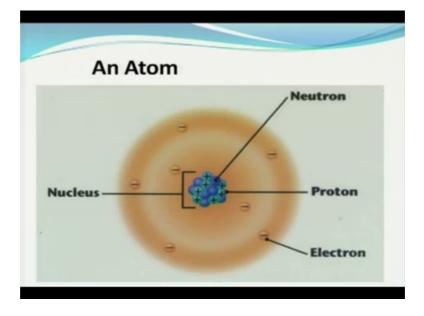
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Module – 01 Overview of Nuclear Properties Lecture – 02 Nuclear Properties

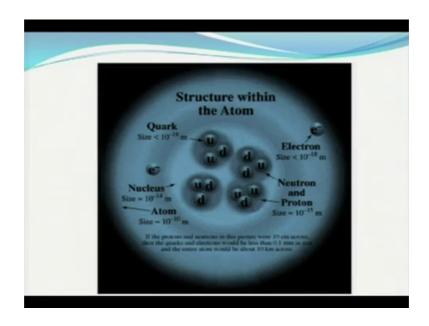
Today we will look at the structure of the nucleus and try to understand some of its basic properties.

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You are all familiar with the structure of an atom the modern picture of an atom tells us that there is an core called the nucleus, which contains all the positive charge and almost all the mass of the atom. This nucleus has positively charged particles called the protons in it, and electrically neutral neutrons and in the first part of this course it is this core called the nucleus that will occupy our time. And in an atom which is electrically neutral there is equal amount of negative charge in it, and the negatively charged electrons are all distributed in different atomic orbitals around this nucleus.

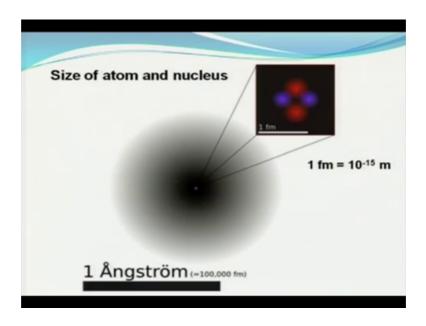
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When we probe the interiors of the nucleons by nucleon I mean either a proton or a neutron. Neutrons and protons together we will call nucleon. So, when we look at these nucleons closer, we will see that or we they are made of smaller particles or they have further structure, and they are further made of what is called the quarks. For example, a proton has 3 valence quarks which are denoted by u, u and d in this picture and similarly neutron has 3 quarks again, but there are 2 of d type and one of up type.

In the second part of this course when we discussed properties of elementary particles, we will discuss more about the quarks and to their interactions and other bound states that the quarks form etcetera. Here in the first part of this course where we discussed the properties of the nucleus, we will be discussing the nucleus at the level of nucleons and their interactions. Electrons on the other hands are structureless as far as we know.

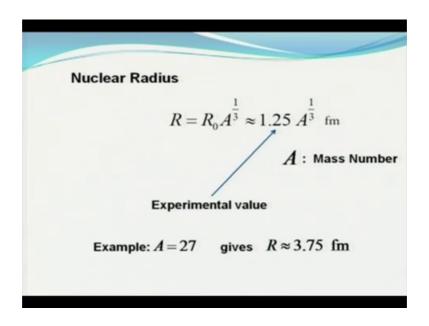
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Let us first look at the size of these objects; you may be familiar with the size of atoms the atom has something like an angstrom size of radius. An angstrom is 10 power minus 10 meters. So, it is a very tiny object when we compare with the usual world objects, but when we compare the nucleus with the atom its much much tinier compared to this atom. So, the size of a typical nucleus is about 100, 1000 times smaller than the size of an atom. So, it is in meters about 10 power minus 15 meters or a few times 10 power minus 15 meters in size.

Protons are slightly smaller than an Fermi or slightly smaller than 10 power minus 15 meters, similarly neutrons slightly smaller than 10 power minus 15 meters, something like 0.8 to 0.9 times 10 power minus 15 meters. Electrons are very tiny again something like 100 times smaller than the protons and neutrons.

