Nuclear and Particle Physics
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Module – 08
Hadron Structure
Lecture – 01
Nucleus

Today we will look at structure of the nucleus in the sense that we have been talking about the nucleus and then later on to the nucleons, the protons and neutrons. Further we went on to the sub structure of the protons and neutrons and then discussed the quark model which tells us that it is possible to consider quark as the basic constituents of firmata along with the electrons of course.

And especially considering the nucleus, nucleus is made of quarks and these quartz can form bound states or they are found only in bound states in for that matter no free quarks are found. And quarks can be formed in two different type mean different types of bound states one is basically consistent consists of 3 quarks which are called baryons like protons and neutrons, and then we also said there are particles which nonbaryonic which means that he is slightly different from the protons and neutrons etcetera. They are made of quarks and antiquarks, one quark and one antiquark and they are called mesons. Examples are pions, kaons etcetera.

So, we had all these theoretical description and we had a consistent picture and we introduced ourselves to this notion of quarks, how do we actually test our ideas experimentally. At some earlier stage we heard in fact, mentioned that it is basically the scattering experiments that gives us information about the structure of the nucleus and structure of the sub nuclear particles and even particles inside the nucleons which are quarks. We also said that the scattering experiment is basically the kind of experiment that Rutherford and his collaborators had performed giving us the information about a hard nucleus consisting of all the positive charged positive charges in an atom.

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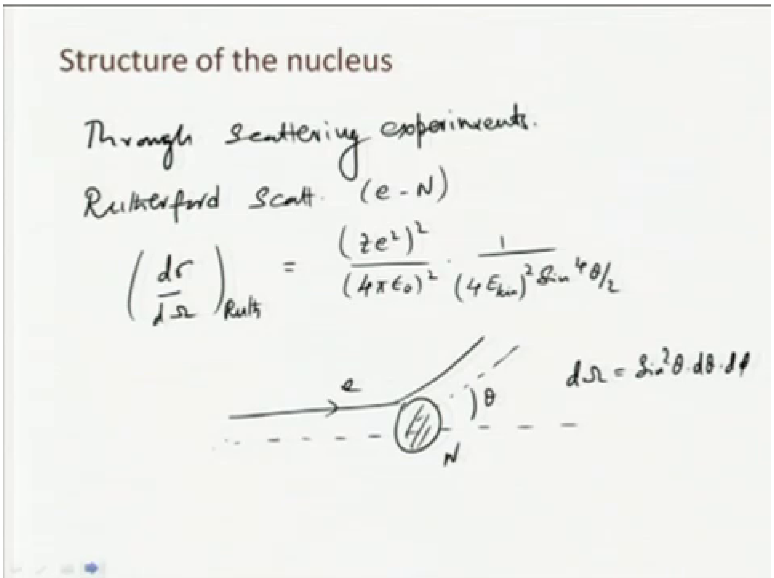

So, we have to in understanding the 7 sub structure the quark level picture of the protons or other hydrons we have to rely on again scattering experiments. So, first we will spend a little time discussing the structure of the nucleus or how do we actually come out with the interpretation that the nucleus itself, I mean going beyond the Rutherford's experiment, going beyond Rutherford's force experiment say that there are actually sub structures in the nucleus like protons and neutrons.

So, let us look at this the Rutherford's let me just remind you what the Rutherford experiment cross section is. So, let us consider Rutherford's scattering. Now, let us consider electron scattering on the nucleus rather than alpha particles just for somewhat giving a little simpler picker. We define what is called the scattering cross section. Let me call in this particular case this Rutherford scattering cross section d sigma or d omega which is related to the number of particles that you will observe at an angle theta in a solid angle d omega. This is basically in the case of electron nucleus with a z number the atomic number Z, we have Ze square whole square 4 pi epsilon 0 square 1 over 4 kinetic energy of the electron square sin 4 theta by 2, and theta is essentially the scattering angle.

So, if you have a nucleus and electron sent towards this one it will see the electrostatic potential and scatter at an angle theta. So, this is the theta angle and corresponding cross section is given by this and solid angle just to remind you in case some of you do not have forgotten what is the solid angle its basically sin square theta d theta d phi in polar coordinates or it is basically the area of a cap of radius r divided by r square.

