Nuclear and Particle Physics
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Module – 09
Electroweak Interactions
Lecture – 06
QED - interaction of charged particles

Come to the interaction of charged particles earlier discussions.

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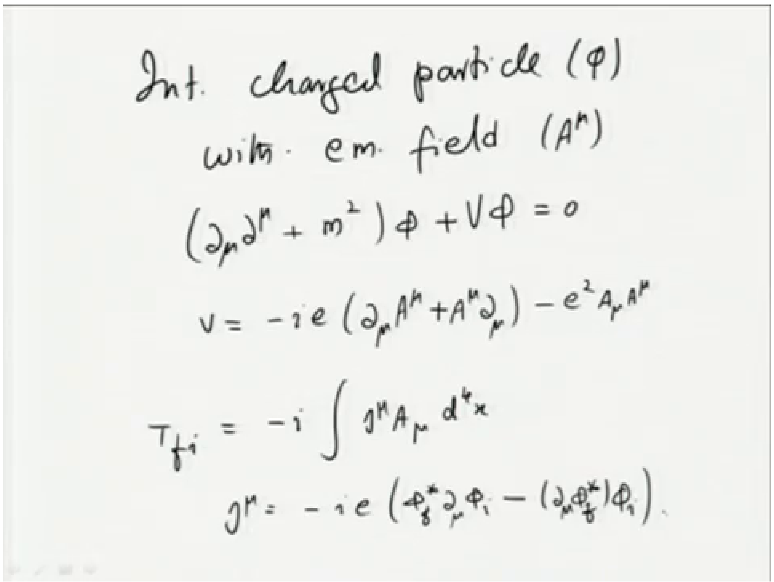

We had considered interaction of interaction of charged particles with some electromagnetic field represented by say A mu charged particle, represented by wave function phi. And we had written that the equation Klein Gordon equation satisfied by this charged particle then is dou mu dou mu plus m square phi plus V phi is equal to 0 where m is the mass of the particle corresponding to the wave function phi, and v is the potential due to the electromagnetic field, which can be written as v minus v equal to minus i e dou mu A mu plus A mu dou mu minus e square A mu A mu.

Then we said the transition amplitude is in minus i integral j mu A mu d 4 x, where j mu is the 4 vector current of due to the particle minus i e phi f star dou mu phi i minus dou mu phi f star phi, where phi i denotes the wave function of the particle before the interaction, phi f denotes the wave function of the particle after the interaction is the electric charge.

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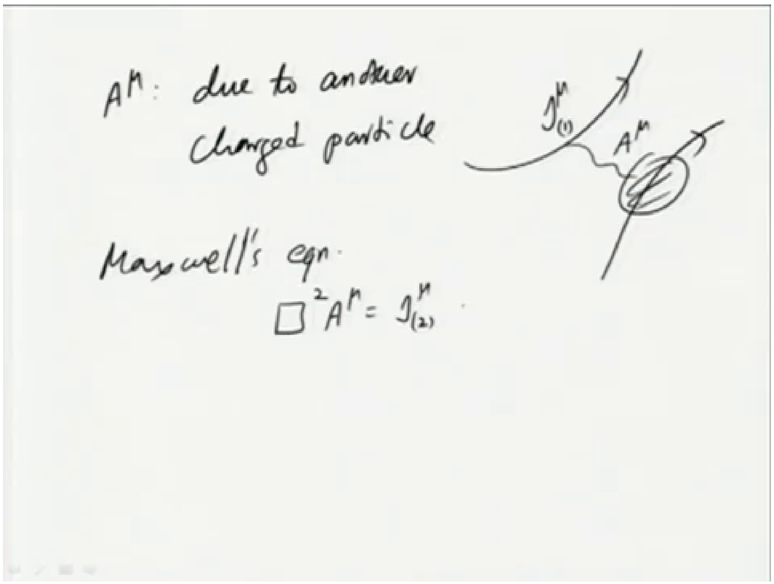

Pictorially this can be written as a current j mu interacting with, and electromagnetic field represented by A mu. In particular this A mu could be due to another charged particle, in that case we can associate A mu with this mu charged particle, and let us say A mu is the electric field due to that charged particle and therefore, Maxwell's equation will tell you that box where A mu equal to j mu, where this j mu is not the earlier current, but this is the second current due to which the first current, charge type or the first particle sees an electromagnetic field m.

