Nuclear and Particle Physics
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Module – 07
Elementary Particles
Lecture – 03
Quark Model-continued

So, we will continue our discussion on the Quark Model. So, yesterday we discussed the grouping of mesons, like pions under what is called the isospin. So, with said that we can consider pions was made of quarks and anti quarks and think about an isospin symmetry or isospin grouping under which a u quark and a d quark belongs to isospin equal to half. So, I equal to half, with u having I 3 equal to plus half and I 3 of u half u is plus half and I 3 of d quark is minus half.

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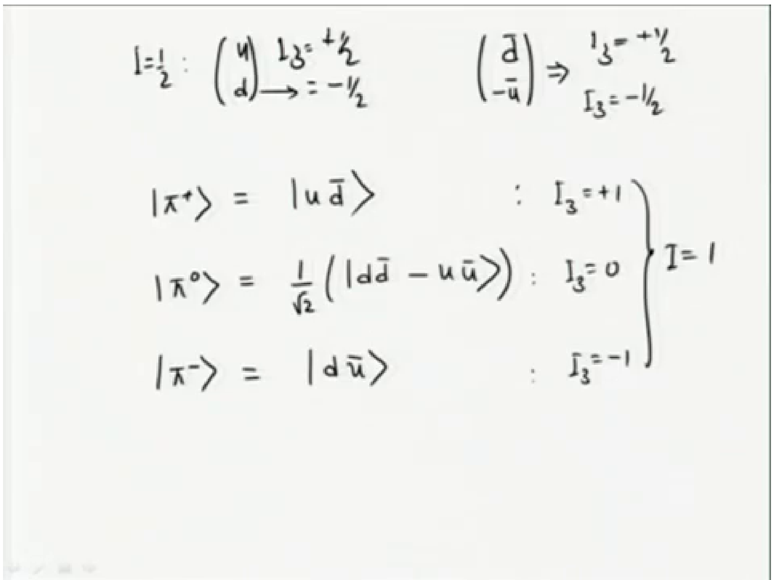

And similarly we can think about the anti quark isospin half group d bar and u bar and we said we will put a minus sign in whenever the wave function of u bar is considered.

This will give you I 3 equal to plus half for the d bar quark and I 3 equal to minus 1 by 2 for the u bar quark. With this we worked out that pi plus is actually a combination of u and d bar and pi 0 when you actually get from the pion wave function thought of us isospin 1, isospin 1 and isospin 3 1 by plus 1. Then we can actually in use step down operator to get the wave function corresponding to pi 0 and then that is what we did in yesterday and we got 1 over 2 d d bar minus u u bar. The minus sign is because of the minus sign associated with u bar in the I 3 equal to half multiplet and pi minus is again going to be d and u bar only one such combination possible.

So, this first one is our I 3 equal to plus 1, second one is I 3 equal to 0, and third one is I 3 equal to minus 1 all belonging to I equal to 1. This is what we discussed yesterday and now we said we can also think about other masons like pions.

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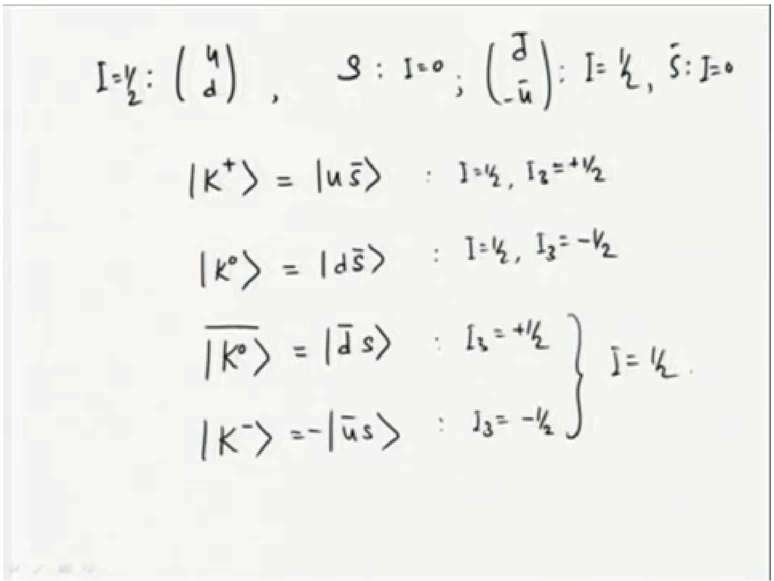

