**Nuclear and Particle Physics**
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**Module – 08**
**Hadron Structure**
**Lecture - 03**
**Deep Inelastic Scattering**

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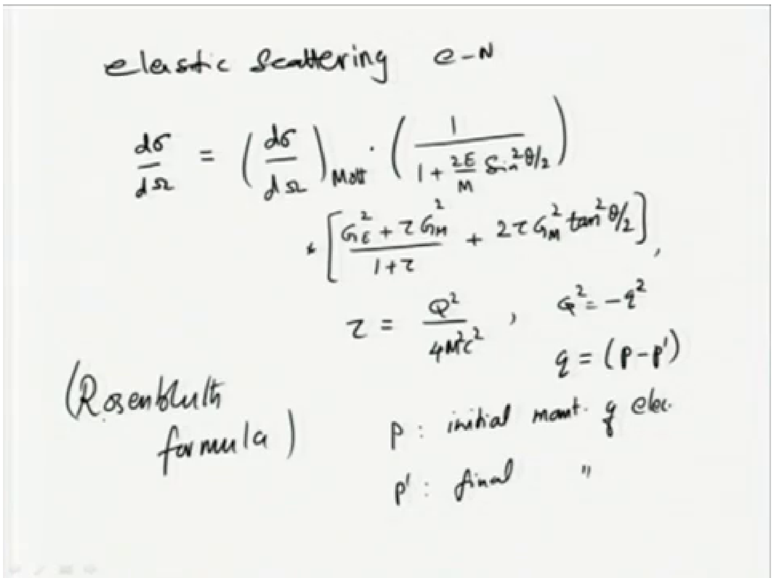


In case of extended objects, and we considered in fact the elastic scattering of the electrons on hadrons like proton, and then we said in the case of elastic scattering e-N scattering electron nucleon scattering in particular we can consider electron proton scattering. The differential cross section can be written as the differential cross section corresponding to the point particle scattering without the nuclear spin effect times and the recoil effect of the electron has of the proton has to be added to this 2 E over M sin square theta by 2 in the denominator times the charge and charge distribution and the electromagnetic interaction part, so electrodes the magnetic interaction part which is for a point particle would correspond to the magnetic dipole interaction.

Here, in the case of particles which are not point particles, we will have to consider form factors. So, there are two form factors, one is the electric form factor, and magnetic form factor. And they appear in the expression for the differential cross section in this fashion plus 2 tau G m square tan square theta by 2, where tau is equal to Q square over 4 M square C square where Q square is equal to minus Q square and Q itself is basically the realized the for momentum transfer. So, p corresponds to the initial momentum energy and momentum of electron p, and p prime is the final momentum of the electron, this we discussed yesterday.

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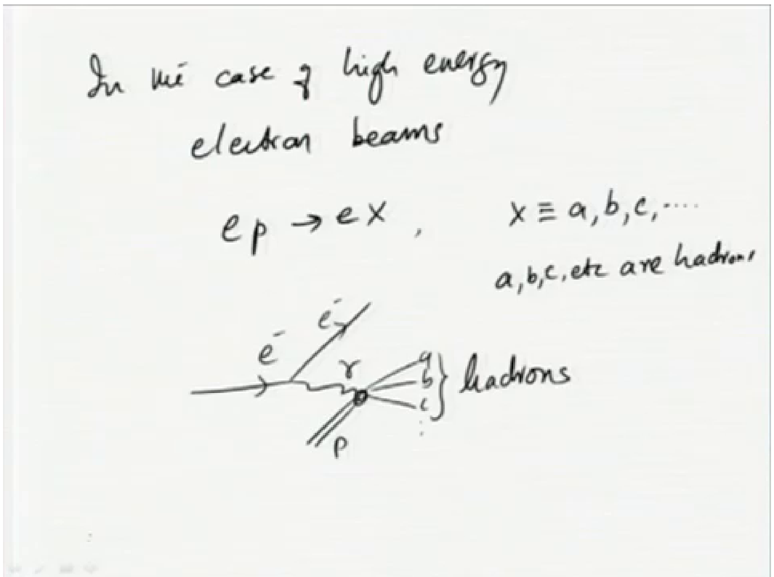