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So, put it in your perspective, I will give an analogy comparing it with blowing it up and then comparing it with the sizes of objects around us.

So, if you blow up the size of the nucleus to the size of a football, which is something like 10 to 20 centimeters in size, a few times 10 centimeter. Then the size of the atom will be 10 kilometers these 100, 1000 times the size of the nucleus which is 10 kilometers which is the size of a city.

So, if you consider the atom to be the size of a city then the nucleus is a very small football sitting at the center, and this contains all the positive charge and almost all the matter in it. So, we can actually say that the atom is or atom has a lot of empty space in it, and almost all the matter is concentrated in a tiny volume and the rest of it is all empty space with electrons distributed in that empty space, and the electrons in this picture will be of the size of one or ones millimeter or smaller.

Another important thing that we like to realize when we discuss the size of these objects is that we are talking about quantum objects with quantum mechanical properties. Therefore, with the inner and uncertainty of the quantum mechanical measurements or quantum mechanical definitions of size etcetera, we will have to be careful in defining what we mean by the size of an objects. For an atom it is relatively easier to define the size, we know the electrons are distributed around the nucleus in an atom. So, take the outermost electron find out what is its average position compared to the center of the atom or in a coordinate system with origin at the center of the atom, we can ask what is the expectation value of the position of the outermost electron and that will give us an estimate of the atomic size.

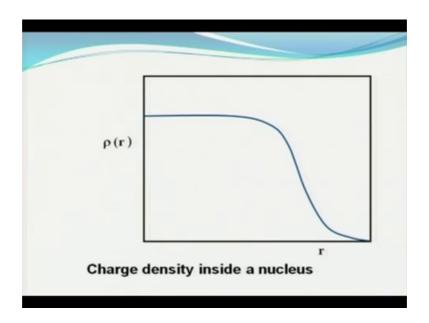
In the case of a nucleus, we do not have such conveniences. So, we will have to actually depend on other ways to define the size of the nucleus. So, what is usually done is to consider the matter distribution or the n charge distribution, electrical charge distribution of the nucleus and ask the question what is the size of this distribution which will give us an estimate of the size of the nucleus and experimentally we can probe the charge distribution of the nucleus by for example performing scattering experiments.

If we perform a scattering experiment with negatively charged electrons at high energy with a large energy, bombarding on the nucleus then there is a possibility that these electrons will go into the nucleus probe inside the nucleus and the give us information about the charge distribution there similarly we could also find out the mass distributions using the scattering experiment and such experiments reveal that for almost all the nuclei for a large range of the nuclei there is a relation between the radius and the mass number that we could establish.

So, the relation is this, the radius R is found to be proportional to one third power of the mass number, which we denote by A. So, we could write R the size of the nucleus is equal to some constant R 0 times A power 1 by 3, and experimentally this constant is found to be something of the order of 1.25 into 10 power minus 15 meters or 1.25 femtometer. Femtometers incidentally is also called a Fermi. So, 1 Fermi is equal to 10 power minus 15 meters.

So, here if you take an example with example of a nucleus with mass number A equal to 27 cube root of that is equal to 3, and that gives the radius r equal to 3 times 1.25 Fermi which is equal to 3.75 Fermi.

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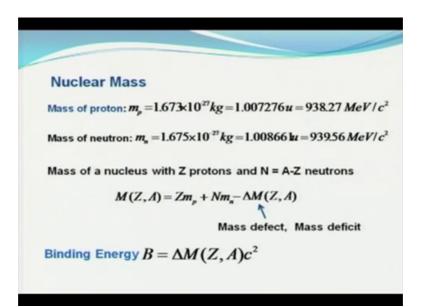
Another observation that we could make using the experimental determination of the charge density is the following. It s found that for most of the nuclei the charge density is found to be a constant in for smaller values of r. So, in the integers of the nucleus charge density is more or less constant for almost all the nuclei, and this charge density will fall off as r approaches the radius of the nucleus and follow rather fast, but smoothly.

