2, P 3 plus alpha 2 beta mP 3 acting on psi alpha 3 alpha 1 P 1 actually P 3 does not matter here, P 3 P 1 alpha 3 alpha 2, P 3 P 2 plus alpha 3 beta mP 3 psi.

Earlier 1 is actually P 2, then 1 more time beta alpha 1 P m P 1 beta alpha 2 mP 2 plus beta alpha 3 mP 3 psi. So, these are the terms when you expand it deliberately expanded it in explicit fashion. So, that we do not confuse ourselves, and we also did not assume that alpha 1 alpha 2 is equal to alpha 2 alpha 1, P 1 and P 2 are momentum operators which we know commute P 1 and m also confused commute.

Similarly, other components of P 1, P also commute with each other and also can meet with other commute with mass m, but the other coefficients alphas and beta, we are not assuming to be commuting, and we will see what are the relations that it should follow say for example, if you take it to be commuting what happens. If you take they are commuting, and then it will be a problem.

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$$
\begin{aligned}
H^2\psi = {} & \left[\alpha_1^2 P_1^2 + \alpha_2^2 P_2^2 + \alpha_3^2 P_3^2 + \beta^2 m^2\right]\psi \\
& + \left[(\alpha_1\alpha_2 + \alpha_2\alpha_1) P_1 P_2\right]\psi \\
& + \left[(\alpha_1\alpha_3 + \alpha_3\alpha_1) P_1 P_3\right]\psi \\
& + \left[(\alpha_1\beta + \beta\alpha_1)\, m P_1\right]\psi \\
& + \left[(\alpha_2\alpha_3 + \alpha_3\alpha_2) P_2 P_3\right]\psi \\
& + \left[(\alpha_2\beta + \beta\alpha_2)\, m P_2\right]\psi \\
& + \left[(\alpha_3\beta + \beta\alpha_3)\, m P_3\right]\psi
\end{aligned}
$$

$$H^2\psi = (\vec{P}^2 + m^2)\psi$$

So, let us look at it H square psi equal to alpha 1 square, P 1 square plus alpha 2 square P 2 square plus alpha 3 square P 3 square plus beta square m square psi, plus P 1 and P 2 and P 2 and P P 2 P 1 and P 2 P 1 P 2 are the same they commute. So, I combine alpha 1 alpha 2 plus alpha 2 alpha 1, P 1 P 2 acting on psi plus alpha 1 alpha 3 alpha 3 alpha 1 P 1 P 3 psi alpha 1 no 1 no alpha yes.

Alpha 1 beta plus beta alpha 1 mP 1 psi plus alpha 2 alpha 3 plus alpha 3 alpha 2 m no m P 2 P 3 psi, plus alpha 2 beta plus beta alpha 2 mP 2 sin plus alpha 3 beta plus beta alpha 3 mP 3 acting on psi, these are the only terms.

Now, let us look at the relation energy momentum relation that let me write it here, again situation to that we had written earlier H square psi is equal to P square plus m square psi, that is equal to P 1 square plus P 2 square plus P 3 square plus m square psi, but now the first line here, in a square psi what we have is alpha 1 square P 1 square plus alpha 2 square P 2 square plus alpha 3 square P 3 square plus beta square, and square psi, and there is no other term with P 1 square P 2 square P 3 square or m square. So, the coefficients of P 1 square P 2 square P 3 square, and m square should all be equal to 1 that is 1 condition let us put that there.

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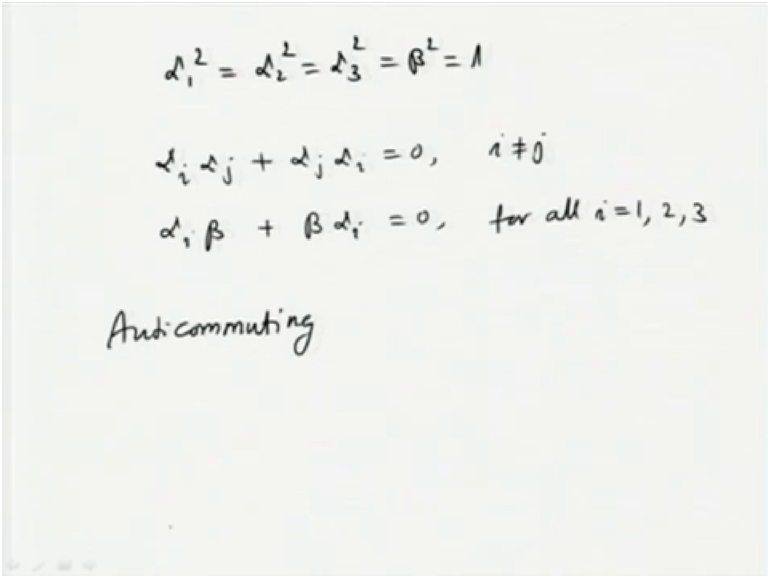

So, alpha 1 square equal to alpha 2 square alpha 3 square equal to beta square equal to 1, all of them are the same and equal to 1, then other thing is alpha 1 alpha 2 plus alpha 2 alpha 1. Suppose we put and there is no such term, in the H square, because H square is P square plus m square and that is already accommodated now.