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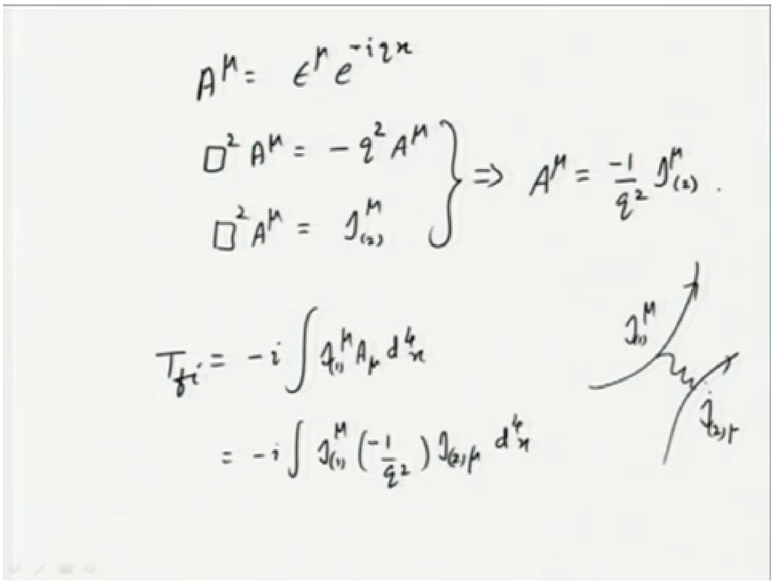

In this case we can if you write A mu as epsilon mu function of q, e power minus i q x, then box square A mu is essentially minus q square A mu, but then now we also have del square A mu equal to j 2 mu together, we then can say that A mu is minus 1 over q square j mu. If you observe carefully, if you are careful with our earlier interpretations of q square in the Maxwell's equation the way we had written, q square equal to we had q square equal to 0 for case which represents a photo.

But now here if we take q square equal to 0 this will not work out, there will not be any current q square equal to 0 works for 0 current, that is if you put j 2 mu equal to 0, then it is 0 by 0 in the limit are you will get some image. We cannot we do not take that. So, what happens to the mass of the photon, remember we are not having a free electromagnetic field. So, it is not a free photon which is propagating. Free photons will satisfy q square equal to 0 or mass is equal to 0.

But in the presence of matter or in when it is interacting with other things, it can have an effective mass either an effective positive mass, or it can even have a negative, it can be interpreted as taking away energy from the system, or giving the energy to the system, and remember also that we are working with quantum mechanics, which allows us to violate even the energy conservation, as long as it happens for a short period of time.

Because delta a delta d is approximately equal to h or it should be larger than or equal to x for any transformation. So, you can actually give some energy to some particular system, in a very short time from a massive particle, and then can remain mass less than the earlier original actual mass of the particle, for a short period of time as long as the uncertainty principle allows you to do that.

So, actually it is going the other way around it says that we will not really be able to measure such deficits or such other things. So, this is one way of when we actually try to interpret it in a in that way. And then the possible confusion is that q square was taken to be 0 earlier when we associated that with A and why is it not 0 here, understood in this fashion that when photon is interacting with matter, or when we are considering the photon associated with a charged particle, then we do not really have cannot take q square equal to 0 for that particle, rather A mu there is then minus 1 over q square j 2 mu, which is the current associated with that particle.

Now, look at T f i it is minus i j 1 mu A mu d 4 x, which is equal to j 1 mu minus 1 over q square j 2 mu d 4 x that is there is a j 1 mu interacting with a another current, j 2 mu interaction is facilitated by transfer of something say the photon..

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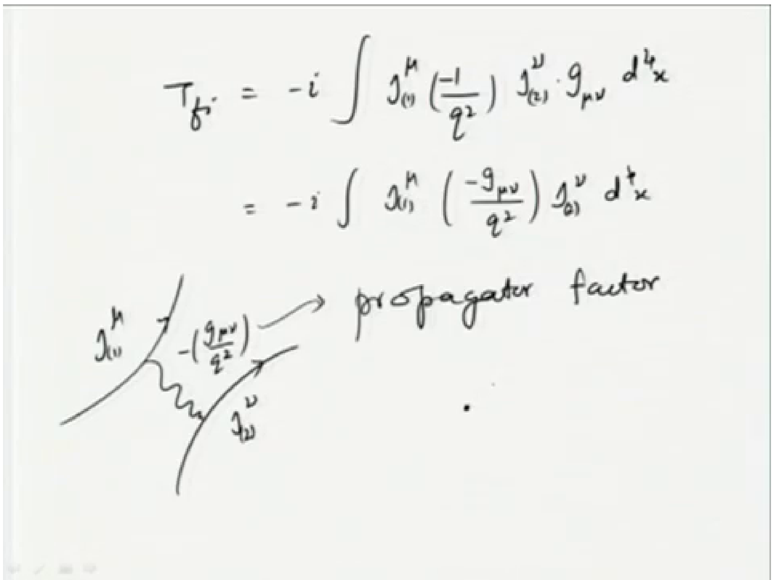

