Nuclear and Particle Physics
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Module – 09
Electroweak Interactions
Lecture – 05
QED – continued

So, we will continue our discussion on the interaction of charged particles with electromagnetic field in a quantum picture.

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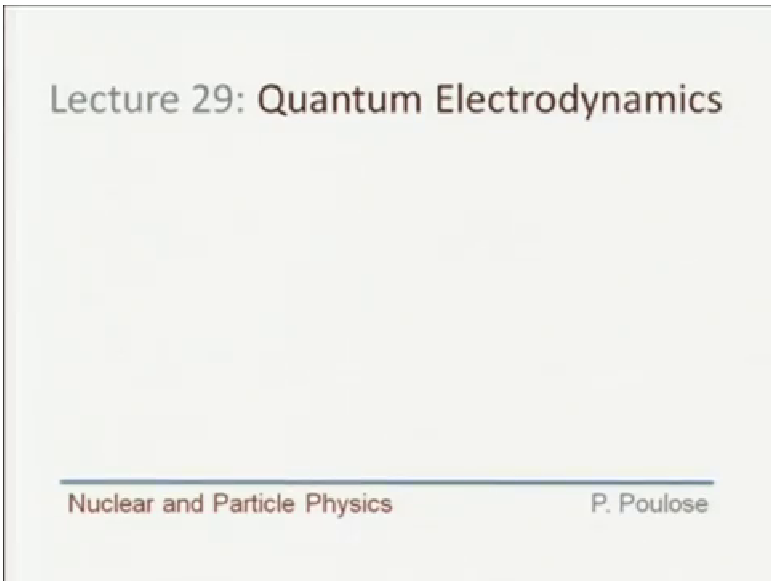

And in relativistic case, essentially this is what we call a quantum electrodynamics, but we will come to the formal quantum electrodynamics as a field. The quantum field theory at a little stage, but this is basically to build or give way to such a quantum field theory how to think about that.

So, we will consider the discussion that we have been doing in the past couple of lectures forward.

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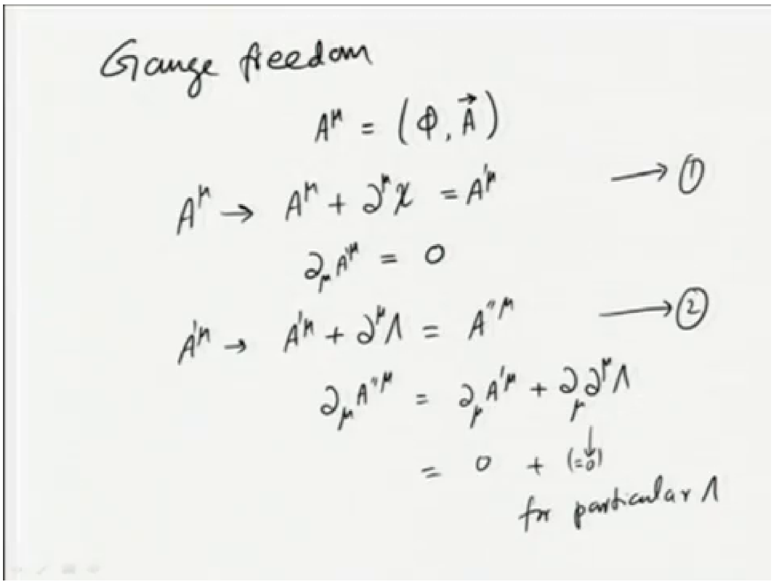

So, we were discussing the gauge freedom associated with the 4 vector potential picture of the electromagnetic field. So, we said we could express the electromagnetic potential A mu 4 express, the electromagnetic field in terms of the scalar potential and the vector 3 vector potential putting them together as components of A 4 vector, we can consider A mu the 4 vector as the 4 vector potential representing the electromagnetic field.

And we said, if we consider the measurable quantities electric and magnetic field E and B associated with this A mu, then we will see that the A is not uniquely defined or fixed we can actually take A mu to A mu plus derivative of some scalar function chi which will still give you the same electric and magnetic field and this freedom to choose any mu related by this relation is basically called gauge freedom and that is a kind of an arbitrary defining the potential.

But it is taken as an advantage in quantum electrodynamics we will come to that a little later when we discuss the gauge symmetry, but at the moment this lets us fix the A mu. So, that dou mu A mu is equal to 0 that is A mu can be made divergence less 4 divergence a is equal to 0 by appropriately choosing the scalar function chi.

Now, this is what we saw yesterday and even after fixing this there still we can actually transform A mu to A mu plus dou mu lambda the secondary A mu is the now A mu now mu that satisfies the. So, let me call this A mu prime to avoid any confusion. So, A mu prime is the new field and A mu prime can be transferred now to A mu prime plus some dou mu lambda and then of course, this should also give you the same electric and magnetic field because it is similar to the earlier transformation.

But we have to check whether the condition the divergence condition of a prime mu is also conserved if you take the divergence of the new field a double prime you then we will have divergence of a prime mu plus dou mu of lambda. So, this first term is equal to 0 and second term can be made 0, if we choose a lambda appropriately. So, this is equal to 0 for particular lambda.

So, first one transformation fixes the diver a fixes A mu. So, that it is divergence less and then we see that still there is a freedom to choose a the potential and we see that there is always possibly this always possible to choose a double prime by adding a derivative term of a scalar function again to the yet prime as in the case of equation 2 this will not change anything, it will not change this electric and magnetic field the physical quantities neither will it change the condition that the a prime is divergence less or it therefore, a double prime is also divergence less.