In that case what we do is we allowed to consider the fact that u d let me write that again belongs to an I equal to half group and S, the strange quark S quark belongs to an I equal to 0 group and similarly d bar u bar minus belongs to a and I equal to half and s bar belongs to an I equal to 0. So, with the this u d and s bar for example, we can consider u s bar combination.

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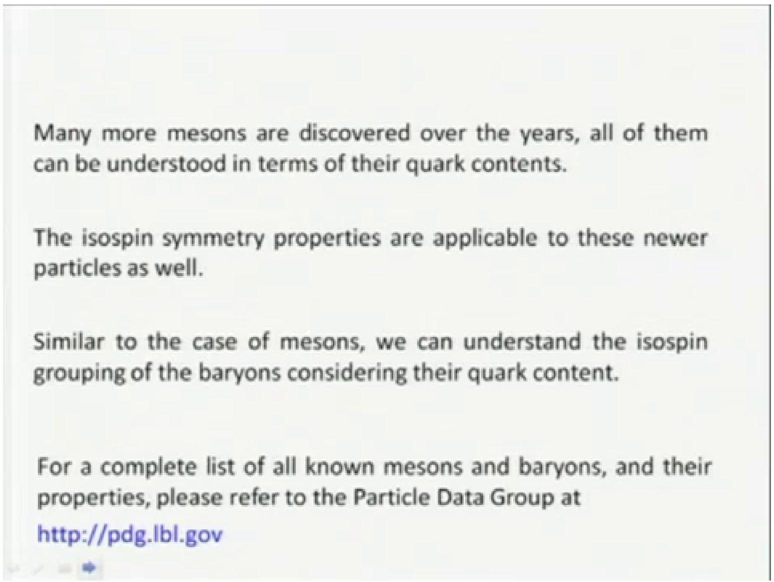

This will have an isospin half because S has and S bar as isospin equal to 0 u has isospin equal to half. So, the only possible result and isospin of the combination is half and I 3 similarly is plus half. So, we half I equal to half, I 3 equal to plus half for the combination combined u s bar system as well we associate this or we identified this as the k plus measure.

And from the other member of the quark doublet u isospin doublet u d be how d s bar which is again I equal to half, but I 3 equal to minus half because d has isospin third component d equal to minus half. And this the if you look at the electric charge is, it is a neutral particle we denote that as k 0 if you look at the mass of this you see that experimentally they are how the same must. We will come to some mass relations as we progress the quark model ok. So, this is when they, but then you also have a antiquark a d bar and u bar combination and the s quark. So, with that again you can make u bar s or let me take since we are talking about the isospin half I 3 equal to half first plus half first.

Let me take the d bar s. So, now, electric charge of d bar s combination is also neutral. So, I should be denoting this also as a neutral particle with a 0 there this means in the place of charge, but then this is different from k 0 it is actually the antiparticle of k 0. So, if you look at the quark content it is clear, that d goes to its antiparticle s bar goes to its antiparticle he said that the antiparticle is the relative concept in some sense. So, together d s bar goes to its antiparticle, so k 0 goes to its antiparticle if you take this change, and this is again I equal to I 3 equal to plus half system. And similarly you can half a k minus with u bar s you can put a minus sign because u bar is always there is a minus sign associated with u bar wave function. So, I 3 equal to minus half belonging to and isospin doublet.