And I can actually write the same T f i, as we already known by raising the index on by raising the index or j 2. So, j 2 nu I want to write, but then I should have a g mu nu. So, that everything is proper integrated over for moment for position coordinates.

So, I will take it as integral j 1 mu, let me take this g mu nu with this q square and write it as g mu nu minus i minus g mu nu over q square j 2 mu. Now look at the picture again. So, we have a current j 1 mu, interacting with another current j 2 nu through exchange of something, which is represented by g mu nu over q square. So, these are 2 currents j 1 and j 2, and the other 1 the factor is basically propagation of the effect of j 2 as far as j 1 is concerned.

When j 1 looks at it, it sees the effect of j 2 propagated to j 1s, and the factor corresponding to that coming in the transformation matrix probability transition probability sorry is minus g mu nu over q square we call this the propagator factor. So, we have in T f i a current interacting with another current, a current j 1 interacting with another current j 2 through a propagator, then you sum over all the possible values of x corresponding to this. And then that will give you the transition amplitude.

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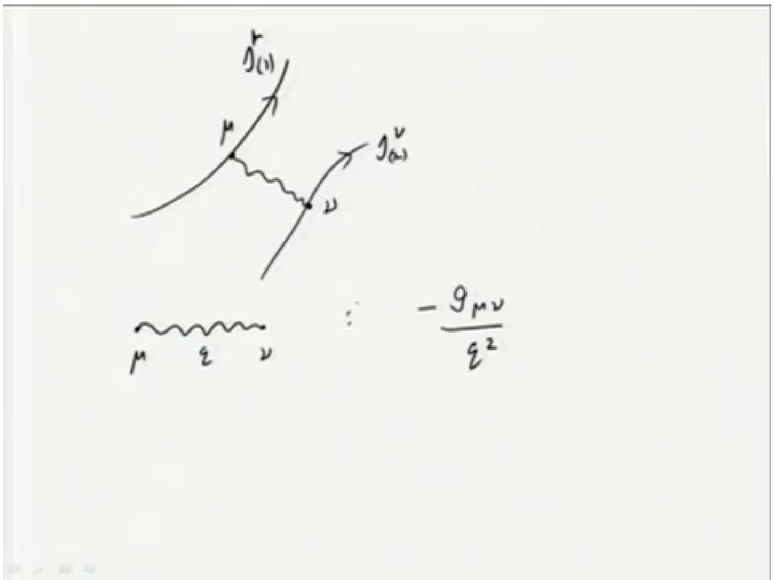

I can actually say that let me do it in a proper way at a particular point. Corresponding to that I have A mu, and another point corresponding to that I have an index nu. And this is j mu, and this is j nu, and so the propagates from some point to another point, and if the indices are nu, and mu the momentum associated with that momentum transfer, which means momentum that the propagator carries is q, then the factor associated with that is g mu nu over q square this is 1 way of diagrammatically representing it.

It use as a feeling that we understand clearly what is happening there, but I would advise that you should take it not as a real picture or realistic case in that sense, this is only an aid to get a feeling get an understanding of what might be going on, but in quantum picture you cannot actually trace the particle perfectly as in a box therefore, a current it is a very hazy picture in quantum case. And then it is not possible to actually think about a particular time a photon is emitted by something, and absorbed by a particle decision.

But what is but that does not say that what we are discussing is a half cooked idea. It incorporates this uncertainty in it. So, when we actually look at the transition probability it says the probability for the transition to occur in one area. In a particular interaction case it may or may not happen one thing, second thing is that the expression itself is typed, and mathematically correct up to whatever order of perturbation that we are considering, it is only that the way we want to see it perceiving, picturize this, in our mind by actually looking at a see if a particle is going in a current, and have wiggled photon emitted by that which is absorbed by it is interacting with another particle current etcetera is not completely in that way.