If lambda satisfies the condition that dou mu dou mu lambda is equal to 0. So, the second transformation is not for any scalar function unlike the first transformation where chi could have been any scalar function lambda is not any scalar function lambda is the scalar function which satisfies the condition dou mu dou mu is equal to 0. So, that is basically the thing we will return back to this when we discussed explicit expression for A mu later.

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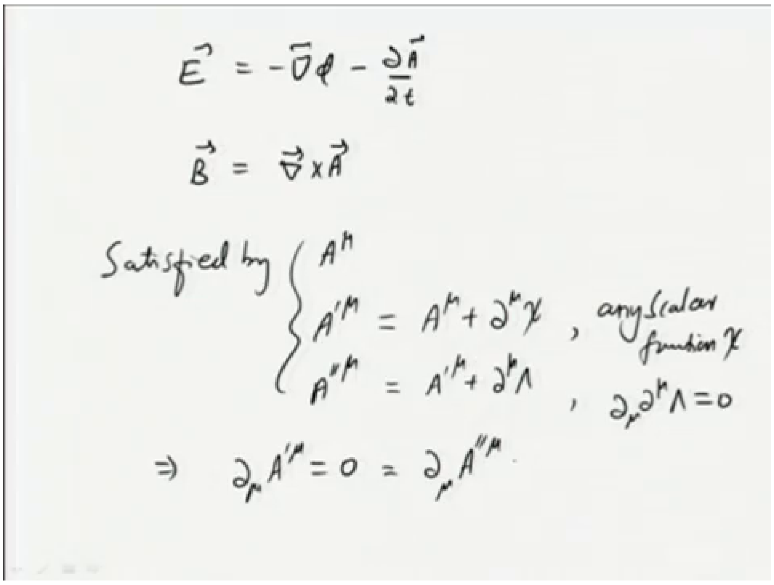

Now, this actually says E is equal to minus del phi minus dou A over dou t and B equal to curl of A as satisfied by A mu or a prime mu which is essentially equal to mu plus dou mu chi any scalar function chi and A double prime mu which is a prime mu plus dou mu lambda where lambda is a scalar function. So, that dou mu dou lambda is equal to 0 and this gives dou mu A prime mu is equal to 0 and also dou mu a double prime mu equal to 0 divergence.

So, we have the same physical situation for any of these fields and then in air double A, we will see that A double prime mu, we have fixed the arbitrariness. So, that unique in the sense that I mean that fix that is an in the sense that you cannot add any more terms in the other thing because lambda is already lamb the choices of chi and lambda will fix this thing for the condition for A mu prime divergence less mu a prime is satisfied here.

Let us look at the Maxwell's equations with all these conditions applied to the potential.

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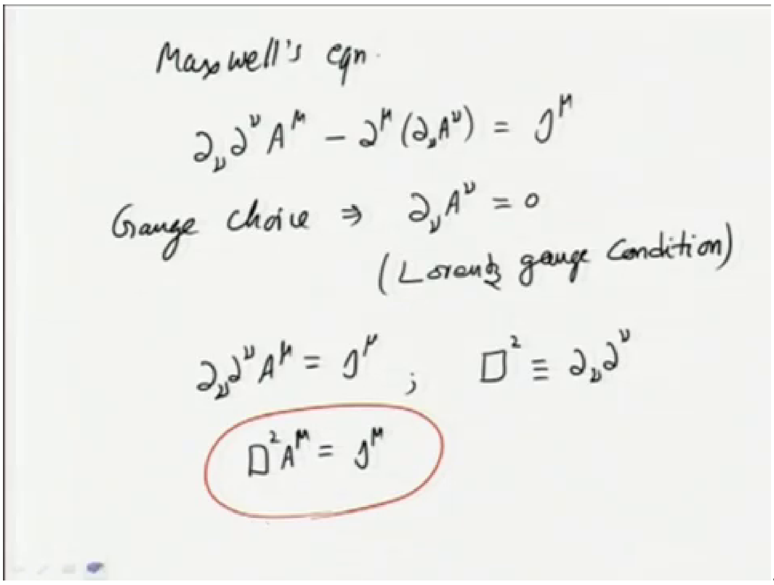

Maxwell's equation written in terms of the potential was dou nu dou mu A mu minus dou mu dou nu A nu for divergence equal to j mu. So, if we have a condition that gauge choice is such that A nu dou nu; A nu is equal to 0 as we discussed in earlier case this condition is called Lorenz gauge condition.

In that case the second term from the Maxwell's equation drops out and then we have dou nu dou nu A mu equal to j nu j mu or in a slightly more compact way we can write it as box square which is equivalent to dou nu dou nu or dou mu dou mu where nu is summed over and in that case, I can write the box square A mu equal to j mu slightly in need a way of writing it meaning the same thing in any case.

Now, let us come to solutions of this equation this is the Maxwell's equation with Lorenz gauge condition applied maybe I should highlight this. So, this is our Maxwell's equation with Lorenz condition. So, let us look at it. So, we have del square A.

