Nuclear and Particle Physics
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Module- 08
Hadron Structure
Lecture-02
Proton structure

Last class, we discussed the structure of a nucleus, experimentally how to look for this structure what is the signature that we can look for to see whether the nucleus has a structure or if it is a point like particle.

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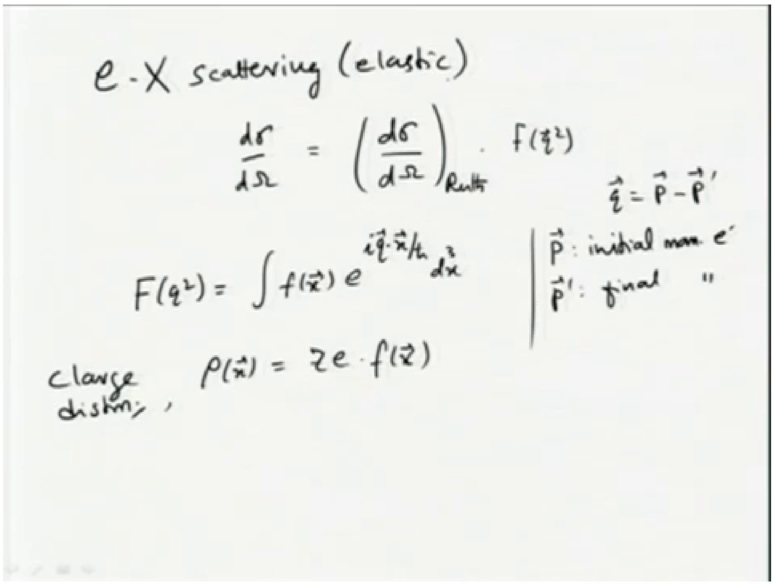

So, we said that in the case of electron scattering of a nucleus let me denote the nucleus by X. So, e X scattering the cross section d sigma over d omega the differential cross section, which actually can be related to the number of particles scattering into a solid angle d omega is equal to d sigma over d omega as per Rutherford formula, which corresponds to considering the nucleus as a point particle; times a form factor denoted by F which is a function of the square of the momentum exchange, q is basically the momentum exchange.

So, q is equal to p minus p prime, where P is the elect initial electron momentum, initial momentum of electron and P prime is the final momentum of that electron and this form factor F is reverted to a function f times i q dot x over h cross or is it is actually the

Fourier transform of the function f x, where f x is related to the charge distribution rho which can be now written as Z ez is times Fx. So, f x actually tells us how the charge is distributed, z d is the total charge in the nucleus. So, we are considering a nucleus of atomic number z or z number of so this is the charge distribution. So, experimentally then we can measure the cross section d sigma over d omega from counting, the number of particles getting into d omega solid angle and comparing it with the right hand side we should be able to find out what f q is like, if it deviates from 1 which corresponds to Fx delta function.

Then which is the distribution function for a point like charge. So, if F the form factor F q square deviates from 1 unity, then we can say that it is not a point particle, so all these actually elastic scattering the cells.

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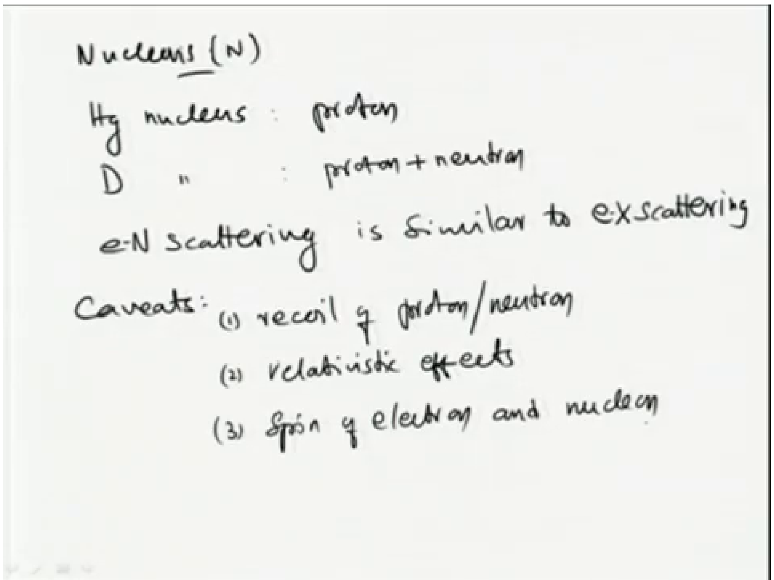