So, this behavior that the charge density is almost constant inside the nucleus for most of the part except when it reaches the periphery has its implications on the nuclear forces. For example, the nuclear force has to be short range in this case, if it is an long range force all the nucleons inside the nuclei will be interacting with each other.

And therefore, as we go to the integers of the nuclear, we will expect that the charge density is larger there, but unlike that behavior is found that it is constant and therefore, the charge density is constant and therefore, we could say that the nucleons inside the nucleus interact with near space and not with all the pairs in the nuclei, this also is reflected in the binding energy which we will discuss in a short while.

So, the next property of the nucleus we will consider is the nucleus a nuclear mass.

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The proton has a mass of 1.673 times 10 power minus 27 kilogram very very small there are ways to find out mass of the proton, we will not go into the experimental details of that, but let us take this value and assume that the proton has this tiny must here. In to express such small masses we actually usually use other units similar to small distances the small length scales are expressed in terms of angstrom rather than meter. Here we have other units to express the masses of protons and objects of similar mass.

So, in another unit the proton mass is 1.007276 u, u is a unit at which is called a atomic unit in this unit carbon atom is taken to be with mass told you and everything else all other nuclear masses and proton and neutron masses are defined with respect to this standard it also roughly agrees with the atomic mass number and in another unit which is more often used in the particle physics studies.

We can express the mass of the proton in electron volt for c square where c is the speed of light in vacuum. Electron volt is actually the energy as a unit of energy; one electron volt is equal to 1.6 times 10 power minus 19 joules. This is also the energy that an electron with charge one points times 10 power minus 19 coulomb gains, when it is taken across voltage difference of 1 volts.

So, in this unit the proton has a mass of a 938.27 MeV, where MeV means mega electron volt which is 10 power 6 electron volts per c square. And mass of the neutron is similarly 1.675 times 10 power 20 is minus 27 kilogram or 1.008661 u which is equal to 939.56

MeV per c square. They are almost the same us, but neutron is a little bit heavier compared to the proton in a tiny bit.

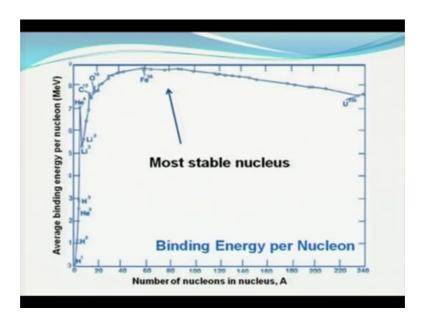
Now, when we consider a nucleus with say Z number of protons and n number of neutrons in it, we can estimate roughly the mass of the nucleus to be Z times mass of the proton plus n times mass of the neutron. Experimental determination of these masses differ a little bit from this slightly from experimentally, we can find out the mass of the nucleus by using spectrometer mass spectrometers. This is an arrangement with magnets arranged. So, that there is a high magnetic field in a region and beam of a particular nucleus or a mixture of many nuclei, with some kinetic energy some speed is passed through this region.

Now, we know the nuclei have charged electric charge and then electrically charged objects when passed through a magnetic a region with magnetic field will experience Lorentz force and therefore, take a curved path and so, are these bmo nucleus. And the radius of the path depends on the charge as well as the mass of the subjects and from this knowing the charge and experimentally measure finding and the radius, we will be able to find out the mass of the nucleus.

We will not go into the details of such a little mass determination, but we will take it that experimentally it is possible to measure the masses very accurate. And these measurements differ a little from the total number of the mass the total mass of the proton and neutron inside the nucleus and this difference if I denote by delta M is called the mass defect or mass deficient.

And there is a corresponding energy that we could reconsider delta M times C square which is called the binding energy and we will denotes this binding energy by B. So, of course, there is the nucleus with slightly smaller energy compared to the Z number of free protons and N number of free neutrons and that certainly is making it a more stable configuration compared to the free protons and neutrons of same number.