Now, the rest of it all the terms should either cancel with each other, or should vanish identically. So, the first term is proportional to P 1 P 2, second one is proportional to P 1 P 3, then m P 1 P 2 P 3 mP 2 mP 3. Either some of these components are 0s. So, if P 1 is equal to 0 P 2 equal to 0 P 3 equal to 0 m equal to 0 then everything is fine, but that anyway we will give you another set x square is equal to 0.

So, that cannot be, but then the other thing that can happen is coefficients of P 1 P 2 identically goes to 0. It should be valid for again general momentum any momentum it should be valid it does not matter, what particular case P 1 or P 2 equal to 0 1 particular term will vanish, but in general if you want this to be so, and then each of these coefficients alpha 1 alpha 2 plus alpha 2 alpha 1 should vanish, and they will vanish like that only if either alpha 1 alpha 2 r or either of this is equal to 0 that will also not give you a general case.

Because otherwise there would not be either a P 1 square or a P 2 square term there, in a square which is not allowed, which is not a general case. So, now, alpha 1 alpha 2 plus alpha 2 alpha 1 is equal to 0 means they are objects which do not commute. So, these are no ordinary numbers, or ordinary functions that tells us that alpha i alpha j plus alpha j alpha i equal to 0, if i is not equal to j, i equal to 1 j equal to 2 it is alpha 1 alpha 2, alpha plus alpha 2 alpha 1 which is equal to 0.

And similarly you have alpha i beta plus beta alpha i equal to 0, for all i equal to 1 2 3 these subjects therefore are called anti commuting objects, because if you switch the order in which they are multiplied that will give you a change of sign. So, that is one thing. Now so, that is 1 thing that we notice.

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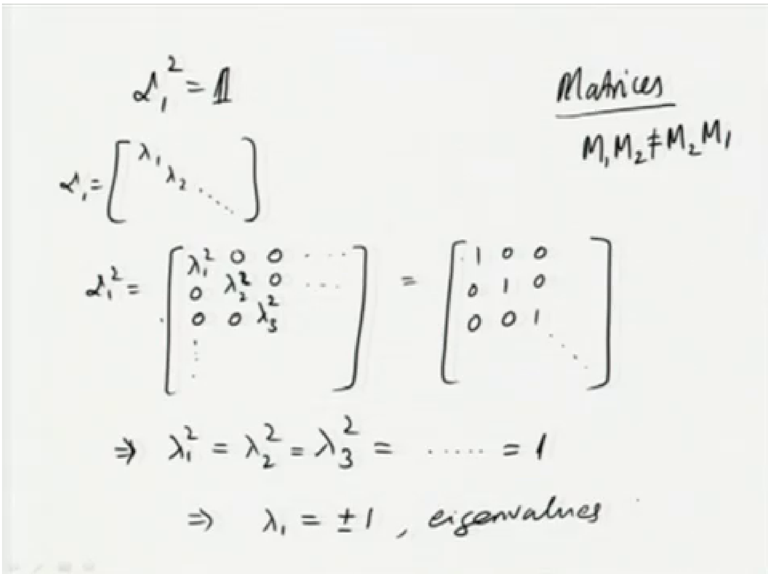

Now, let us say we said alpha 1 square equal to 1, it is not half an ordinary function we know we are familiar with one, but one type of this anti commuting objects matrices in general do not commute. If you take a matrix A and the matrix B multiply it together. So, that you have first element A, and then B matrices A or M 1 and M 2 in general is not equal to M 2 M 1 that is clear to you, because you are familiar with these matrices.

So, it could be a matrix one of the ways this can be represented is through matrix, and we will certainly be doing that. So, at least some mathematical entity we know, which actually are not which can be non-commute. Now if you have a matrix form say, and then suppose you have diagonalize this matrix. If you diagonalize this matrix, then you have only diagonal elements for this lambda 1 lambda 2 etcetera, then alpha 1 square is essentially lambda 1 square, lambda 2 square, lambda 3 square etcetera.

So, the other elements are all 0s only the diagonal elements exists, depends on how many what is the dimension of the alpha. So, when I write it as lambda square equal to 1 i actually mean a unit matrix. So, this is a unit matrix, and a unit matrix is a diagonal matrix with diagonal entries equal to 1 etcetera, which means that lambda 1 square is equal to lambda 2 square equal to lambda 3 square equal to 1 so, etcetera all of this.

Diagonal elements should be equal to 1, this 1 is actually just 1 in the number 1 not in a matrix, and that then tells you that the eigenvalues lambda, lambdas 1 lambda 2 lambda 3 etcetera of alpha 1 is either plus 1 or minus one. So, the lambda eigenvalues can be either plus 1 or minus 1 nothing else. So, it cannot be 0 it cannot be plus 1, it cannot be minus 1, or any other number it can be either plus or minus 1, that is one consequence.