And we also discussed that in the case of high energy electron beams, there are we have seen e p not just elastically scattering to e p, but can go to some e x other particles, x could in general be any number of hadrons. So, in one particular case, we saw p the proton itself and pi over in that were in general it could be any different hadrons a, b, c etcetera are hadrons either mesons or bariums. So, this is inelastic scattering such an elastic scattering let us represent in fashion as e minus in the initial state and e minus in the final sate and proton in the initial state, but going into different hadrons in the final stage right.

So, I can picturize it in this fashion. We already said that the effect of electron interaction with proton can be picturized in terms of exchange of photons and that is what we have here. Now, let us look at this inelastic case and then see what are the changes that we had to make the earlier formula that we had here is basically the called the Rosenbluth formula and G E and G M are the electric and magnetic form factors which depend on Q square.

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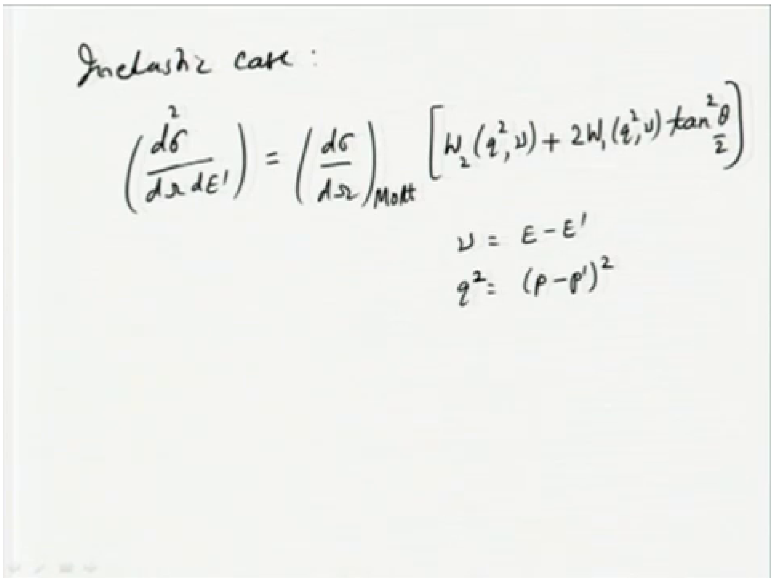


So, we take in the case of inelastic case, we consider an expression similar to that of the Rosenbluth formula and write it as d omega d sigma over d square sigma over d omega d E prime. One difference here is compared to the earlier elastic case is that we are also differentiating it with respect to or taking the different varying the final electron energy. So, remember our earlier discussion, we had said that in the elastic case the final energy of the electron is fixed, you get a particular value for the electron. And if you look at the experimental spectrum, you indeed get a large number of points at E prime is equal to whatever the kinematically allowed value you a fixed value. But in the case of inelastic collisions, this electron transfers the energy some of the energy to the other particles, and then the energy of the electron because the weight goes x.

If you look at the earlier expression earlier picture, e p going to e x, x need not to be a particular particle right it can be any many different hadrons. In that case, the energy of the electron will not be fixed; it will vary depending on what are the other particles, what their masses are and what their energies are etcetera. So, we will get an energy distribution for the electron in the final stage. So, therefore, we can consider their electron energy spectrum and see that it will now depend on the form factors similar to the earlier case, but the expression is similar to the earlier case, by earlier case of elastic scattering, but the structure the form factors are now slightly different. So, they are called actually the structure functions.