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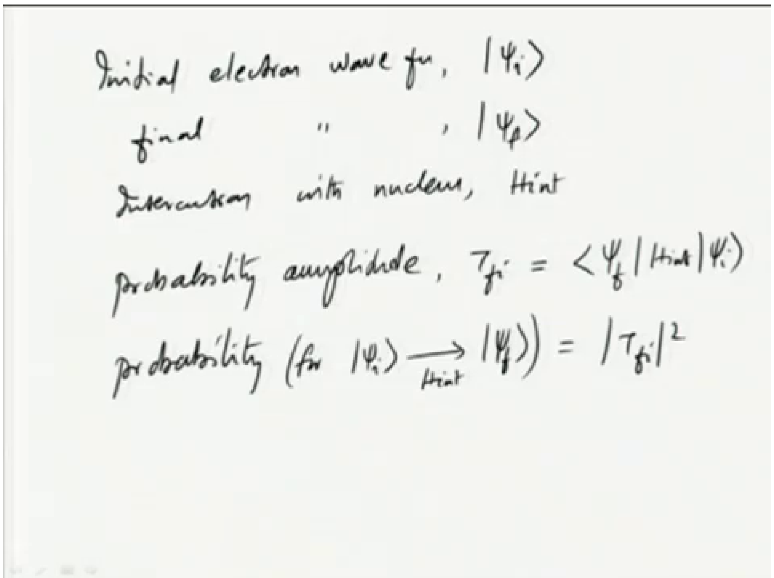

Now, let us have some theoretical understanding of this, whole thing. Basically let us consider the initial electron wave function as, k is a wave function is psi i let me denote it by that and then final electron wave function after the scattering let me denote it by psi f. And between these two the electron undergoes an interaction. So, interaction of the electron with nucleus let me denote that by a Hamiltonian power H interaction, so interaction Hamiltonian the potential essentially in quantum mechanics the corresponding operator.

So, we can say that the probability amplitude for an electron in psi i wave and state to interact with H, in a H interacting interaction to transform to stage psi f is let me denote that by T fi initial to final as basically H int sandwiched between H i and H f this is the quantum mechanical probability amplitude for this particular thing to happen. And the corresponding probability itself for let me write that down for psi i to go to psi f through H interaction is equal to now, T fi squared.

So, this is basically the quantum mechanically schematic way of writing the transition of the electron state when it scatters off a nucleus or encounters any such nuclear potential or inside potential in fact. Now, usually in an experiment you do not have a particular electron which is sent on a particular nucleon. Rather you will have a beam of electrons like beam of alpha particles sent to a target, system of a target nucleus a nuclei many such as thing like alpha particles into a gold foil.

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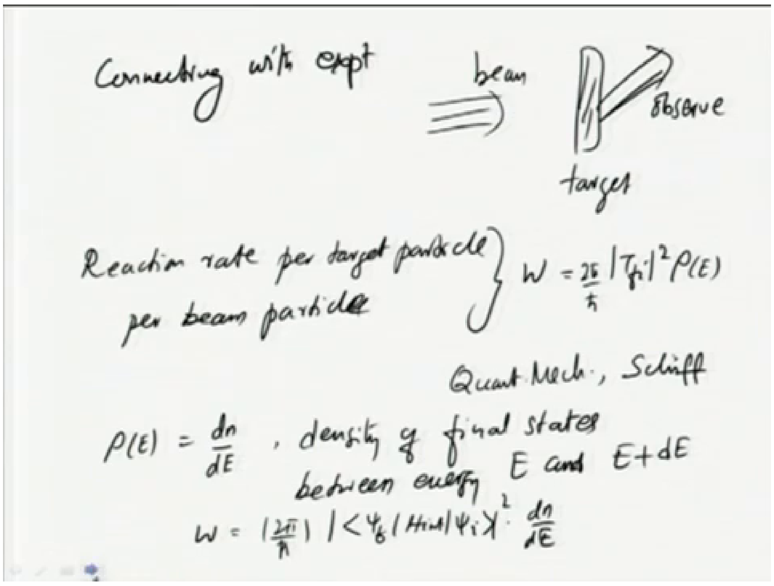

So, let us connect with experiment like others where we have a beam sent to a target and then we observe these scattered particles. So, in that case what is detected a number of particles which are scattered into certain areas or you go over around the whole of the target and then get the all of the target having the scattered particles or if you keep a detector at some particular angle it will detect the particles coming scattering into that angle.