So, if you look at again k minus it is the antiparticle of k plus because u has gone to u bar s bar has going to s. So, in a similar fashion we can actually think about other mesons also. So, there are many mesons which are discovered over the years, and all of them can be understood in terms of their quark contents hm. And we can apply the isospin symmetry properties the way we have done to all of them only thing is now you have to consider the isospin along with this one you have to also consider the c quark, b quark, c quark and b quark essentially t quark top quark does not really form about centuries discuss that.

So, basically considering the isospin of c quark equal to 0, b quark equal to 0 and s quark equal to 0 and an isospin of u quark is plus half with along with the d quark we can work out the isospin of any of the mesons and also different combinations which will combine together to give isospin multiplet of any mesons.

In exactly similar way we can also look at the baryons which are now made of only quarks not anti quarks. So, for example, if you look at proton you can see that if you look at u u u then sorry u quark and a u quark and a d quark then you will have a proton. And if you work out the isospin then you will see that u quark has an isospin third component is equal to plus half. Another u quark, so together they will have an isospin combination of plus 1 and then you have a d quark which has an isospin third component minus half.

So, together they have an isospin third component plus half. And you will see that you can actually work it out to get both proton and neutron belonging to one isospin group with I equal to plus half and I 3 equal to plus half for proton and I 3 equal to minus half for the neutron.

Similarly, for other baryons we can look at the isospin grouping and in fact, if you are interested a complete list of all the non mesons up to today is available at the particle data group which actually compiles all this information from various different experiments and keeps it in one place for our easy reference. The URL of the particle data group or pdg is given here. So, you can take a look at that then you will see that different mesons what there are other properties, data charges are what the quark content is, what they are isospin is, what they are other quantum numbers and other properties are when they decay what they do decay into etcetera. So, the lot of information including what we have discussed here and many more information; that we could not discuss in here also there so, please take a look at it.

Underlying these isospin we kept calling saying that the isospin symmetry at some point I said that there is some isospin symmetry and then we can group these particles under the symmetry. What is the symmetry? The symmetry is basically this and in fact, the whole thing originated from the fact that from the observation that neutrons and protons, although are electrically different and then will be behaving differently under electromagnetic interactions because proton has charged plus and neutron has no charge electrical charge. Although they behave differently under electromagnet interactions they behave the same way under the strong interactions this was one observation that people do found people made.

Now, this then said that in some sense they are they can be thought of as 2 different states of a single kind of an entity called nucleon as far as strong interactions are concerned. So, we can actually think about proton and neutron belonging to a nucleon the 2 states of a nucleon, in a sense this is similar to the spin up and spin down states of an electron.

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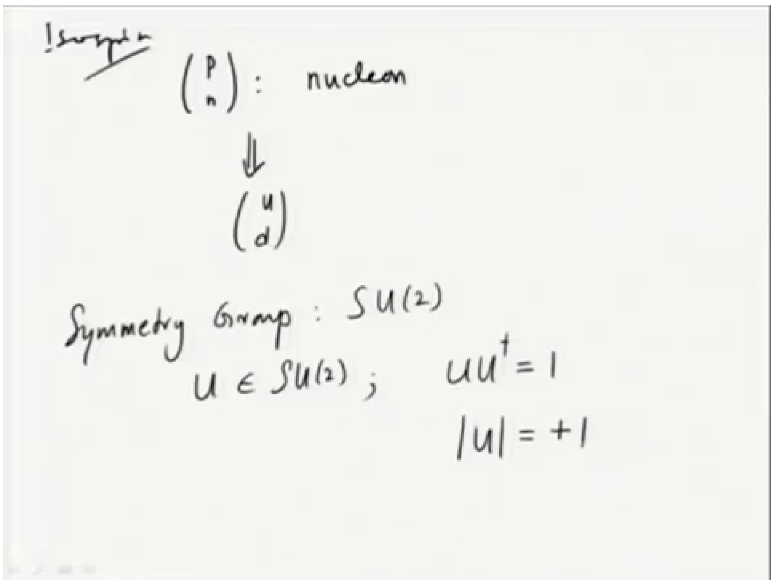

If you forever the spin part k spin splitting etcetera or the magnetic interactions of including the spin. Then electron whether it has an up type it is it has a spin orientation in the up direction with respect to some quantization direction or down direction opposite or along to the quantization direction. It behaves the same way for all purposes except for a for some special cases except in some special cases like magnetic interactions etcetera.