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$$\alpha_1 \beta + \beta \alpha_1 = 0$$

$$\alpha_1 \beta = -\beta \alpha_1$$

$$\beta \alpha_1 \beta = -\beta^2 \alpha_1 = -\alpha_1$$

$$Tr(\beta \alpha_1 \beta) = -Tr(\alpha_1)$$

$$\Rightarrow \quad Tr(\beta^2 \alpha_1) = -Tr(\alpha_1) \qquad Tr(ABC) = Tr(CAB)$$

$$Tr(\alpha_1) = -Tr(\alpha_1)$$

$$Tr(\alpha_1) = 0$$

Then let us see, let us look at the property of the anti-commuting property of alpha I, let us specifically take alpha 1, and then or I can write as alpha beta is equal to minus beta alpha 1. Now let me multiply this by beta 1 from the left side. So, that will give you beta square alpha 1, but beta square is equal to 1 therefore, it is equal to alpha 1.

Let me now take the trace of this, beta alpha 1 be down trace off or minus the trace of alpha 1, but now we have a property of the matrices trace of a chain of products of matrices say ABC is equal to unchanged, if you cyclically rotate this CAB that is an exercise, which you can prove later on. That will tell you that trace of beta alpha 1 beta is also equal to trace of beta square alpha 1, that is equal to beta square is equal to 1. So, that will give me trace of alpha 1 is equal to minus trace of alpha 1, this is possible only if trace of alpha 1 is equal to 0.

The quantity is negative of the same quantity; only if that is identically equal to 0. So, there are 2 such properties that before, similarly for other alpha values and even for beta. So, we can replace alpha 1 by alpha 2 or alpha 3. So, that will immediately tell you alpha 2 trace is also equal to 0, and instead of multiplying the second line here, by beta if I multiply by alpha 1, then again I will get alpha 1 square equal to 1 will give me trace of beta is equal to 0. So, trace of all of this is equal to 0.

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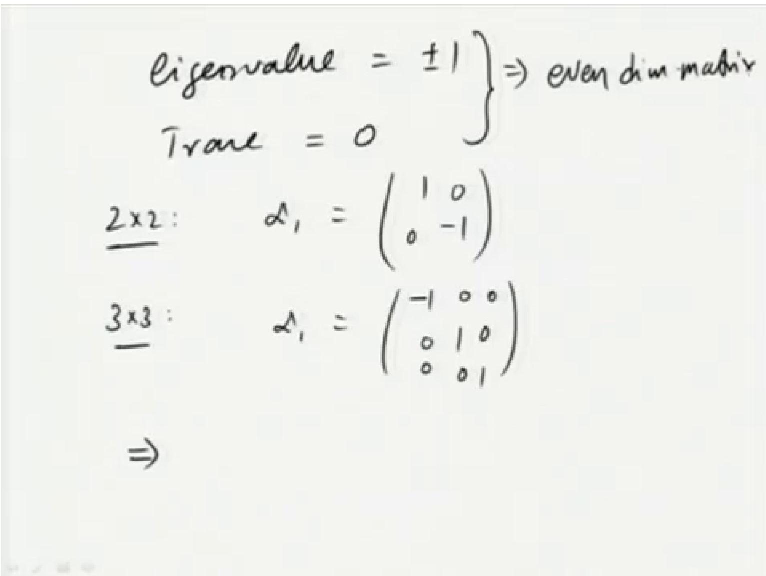

Now, what does that tell us 1 is Eigen value either plus 1 or minus trace. So, let us say let us take an example, if I take 2 by 2 case dimensional matrix, then say alpha 1 can be in the diagonal form 1 say minus 1 this is an example, or let me take alpha 1. This is now trace less eigenvalue is either 1 or plus 1 or minus 1.

But now let me take A 3 by 3 case. So, let me take alpha 1, 1 possibility is to take 1 1 1, because then that is a unit matrix so; obviously, the trace of that is not equal to 1, but can I make one of them minus 1 other is plus 1, and then also trace is minus plus 1. This is not possible to have a trace less you cannot have eigenvalues either plus or minus 1, and at the same time trace is equal to 0 for an odd dimensional case that, actually tells you that. In fact, these 2 tells you that it is an even dimensional matrix.