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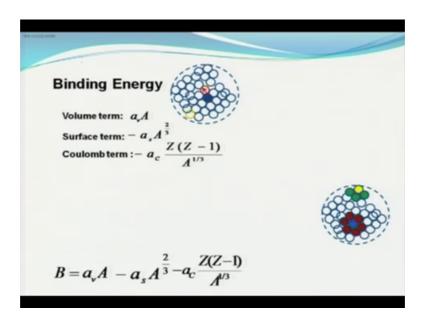


So, let us look at this binding energy a little more detail experimental determination of binding energy or the mass of the particles, we will tell us about the binding energy. And if we plot the binding energy rather the binding energy divided by the mass number A which will give us the average binding energy per nucleon. That has a behavior as shown in this plot.

So, for light nuclei with small a values binding energy per nucleon is small and it actually rises very first to a value of something like 8 MeV and then on it rises slowly from value of say a equal to 22 50 years 55 and then gradually falls down to value of 8 or. So, 8 MeV or. So, by the time it the mass number reaches a value of 200.

So, there is a peak in this binding energy per nucleon and that will tell us that there is a most stable there is a very stable configuration that we could find, well there is a very there are with a very stable configurations around the peak. So, it is found that the most stable nucleus first what I saw top of iron with mass number 56, and the binding energy per nucleon corresponding to this is about 8.8 MeV. So, we could see that for almost all the nuclei with a value starting from 20 onwards or 20 to 30 onwards, is almost a constant it is around 8 MeV and this tells a lot about the behavior of the nuclear force.

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So, the binding energy as I said this proportion is binding energy per nucleon is A constant and therefore, we could say the binding energy b is proportional to a and the proportionality constant I denote by a v which is a standard way of denoting this and this term is called the volume term. So, by and large we could say that the binding energy is equal to a b a constant, times a; the mass number. This is called the volume term because as we already said radius is proportional to 1 over 3 or 1 over third power of a and therefore, a is proportional to r cube which is the volume of the nucleus when we consider it as a sphere. So, this time is called the volume term.

Now, this proportionality to the mass number also tell us something about the nuclear forces, as I already mentioned when we talked about the charge density. This binding energy arises due to the binding forces or the nuclear strong nuclear force in between the nucleons inside the nucleus. Now let me concentrate on one particular nucleus nucleon inside the nucleus either a proton or a neutron; it interacts with strong interaction or strong nuclear force with other nuclei nucleons around it.

Now, let us take the nucleus another nucleon very near to this nucleon that we have considered. So, particular nuclear nucleon we consider will be taken here as the one with solid blue color, and that let us say interact with the nearest neighbor which is denoted here by a red circle, and in principle it could also interacts with other pairs say for example, nucleon denoted or represented here by an orange circle a bit away from this w

under consideration. So, if you consider all such pairs then we expect the number of such pairs for a number of nucleon is going to be A times A minus 1, and then we expect the binding energy the strong force of interaction or energy due to that is proportional to a times A minus 1.

But that is what is seen. What is seen is that the volume term for the binding energy is by and large proportional to A note A times a minus 1 and this tells us that maybe only the nearest neighbors interact in the nuclear interaction or take part in the nuclear interaction. So, it is a very short range interaction in that case the number will be proportional to a and that is what we see largely, but that is not the whole story there are corrections to this that we had to consider for example, one correction that we could think of is the following. Consider an in a nuclear in the integers that will have a certain number of nearest neighbors and it will interact with all those and give rise to binding energy.

But now when we consider the binding energy to be proportional to a we are thinking that the number of nearest neighbors for all the nucleons are the same, but that is not true when we consider a nucleon sitting at the periphery, at the circles we will see that that has slightly smaller number of nearest neighbors and this adds a correction to the binding energy.

This reduces the binding energy because the number of nearest neighbors is smaller than what we thought it is because of the surface effect. And it is surface effect therefore; we expect it to be proportional to the surface area and proportional to r square. And r square is proportional to a power 3 by 2 sorry 2 by 3 and therefore, we consider the surface term to be some constant as times a power 2 by 3 and this is a reducing effect therefore there is a negative sign to it.