So, now it is this analogy that we can take again here and then this additional quantum number that will distinguish the nucleon thus called the isospin. And we can make a statement that the strong interaction since it does not distinguish between p and n, it is symmetric under isospin transformations, the sense that if you make a transformation from one state to another which differs in isospin states. It is not going to make any difference as far as the strong interactions are concerned. This observation led to is the underlying theme that brings out the concept of isospin symmetry.

And in the same way we can actually talk about u quark and the d quark and then say that the strong interaction does not really distinguish between the u quark and d quark in the sense and whenever there is a symmetry in particle physics or especially in physics in general, in the in physics we try to associate that symmetry with some groups it is an easy mathematical way of understanding symmetry.

For example, rotation can be understood through what is called a special orthogonal group rotation in 3 dimension can be understood with this case of with the help of special orthogonal group of dimension 2, s o 3 and the symmetry that is associated with the isospin is, the symmetry group of SU 2. What is SU 2? SU 2 is basically the following.

If you take the element an element of SU 2 any member of SU 2 this member is unitary, if you take that member and admission conjugate of that member say for example, in a matrix representation you take a matrix complex matrix the Hermitian conjugate of the same matrix and then that is going to be equal to the multiplication the product of these 2 is going to be equal to 1. They are called unitary remember unitary matrices.

So, the members of SU 2 are unitary in matrix representation it is a unitary matrix they are unitary matrices and special unitary is what the SU stand for and the special here means that the determinant of the member element is equal to plus 1. You can show that the determinant of any unitary matrix is going to be plus or minus 1, and if you take only those with determinant equal to plus 1 then you will get the special unitary group. So, this is the symmetry group of isospin symmetry.

So, as we said the if you take p n or u d they belong to a doublet of p n or u and d they belong to the doublet of SU 2 isospin. And the strong interaction do not distinguish between the members of a particular multiplet whether it is p or n does not know, u or d you cannot distinguish and the iso in a strong interaction. So, let me write that here for you.

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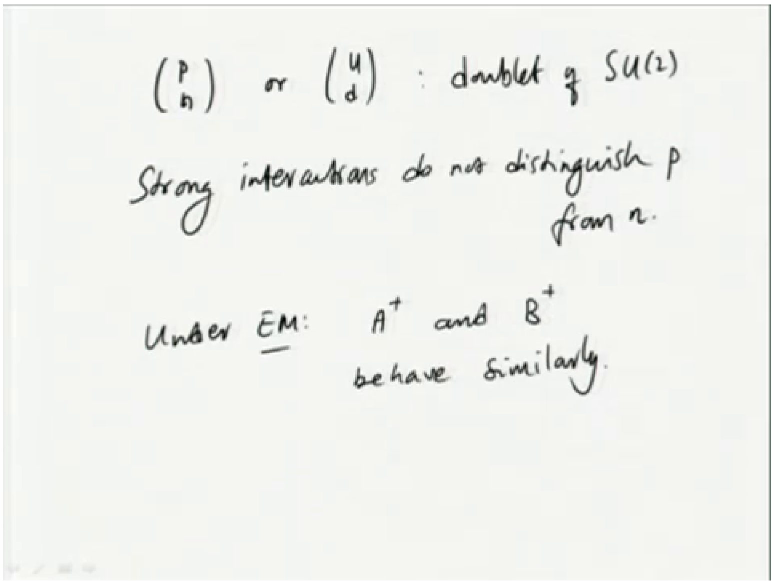

Strong interactions do not distinguish p from n or u from d. In fact, if you take a particular multiplet it cannot distinguish one member from the other under SU 3, I mean under the strong interaction ok. This is actually similar to the case as we said that the spin case or the other way of saying is that another example is if you take charged particle electromagnetic interaction of charged particles, if a particular particle say a has charged plus 1 and some other particle b has charged plus 1 their interaction electromagnetic interaction will be exactly the same with something else, ok.