Now today we will discuss the case of nucleons, in the case of nucleons like protons and neutrons basically, if you take hydrogen nucleus it is essentially a proton and if you take a deuterium then it is a proton plus neutron. So, if you want to study the neutron we can proton, for example then we can take hydrogen nucleus and apply the formula that we had in the previous slide. So, it is a similar to e x scattering therefore.

So, e nucleon nucleons I will denote nucleons either proton or neutron by p sorry by N. So, e-N scattering is similar to e-X scattering, but we have to be careful about a few points one is that here we are not considering very light very heavy nucleus, but very light a nucleus or even just 1 nucleon. The other thing is that the sizes are going to be smaller compared to the normal gold nucleus or heavy nucleus; which means that we need higher energy probes higher energy electrons to probe these particles, when you have higher energies larger energies then the recoil becomes important. In the earlier expressions we did not consider recoil, we had neglected the recoil in the calculations in the Rutherford cross section expressions, but here we cannot neglect that. So, 1 is the recoil of proton or neutron second is that as we said we had to go to higher energies GeV energies.

Then for the projectile motion so therefore, relativistic effects has to be considered and then we also did not consider the spin of the electron and the spin of the protons or the nucleons nuclear in the case of Rutherford expression. So, you have spin of both the projectile and the target that has to be considered, so now we will not go into the details of how exactly arrive at these expressions but rather give the results for spin of the electron.

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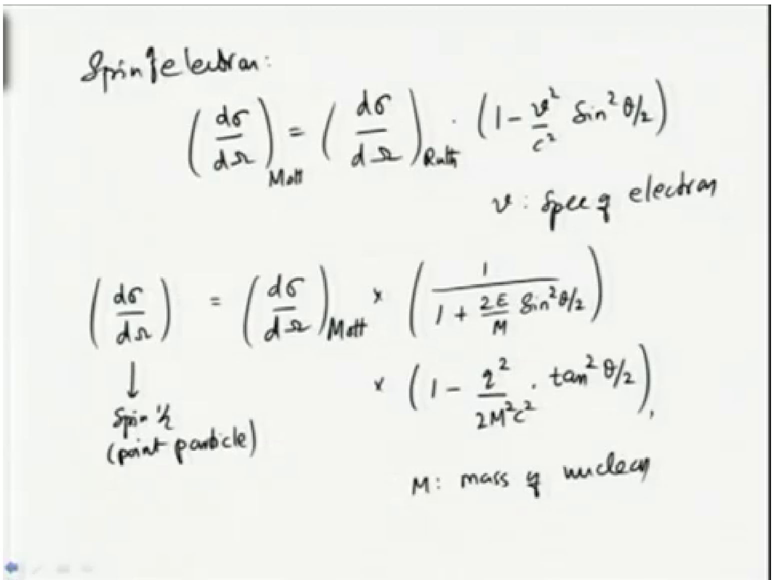

We have to modify the cross section expression compared to the Rutherford scattering cross section expression; let me denote that by d sigma over d omega Ruth r u t h for Rutherford, we have to add a factor 1 minus v square over c square sin square theta by 2 and this is due to mott and then we call this the mott cross section here v is the speed of electron. In fact, when you have relativistic highly relativistic electrons very large or

GeV energy electron beams, then v is almost c and then the this additional factor is then 1 minus sin square theta by 2, which is cos square theta by 2.

Now, considering the other effect the recoil effect and the spin effect we have sorry spin of the neutron the most fun. So, let me first consider the recoil part, so for that then d sigma over d omega mott cross section will have to be modified by, see what happens in this case is that the energy expression X the energy of the electron is now different from the energy of the electron, when you consider the proton recoil. So, the kinematics is different there. So, there is actually come the whole effect can be taken into account if you consider the energy expression or consider the factor 1 over 1 plus twice the energy of the initially, left beam energy of the electron sin square theta by 2 into this and then to take into account the spin of the proton or the neutron what does it do? We look at a wider therefore, actually what is the potential that we used in the Rutherford case where we considered only the coulomb interaction only the electrostatic interaction. Now we know that these are not spin less particles, they are spin half particles the proton as well as the electron. So, spinning effect of the electron is taken care of and now the spin effect of the proton what it does is effectively, the electron which is flying into the proton will see in addition to the coulomb interaction coulomb field, it will see a magnetic dipole field; this magnetic dipole is due to the spin of the proton. So, this has to be taken into account.