So, we can now, think about reaction rate which means the scattering rate. So, per target particle per beam particle yeah let me denote that by what is called a letter W is equal to T fi square the probability into what is called the number of or density of final state which is available and 2 pi over h cross. This is an expression which can be actually derived considering the quantum mechanical process of scattering. We will not go into the details of how to get this because that is that will take us away from our discussion, rather I will point you to where you can read this and understand it if you are not already familiar with this. You can take any book on quantum mechanics to understand this usually there is a discussion on scattering in any basic quantum mechanics book, specifically if can you look at the quantum mechanics by the classical book by Schiff.

So, where the; here rho is basically the number density, number of states density of final states that this beam can go into number of available states density of final states between energy E and E plus dE. So, with the T fi described in the earlier slide we can write. Now, W as 2 pi over h cross 2 pi over h cross psi f, H interacting psi i square dn by d which is basically E, rho E, fine. Now, we will see what is this number of final state that we are talking about. Let us consider a particular particle and what is the minimum space that it will occupy in a way in its phase space.

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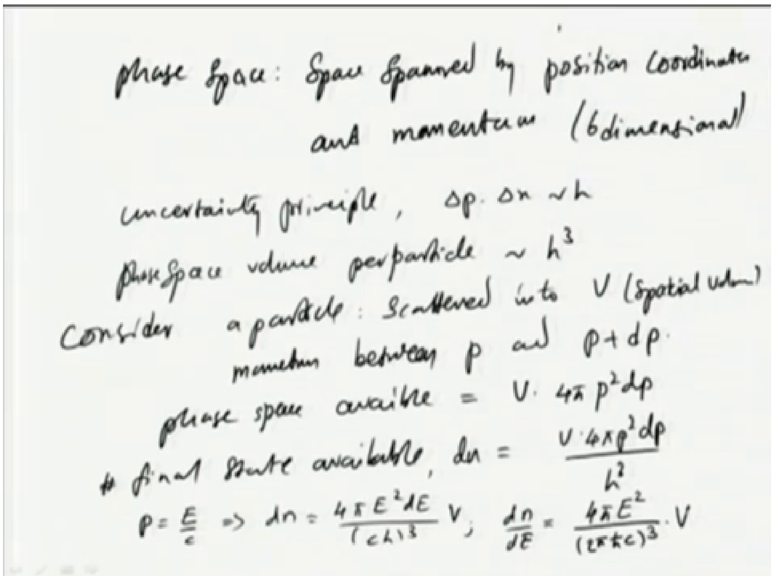

When I say phase space that is the space spanned by coordinates position coordinates let us say or the position coordinates and momentum of momentum. So, it is an 3 dimension for the coordinates and 3 dimension for the momentum together 6 dimensional space, 6 dimensional. Idea is, idea in why we are talking about phase space is because essentially when we talk about a particle the kinematics of the particle what we need to know is where the particle is and what the energy of that particle or the momentum of that particle is.

This is what basically we do when we say that go back to the Newtonian mechanic classical mechanics or any such other thing there also when we say that we understand what the dynamics is basically to understand where are the particle is and how it is actually traversing in space and with what momenta at any time. So, therefore, the phase space is one of the important things to know. So, if you know where the position what the position of the particle, position meaning where the particle is in the momentum and coordinate space together which is called the phase space then we know what it its kinematic, I mean what its energy is, what its position is etcetera. So, that is what and the trace of this will give you the evolution of the particle and then give you a lot of information about the dynamics again.

Now, let us consider a particle in quantum mechanics the particle momentum can be determined with some minimum and certainty, you cannot go on to determine the position to finite very extra infinite accuracy and the uncertainty principle tells us what is the accuracy with which we can actually go. So, it is essentially that delta p the uncertainty in the momentum times the delta x the uncertainty in the position for any particle is of the order of H minimum. So, that is the minimum space needed for a particle if we consider a one-dimensional system and in three-dimension it becomes 3 such a coordinates and 3 such momentum components.