If you take some background electric field put a charged particle of charge plus 1 and another particle with charge plus 1 say particle under electromagnetic interaction a particle a with charge plus 1 unit I mean and another particle with charge plus 1 unit behave in a similar fashion, which means that if you take f a put it in an electric field and take b and then put it in an electric which we put it in the same electric field they will experience exactly the same kind of force ok.

So, this is not a strange thing and then this means that there is a particular quality or quantity of quantum mechanical or I mean property which called charge electric charge and then that is the only thing which will distinguish the particle nothing else us such. Similarly there has to be something which actually is associated with the strong interaction we call this the charge strong charge over the strong color charge and that is the only thing to distinguish this vertical and then it is not the electric charge say there that will distinguish this. So, as far as that is concerned in a u quark and the d quark behave in a similar fashion. We will come to the color charges later.

Now, there is another way of grouping these quarks which is called a flavor symmetry ok,.

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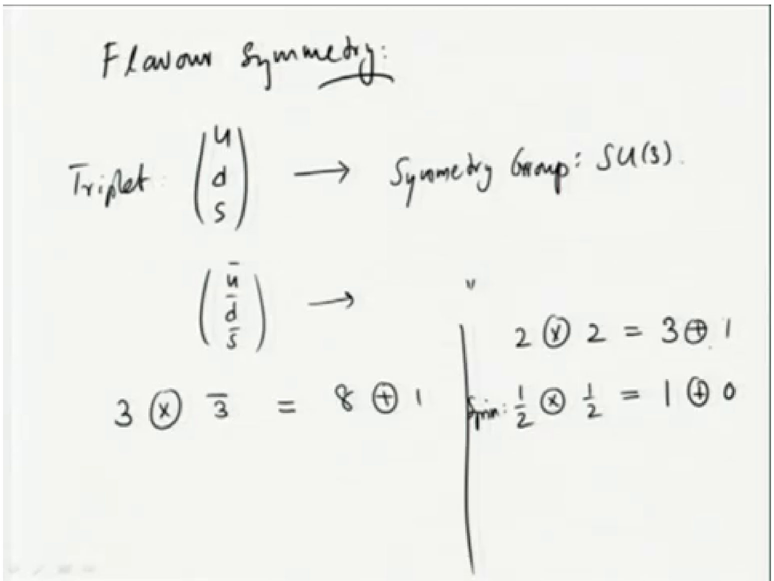

Which says that the if you take a u quark a d quark and an s quarks, you can put them in a particular group ok, and this flower symmetry I mean the transformation within this it is not going to make any difference as far as the strong interactions are concerned. With this we can associate the symmetry group of SU 3.

Now, in a similar way we can also take u bar, d bar and s bar and then put it under SU 3, and this is called a triplet under SU 3. Similarly a this is a a a u bar, s bar, d bar is a triplet under SU 3. Now, when we combined this to make quarks if you take a 3 triplet let me denote the triplet by 3 and an antiquark triplet by 3 bar under the SU 3 we can understand this, as a combination of an 8 under one. The objects that you will get the result of this combination will actually be can be classified under a group of 8 and 1 standing alone. What do I mean by this? This is something which we just now earlier said think about the spin half object which is a doublet in a spin half of the multiplet, you have spin up or spin third component of the spin as plus half or minus half there are 2 members. So, that is why it is a doublet.

And take another spin half combine them you will get one spin one particle which belongs to which is actually a triplet. So, this is spin half, cross half another way of writing it has spin here in the first case we are writing it as 2 cross 2 equal to 3 which means that 3 plus 1 ok, which means we are denoting this by the multipliers number of members in a particular group. And second way of writing it is basically denoting it by the spins spin half as a doublet for spin half cross spin half will give you a spin 1 and then spin 0, is ok.