And they will be usually written as W 2 q square nu WW 1 q square nu functions of q square nu q square and nu I will tell you what nu is in a moment times tan square theta by 2 the theta dependence is similar. The same there is a term, which does not depend on theta square and there is a term which depends on tan square theta by 2. In addition to that, of course, d sigma over d omega mott has some theta dependence, but apart from the mott scattering differential cross section which corresponds to the point like particle without the spin considered in a same side this one, spin of the target which is considered or without the magnetic interaction.

So, this structure of function this W 1 and W 2 are called the structural functions and there is no recoil factor that we will consider in these cases, because there is no final proton that we are it is not the elastic scattering that we have considering, it is an inelastic scattering that we are considering. And nu itself is basically E minus E prime and q square of course, is similar to the earlier case p minus p prime square. So, in addition to the q square, the E minus E prime is another variable which can independently vary compared to q square. Let us see how that goes. So, the difference therefore, is that compared to the earlier case is that the structure functions are now functions of not only q square, but also nu which is basically the difference in the energy of the final and initial electrons.

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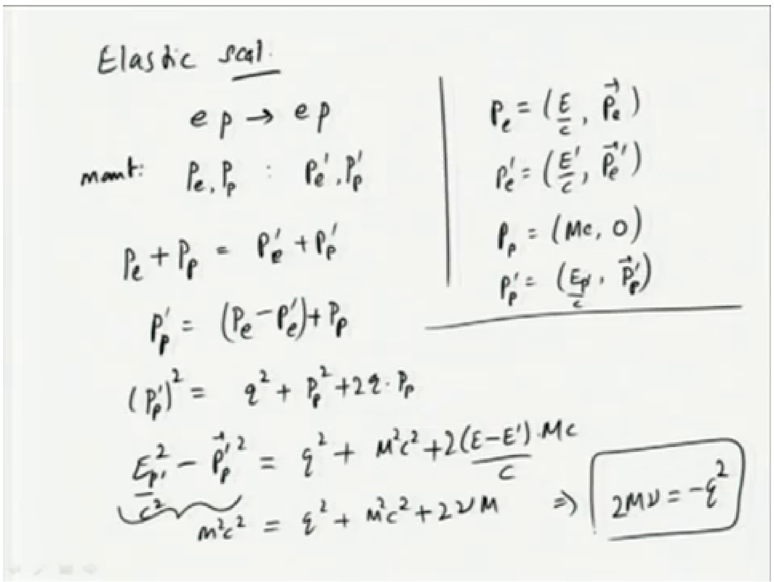

So, let us look at why that nu is now not now an independent variable unlike in the case of elastic case. So, let us consider in the elastic case first elastic scattering say e p go into e p let me denote the momenta as p e for the initial electron p p for the final electron; and p prime for the final electron, and p p prime for the final proton. In these are four momenta, which means that the first or the zeroth component is energy of the electron. In fact, E by c, and the three momentum; and p prime similarly is E prime over c, P e prime. And p p is at rest initially, so it only has the mass of the proton and no three momentum p p prime is let us say some E p prime and P p prime all right.

So, energy momentum conservation will tell you that the four momenta addition of the four momentum in the initial case is equal to P e prime plus P p prime the sum of the final momentum squared. This includes both energy conservation and momentum conservation. Zeroth component will give you the energy conservation, and the one, two, three components will give you the momentum conservation.

Now, let me write it in a different way P p prime is equal to P e minus P e prime plus P p. So, if I square this P p prime square is equal to what is there in this bracket is basically q, so it is q square plus P p square plus q times P p. This is essentially equal to well left hand side is mass of the proton square the four momentum square is equal to the mass square which we saw yesterday let me actually write it again.

So, this is equal to E p square E p by c in fact, E p square by c square minus P p prime square is equal to q square plus similarly this only has the P p only has the zeroth component which is M square C square plus q dot P p. So, there is only where the zeroth component. So, here Q b is basically E minus E prime into so this is twice this thing so two times this into mass into C. And this is M square c square energy momentum relation, M is the proton mass is equal to q square plus M square C square plus 2 nu E minus E prime I denote by nu and M C.