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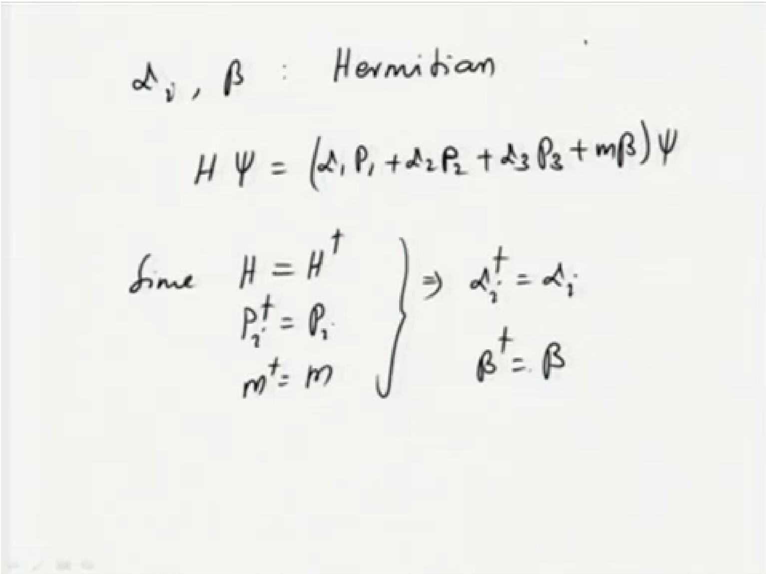

Now alpha the other property of alpha i and beta is that they are hermitian, and how do we see that H psi is equal to alpha 1 P 1 plus alpha 2 P 2 plus alpha 3 P 3 plus m beta psi. Since H is hermitian, P is hermitian, and M is anyway constant hermitian that will tell you that alpha i is hermitian, and beta is hermitian.

So, this is another property. So, alphas and betas are hermitians objects dimensionality is it should be an even dimensional racing either, 2 by 2, 4 by 4, 8 by 8, 6 by 6 or whatever cannot be odd dimension 3 by 3 is not possible, Eigen values are either plus 1 or minus 1 and they are traceless.

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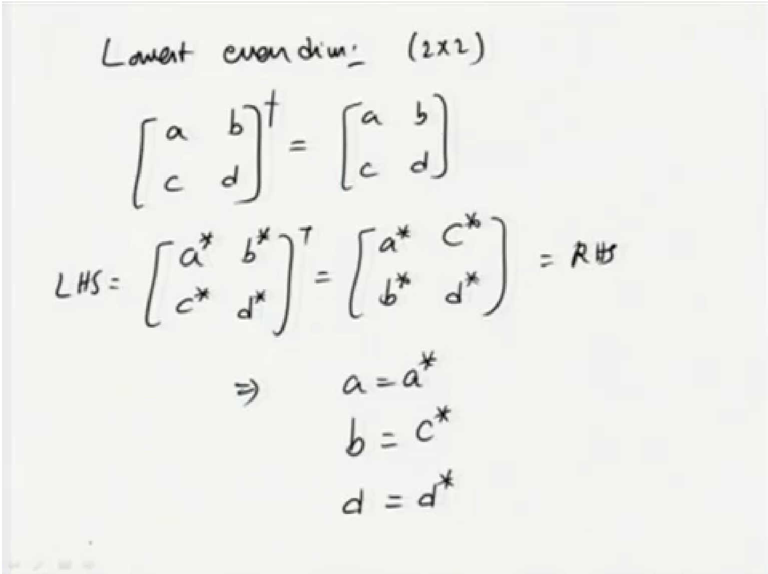

Let us take 2 by 2 case lowest even dimension 2 by 2, let us take a general matrix a b c d, hermitian will tell you that a b c d e a dagger is equal to a b c d itself, and that will tell you that. So, the hermitian thing is left hand side is a star, b star first the conjugate transpose. So, LHS is equal to equal to a star, c star, b star, d star. So, that should be equal to RHS tells you that a is a real number b is equal to c star, and d is equal to d star the real number.

So, the diagonal elements are real off diagonal elements are complex conjugates of each other, symmetrically that is what will happen if you have higher a larger than our dimensions. So, let us take 2 by 2 and then this is how it.

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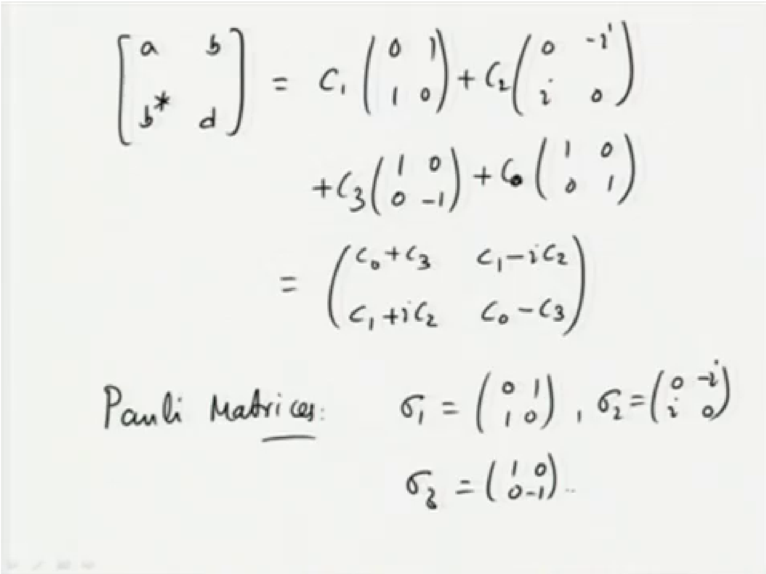

So, any hermitian matrix you can write as a b, b star d fine. So, now let me see if I can write it in terms of 0 1 1 0 plus 0 minus i i 0 plus 1 0 0 minus 1 plus some c 0 for whatever reasons I am denoting it by 0.