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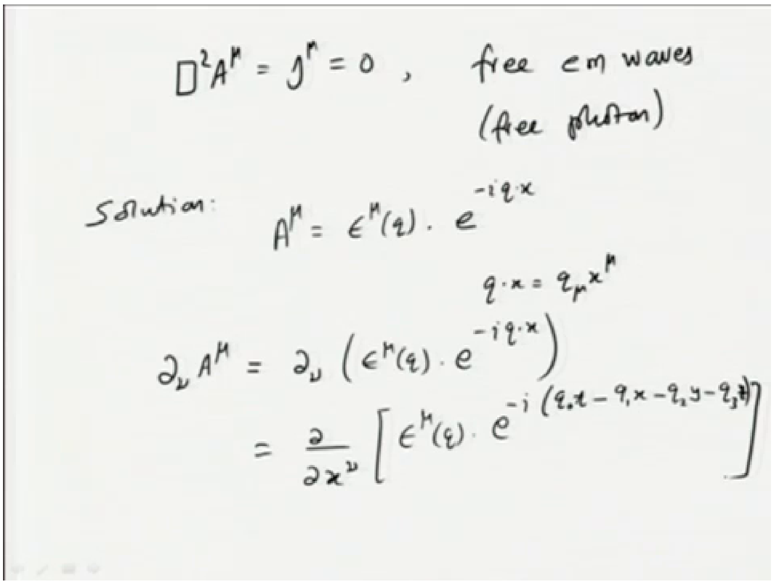

Mu equal to j mu equal to 0 for free electromagnetic waves or I would say in the quantum picture free photons, we are not quantized it yet, but let us say that we can do that and then think about electromagnetic field as collection of photons in that case A mu represent the photon. Now we are actually deviating a little away from the earlier interpretation or earlier picture of giving more physical meaning to the electric field and the magnetic field and then considering the 4 potential as an auxiliary and function that was the earlier picture and then we said that we can actually how different the potential is even which will give you the same physical situation.

Now, in the quantum electrodynamics we will deviate from this picture and then say that photon which is the quantum of the electromagnetic field is more meaningful physically and the field associated with that is the potential where is equal to the potential A mu and therefore, A mu is the one which represents the photo associated to that any photon we can actually think about the electric field and the magnetic field.

When you come down to a fine, see the picture of the photon in the non relativistic case we will see the electric field and magnetic field associated with photons with certain relations between them, but A mu therefore, takes a more physical meaning in quantum electrodynamics and arbitrariness can be fixed by choosing the gauge conditions and then it is somewhat uniquely fixes the potential corresponding to the or the yeah the potential corresponding to the photon or the wave function corresponding to the photon or the field corresponding to the function.

At this moment I will be a little vague in defining what is wave function what is the field what is the potential etcetera basically I will interchange I called A mu sometimes as feel sometimes as the wave function corresponding to the photons sometimes as the potential. So, all these are the same thing, but looking at from different pictures accordingly the interpretation is slightly different, but basically it is the same entity. So, for the time being we will not in distinguish it clearly.

As we come to the clear representation of the quantum field corresponding to the photon we will assay as an operator that is the correct picture in quantum field theory or quantum electrodynamics we will talk a little more about what is the quantum field a mu, but essentially that is the same as the potential that we are considering in a relativistic electron dynamics. So, this does the box square is essentially called DLM version. So, DLM version A mu acting on DLM version is an operator which acts on A mu here to give you 0 for 3 photons.

Solution well this is basically an easy guess if I take A mu to be some constant function constant for vector as far as x is concerned DLM is an operator which acts on the coordinates in order on the momentum. So, I can take this for some constant function in a constant in x function which is constant in x, but could depend on momentum A for vector and explicit coordinate dependence in an oscillatory exponential minus i q dot x where q dot x is for vector dot product q mu x mu where mu is summed over ok.

Let me convince you that it is actually indeed a solution or what are the conditions that anything should satisfy for this to be a solution. So, let me act on it by nu mu dou nu A mu is equal to dou nu acting on epsilon mu q E power minus iq dot x which is equal to dou by dou x nu acting on the same thing epsilon mu q E power minus i q 0 p minus q 1 x minus q 2 y minus q 3 z ok.

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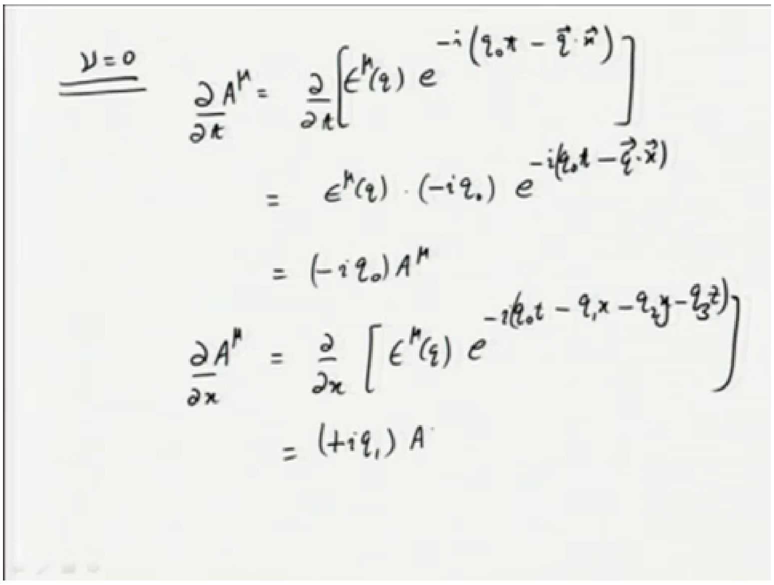

So, now let us take case by case nu equal to 0 which is dou by dou t of A mu which is equal to dou by dou t of epsilon mu q E power minus i q 0 t rest of it is 3 momentum dot product with 3 coordinate x mu x vector. So, there is only one particular place where t appears on the right hand side it is exponential minus i q 0 t. So, when I take the derivative epsilon mu q is independent of t comes out E power minus i q 0 t is minus when the derivative acts on its minus i q 0.