This tells you that q square E E by C there is. So, M square cancels with the M square on the left hand side and then this will give you the relation that 2 M nu is equal to minus Q square. So, nu is not independent of q square, given q square nu is given by minus q square by 2M. So, in the elastic case these are not two independent variables.

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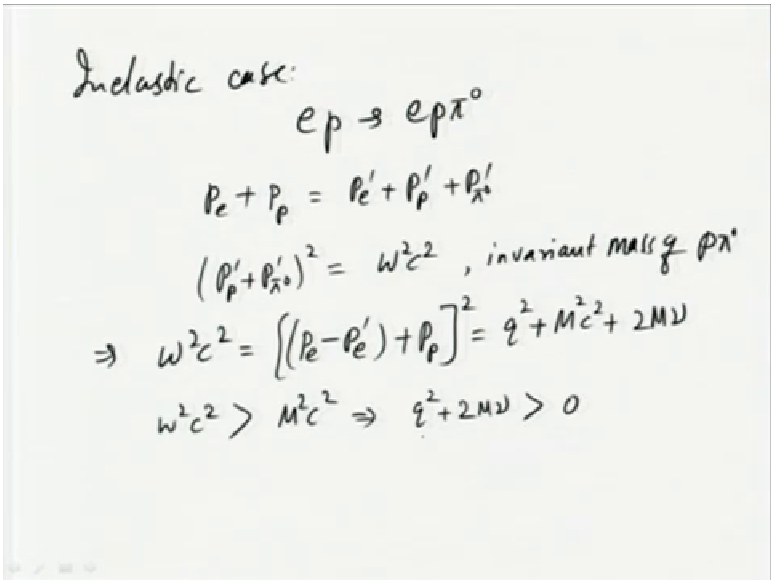


But now for the inelastic case, let us consider the example of E p go into E p pi 0 all right. So, now, you will see that the invariant mass, let me write down the four momentum conservation P e P p is equal to P e prime plus P p prime plus P pi 0 prime, where P pi zero prime is the momentum of the final pi zero and rest of it is similar to the earlier case. So, here again when we consider P p prime plus P pi 0 prime square this is what we call the invariant mass of the system this is equal to W square C square invariant mass of P pi 0 system.

So, this tells you that W square C square is equal to P e minus P prime plus P p square which is exactly what we had earlier on the right hand side which is q square plus M square C square plus 2 M nu. Since, W square C square is always larger than M square C square because it is the sum of P prime P p prime plus P pi 0 p prime square which means P e P p prime that is the four momentum of the final proton square itself is M C square or M square C square. And in addition to that you have the part coming from pions moment for momentum. So, W square C square is always larger than P square C square.

And along with this, so therefore, that will give you that the right hand side 1 square plus 2 M nu cannot now be 0 it can be it has to be always larger than 0 in this case. And there is no such relation and there is no such relation between 1 square and nu as we had in the case of elastic collisions. In the elastic collision case, it was actually q square W was replaced by m p the mass of the proton. So, we had q square plus 2 M nu exactly equal to 0; here it is not so. So, these two can 1 square and nu can vary independent of each other alright. So, this is why we have in the expression for differential cross-section in the case of inelastic scattering, the form factors or the structure functions as they are called r functions of two variables 1 square and nu unlike in the case of elastic collisions elastic scattering. Where it is only a function of 1 square and nu is not independent of q square there.