So, spin 1 has 3 projections plus 1, minus 1 and 0, there are 3 values plus 1, minus 1 and 0. In the case of spin 0 there is only 1 such that things. So, that belongs to a singlet, one multiplet of member one and spin 1 belongs to multiplet of 3 which is triplet. In a similar region fashion if you do the algebra and consider the SU 3 grouping then 3 cross 3 bar is going to give you an 8. Remember a groove or a multiplet of 8 members and 1 singlet. This should add up 3 into 3 is 9, so that is equal to 8 plus 1. Similarly for the spin case 2 into 2 is 4 and 3 plus 1 is also equal to 4 ok.

So, now, let us take u d s find out what are the different type of mesons you can make with a u bar and d bar and s bar under the flavor.

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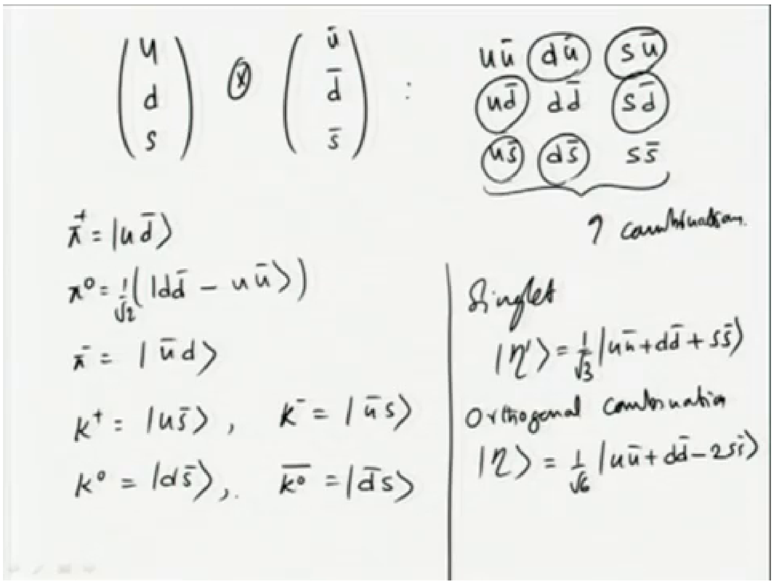

So, this tells you that you can have a uu bar you can have a ud bar and you can have a us bar, you can have a dd bar, you can have a ds bar, you can have an s s bar, you can have d bar u sorry du bar du bar and you can have a su bar and an sd bar. There are 9 such possibilities of course, so 9 combinations.

Now, how do you make it an 8 and a 1? We saw that ud bar is nothing but a pi plus and the pi 0 is 1 over root 2 uu bar, dd bar minus uu bar and pi minus is, pi minus is a u bar d ok. So, these are taken up actually then you see that ud bar is taken over d bar is taking let me circle that and then u bar d is also taken by pi minus dd bar and a combination of dd bar and uu bar is also considered in that this things. So, there was the another combination of uu bar and dd bar or along with something else which will come up that is another one.

And you have a k plus which we had considered, which is us bar us bar and k 0 was ds bar then you had k 0 bar was or k 0 k minus u bar s k 0 bar is equal to d bar s ok.

So, these are already identified you know this thing now we already have one particle which has electric charge equal to 0 which is on one another one when particle pi 0 which is taking up the combination dd bar and uu bar there is one singlet that we had to make as per our split of 3 cross 3 equal to h plus 1 using uu bar, dd bar and ss bar so that singlet should be treating us and d in a similar fashion. So, let me call that singlet us eta bar and write it as uu bar plus dd bar plus ss bar in a taking it in exactly the same footing compared to each other. Normalization constant is 1 over root 3, so that eta bar eta bar dot products or be normalized eta bar is normalize the wave function is normalized to 1 over 3 root sorry 1.