E power minus i q 0 t and other things remain the same minus q dot x. So, this is essentially equal to minus i q 0 A mu all right. So, now, now similarly dou by dou x A mu is equal to dou by dou x epsilon mu q exponential minus i q 0 t minus q 1 x q 2 x y minus q 3 z. So, that will give you minus of minus plus i q 1 A. So, that is one thing.

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$$\partial_\nu \partial^\nu A^\mu = \frac{\partial^2}{\partial t^2} A^\mu - \frac{\partial^2}{\partial x^2} A^\mu - \frac{\partial^2}{\partial y^2} A^\mu - \frac{\partial^2}{\partial z^2} A^\mu$$

$$\frac{\partial}{\partial t} A^\mu = (-i q_0) A^\mu$$

$$\frac{\partial^2 A^\mu}{\partial t^2} = (-i q_0) \frac{\partial A^\mu}{\partial t} = (-i q_0)(-i q_0) A^\mu = -q_0^2 A^\mu$$

$$\frac{\partial^2}{\partial x^2} A^\mu = -q_1^2 A^\mu, \quad \frac{\partial^2}{\partial y^2} A^\mu = -q_2^2 A^\mu$$

$$\frac{\partial^2}{\partial z^2} A^\mu = -q_3^2 A^\mu$$

Now, what we want to have A dou nu dou nu acting on mu which is equal to dou square by dou t square acting on mu minus dou square by dou x square acting on A mu minus dou square by dou y square acting on A mu minus dou square over dou z square acting on A mu say mu and we had dou by dou t or A mu is equal to minus iq 0 A mu.

So, dou square A mu by dou t square is equal to minus iq 0 independent of t and dou by dou t acting on A mu which is another q 0 minus i q 0 A mu which is equal to minus q 0 square A mu and dou square by dou x square A mu will give you minus q 1 square A mu and dou square by dou y square A mu is minus q 2 square A mu and dou square over dou z square A mu is equal to minus q 3 square A mu putting it together dou nu.

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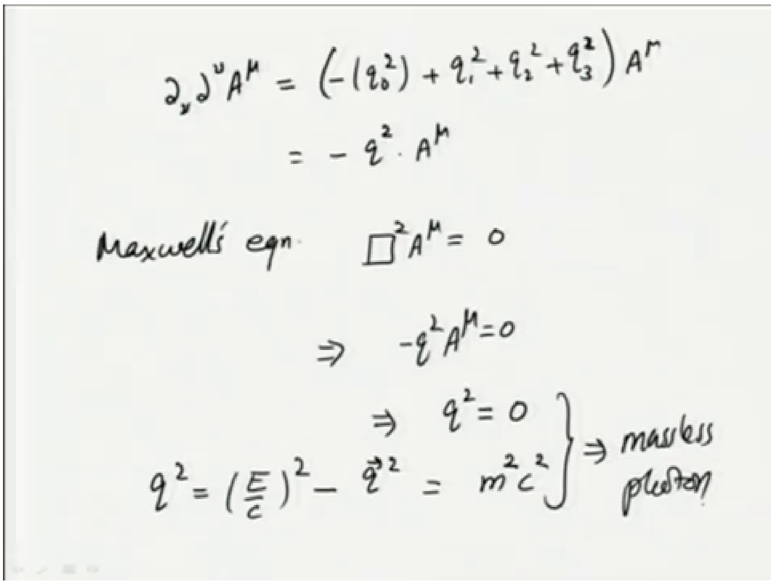

Dou nu A mu is equal to minus q 0 square plus q 1 square plus q 2 square plus q 3 square which is equal to minus 4 vector square q square A mu.

And we have Maxwell's equation with Lorenz gauge condition del square I mean the box square A mu equal to 0 and that gives us minus q square A mu equal to 0, we essentially q square need to be equal to 0 and what is q square q square is equal to q 0 square which is essentially E square minus p square or 3 vector minus 3 vector square which is equal to m square c square and when we say q square equal to 0 m square is equal to 0 q square for vector square is equal to 0.

Now, as we said earlier when we consider the A mu as representing the photon and we associated f for momentum q with A mu like in this case A mu is equal to epsilon mu q exponential minus i q dot x is what we considered, right. So, q is essentially then interpreted as the momentum for momentum of the photon and Maxwell's equation gave us for momentum square is equal to 0 for the photon of course, you can ask the question like this was in the case of A mu satisfying the Lorenz condition what about if I consider A mu which is not divergence less then still can I interpret this as the photon with mass equal to 0.

So, this essentially gives you mass less photon I mean when I say mass less photon in the sense it says that photon has no mass, but then you remember this gauge condition is not an additional condition that we can apply we could have applied the gauge condition and chosen any condition any kind of gauge any conditions that satisfies the gauge transformation.