Anything else yes when we consider the nucleus we have to consider one more thing in it, that it contains a lot of protons the protons are all electrically like charged objects, and therefore, they will interacts with the coulomb force and they will have a coulomb repulsive effect. And this repulsive effect will again reduce the binding energy. If we have Z number of protons in it in the nucleus then there will be Z times Z minus 1 pairs of this, and coulomb force is proportional to 1 over r and therefore, we expect this tempt to be proportional to Z times Z minus 1 divided by r which is equal to A power 1 by 3

which is proportional to A power 1 by 3 and the proportionality constant here is taken to be a c and effect is and reducing effect therefore, there is a negative sign.

So, the binding energy is or this with this coulomb term added to it, that is not all there are.

Binding Energy	Stable isotopes	z	N
5 57	Carbon(C):	6	6
Weberstein auf	Oxygen(O):	8	8
Volume term: a,A	Sodium(Na):	11	12
Surface terms = a 13	Aluminium(Al):	13	14
Surface term: $a_{,A}$	Calcium(Ca):	20	20
Coulomb term :- $a_c = \frac{a_c (z-1)}{a_c}$	Tungston(W):	74	184
1	A Gold (Au):	79	197
Surface term: $-a_{s}A^{\frac{2}{3}}$ Coulomb term: $-a_{c}\frac{Z(Z-1)}{A^{1/3}}$ Symmetry term: $-a_{p}=\frac{(A-2Z)}{A}$	Mercury(Hg):	80	202
Pairing term : $\delta = \pm a_p / A^{3/4}$			

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Other observations that we have made when we look at lot of atoms and many isotopes of this atom, incidentally and isotope I believe you are familiar with is atoms with same number of Z values, but with different number of neutrons in it are called isotopes. They do have the same chemical properties, but the number of nucleons or the mass number is different.

So, for a particular element say carbon we could think of different isotopes with the same number of protons, but different number of neutrons in it. Its proton number is 6, its neutron number could be 6, it could be 5, it could be 7 or some other number. And when people consider all these possibilities naturally occurring carbon atoms are found to be mostly the isotope with 6 number of protons and 6 number of neutrons in it, we face that that is the most stable isotope and that is the most favored configuration.

So, the binding that may be translated into a expression in the binding energy. This is not a single instant if we consider a large number of nuclei for atoms, we will see this behavior that for a particular Z value n is more or less equal to the Z value for stable nucleus. This is true for light and medium weight nuclei, but when we come to heavier elements. So, heavier nuclei say for example, something like tungsten with atomic number Z equal to 74 and number of neutrons equal 184 or number of neutrons equal to 180 or something of that around that value.

Again here we see that the Z value is not the same as n value, it is the number of neutrons is almost double the number of protons in it. This is true with almost all the heavy nuclei. So, to take care of this to take into account this observation term, which is usually which is denoted or called as standard symmetry term, and denoted or expressed as a minus 2 Z power 2 divided by A.

So, the numerator takes care that this term is non 0, when n equal to Z or a equal to 2 Z or any deviation from this symmetry between the number of neutrons and number of protons will add to this symmetry term will contribute to the symmetry term and we want this term to be less significant when we will take larger and larger values of a therefore, a one over a term is added to it. So, this term added the binding energy looks like this is there anything else that we want to take here.

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Binding Energy Volume term: a A Surface term: - a, A Z(Z - 1)Coulomb term :- a. Symmetry term : : Z and N are odd Pairing term : $\delta = \pm a_p / \Lambda^{3/4}$ + : Z and N are even 0 : odd A $B = a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z(Z-I)}{A^{1/3}} - a_{row}$ $(A-2Z)^2 + \delta$

There is one more term which is called pairing term this comes out in this fashion. When we consider a large number of nuclei when we study an odd number of isotopes, then it is found that the nuclei with even number of protons and neutrons in it are found to be exceptionally stable they are very very stable compared to other nuclei. So, something is actually telling us that if the number of neutrons are even and the number of protons are even, or the number of nucleons in general are even then there is going to be a strong binding energy there, and it is also found that if one of this is odd then that is still stable, but not as stable as the even nucleus. And if both are odd both the Z and n are odd values then they are very likely to be unstable, they are very unstable compared to the other nucleons. This behavior is incorporated into this binding energy expression by adding what is called a pairing term.