So, the last 1 here is a unit matrix 2 by 2, and you recognize the first 3 has the pouli matrices, if I do this first element only c 3, and c 0. So, let me write it in that order c 0 and c 3 c 0 plus c 3, then second c 1 minus ic 2 c 1 plus ic 2 and c 0 minus c 3, indeed now off diagonal elements are complex conjugates of each other diagonal elements are real.

Now, provided the c s are diagonal I mean c s are real numbers, what does this mean otherwise this is arbitrary c 1 can be any number, c 2 can be any number, c 0 can be in number, c 3 can be any number, which says that a 2 by 2 in general a 2 by 2 hermitian matrix can be written as a combination of these 4 matrices, they are called the pauli matrices.

Pauli matrices is a sigma 1 is 0 1 1 0 off diagonal, sigma 2 is 0 minus i i 0 off diagonal, sigma 3 is diagonal 1 minus 1. So, the Pauli matrices along with the unit 2 by 2 unit matrix is enough to describe any arbitrary hermitian matrix of dimension 2 by 2, and you can also see that sigma 1, sigma 2, sigma 3 do not depend on each other or they are linearly independent of each other.

You cannot get one in terms of another one by observation, you can see that also I mean just multiply 1 by some number you will not get the other 1, then the and the unit matrix is also linearly independent of this. So, we have 4 uni linearly independent matrices, in terms of which we can write any other matrix.

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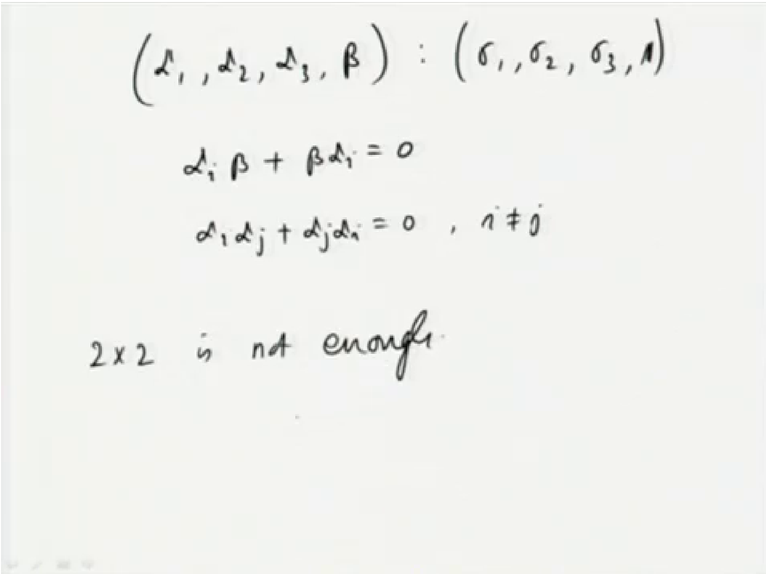

This means that if we have to consider alpha 1 alpha 2 alpha 3 and beta other direct matrices. In 2 by 2 matrix form it is enough to actually consider sigma 1 sigma 2 sigma 3, and beta and the unit matrix, because any other matrix you consider would be can be written in terms of this anyway. And now if you look at the anti-commuting properties, and alpha I, alpha j, plus alpha j alpha i equal to 0 for i not equal to 0, all of them anti community with all the others.

But in the set we have along with the Pauli matrices, we have a unit matrix and that commutes with everything. So, we cannot have 4 linearly independent matrices, which anti commute with each other in case of 2 by 2 matrices. So, 2 by 2 cannot is not enough?

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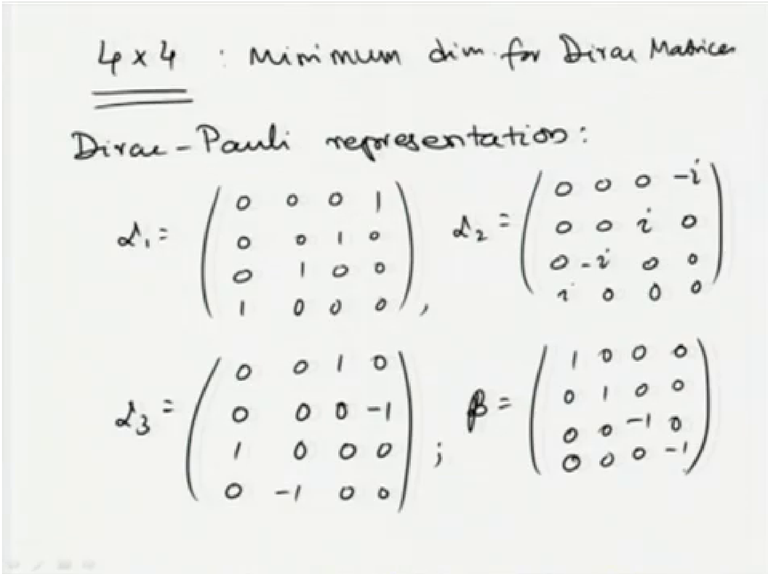