There are a lot of issues with that and then therefore, you should take it in that spirit. So, I just mentioned this, because one should not go away with a wrong picture of what is happening there as such. So, it is only a pictorial aid to let us write down the expression for example, correct expressions for the transition amplitude in this case. Although we call it by either propagator the current etcetera so, all these things are understood in that sense.

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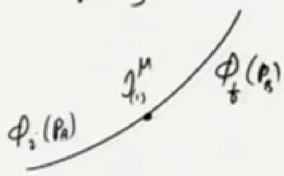
$$j^{\mu}_{(1)} = -ie\left[\phi_f^*(\partial^\mu\phi_i) - (\partial^\mu\phi_f^*)\phi_i\right]$$

Consider

$$\phi_i(x) = N_A\, e^{-iP_A x}$$

$$\phi_f(x) = N_B\, e^{-iP_B x}$$



$$(\partial_\mu\phi_i) = (-iP_A)_\mu N_A\, e^{-iP_A x} = (-iP_A)\,\phi_i$$

$$(\partial_\mu\phi_f^*) = (+iP_B)_\mu N_B\, e^{+iP_B x} = (+iP_B)\,\phi_f^*$$

Let us come to more proper understanding, let us consider j mu 1, I can write it as minus ie phi f star dou mu phi i minus dou mu phi f star phi i. So, this is the current due to a particle let us say this is the interaction point. So, before that is phi i let me associate the momentum to that particle as P A, and at the end after the interaction it has it is represented by phi f, let us say the momentum is P B.

So, the current that we are talking about is j1 mu is this. Now consider phi i equal to some normalization exponential minus i P A x. So, we have considered a plane wave solution, plane wave solution the box normalization or some appropriate normalization has to be considered. We had to constraint the whole thing in a finite volume in that sense. So, N A is the normalization constant P A is the momentum associated with this thing, and phi f similarly is N B e power minus i P B x.

And dou mu phi i is equal to minus i P A mu N A exponential minus i P A x, and dou mu phi f star is equal to plus i P B plus, because it is a phi star that I will consider not phi N B e power minus not minus plus i P B x.

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$$J_{(1)}^{\mu} = -ie \left\{ \phi_f^{*} (-iP_A) \phi_i - (+iP_B) \phi_f^{*} \phi_i \right\}$$

$$= -e \left\{ P_A \phi_f^{*} \phi_i + P_B \phi_f^{*} \phi_i \right\}$$

$$= -e \, (P_A + P_B) \, \phi_f^{*} \phi_i$$

$$= -e \, (P_A + P_B) \cdot N_A N_B \, e^{-iP_A x} \cdot e^{iP_B x}$$

$$= -e \, (P_A + P_B) \, N_A N_B \, e^{-i(P_A - P_B) x}$$

Putting this together J 1 mu is equal to minus ie dou mu phi star phi f star dou mu phi i. So, dou mu phi a is actually equal to minus iP A phi i, and dou mu f phi f star is plus iP B phi f star. So, j mu is equal to phi 0f star minus iP A phi i minus plus iP B phi star phi i, this is equal to minus i minus i it is minus e, then I have P A phi f star phi i, second one I have minus i, and minus i that is what is taken out.

So, it is plus P B phi f star phi i, which is equal to minus e P A plus P B phi f star phi i. Now this is equal to minus e P A plus P B N A N B e power minus i P A x e power plus i P B x, or I can write it as minus e P A plus P B N A N B e power minus i P A minus P B y x. So, that is the current that I have j 2.

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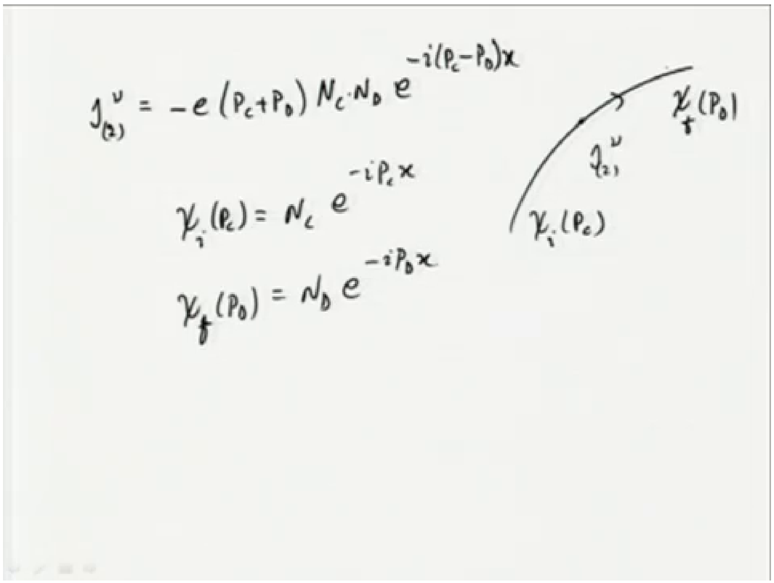