Now, an orthogonal combination of this is given here pi 0. If you look at pi 0 and eta prime they are orthogonal to each other ok. Now, if you take you have to may one more orthogonal combination and that can be written as, so orthogonal combination which is orthogonal through both eta prime and pi 0 is you have to have a y bar u uu bar and dd bar plus with a plus sign because that will make sure that it is orthogonal to pi 0.

And to make sure that it is orthogonal to ss bar, now that tear across the u and d at the same that will give you a 1 plus 1 which is 2, here to cancel that by 2 ss bar. So, that the whole thing is orthogonal to each other uu bar that will give you a 1, dd bar dd bar will give you another 1 and minus 2 ss bar has put into 1, ss bar will give you minus 2. So, that is orthogonal to the combination eta prime.

So, thus we have 9 combinations all right. So, 9 combinations out of uu bar, u d s and u bar, d bar, s bar. And since we have put them in different multiplet us let me collect those things and then put it in a group of 8 and a group of 1. How and a conventionally what is done is you actually put it in a plane of in a plane of I 3 and S, where S is called the strangeness ok.

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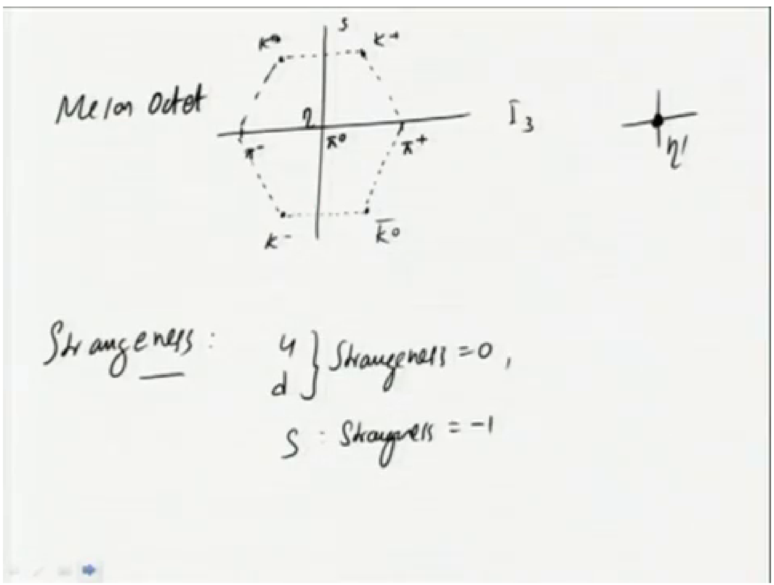

So, since we did not make this especially a explicitly earlier strangeness, similar to the ss been you will also assign what is called a strangeness quantum number. So, the strangeness is a sign so that u quark and d quark has strangeness equal to 0 and s quark has strangeness equal to plus actually minus 1. So, with this we will see that if you put in a plane of I 3 and S the pions are made of u quark and q quark alone. So, strangeness is 0, so they should be coming in the I equal s equal to 0 plane line and I 3 equal to plus 1 0 and minus 1.

If you take k plus, k plus is an a the quark content of k plus is us bar. So, the strangeness of this is plus 1 because strange quark has strangeness is equal to minus 1 and s bar the strange anti quark has strangeness equal to plus 1. So, k plus will have a strangeness plus 1 and k 0 will have a strangeness plus 1. And they have isospin third component of the isospin equal to plus half and minus half respectively. So, we have k plus and k 0 and similarly you have a k minus and k 0 bar with strangeness is equal to minus 1 because the quark content has an S in it. So, they have an S in it. So, let me put a dotted line to join these.

So, you if you have 4 plus 3, 7 members here and you put the eta which is not a singlet along with it. And this makes a group of 8 and then this is called the meson octet. And the singlet eta prime in the I 3 s plane stays alone at I equal to 0 I 3 equal to 0, s equal to 0 position because if you look at eta prime it has uu bar, dd bar strangeness equal to 0 and a term with ss bar again strangeness is 0 there because s quark a strangeness minus 1 and s bar has strangeness plus 1. So, this is how this grouping goes.