So, that electric field and magnetic field remains the same from that point of view the physics should not depend on whether I have taken Lorenz gauge or not Lorenz gauge condition or not which means the A mu that satisfies the Maxwell's equation, the way we had written with the gauge condition or some air prime which is not divergence less and do not satisfy the Lorenz gauge condition.

But is derived were related to this A mu through a gauge condition gauge transformation by adding A or differ by A dou mu chi kind of a term then chi is a scalar fit in that case they are physically the same equivalent to each other and therefore, all of those when we consider as photons if in one case, it is mass less, then in all other cases also it should be mass less.

So, this represents the same this thing and then photon in any case whether there is a Lorenz condition applied or not it is mass less only that when you take it in this pattern it is manifestly clear its manifested, there it is very apparently there clearly there. Now let me look at it a little more and then see; what is the interpretation that we can give to epsilon.

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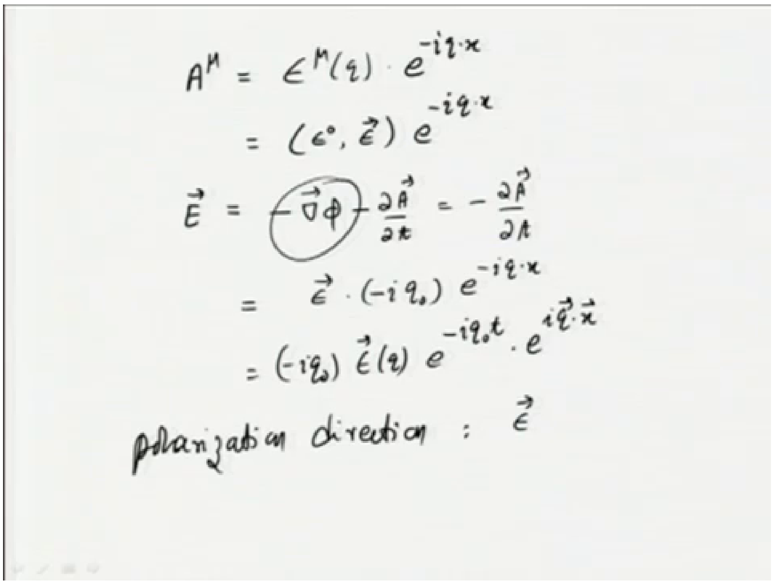

So, we had we had the solution if A mu equal to epsilon mu q E power minus E power minus i q dot x for the photon field satisfying the Maxwell's equation.

Let me write this epsilon mu as epsilon 0 and 3 vector epsilon E power minus i q x. So, I have not explicitly written they q dependence on the epsilon now it is understood that it depends on q, it is just for the sake of clarity that we are not adding the bracket with q they array what is E electric field is equal to minus grad phi minus dou A by dou t.

supposing we do not have any electrostatic potential phi and electric field is only due to the changing magnetic field like in the case of electromagnetic waves. So, for a free photon we can imagine that there is nothing else, but the changing electric field and changing magnetic field one inducing the other. So, in that case this is equal to 0. So, you have minus dou A by dou t as the electric field and that is essentially epsilon exponential dou by dou t acting on this thing will give you minus i q 0 E power minus I qx or minus i q naught epsilon which depends on q exponential minus i q naught t exponential i q dot. So, this is basically the electric fields with explicit dependence on the time and x narrator.

We are familiar with this and the amplitude is given by a complex amplitude here minus i q naught epsilon 3 vector direction of the electric field is the same as the direction of the epsilon with 3 vector and therefore, we can say that polarization direction is of the photon is that of the epsilon.

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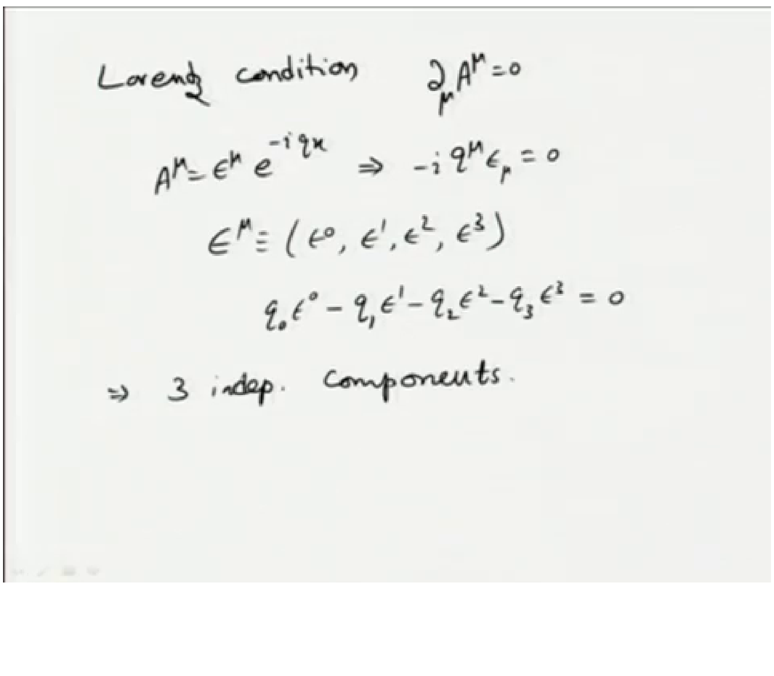

Now, let us see what happens to the polarization vector in the case of Lorenz condition, let us look at Lorenz condition which is dou mu A mu equal to 0 A mu equal to epsilon mu E power minus i q x will immediately give you q mu or minus i q mu epsilon mu is equal to 0 when taking when the when dou mu a is acted on A mu. So, this says that Lorenz condition says not all the components epsilon 0 1 2 3 are independent of each other.