So, we will have an expression 1 minus q square over twice mass of the proton square or the neutron c square tan square theta by 2. So, as I said M is the mass of nucleon and q we know is the momentum transfer and this is for spin half particle, still point particle. We had not added any other thing we are still considering point particle like for the there is no distribution of the charge that we are considering or the form factor has to be considered along with this over and above this. Now we come to what are the forms factors that we need to do. So, earlier when we had only the coulomb interaction, what we had to worry about when we go from a point particle to a extended charge distribution is just consider the effect of the charge distribution, which was incorporated through the charge distribution integrated over the volume of the nucleus and in turn it becomes a form factor which depends on the momentum exchange q square; here we have not only the charge distribution, but also the magnetic dipole moment that just now we talked about, therefore we need 2 form factors to take this into account.

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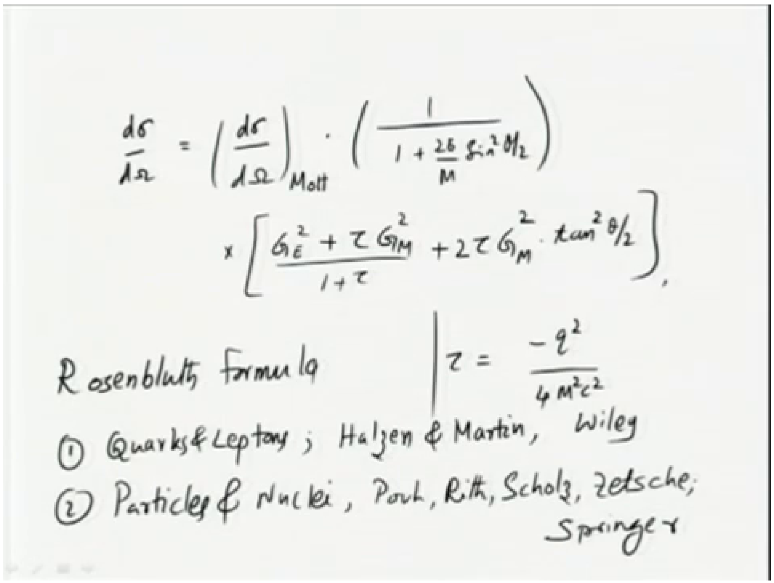

So, when you actually have an extended particle which scatters the electron, then we have to consider the cross section as mott cross section times 1 over 1 plus 2 E over M sin square theta by 2, to take into account the recoil spin part is already taken into account, we are considering the mott cross section and now the charge will actually take up a distribution where we will have a form factor corresponding to the electromagnetic interaction and we have another form factor to take into account the magnetic interaction.

So, G E and G M are 2 form factors and the expression again without deriving it, not even justifying it fully we will just reproduce it here; I will give you the reference in a minute. So, this G M square tan square theta by 2. So, as you can see the G M the magnetic form factor is the 1 which comes along with this tan square theta by 2 which we added to take into account the magnetic dipole moment interaction. So, and tau is basically just for the sake of simple notation we have used q minus tau as minus q square over 4 M square C square; this expression is called Rosenbluth formula and I will give some references 2 references. In fact, one is a book called named Quarks and Leptons an introduction to vertical physics by Halzen and Martin publisher is Wiley, another useful book is Particles and Nuclei an introduction again, this is by the authors Povh Rith Scholz and Zetsche Springer. So, the details of this calculation some of the details they also give some justifications for various things and then also give some amount of details of this expressions and then give further references there.

So, please look at it if you are interested in going into the details and understanding how you arrive at this expression the Rosen bluth formula. But essentially the from here we have 2 form factors that we had to worry about, when you consider the spin of the particle as well as the charge distribution. The other effects like the recoil and relativistic effects have to be also considered, since these are not negligible in this case compared unlike the case of heavy nucleus. Value of G E, I said G E corresponds to the electromagnetic this interaction.