So, the space, phase space volume per particle or needed for a particle is about H cube in 6 dimensional phase space. Now, consider a particle scattered into a volume spatial volume V and has momentum between p and p plus dp and this will tell you the phase space available in this particular case is equal to V times 4 pi p square dp, 4 pi p square dp is the volume in the momentum space for momentum between p and p plus dp. That is only the magnitude that we are considering. So, we had to integrate over the spatial angles and then that will give you a 4 pi factor 4 pi p square factor.

So, this is the volume phase space available for available total. So, per particle now, or the number of final state particles or number of final state available for which means that which means how many particles can be accommodated in this volume as what we are considering dn is basically available phase space divided by the phase space volume occupied by one particle which is H cube.

So, we can actually write it in terms of energy by considering the fact that p is equal to E by c relativistic relation. So, that will give you dn equal to 4 pi E square p square becomes E square by c square. So, we will take into account what c the factor of c will come down dp is dE divided by c. So, there are 3 c factors c H cube into V or I can write dn over dE which is what we considered as rho earlier which is basically the number density between energy E and E plus d is equal to 4 pi E square let me write H in terms of h cross which is 2 pi h cross c cube into volume spatial volume. Keep that in mind we will take that information at a later stage all right.

So, we have, we will now connect this with the cross section.

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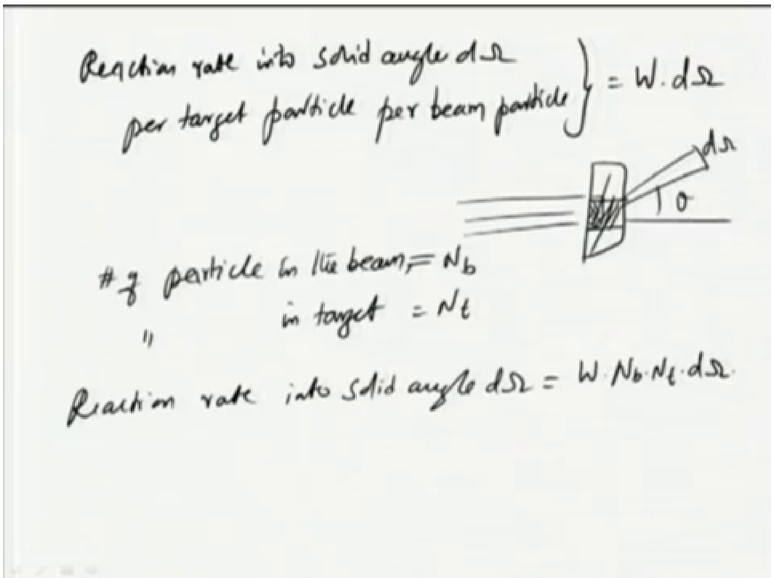

So, what is this? Let us consider again the rate of reaction rate in the solid angle d omega per target particle per beam particle this is basically W, but not the whole thing only in do solid angle d omega.

So, what we mean here is, what is the rate of reaction there is only one particle coming in into a solid angle d omega and at angle say theta, whatever. So now, if number of particle in a beam if this is equal to N b and number of particle in target is equal to N t when I say a number of particles in the target it is in the within the cross sectional area of the beam which is what is relevant. So, basically the region here, the region here, well all through that thing.

If this is labelled like the in bad (Refer Time: 23:49) we can say that the rate of reaction or reaction rate into solid angle in this case with N b particles in the beam and N t particles in the target is W N b N t d omega and which means that if you have such a reaction such a scattering experiment where there are N b particles in the beam and N t particles in the target then scattering into an the number of particles that is going to scatter into the solid angle d omega per the rate of this particle the particle which is going to get into this thing per unit time is basically W d omega which is essentially the reaction rate per target particle per beam particles. So, you have to multiply it with the number of particles in the beam times number of particles in the target to get the actual array reaction rate.

Now, let us connect it with cross section.