So, the next thing to consider is the 4 by 4 3 by 3 is an odd dimension. So, we will not will to look at that. Now looking at the 4 by 4 matrices let us look at that is the minimum dimension for Dirac matrices, there is 1 particular representation called Dirac pauli representation. This is alpha 1 is 0 0 0 1 1 0 0 1 1 0 0 0 0 0, alpha 2 0 0 0 minus I, i 0 alpha 3 equal to 0 0 1 0, 0 minus 1, 1 0 0 0, 0 minus 1 0 0, and beta equal to 1 0 0 1 0 0 0 0 0 minus 1 minus 1.

This particular representation I would like you to check that, they satisfy all the anti-commutation relation, and also that their square is equal to unit matrix.

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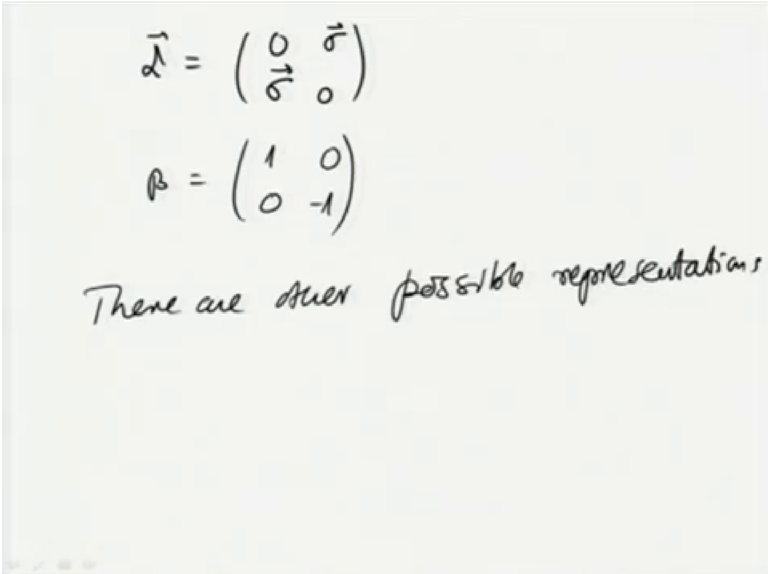

In a compact way you can write alpha as 0 sigma sigma 0, sigma is a pauli matrix, where now you had to well. Now when you consider alpha 1, sigma 1 is considered alpha 2 sigma 2 alpha 3 sigma 3, and beta is a unit matrix and minus of the unit matrix each of these entries here are 2 by 2 matrices. So, it is written in block form, and you can check this thing. This is only 1 representation, and there are other possible representations poly too which will give the same physics.

So, there is no unique representation, it can be this the rack coordinate representation it could be some other also, we might mention some other representation, which is interesting in particle physics.

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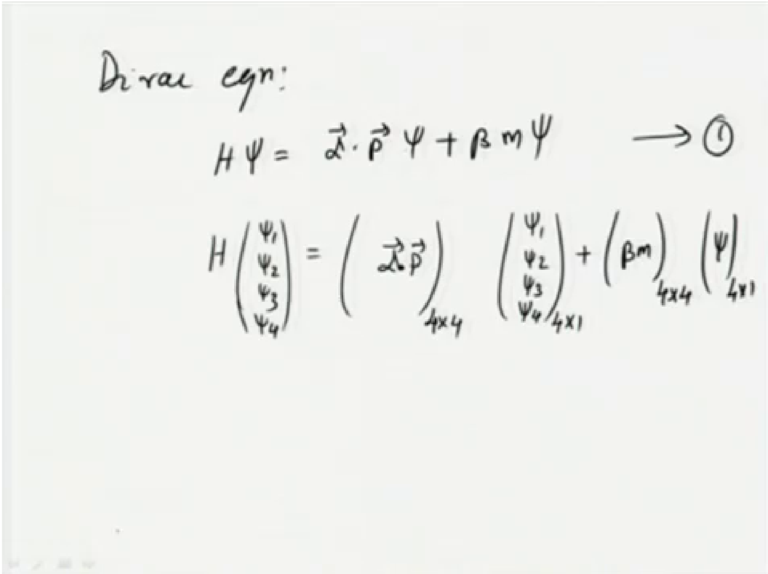

So, now, let us consider the Dirac equation again H psi is equal to alpha dot P psi plus beta m psi, where now alphas and beta are 4 by 4 matrices and therefore, psi should be a 1 by 4 by 1 matrix. So, H I will have actually psi 1 psi 2 psi 3 psi 4 A 4 component object, alpha is A 4 by 4 matrix dotted with P, and then you have a so, alpha dot P itself is a then A 4 by 4 matrix, then you have psi 1 psi 2 psi 3 psi 4 4 by 1 matrix. So, that they can be multiplied plus similarly beta m psi 4 by 4 matrix, and you have psi 4 by 1 matrix this should be kept in mind.