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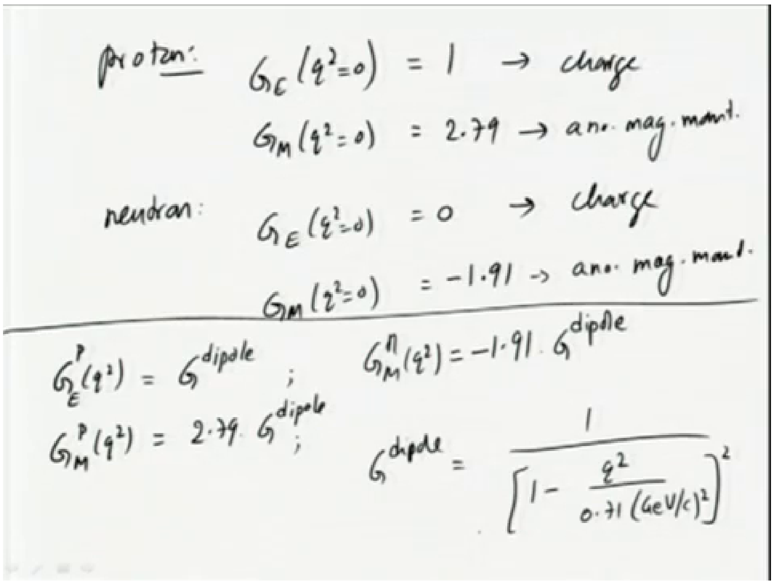

So, if you have very low energy this one interactions then say for example, q square is small very small or equal to 0, then this form factor should go to unity this corresponds to the charge of the electric charge of the proton and G M q square equal to 0 corresponds to the magnetic dipole moment.

In fact, the anomalous magnetic dipole moment for spin half particle we already have considered the magnetic moment. So, compared to the point like the spin half particle if it is an extended object or structured particle, then we have additional or anomalous magnetic dipole moment which would be (Refer Time: 19:43) the anomalous part of the magnetic moment is equal to 1 for spin half particle, which is a result of what the Diracs equation and Diracs theory of relativistic quantum mechanics and we may come to that at some point of time later. So, this is basically the anomalous magnetic moment of the proton, this is an experimentally measured quantity after this thing and for neutron we have G E equal to 0 for neutral particle and G M at q square is equal to 0 is minus 1.91 this is the anomalous magnetic moment. So, now when q square is not equal to 0 we will have to, so for example if you take G E of proton this is not equal to 1 now for q square not equal to 1 0.

So, this let me denote by some G dipole similarly G M P q square is 2.79 G dipole and G of G M of neutron q square is minus 1.91 G dipole. So, all of these are given for expressed in terms of a particular form factor, which is the dipole form factor, which is essentially equal to 1 over 1 minus q square over 0.71 GeV over c square power 2. So, you can see that when q square is equal to 0, G dipole is equal to 1 and we get back the anomalous magnetic moment and the charge of the respective particles. So, this particular dipole form factor is obtained by fitting the experimental data.

So, it says that this 0.71 is the 1 which comes below q square and as well when we take q square in Gv per c, cq in Gv per c. So, this has this is the case of elastic scattering, so now we will see what about the case of inelastic scattering.

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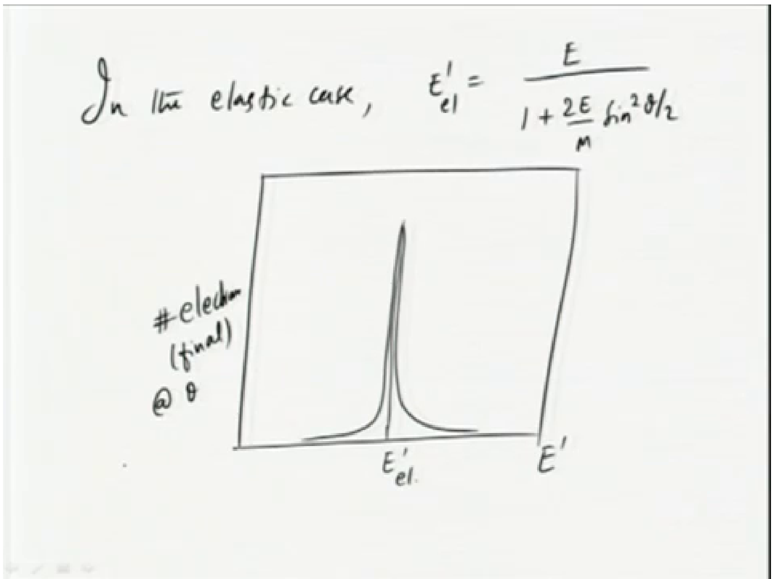