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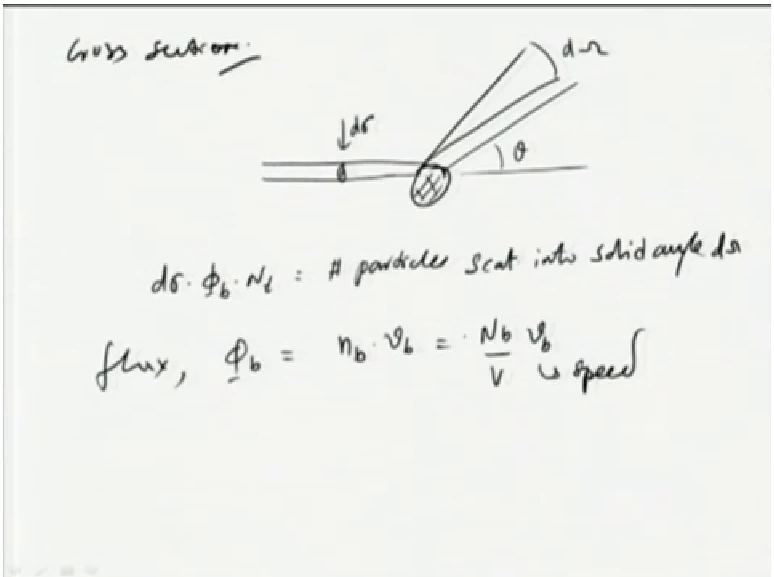

For that we had already discussed earlier what the cross section is etcetera in the when we discussed the Rutherford experiment. So, let me remind you what we heard there if you have a nucleus like this or take a hard sphere scattering we just what we considered as an elementary scattering experiment. So, if the particle in a small cross section scatters into a solid angle d omega at an angle k theta along with the beam direction. Then the d sigma which is the cross section of the beam relevant portion of the beam times the flux of the beam number of particles crossing cross sectional area in unit time, a per unit time and per unit area of the beam cross section. Times the number of target particle is going to be either a number of particles scattered into solid angle omega, a scatter angle solid angle d omega.

Flux itself, k flux can be written as number density into the speed all right. So, that will give you the number volume number density n b is the number of particles divided by volume into v b is basically the speed of a particle. And so, this n b is what we had defined earlier.

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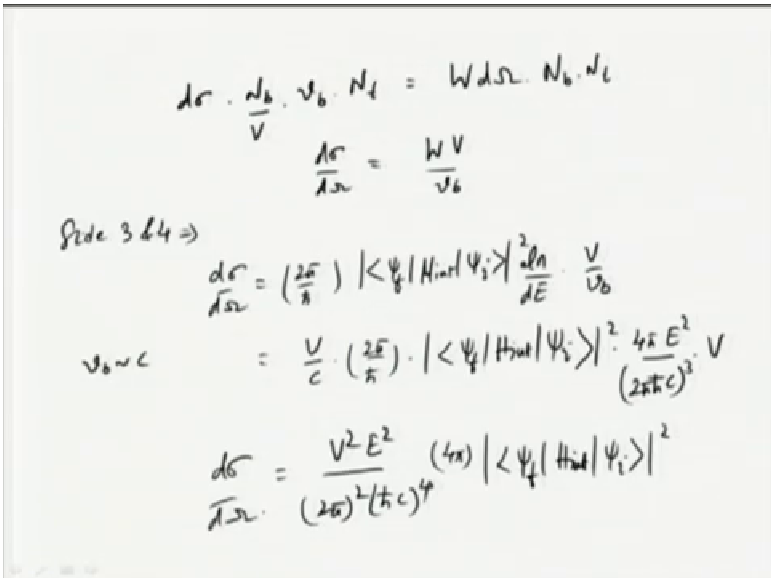

So, this now tells you that T sigma flux N b by V into v b into N t is equal to number of particles scattering into solid angle d omega which is W d omega N b N T or we can write d sigma over d omega equal to W V by v b.

Now, go to slides 3 and 4 to get W and write it as write then d sigma over d omega as 2 pi over h cross whether is this, there is W into psi f H interacting psi f square d N over d E into V by v b and dn by dE was given in slide 4, and let us approximate v b to c. So, that will give you V over c 2 pi over h cross psi f H interacting interaction Hamiltonian this should be psi i psi i square 4 pi E square dn by d is 4 pi E square 2 pi h cross c q c power 3 into V and the whole thing then becomes V square E square 2 pi square h cross c power 4 and a factor of 4 pi psi which we will worry about later H interaction psi i square. So, this is your d sigma over d omega what we call the differential cross section.