So, this term is equal to in magnitude ap some constant divided by a power 3 by 4 some kind of value empirical estimate and it is positive when both Z and n are even. So, we want to have we have observed that the binding energy is larger when both are even. So, it is a positive than, then adding constructively to the binding energy, and it is negative when both are odd and it is equal to 0 when a is odd or only one of the Z or a n is odd and the other is even.

So, that gives us the estimate of the binding energy, in this we have to find to get this expression we have relied on some kind of theoretical arguments as well as a lot of experimental observations. So, this formula what this expression is called a semi empirical relation for the binding energy.

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Binding Energy

$$B = a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z(Z-1)}{A^{4/3}} - a_{sym} \frac{(A-2Z)^2}{A} + \delta$$
Experimentally ...

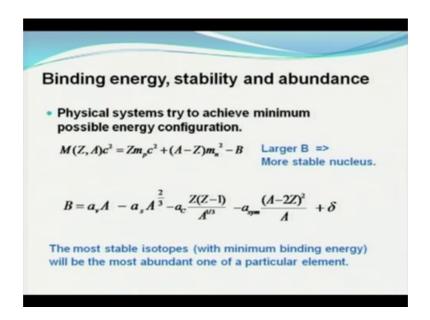
$$a_v = 15.6 MeV \qquad a_c = 0.72 MeV \qquad \delta = \pm a_p / A^{3/4}$$

$$a_s = 16.8 MeV \qquad a_{sym} = 23.3 MeV \qquad a_p = 34 MeV$$

In fact, we could take that and then add to the mass the as expressed earlier, and that will give us a semi empirical mass relation or mass formula.

Now, experimental dissemination of these constants tell us that they have these values, av the volume term the constant in the volume term is equal to 15.6 MeV all of these calls energy units, a s the surface term has 16.8 MeV value, s e is approximately or more or less equal to 0.72 MeV small and symmetry term is 23.3 MeV and the ap constant which appears in the delta the pairing term is 34 MeV.

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Now, the binding energy and stability of the nuclei we already mentioned this that so, but let us discuss that again to make it clear any physical system that we have seen has a tendency to go to the minimum energy configuration, and that is also some kind of a behavior of many of the human beings also.

We want to minimize our energy by sitting idle somewhere, almost all the physical systems I mean. In fact, all the physical system has this tendency. So, if you look at the nucleus energy is mass times C Square which is equal to m c square equal to Z m p C square plus number of neutrons a minus Z times mn c square means to see there. So, it is mn c square minus the binding energy. So, this formula we already saw earlier.

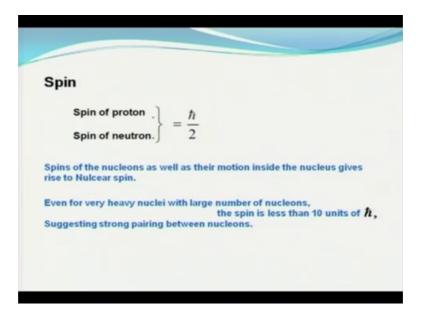
So, mn is smaller if B is larger; B is larger main larger binding energy server and we already said the physical system tries to achieve minimum energy configuration as much as much as possible and therefore, larger the binding energy the more stable the nucleus is. So, binding energy here we express again is given by this and we can. In fact, find out for a particular value of a what is the minimum, what is the value of Z that will give the

maximum value of B or the minimum energy configuration for the nucleus and the most stable configuration for the nucleus.

So, for a given a which element will have will be the most stable one we can actually take this expression of b take the first derivative of that equate that to 0 and that will tell us that for the value of Z corresponding to that we will tell us that that is an extremum and if we find that the second derivative of b with respect to Z is negative corresponding to this Z value then we could say that that corresponds to the maximum b value.