Let me, but now take this first equation here, 1 and multiplied by or first let me take the so, let me take this equation.

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$$i\frac{\partial}{\partial t}\psi = \vec{\alpha}\cdot(-i\vec{\nabla})\psi + \beta m\psi$$

$$i\beta\frac{\partial\psi}{\partial t} = (-i)\beta\vec{\alpha}\cdot\vec{\nabla}\psi + \beta^2 m\psi$$

$$\left(i\beta\frac{\partial}{\partial t} + i\beta\vec{\alpha}\cdot\vec{\nabla}\right)\psi - m\psi = 0$$

$$\gamma^\mu \equiv (\beta, \beta\vec{\alpha})$$

$$\Rightarrow \left(i\gamma^0\frac{\partial}{\partial t} + i\gamma^i\partial_i\right)\psi - m\psi = 0$$

$$\partial_\mu = \left(\frac{\partial}{\partial t}, \vec{\nabla}\right)$$

The Dirac equation and write it in the operator form i dou by dou t, the hermitian at the hamiltonian energy operator acting on psi is equal to alpha dot minus i gradient, which is the momentum operator acting on psi plus beta m psi, multiply this by beta from the left will give me, i beta dou by dou t of psi minus i beta alpha dot grad psi plus beta square m psi beta square is equal to 1.

Now, I can bring this to 1 side. So, I will have i beta dou by dou t plus i beta alpha dot del acting on psi, minus m psi equal to 0 beta square is equal to 1. Let me introduce the notation gamma mu equal to beta, and beta alpha. So, gamma 0 is beta gamma 1 is beta alpha 1 gamma 2 is beta alpha 2 gamma 3 is beta alpha 3.

In this notation now I can write the Dirac equation as i gamma 0 dou by dou t minus i gamma i dou i dou psi minus m psi equal to 0 here, dou mu is equal to dou by dou t, and grad minus sorry this is plus. So, dou mu so, this will be then plus.

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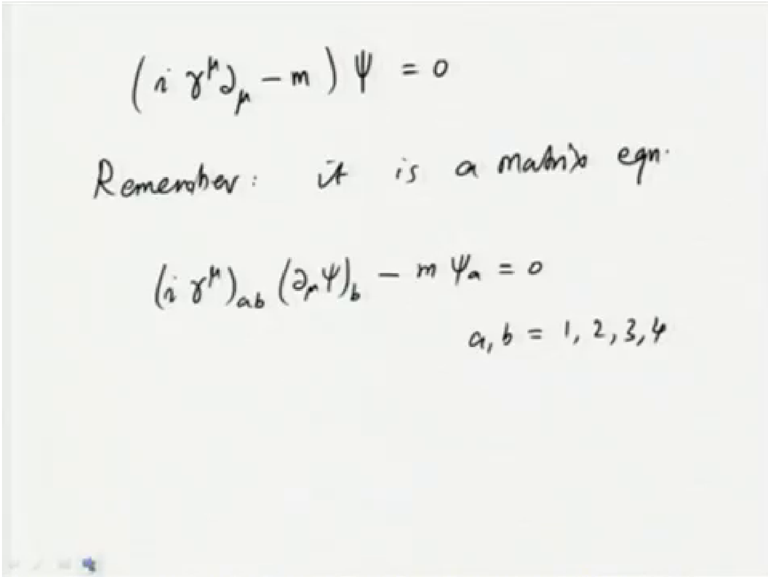

Together I can write it as gamma mu dou mu minus m psi is equal to 0, just to remind you that it is a matrix equation.

We will write this matrix indices explicitly a b element of i gamma mu, multiplied dou mu psi element b element of this is mu psi, and subtract so, that will give you a element of gamma mu dou mu psi, and multiplied by m psi a will give you 0, where a and b are Dirac indices 1 2 3 4. So, this a and b sums I mean runs through the direct components the components of gammas and components of psi.

So, this is not this has nothing to do with the Lorenz index mu or the components of vector in that sense. So, this is what we have here for this. And now let me look at the properties of alpha, and beta written in terms of gamma now.