So, we have a meson octet which we have somehow supported the why we put it in an octet in some fashion we have sub somewhere of a support from the quark model where we can think of the s, u and b quarks belonging to a triplet of SU 3 and then similarly for the anti quarks and then make a combination of these v quark and anti quark and then see that they actually can be thought of as a group of 8 and a group of 1.

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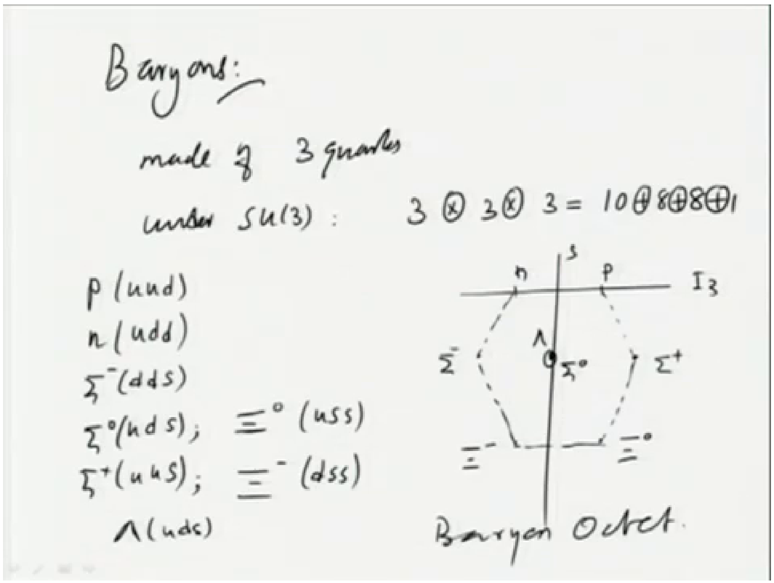

Baryons, we can actually think about the other mesons also in a similar fashion put them in different groups multiplet us is different multiplet us of under the SU 2, SU 3 a grouping. But now coming to the baryons we will be able to behalf they are made of 3 quarks. So, we should be taking under SU 3 of flavor you should be taking a member from a triplet, and a member from a triplet, and a member another member from another triplet.

Then see that group theoretically this 3 cross 3 cross 3 will go to a 10 and 8 and then 8 and then one. So, you will have a group of 10 a group of 8 a group of another group of 8 and a singlet ok. So, with u d s groupings and we can actually work it out in exactly the same way as we have done earlier and we will see that.

So, I will write only the quark condense here not exactly the wave functions. So, quark content of p the proton is uud and quark content of neutron is udd. So, again in a plane of I 3 and strangeness group, so they will come at ok. So, let me, so this is your I 3. So, proton and neutron are spin isospin half and I 3 equal to plus half for proton I 3 equal to minus half for neutron strangeness is equal to 0 for this thing. And then you can actually you have another set of these baryons which are called sigma baryons there are 3 different types of them sigma d sigma minus sigma 0 uds and sigma plus which is uus.

All of them has one strange quark in it and therefore, the strangeness is minus 1. So, strangeness minus 1, isospin half isospin minus half is isospin 3 third component of this is minus half a sigma minus and sigma 0 has isospin 0 and sigma plus has isospin third component of isospin equal to plus 1.

And then there are 2 other baryons which are called cascade 0 which can be thought of as made of a u quark and the s quark, ss u; uss and cascade with a charge minus 1 unit which is dss. And they have isospin their component plus half which is cascade 0 and minus 1 unit, which is cascade minus. And along with that you have a another this one called with I equal to 0 which is u d s the lambda quark. So, these things together ok, so lambda will also come there, can be thought of as a multiplet of 8 ok, and when you put it in a I 3 s plane they will are not be arranged in this fashion. This is called the baryon octet.

We indeed have another baryon octet and the 10 is also there we will discuss that in the next class.