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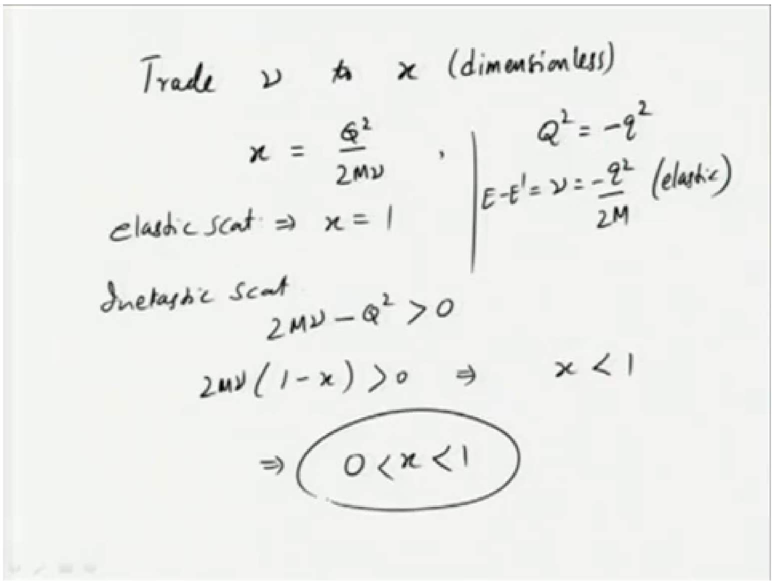


Now, usually we can actually write the variable nu in a different fashion or nu and Q square in a different fashion. You can actually trade it to or trade it with a variable x, which is dimensionless, nu is E minus E prime, so the dimension of nu is dimensions of energy, but we can define a variable called x as Q square over 2 m nu. From now on, we will consider capital Q square instead of small Q square, because we saw that nu is equal to minus Q square over 2 M in the elastic case.

And nu is E minus E prime usually the electron gives energy to the proton, and therefore the energy of the final electron is always smaller than the energy of the initial electron. So, E minus E prime is positive nu is positive and Q square is always negative. So, minus Q square is taken as a positive quantity Q capital Q square.

And elastic scattering gives x therefore, equal to 1; For inelastic scattering, we have 2 M nu minus Q square a larger than 0, so that is equal to or that is equivalent to 2 M nu 1 minus x larger than 0. That tells you that x has to be less than 1, because M is always positive nu is always positive therefore, x is always less than 1. And minimum value of x is 0, where Q square is 0, so that gives the range of x as 0 between 0 and 1. So, this is these are the two variables that we should consider Q square and nu, or x and Q square.

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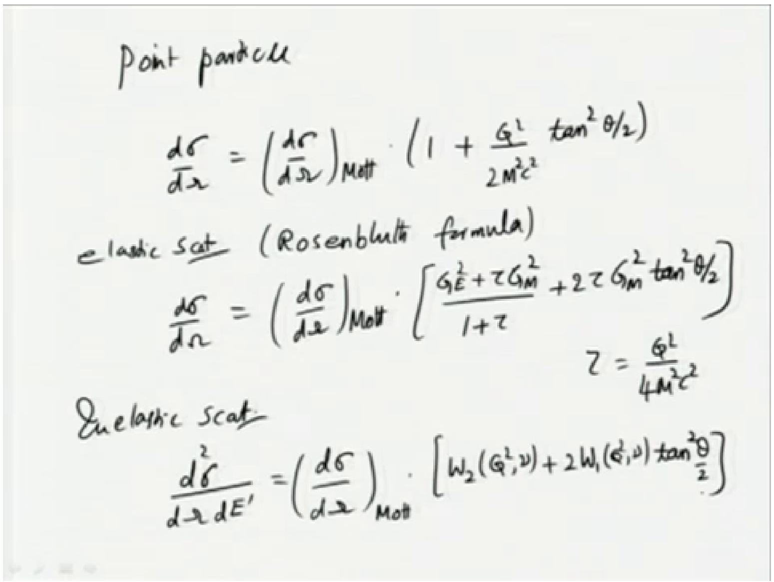


Now, consider the let us have a summary the three different cases. We had for point particle case, d sigma over d omega equal to d sigma over d omega mott times 1 plus now Q square divided by 2 M square c square tan square theta by 2. And elastic scattering case the Rosenbluth formula d sigma over d omega equal to d sigma over d omega mott times G E square plus tau G M square over 1 plus tau plus 2 tau G M square tan square theta by 2. Where tau is Q square over 4 M square C square. And in elastic case, we have now e prime not fixed, so therefore, we consider d square d omega d E prime where E prime is the energy of the final electron equal to d sigma over d omega mott times W 2 Q square nu plus twice W 1 Q square nu tan square theta by 2.