So, this was j 1 and j 2 nu. Let us consider this to be some chi i initially with momentum P C, and chi f with momentum P D. In that case j 2 nu in his exactly similar fashion as the j 1 mu can be written as minus e P C plus P D N C N D e power minus i P C minus P D x, where chi i P C is equal to N C e power i P C x, and chi f P B the final state is N D e power minus i P B x.

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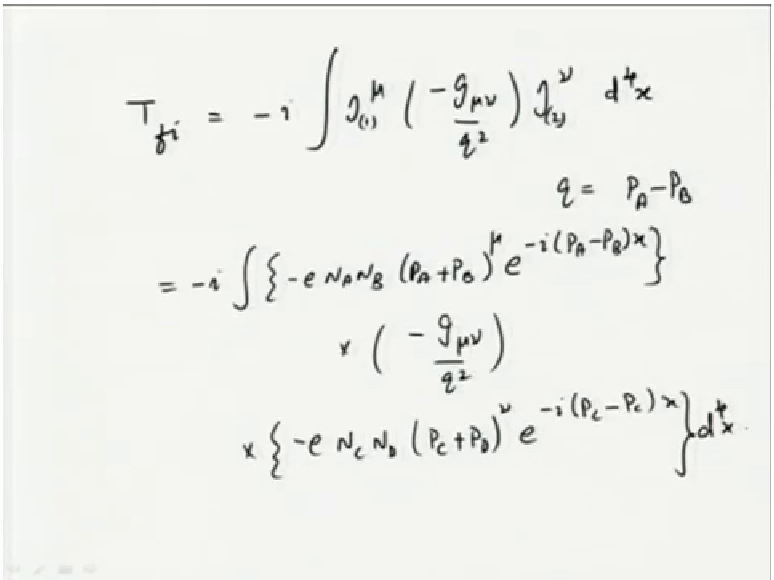

So, we have j 1 and j 2 putting together in transition amplitude T f i, this gives us minus i j 1 mu minus g mu nu over q square j 2 nu d 4 x, this is our transition amplitude, where q is essentially the momentum transfer. So, that is let us say from the point of view of the current 1 particle 1 it is P A minus P B the for momentum terms, and in terms of the wave function explicitly written, we have integral minus e N A N B P A plus P B e power P B mu alright.

So, in all these cases there is mu which is missing. So, I should be similarly for j 2 there is a nu there e power minus i P A minus P B x times minus g mu nu over q square, then the other current minus e N C N D p c plus P D nu e power minus i P C minus P D x integrated over for volume d for x. This is the transition amplitude written now, in terms of the wave function explicitly written as a function of the momentum and the position.

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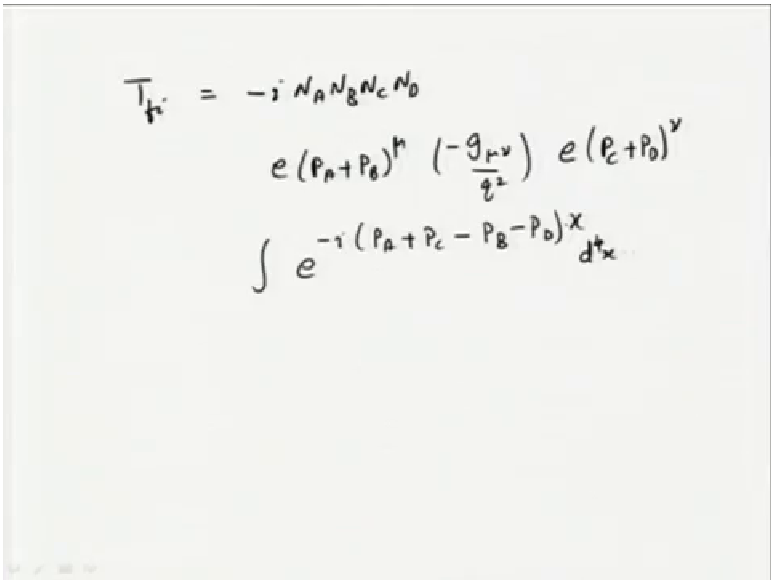