Unless we know what H interaction is we will not be able to proceed. So, let us look at the case of electrostatic interaction.

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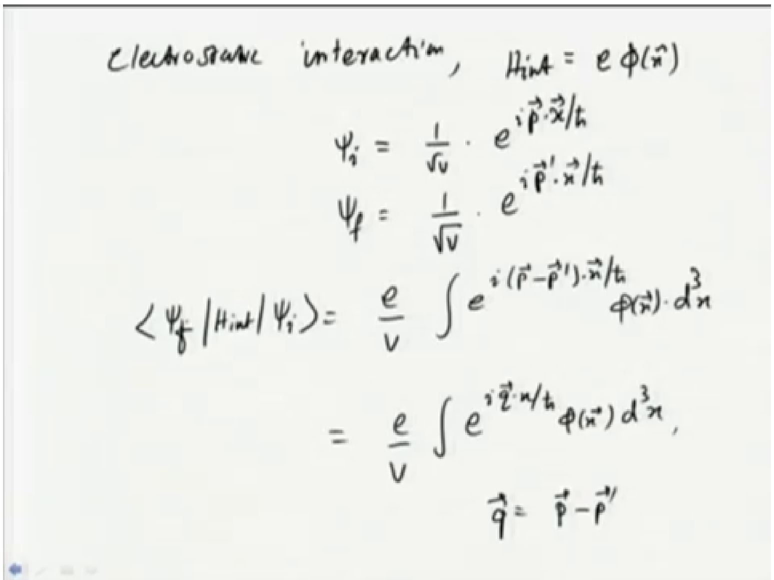

Electrostatic interaction between the electron and the nucleus say. In that case interaction Hamiltonian is e times the electrostatic potential and in quantum mechanics it is the corresponding operator file and we can write psi i as 1 over root V exponential i p dot x over h cross and psi f the wave function in the final state as exponential i p prime let us say the momentum is denoted by p prime in the final stage and that will give you psi i sorry psi f H interaction psi i equal to e over there are two under root 2 V's in the denominator we will give you one over V. And the transition matrix element in the coordinate representation is exponential i p minus p prime dot x over h cross phi x, d 3 x or I can write this as e over V integral e power i q dot x over h cross phi of x d 3 x where q is the momentum transfer, change in the momentum.

Now, we have to understand what this phi is or we will actually do some algebraic manipulation to get it in a better form.

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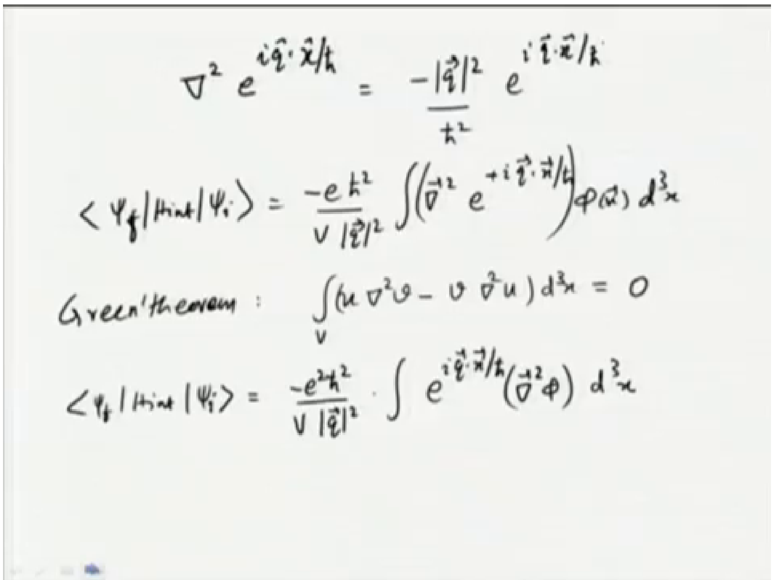

Let us consider the del square operator acting on a power i q dot x over h cross this will give you minus q square minus because of the i, there in the exponent over h cross square then, the same i exponential i q dot x over h cross. This then what we can do is to write E power i q dot x over h cross as h cross over q square with a minus sign times del square acting on the same object. That is what we will do to be, clear in a moment why we are doing this.