If you look at the these expressions, you see that G E and G M are dimensionless and because the d sigma over d omega and d sigma over d omega mott both have the same dimensions. Whereas, W 2 and W 1 have dimensions of 1 over energy, because on the left hand side we have d square sigma over d omega d E prime. So, there is a 1 over these structure functions are dimensionless have dimensions of inverse, inverse dimensions of energy.

Now, let us take actually the inelastic scattering case as kind of a general expression supposing this is the general case. So, the expression is written in a in terms of W1 and W 2 in the general case then W use in the elastic and inelastic cases can be considered in this fashion.

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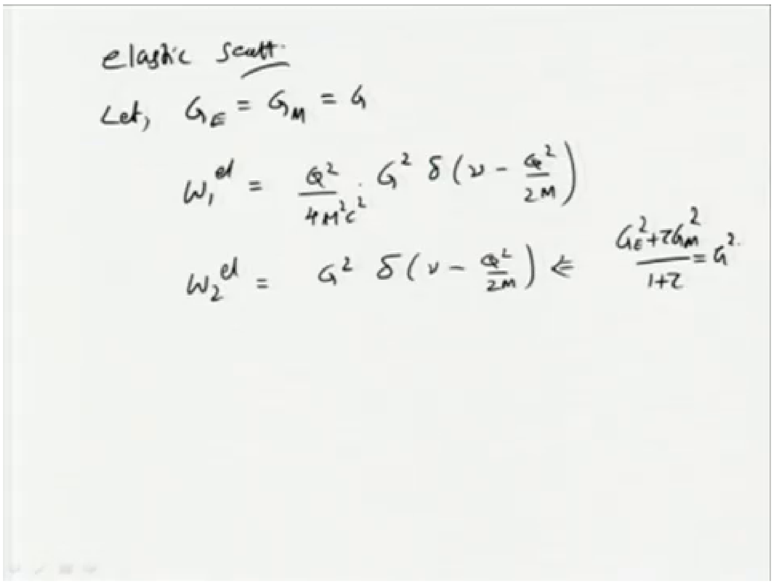


Let us consider the elastic scattering case not point particle, but particles like protons with extended charge distribution and composite particle, compositeness itself is not coming in here the extended particle not point like particle. We will consider an assumption that assume that G E is equal to G M just to illustrate this and make it a simple case. So, G E is equal to G M and denoted by some common G. In that case, if you compare these expressions Rosenbluth, and in elastic case expressions, then you can see that W 1 let me denote it by W 1 elastic e corresponds to Q square over 4 M square C square into G square into delta nu minus Q square over 2 M.

You remember the elastic case nu is always equal to Q square over 2 M, so that is what we have done we have taken this expression and that 1 over 1 plus tau in the Rosenbluths expression say here is cancelled by the 1 plus tau times G E square coming from the numerator when G E is equal to G M. And then 2 M elastic is equal to G square delta nu minus Q square over 2 M, this is because G E square plus tau G M square over 1 plus tau is essentially equal to G 1 plus tau cancels, now this is one thing. And G square is a function of Q square. So, we have this expression here keep that in mind.