Now, let me do some simplification. Firstly, there is a minus i N A N B N C N D integral e P A plus P B mu minus g mu nu over q square e P C plus P D nu, none of this depend on the position. So, integration comes out of the integral; integral the exponential minus i P A plus P C minus P B minus P D x D for x.

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$$\int_{-\infty}^{\infty} e^{-ipx}\, dx = (2\pi)\,\delta(p)$$

$$\int e^{-i\vec{p}\cdot\vec{x}}\, d^3x = \int e^{-i(p_1 x + p_2 y + p_3 z)}\, dx \cdot dy \cdot dz$$

$$= \int e^{-ip_1 x}\, dx \cdot \int e^{-ip_2 y}\, dy \cdot \int e^{-ip_3 z}\, dz$$

$$= 2\pi\,\delta(p_1)\cdot(2\pi)\,\delta(p_2)\,(2\pi)\,\delta(p_3)$$

$$= (2\pi)^3\,\delta^3(\vec{p})$$

What is integral e power minus i p x d x minus infinity to plus infinity full right, this is nothing but the Dirac delta function. Now if I have more than 1 variable, I can write e power say 3 dimensional case P dot x d 3 x, I will write it as exponential minus i P 1 x plus P 2 y plus P 3 z dx dy dz. So, each of these are independent of each other.

So, I can take it as integral e power minus i P 1 x dx integral e power minus i P 2 y dy integral e power minus i P 3 z dz. This is equal to first one is a delta function 2 pi delta P 1, 2 pi delta P 2, 2 pi delta P 3 together as a compact notation, I can write it as 2 power 3 delta 3 vector P, this meaning of the last line is actually explicit form in the explicit form the line just before that.

In a similar fashion look at what we have in T fi last line integral e power minus i P A plus P C minus P B plus P D that can be taken as some single entity q or some capital q or whatever as a momentum combination of something, and then total together is a for moment times or dotted with x d for x.

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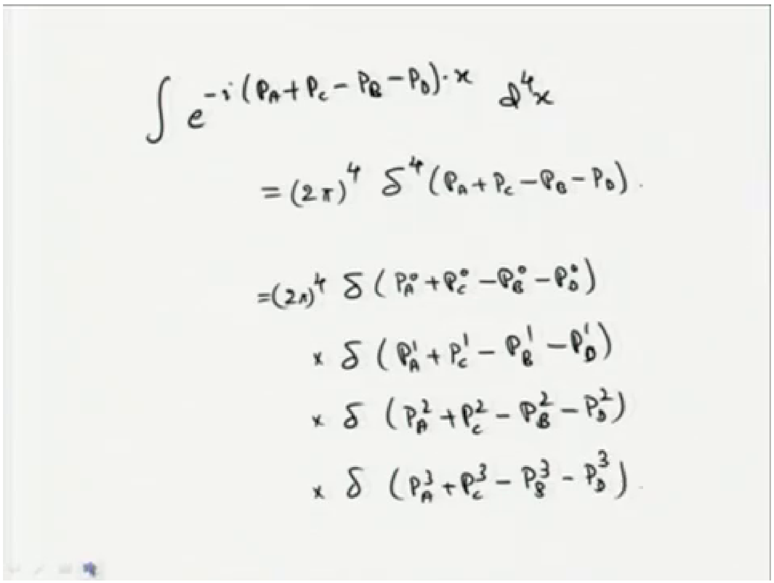

So, that is essentially delta function minus i P A plus P C minus P B minus P D x d 4 x, where now x is the 4 momentum a 4 vector x x 0 x 1 x 2 x 3 or x 0 x y z all of this together.

So, we have 2 pi 4 delta 4 dimension P A plus P C minus P B minus P D. This actually means that 2 pi 4 1 delta function says P A 0 plus P z 0, P B 0, P D 0, and another delta function for P A 1 P C 1 P B 1 minus P D 1 into delta P A 2 plus P A P C 2 minus P B 2 minus P D 2 into delta P A 3 plus P C 3 minus P B 3 minus P D with that, we can write T fi as minus i N A N B N C N D.