But depends through the relation q mu epsilon mu equal to 0 or q naught epsilon not minus q 1 minus q 1 epsilon one minus q 2 epsilon 2 minus q 3 epsilon 3 equal to 0. So, for a fixed q epsilon all the 4 components of epsilon are not independent of each other. So, there is a relation that they had to satisfy therefore, there are only 3 independent that gives 3 independent components.

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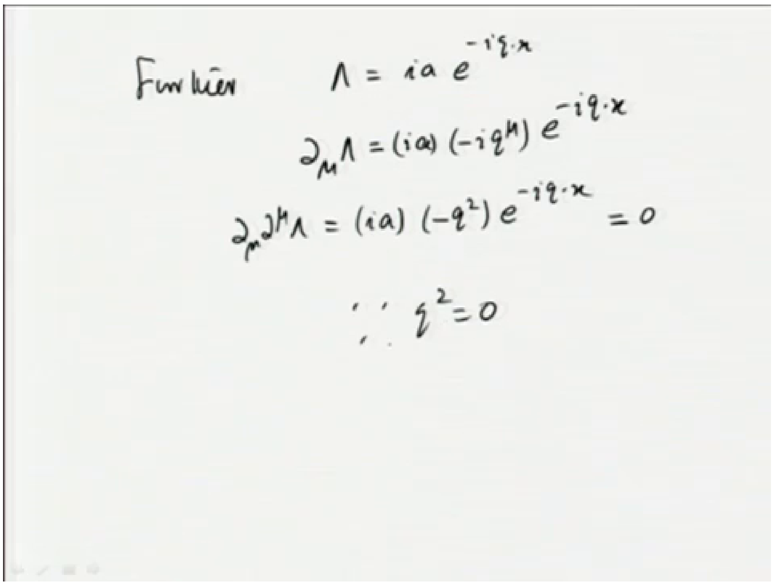

Further we can choose lambda let us choose lambda equal to i a q sorry i a exponential minus i q dot x what is dou mu lambda dou mu lambda is equal to i a minus i q mu E power minus i q dot x and dou mu dou mu lambda is equal to i a minus q square E power minus iq dot x, but q square is equal to 0 says that this is equal to 0.

So, an as a scalar function satisfying dou mu dou mu lambda equal to 0 is i a exponential minus i q dot x well this i a is a fixed is kept like that because of reason that will be clear just now. So, this is 0 because q square is equal to c. So, now, what does that tell you in terms of the change?

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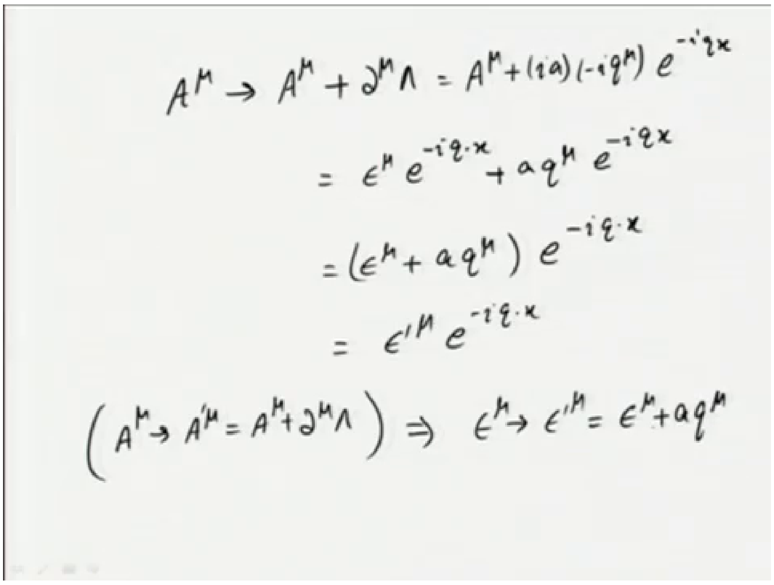

In epsilon corresponding to a change a transformation of A mu to mu plus dou mu lambda this is equal to A mu plus a q. So, we had i a minus i q mu exponential minus i q x.

So, if I write A mu as epsilon mu E power minus i q dot x then minus plus i a times minus i q mu is plus a q mu exponential minus iq x which is epsilon mu plus a q mu E power minus i q x if I denote this by some epsilon prime E power minus iq x as A mu goes to a prime mu equal to A mu plus dou mu lambda with this particular lambda this gives epsilon mu going to epsilon prime mu which is equal to epsilon mu plus A q mu right, fine.