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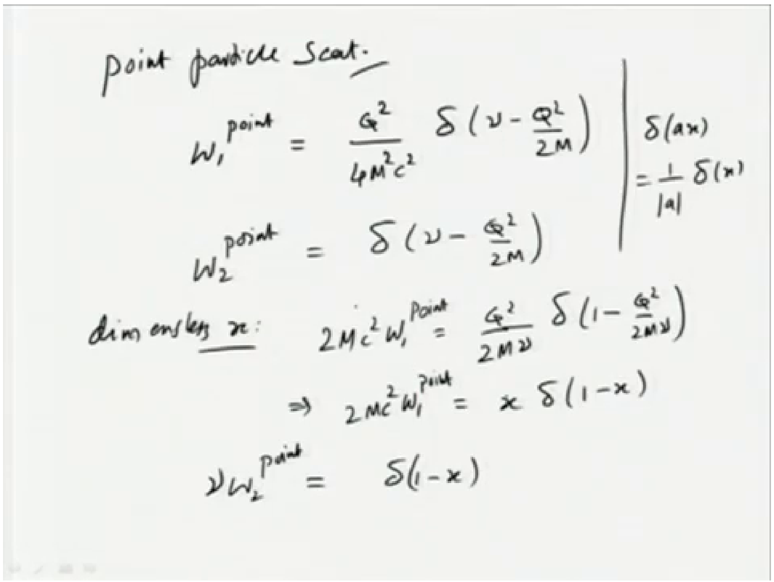


And let us go to the case of point particle scattering then the W 1 point particle is equal to Q square over 2 M sorry Q square over 4 M square C square delta nu minus Q square over 2 M, and W 2 is which is delta nu minus Q square over 2 M. In terms of the dimensionless quantity 2 M C square W 1 point is equal to Q square over 2 M nu delta 1 minus Q square over 2 M nu. Using the property of the delta function that delta a x is equal to 1 over mod a delta x, this is the property of the delta function which can be easily verified. This gives you 2 M C square W 1 point is equal to this is nothing but x delta 1 minus x. And similarly you have W 2 point particle is equal to 1 over nu 1 minus delta 1 minus x or I can write this as nu times W 2 point is equal to delta 1 minus x.

Why are we doing this exercise? What we are saying is that if you look at this inelastic scattering expression d square sigma over d omega d E prime in terms of W 1 and W 2 the structure functions and the mott scattering, then we see that if the particle that it scatters on is point particle, we will essentially see that there is no Q square dependence there in the structure function. It depends only on the dimensionless quantity x. So, there is no particular energy scale associated with this thing. The form factor is not there. It is independent of the energy.

Remember our earlier discussions on the form factor, we started with saying that if the particle were not actually if the charge distribution was not really point like charge distribution then we will have to consider the charge density not as a delta function, but as some distribution specially varying distribution or specially distributed charge case. In that case, there is a form factor that will come in their expression for cross-section, differential cross-section, which the form factor itself will now depend on the energy exchange between the electron and the proton.

And now what we are saying is that if it was a point if it is a point particle, it did not have any energy dependence, which is something which we had discussed earlier. But at this point what is going on is that if you consider very large energy, very large meaning a mean of the order of 10 G V etcetera for the electron, then the electron may be able to penetrate inside the protons. And then if it sees point like structures there what we can expect as the scattering with structure functions which denote and depend on Q square that is what we have said right.