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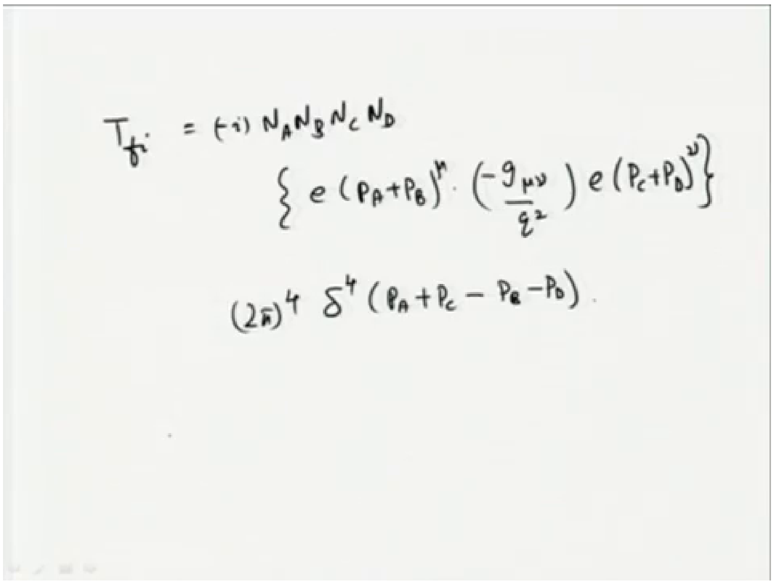

Then you have e P A plus P B mu minus g mu nu over q square e times P C plus P D nu, and you have a 2 pi 4 delta 4 P A plus P C minus P B minus P D.

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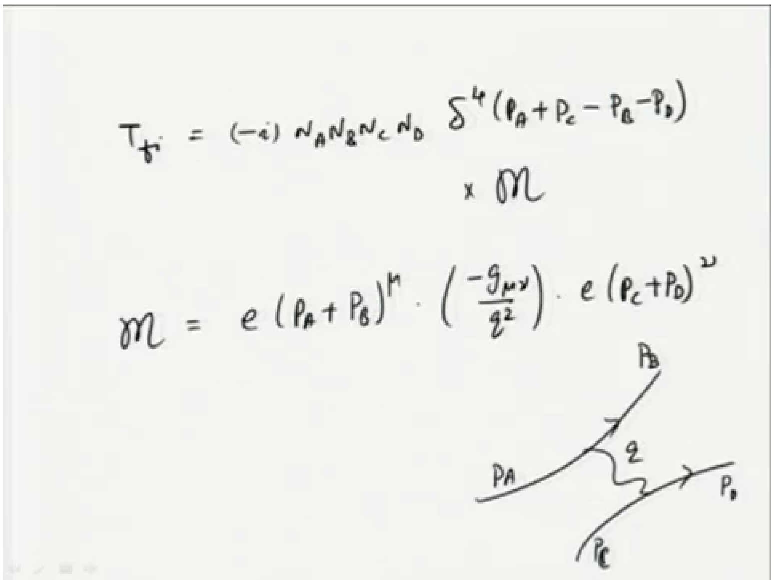

So, we will write it as let me write it here, T f i equal to minus i N A N B N C N D delta 4 P A plus P C minus P B minus P D into m where, m is equal to e P A plus P B mu, minus g mu nu over q square e P C plus P D.

Let us stare at it 4 sometime, what is this. This is the transition amplitude for a particular interaction given by particle, with momentum P A going to a particle, with momentum P

B by after interaction with a particle another particle with momentum P C, and going which will go to I am a particle with momentum P D charges on each of this particle is e.

So, and the momentum transfer is q. This then says that an N A N B N C N D are the normalizations associated with that we will come to that. Look at m a m. It is a Lorenz invariant quantity as we said, this is the dot product between P A plus P B and P C plus P D with a q square which anyway is invariant under Lorenz transformation, and e square which is again invariant under this.

So, the first part e P A plus P B nu is associated with the first current particle current, second part is in the bracket g mu nu over q square is associated with the propagator, and the last part e P C plus P D nu is associated with the third the current due to the second particle, that is 1 part. Essentially that summarizes all the dynamics in it what it is it will give you the charge current interaction. 2 charged particles the currents of 2 particles are interacting with each other.