Let us look at the electron energy in the elastic case; we said E prime is equal to E divided by 1 plus twice E over M sin square theta by 2. In fact, we did not say this explicitly like this, but rather we use this 1 over 1 plus 2 E over M sin square theta by 2; actually that is because the factor that comes as a correction term is E prime by E and then it turns out that it is this where E is the initial electron beam and E prime is the final electron energy.

So, now one can do one thing you perform an experiment and count the number of electrons coming out final electrons and you will see that in this elastic case you plot it against E prime you mean you measure the energy of the electrons and then see plot it against the number of electrons coming and then you will see that it actually comes out at one particular value well experimental uncertainty will certainly give you some kind of a small distribution, but it is more or less highly peaked this one without any number in the tail and this particular value will be E is the one which corresponds to the elastic scattering.

So, let me denote this E prime by elastics. So, you will see that you see that for a particular angle at a but when electrons are considering at a particular angle theta, you will see that they will peak at one particular value of E prime which corresponds to this elastic scattering this is even after considering the recoil, but now it if you take large values of E.

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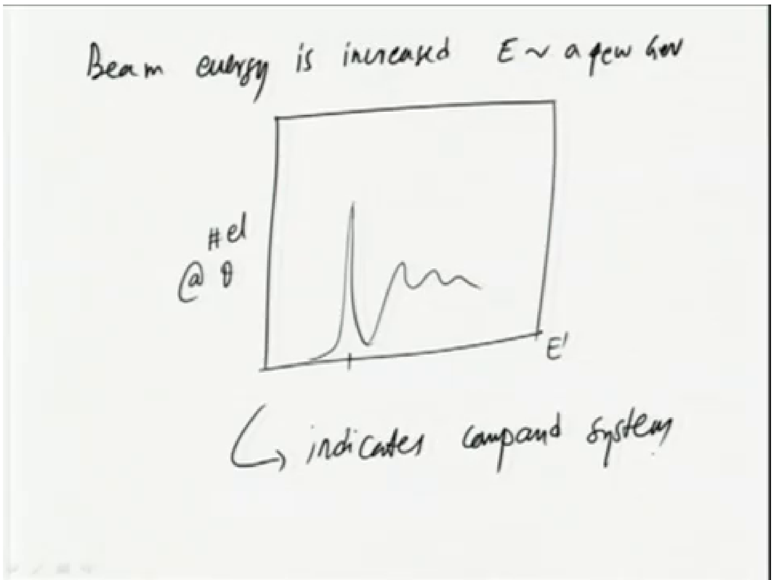

Very beam energy is increased with to some a few or 10 electron, a few gew or a some 10 GeV 5 GeV 10 GeV like that, then you will see and you will again plot the number of electrons at theta and has plot against a E prime then you will suddenly see the elastic peak, but you will also see other peaks; this is characteristic of a compound system. So, this indicates compound system. So, you will see that the nucleus those proton nucleon is not a point particle, but it is a compound particle ok.