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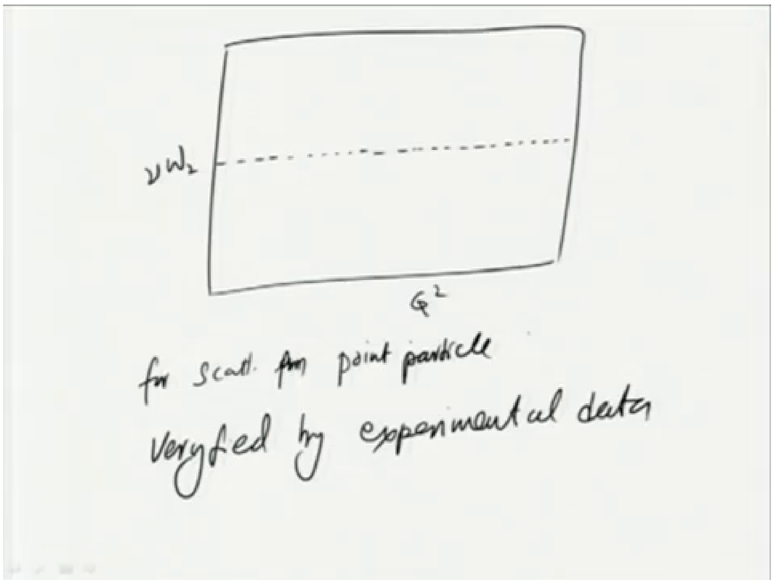


So, what we expect in the case of in such cases. For example, if you plot nu W 2 as a function of Q square for scattering from point like particles or point particles, what we expect is an independent or as what we expect nu w to be an independent function of Q square meaning it is a constant compare our as far as Q square is concerned. It is independent of Q square. So, if you consider if you do an experiment and then plot the W 2 nu, how do you do that, it is exactly like earlier case from the theoretical expression model that you have which is given in the earlier slides. You try to fit to the observed data for various different Q square values and then see that the extracted a W 2 behaves like this that is it is independent of Q square.

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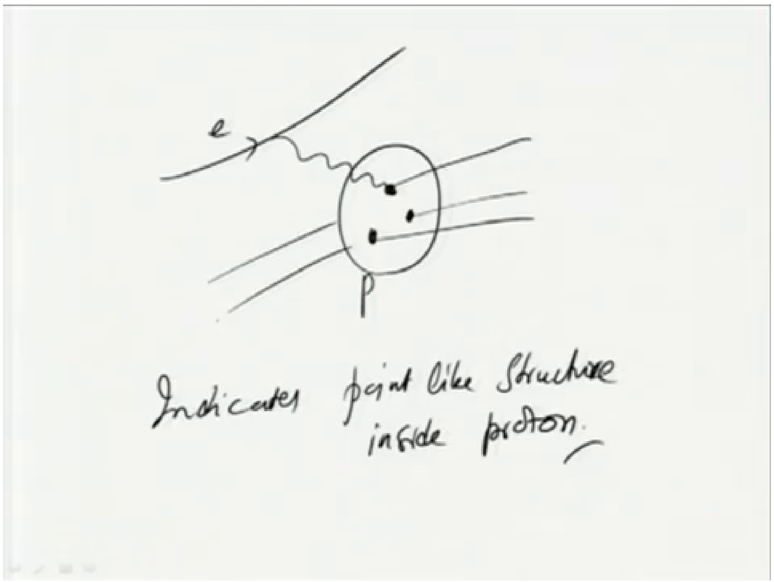


This is the expectation and indeed experimentally verified this one which says that when we actually look at large and high energy scattering of the electron on the proton so this is essentially the proton that we are considering then electrons start seeing it high energies it penetrates the proton say. It starts seeing the inside structure of the proton. And it looks like there are point like structures on which the electron scatters. Say for example, if you take the picture of proton made of three quarks, then you will see that you can imagine that the electron interacts only with a particular quark inside the proton; the other plot other quarks are just spectators in this interaction or spectators of this interaction.

In that case, you will see the behavior of this point like particle scattering reflected in the structure functions behavior of the structure function which means that nu W 2 for example, is independent of Q square. This is seen by experiments and therefore, we can infer that there are structures points like structures inside proton. So, this indicates point like structure inside the proton. Of course, we will have to investigate a little further and then study a little more to have a better feeling and better understanding of this. And to also confirm that there are this point like particles and they indeed correspond to what we call the quark in the quarks in the quark model. And consistently we can build the picture which is which agrees with the experimental results.

So, we will take on that in the next discussion, taking this further to understand these sub structures in a little better way.