Now, the rest of it in T fi is 1 is the normalization factor it is a factor of minus i also, apart from that there is a delta function what does the delta function tell you, when you actually put it in the cross section as we will do in a moment, in the next class as such. So, when we put it in the cross section and try to find the number of particles which are coming out etcetera, then we will have to integrate it over the momentum, when we do that immediately we realize, because of the wave function and the delta function that when you integrate over all the momentum P A plus P C is equal to P B plus P D that is because of the delta function.

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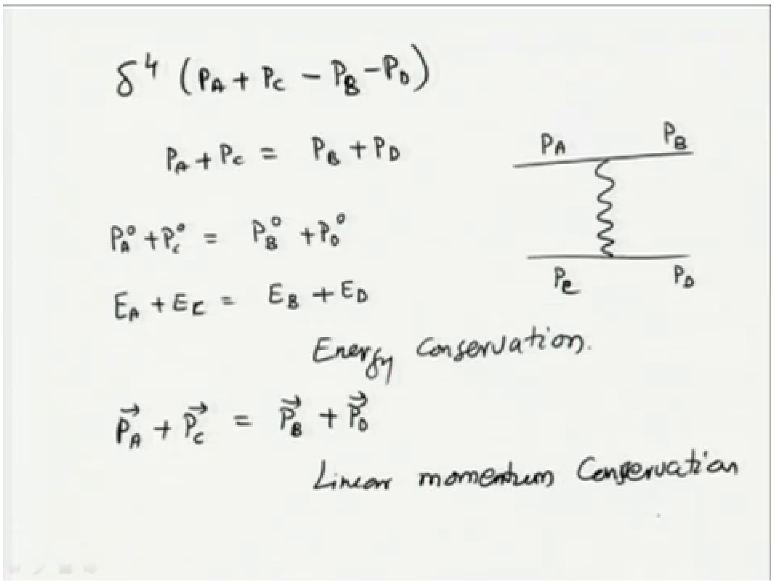

And what does that tell you. So, the delta 4 P A plus P C minus P B minus P D tells us that P A plus P C is equal to P B plus P D, what is P A it is essentially the initial momentum of the 1 particle, and P C is the initial momentum of the other particle. So, if I write it in components these are 4 momentum, I will get P A 0 plus P C 0 is equal to P B 0 plus P D 0 or essentially, when you consider the 4 momentum, this is the energy of the particle E A, plus energy of the particle C is equal to energy of the particle B plus energy of the particle D.

When I say particle D, I mean the corresponding particle in the final set the second particle in the final state. What is this this is basically that the sum of the initial energy is equal to sum of the final energy; this is the statement of energy conservation. Similarly we will when we look at the 3 momentum P A plus P C is equal to P B plus P D, the sum of the initial 3 momentum equal to sum of the final 3 momentum.

So, this is the linear momentum conservation. So, the delta function make sure that the energy of the initial particle, sum of the initial energies is equal to sum of the initial final energies, energy is conserved and similarly Lorenz the linear momentum is conserved. And that only talks about the kinematics that the energy momentum conservation that part.

The rest of it all the dynamics is in the quantity m, and m is the is called a Lorenz invariant transition amplitude. And we should see that note that when we consider these charged particle initially, we have not considered anything any information about this particle other than its momentum and charge, we have in fact represented the particle by a wave function phi i equal to N A e power minus i P A x it has a momentum ok.

It has some charge associated with that that is understood, but supposing we consider it as an electron, then what about the spin of this particle, and underlying equation that we considered was Maxwell's equation. And even before that the Klein Gordon equation and we will see that here with the Klein Gordon equation satisfied by the particle phi, we do not have any provision to discuss the spin of the particle.

So, if we want to incorporate the spin of the particle or the information related to the spin of the particle, then we will have to modify this, and find out slightly different way of dealing with these particles that is 1 thing that we like to understand and then we will come to. The other thing that we are not discussed is what are these normalization constant, and then how does it I mean what at least what are the ways to look at it how does it come in the picture, in the transition amplitude they apparently appear, but then it should be independent of this normalization as such the transition amplitude how does that come about.

Actually we will see that we can relate; this transitional amplitude to the cross section. And the cross section which actually tells you directly which can be directly related to the probability for finding a particle scattered into a particular region etcetera, will be independent of all these normalizations, and we will see how that comes out in the next class.