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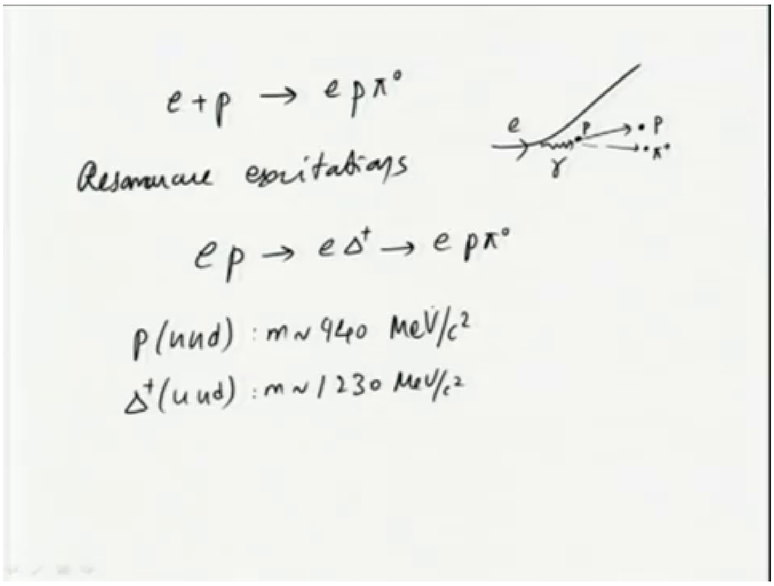

So, that is one thing say for example, if you consider electron proton scattering, then in fact, you will see not only electron and proton in the final set, but some other particles aspect. So, you will see electron going off and there was proton initially and then this goes to proton, and some other particle they pi on in this case. So, this can be understood in some sense in some way as the resonance excitations, what do I mean by that we can then say that e p gets into e excites the electron which is coming in with some energy gives energy to the proton.

In fact, we could rely on our earlier picture of electromagnetic interaction, where we said electrons interact with charged particles through the exchange of photons. So, it gives energy to the proton through photon. So, this is my proton this is some photon and this energy is absorbed by the proton and it could excite itself into a different resonance, this can happen if it has constants like atomic excitations; atom has it is electrons inside that so there may be they release a particular energy level; the atom is in say the ground state if you give more energy to this particle, this atom it can excite from the ground state to a next an excited stage a higher energy level; a similar thing could is happening here is 1 of the ways to understand this say for example, P is made of uud quark and we know that there is another bound state which has mass about 1 to 3 to larger than that of proton.

Which means that the energy of this delta plus is larger than that of P, here when we say energy it is the total energy including the energy equivalent of mass.

It goes into this and then of course, delta plus can decay into proton and P ion. So, this is one way to understand such a resonance formations. So, if you look at the energy corresponding to this thing then you will see that it is indeed so, but. So, what we had to do to identify this resonance or the mass of the resonance corresponding to whatever the energy of the particles coming out, is to look at what is called the invariant mass of the finest particles.

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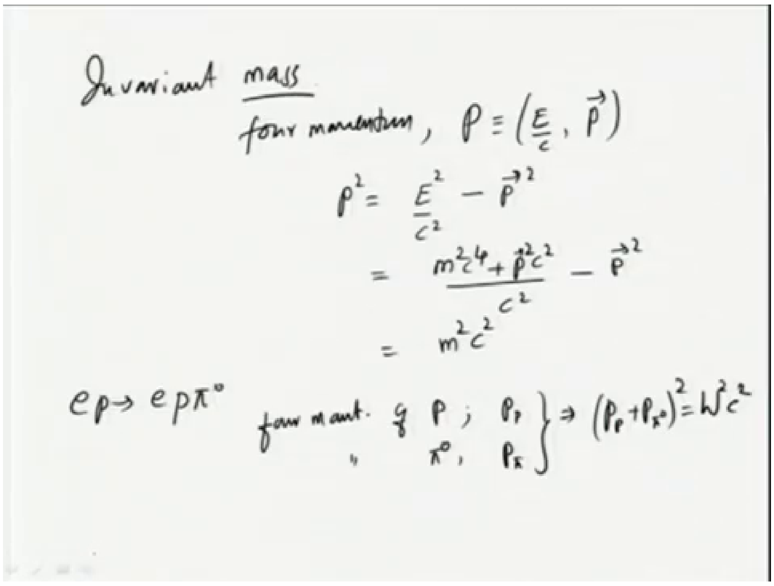

So, let me first tell what is the invariant what is the meaning of invariant mass, let us consider the 4 momentum of any particle say let me denote it by P which is essentially a 4 component object with energy by c as the zeroth component, and 3 momentum as the other 3 components.