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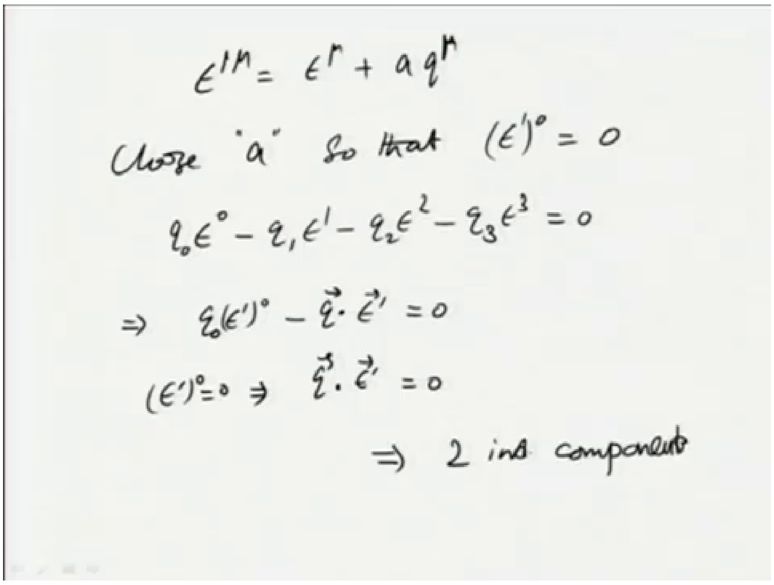

So, I have epsilon prime mu equal to epsilon mu plus a q mu now I can choose a. So, that epsilon prime mu the 0 is equal to 0 all right. So, this is why we had taken the lambda in that particular way. So, I can choose this. So, initially we had a condition on epsilon that epsilon 0 q 0 epsilon q 1 epsilon 1 q 2 epsilon 2 q 3 epsilon 3 equal to 0 now that epsilon prime. So, this is applicable to epsilon primers.

Because its the same this thing satisfying same condition Lorenz condition this is basically the Lorenz condition and changing it by lambda which has dou mu dou mu is equal to dou mu dou mu lambda equal to 0 will not change the Lorenz condition. So, this actually tells you that q 0 epsilon prime 0 minus.

Let me write it as q dot epsilon prime is equal to 0 since epsilon prime 0 is set to 0 by appropriately choosing appropriately choosing a we have q dot epsilon prime 3 vector is equal to 0 epsilon 3 prime has 3 components, but then there is a condition q dot epsilon 3 epsilon 3 vector prime is equal to 0. So, essentially there is a relation between these 3 components and therefore, that gives you 2 independent components.

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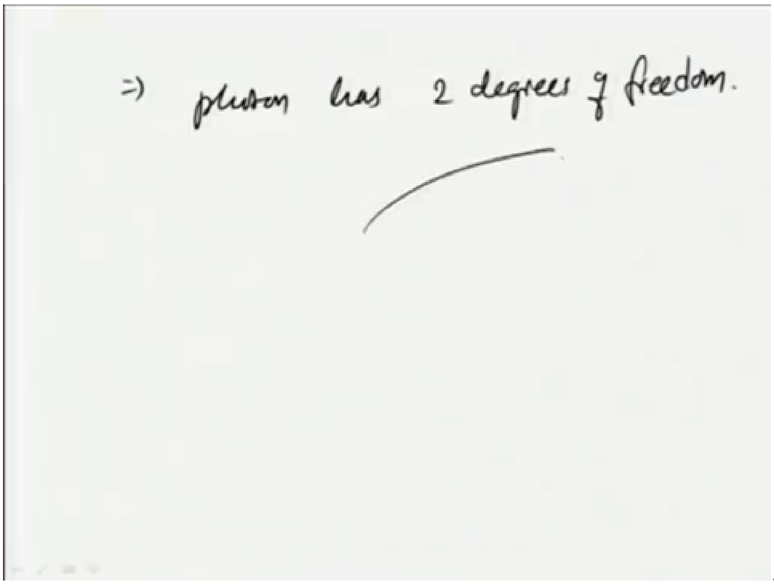

Or we can say that photon has 2 degrees of freedom. So, let us summarize what we have been doing we had the Maxwell's equation

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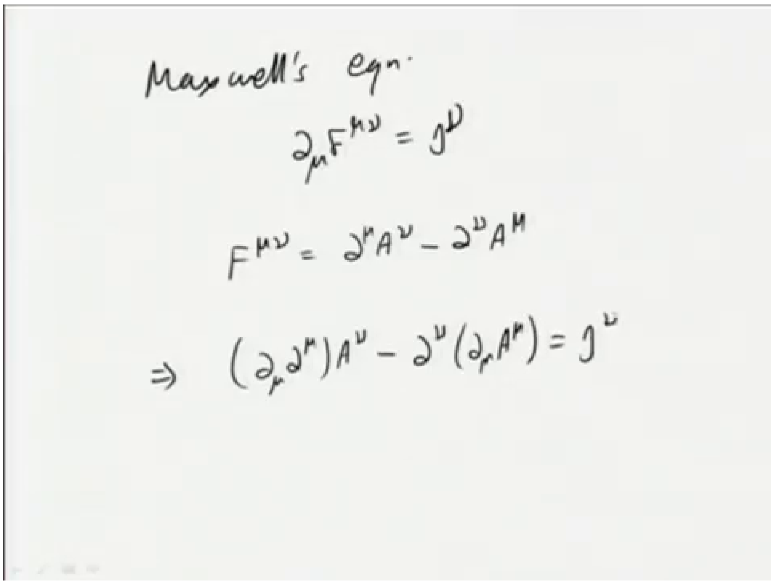

In terms of field tensor f mu is equal to j nu dou mu f mu nu equal to j nu and we could write f mu nu in terms of vector potential as dou mu A nu minus dou nu A mu that gave the Maxwell's equation as dou mu dou mu a nu minus dou nu dou mu A mu equal to j nu.