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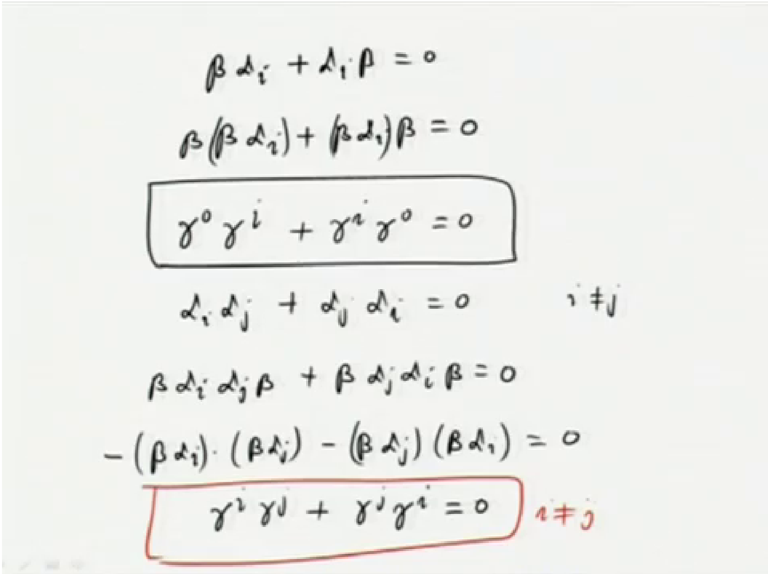

Beta alpha i plus alpha i beta equal to 0, let me multiply it by beta beta alpha plus beta alpha is equal to beta alpha i beta equal to 0. So, that is beta is gamma 0 beta alpha i is gamma i plus, gamma i beta alpha i let me club them together in this fashion, gamma i gamma 0 equal to 0. So, this is one thing.

So, gamma 0 and gamma i for any i commute anti commute take alpha i and alpha j for alpha not equal to j sorry i not equal to j commute, anti commute multiply again by beta alpha i alpha j beta from both sides, alpha j alpha i beta equals, we knew because of the first line here beta alpha is equal to minus alpha i beta, I switch this beta and alpha j in the first term, as beta and alpha i in the second term, that will give you minus i in both cases.

So, I should have added a minus sign here, and then put this as a minus sign equal to zero, but because of an overall way that will give you only an overall minus sign, and the first one is gamma i is gamma j, plus gamma j gamma i equal to 0. So, now that is so we have 2 relations here, one is let me highlight those, one is this that is gamma 0 gamma i plus gamma i gamma 0 is equal to 0, and we have another relation gamma i gamma j, plus gamma j gamma i for i not equal to j equal to 0.

So, gammas anti commute as well. So, this is the same as the relation that we had in that earlier in this case.

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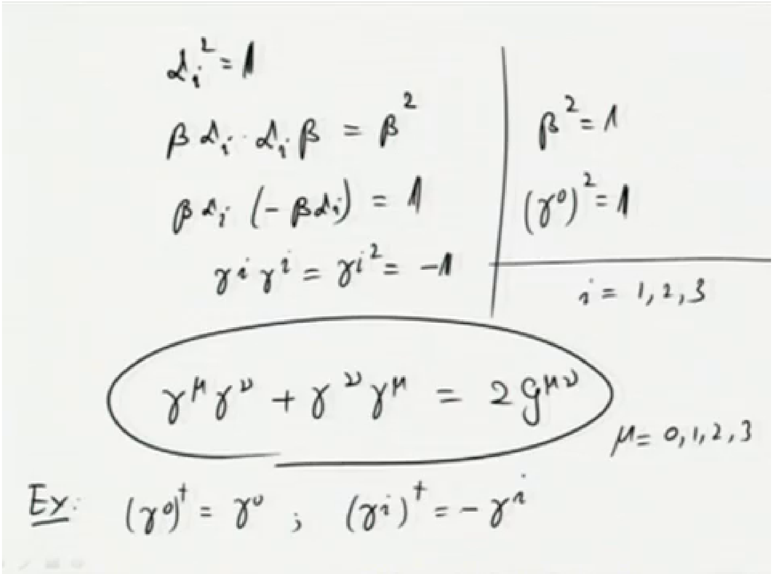

And now some other properties of gammas alpha i square equal to unity, which will give you let me write it as beta alpha i alpha i beta equal to beta square I just multiplied by beta from both sides, and then again and that will give me I have to the first 1 is beta alpha I, second when i switch this beta and alpha. So, I have beta alpha i become minus sign is equal to 1, which is basically gamma i gamma i for gamma i square equal to minus 1, because of the minus sign here.

And beta square is; obviously, equal to 1. So, gamma 0 square is equal to 1. So, beta square gamma square is equal to 1 gamma 0 square is equal to plus 1, and gamma i square is equal to 1 putting together or these the anti-commutation relation, these 2 relations, and this property of the square of the gammas, we can write gamma mu, gamma nu, plus gamma nu, gamma mu equal to 2 g mu nu, I would like you to verify that this in components are exactly the same as what we had a few we have now written here.

So, that is essentially the thing that we want to discuss, there is one exercise that I would like you to have. So, what I will write the exercise here, you just show that gamma 0 dagger is equal to gamma 0, which is obvious because it is beta itself, but more importantly gamma i dagger is equal to minus gamma i, one thing that I had not term mentioned, but it is somewhat obvious the way, we had written is that gamma i runs from

1 to 3 whereas, the Greek indices run from 0 1 2 3 4 dimensions. So, this is another property the hermicity.

So, I will leave it as an exercise for you please check that. So, we will consider other aspects of the Dirac equation in the coming lectures.