Now, if I take a dot product of this 4 momentum with itself I will get P square equal to E square by C square minus P square. If you do not know what is how to take how what is a 4 momentum and 4 vector in general and what is the how to take the product of a dot product of this 4 momentum or how to write it in terms of these components please look at any special theory of relativity notes or any book that deals with this and then you will see that square of the 4 momentum is equal to E square by C square minus P square for any 4 vector it is the zeroth component or first that is written here I call it the zeroth component the rest of the mass 1 2 3 components.

So, zeroth component square here E square by c square minus sum of the 1 2 3 component squares, but then we know that E is E square is m square c square m square c power 4 plus P square C square. So, divided by C square minus p square. So, that will give me m square c square m square c square essentially. So, 4 momentum square is m square c square it is that is then if you have say 2 particles say for example, in the case of ep going to e p pi 0, and if you take the 4 momentum of proton say Pp 4 momentum of pion say p pi that will give you P p plus P pi 0 square is called the invariant mass square of. So, this is called the invariant mass square of the pion proton system. So, that is what we call the invariant mass.

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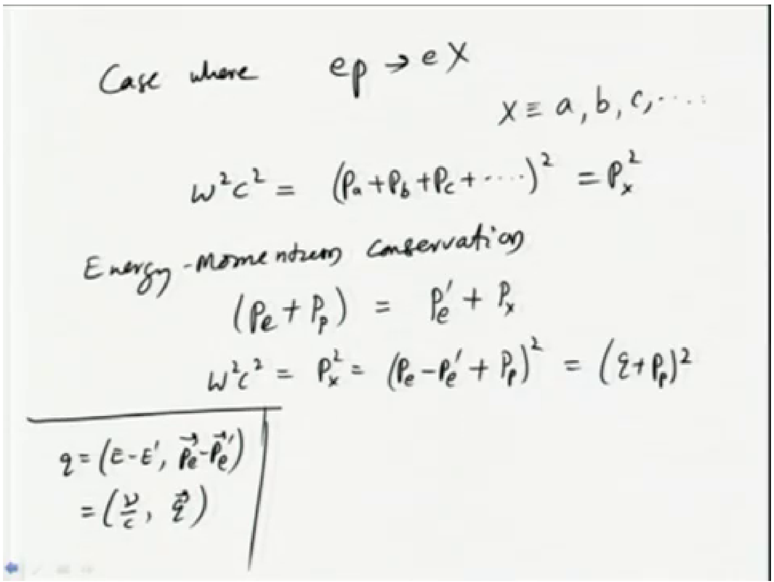

So, let me come to the case where ep goes to e X where X is X denotes many particles abc etcetera in our earlier case it was only 2 particle 1 is proton the other was pion, but in general it could be many particles that is going on inelastic scattering.

So, W square C square the invariant mass square times c square is P a plus P b plus P c plus etcetera all 4 momentum square energy momentum conservation will tell you that initial energy momentum P e plus P p all these are 4 momenta that we are considering is equal to P e prime the final momentum of the electron plus P x. So, let me denote the whole thing by P x square. P x is symbolically P a plus P b plus P c all sum of all the final particle momenta. So, this will tell me that W square C square is equal to P x square is equal to P e minus P prime plus P p square, which can be written as q plus Pp square where q is equal to E minus E prime P e minus P prime and we will denote this by nu by c and we had already denoted P e for 3 momentum minus p prime 3 momentum as q the momentum transfer.

So, this is essentially the. So, let me for clarity write it here as q plus P p square all right. So, what we said is that W square C square is essentially the moment the mass of the combined system, which means that let us go back to the earlier case. If you take pp plus p pi r square let us say it is coming from a delta particle if it is coming from a delta particle then mass of the delta particle square should be equal to P a plus p P b plus p pi square right. So, let me write it in this fashion.