So, I can write psi f H interaction psi i as minus e h cross square over V q square integral del square well del is a vector operator sometimes I will put that vector sign sometime I will not put it, but you just see that that is actually del is a gradient which is basically a vector operator. So, when we say del square you understand that it is del dot del e power i q dot x over h cross times phi x. So, this del square acts only on this phi x d 3 x.

Now, we will consider Green’s theorem which says that if you have two functions scalar functions u and v then you del square v minus v del square u d 3 x over a volume v is equal to surface integral of u some sort of a surface integral which will actually vanish if u and v are 0s at the boundaries. So, let me say that again. The Green’s theorem says that if you have two scalar functions which are functions of coordinates x then integral over the volume u del square v minus v del square u d 3 x is equal to 0 if u and v vanishes at the boundaries, bounding surface any volume is bounded by a surface closed surface and if u and v vanish into the surface this right hand side of this is equal to 0.

In our case we have a situation where we consider everything happening in a large volume. For example, when you consider the tag scattering experiment everything happens in a in a small bound at in a small volume and then outside that all of these activities can be thought of to be 0. The electrons beam, electron beams the electron itself is present only in a finite volume and similarly the target, and the detection is also done enough in that volume. So, in that case if you take a sufficiently large volume that is what we have been considering when we said v as our normalization in psi and earlier also, we had that in mind, that we are confining everything in a large volume v. And outside that none of this or there is no probability for a particle electron or the target particle to exist either initially or finally. And this is why, this is valid for any u and v the only condition is that there should be 0 vanishing at the boundary surface.

So, look at the earlier expression for the probability amplitude. We have an integral del square exponential i q dot x over h cross. So, u can be thought of as exponential i q dot x over h cross and v can be thought of as phi x. So, I can actually swap this u and b and write it as psi f H interaction psi i is equal to minus e square h cross square v q square integral exponential i q dot x over h cross. Now, u del square acting on phi d 3 x this is as per Green's theorem.

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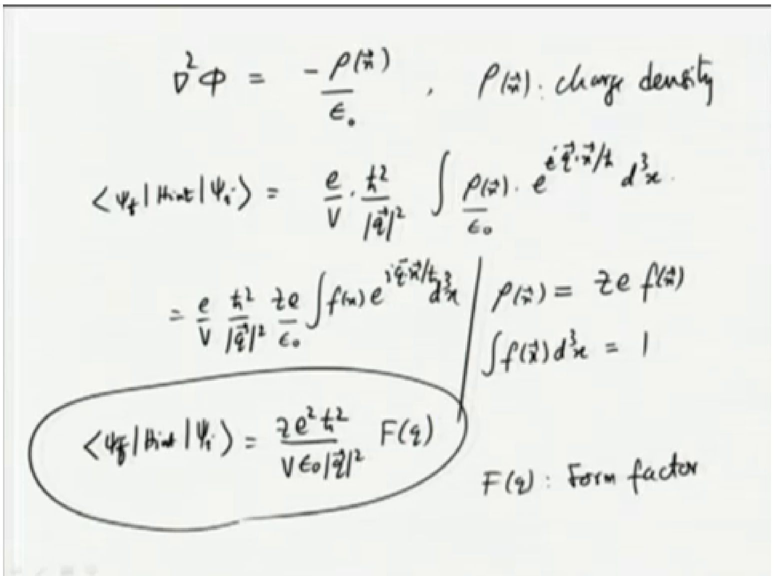

We are talking about electrostatics and therefore del square phi should be equal to minus rho over the charge density over epsilon 0, where rho this rho is different from the rho we considered earlier rho is the charge density electrostatic charge density. For example, for a point particle this rho would be a delta fraction.

So, then we can write psi f H int psi i equal to minus sign in del square f cancels with an already existing minus sign e over V, h cross over square over q square integral del square phi is rho x e power i q dot x over h cross d 3 x. Now, this is the electrostatic potential, the phi was is the electrostatic potential seen by the electron and that is due to the presence of the nucleus in our present case. And we know that the nucleus has a charge Ze. So, rho of x is equal to Ze. Question is whether it is a point particle which means it exists only at a particular point x equal to some x 0. In that case the density is the total charge times the delta function, otherwise we can actually associate a distribution function phi x with this which will tell you how the charge is distributed in space. Since total charge is Ze f of x d 3 x over the old volume is equal to 1, which is the normalization value.