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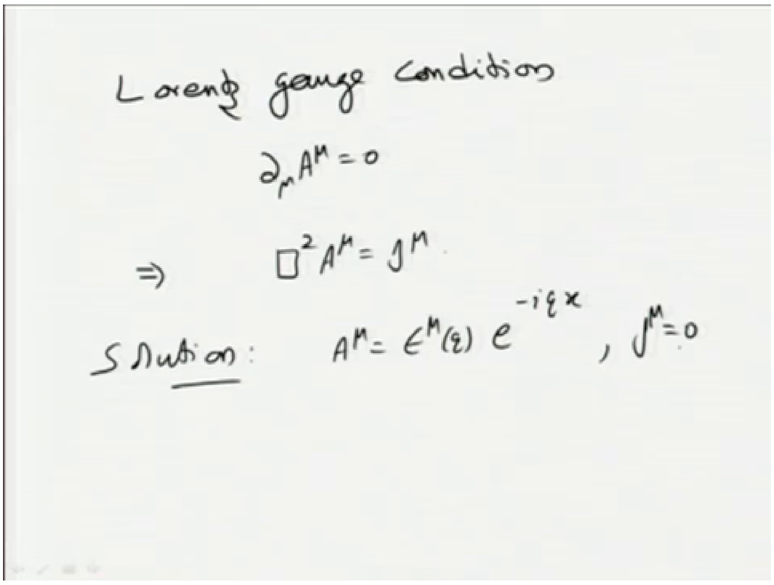

Then we had the Lorenz gauge condition dou mu A mu equal to 0; this Lorenz gauge condition will not affect physics because we can find out any mu which satisfies the gauge condition with the same electric and magnetic field associated with it as the original case and that gives us Maxwell's equation del square sorry the box square A mu equal to j mu, then we considered the solutions A mu equal to epsilon mu q exponential minus i q x for j equal to 0 case which is then interpreted as the as the representing the a free photon without any charge density or current density in the region isolated electromagnetic wave freely propagating in the quantum vector equivalent to a photon represented by A mu.

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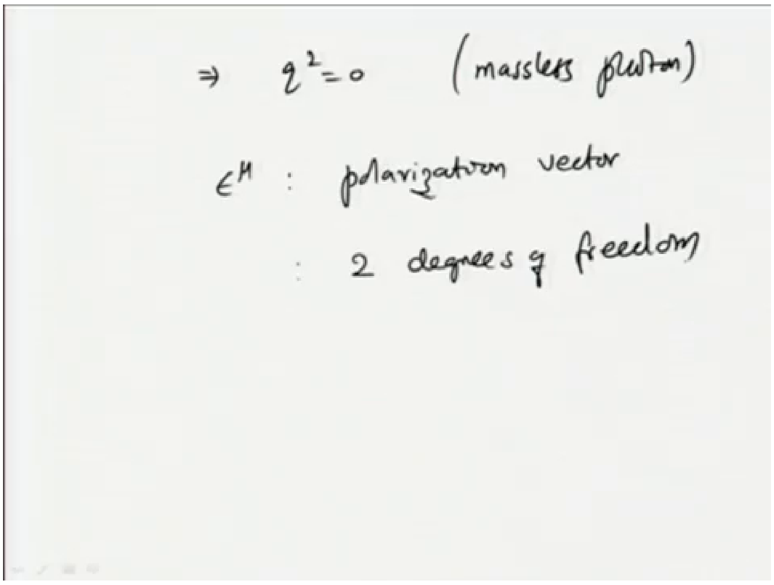

And that told us that we have q square equal to 0 mass less photon and epsilon mu is essentially the polarization vector in the 4 dimensional case and especially in the 3 dimensional case it is the familiar polarization associated with electromagnetic field and we saw that it has 2 degrees of freedom or a 2 independent components maybe I should write it in that fashion.

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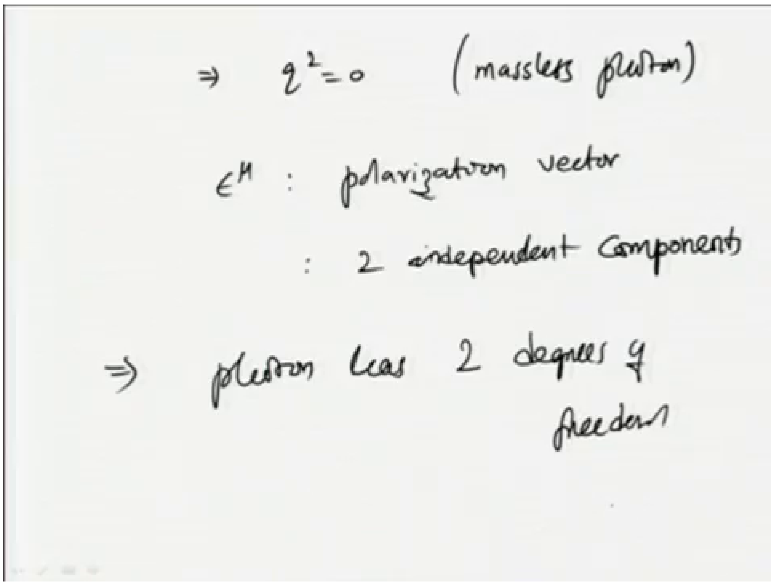

So, this with the Lorenz condition this has 2 independent components that in turn tells us that photon has 2 degrees of freedom we will come to the discussion of the interaction of charged particle with such photons for dolphins in the coming discussion.