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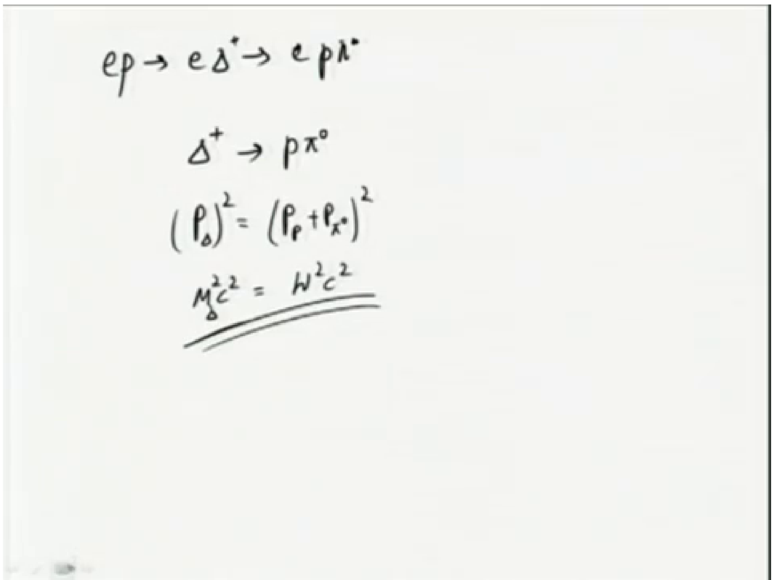

So, in the case of e p going to e delta plus going to e p pi 0, we have actually a intermediate step where the pion the delta particle is formed and then it decays to pi 0 and p. So, let me see if P delta denotes the 4 momentum of the delta particle this should be equal to the final particle p momentum plus p pi 0 momentum and if I now square this and square this.

I will get M square C square M delta square C square which is equal to W square C square as per our earlier notation. So, this is a particular value the mass of the delta particle which is formed supposing this resonance cross section is the which resonance processes what happens.

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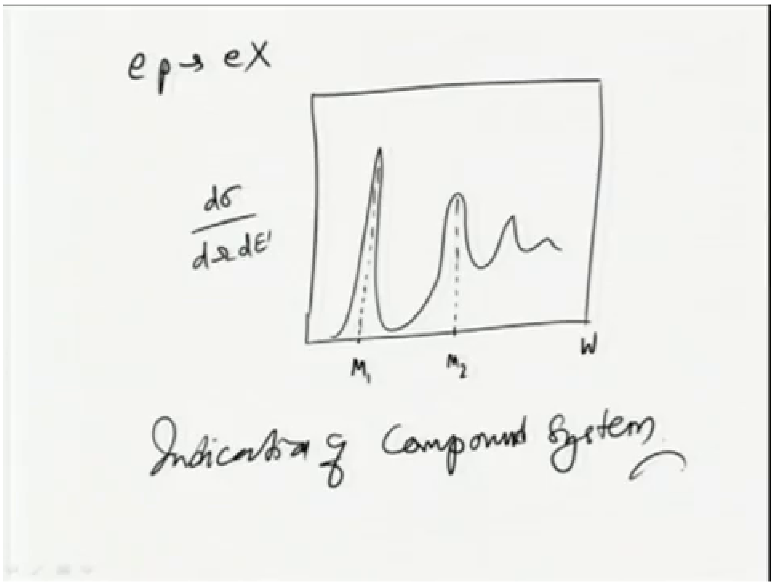

Then if we plot e in the ep to e X process, if we plot the number of particles or number of electrons coming out up into a particular cross section or a particular d omega the solid angle or correspondingly the d sigma over d omega d prime and plot it against the invariant mass W of X the system the final particles apart from the electron. Then you will see that there are resonances like peaks that are visible. Each of this peak corresponds to a particular resonance mass, so M 1 M 2 etcetera.

So, each of this peak as per our earlier discussion correspond to an excited state of this proton it could be. So, or it would be then interpreted as proton having a structure and when energy is absorbed from the electrons these constants are excited to higher energies and therefore, these higher states are formed higher resonances are formed. In fact, if you remember the nuclear scattering experiment the discussion that we had there, there also we had the formation of the resonances that we discussed this is somewhat in a similar fashion. There again we said the resonance formation is possible because the nucleus has constituents and the constituents can get excited to a higher energies, without splitting apart they can actually get into higher energy states.

So, these kind of excitation are possible and then that is actually possible, when in the case of the nuclear as in the case of compound systems. So, this is an indication of as I said earlier compound system. We have to go a little more further to understand the quark structure and then them to pinpoint, there are particles like quarks there, but this itself such inelastic scattering results itself indicates that they are not structureless particles.

So, the next thing that we will take up is basically the understanding of the sub structure in terms of experiments. So, we will study further that in the next discussion.