So, now look at the earlier integral in the probability amplitude, that is that can be written as e over V h cross over q square Ze integral f of x e power i q dot x over h cross d 3 x. This integral, integral f of x e power i q x over h cross d 3 x is the Fourier transform of f x and since x is integrated out it is a function only of q. So, we will write it as psi f H int psi i equal to Ze square h cross square over V of course, I missed an epsilon 0 in all this. So, rho over epsilon 0 was there. V epsilon 0 q square integral over this thing I will write as F q the Fourier transform of this. And this F q is called the form factor of or basically this is the Fourier transform of small effects which is the spatial distribution of the charge density.

And look at the form factor. We know the Fourier transform of delta function as I said if it is a point particle the charge density is Ze times delta function in that case f x is basically delta function and Fourier transform of delta function is 2, integral delta x d 3 x is equal to I mean times sorry integral delta x e power i q dot x d 3 x is equal to 1. So, that is saying that in that case F q is equal to 1 and therefore, probability density comes down to Ze square over h cross Ze square h cross square over V epsilon 0 q square and the corresponding cross section d sigma over d omega is equal to V square over in E square 2 pi square h cross c 4 we had a factor of 4 pi which can then the probability amplitude square Z square e 4 h cross over power 4 divided by epsilon 0 square V square q square F q square.

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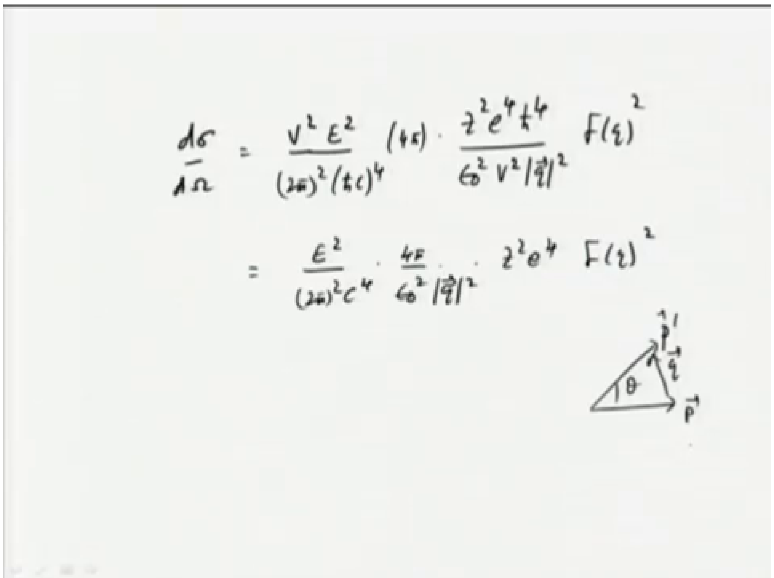

V factor cancels out h power 4 cancels out then and all this factors which will cancel out. So, you essentially have this as E square over c power 4 2 pi square c 4 4 pi epsilon 0 square q square and Z square E 4, F q square. And if it is a point particle if you consider it as a point particle then you will have F q equal to 1 and the rest of it will give you the Rutherford scattering expression, once you identify or rewrite the q in terms of the scattering angle. Remember q is p minus q this thing, so we can in fact, get that from there is this initial momentum and final momentum and q is p minus p prime. So, it is p minus p prime and from the reduction between these we will get q square actually equal to 2 p, if we consider the elastic scattering that the magnitude of p remains the same then you can take it will get that 1 over sin power 4 theta by 2 from q square.

I will leave that part to you to work out, but essentially we will get the Rutherford scattering in amplitude scattering cross section expression if we put F q equal to 1. So, what is F q? As we said it is evidently it is very clear that F q is basically the information about the charge distribution.

Now, we had to do an experiment we do not know what that is say a priori and then we do a scattering experiment d sigma over d omega can be experimentally determined and we will have to fit this experimental result for a different momenta etcetera, to the a right hand side of this expression and find out what is F q. And that will give you information about what is the charge distribution. And if there are clumps, there is a clustering there then we will get information about that clustering of charge distribution inside the nucleus. So, that is the idea.

So, we will get to that in the next discussion.