And another related thing is that say if naturally among the naturally occurring isotopes it will be the most stable isotope which will be most abundant natural right all right. So, we will not really talk more about this at the moment you may come back to it at a later stage in the discussions. So, let us go to the other properties of the nuclei.

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Another property of the nucleus is the spin of the nucleus. We know the spin of the proton is half in units of h cross and spin of neutron is also half in units of h cross, where h cross is the reduced Planck constant I believe you are all familiar with this in the from some introductory quantum mechanics course. And you are also familiar with the concept of spin and you may also be familiar with the spin half objects like electrons which are called fermion and which behave collectively in a particular fashion according to the Fermi is statistic for the rules of the Fermi statistics.

And there are other objects with indeed year multiples of h cross for its for their spin, and they are called bosons and the bosons behave in a different way when they are taken in a collectively and they behave according to the rules of Bose Einstein statistics. They are called bosons half integer spin particles are called fermions and therefore, we could say that protons and neutrons nucleons in general are fermions. So, collectively taken together these protons and neutrons in the nucleus will give rise to some total angular momentum.

You know the spin angular momentum addition rules that if you take more than one objects with spins or angular momenta, you can consider the collectively as a system you can think about a total angular momentum of the system. Usually vector addition of the angular momentum will give us the results of the total angular the values of the total possible total angular momentum.

Similarly, here when we consider many protons and many neutrons with spins spin angular momentum the internal angular momentum then they will add according to the rules of the angular momentum addition and you rise to a total angular momentum. This total angular momentum is called can be called a spin of the nucleus or nucleus considered as a single objects, internal spin internal angular momentum it is called the spin of that object.

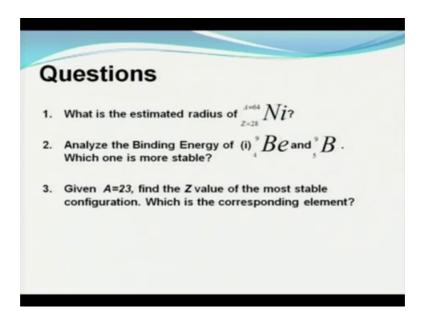
Not only the spins of this protons and neutrons, but also a relative angular momentum of these objects, if they are in motion in the inside the nucleus they may have relative angular momentum and all these relative angular momentum has to be also considered when we consider the total angular momentum of the system. So, all of these together we will give rise to spin of the nucleus. So, we can think about the spin of the nucleus and then define that and then study this and then manifestations of these can be observed.

Now, one particular observation in this regard is the following. We said there are nucleon nuclei with very large number of nucleons for example, if you consider tungsten it was 74 protons in it, and something like 180 or 200 neutrons in it this if you have huge number. And if you add all the spins there is possible to have very large spins very large values for its spin, similarly there are uranium, gold, gold with 79 protons and silver or uranium, led etcetera very very heavy nuclei very very large number of protons and neutrons in the in this nuclei.

So, we may expect very large values for the spins of the nuclei, when we add up the spins of all these protons and neutrons. But what is observed he is otherwise spin of any nucleus that is familiar to us is not more than 10 units of h cross, which is small compared to 200 or 300 which is a possibility in principle, when we take those many nucleons and add their spins and their relative angular moment orbital angular momentum that is what seen. So, this particular observation suggests very strong pairing between nucleons with cancellation of angular moment us cancellation of the spins of the space.

And we will see that some of the nuclear models effectively make use of this particular observation and in fact, to give a theoretical explanation in a very nice way, and this gives rise to magnetic dipole, electric dipoles or quadripoles electric dipole is not usually seen because of because of other reasons, but electric quadripoles they most of the nuclei have electric quadripoles, and we can study them study their behavior find out what the values of this magnetic dipole the magnetic higher multiples are and electric quadripoles etcetera, and that will also tell us a lot about the properties or the dynamics of the a nuclear forces, but we are not planning to get into the details of this in this course ok.

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So, that somewhat concludes our this session. So, let me pose a few questions before we actually conclude this. One as we said where is a relation between the mass number and the radius and we could find out for estimate the value of the radius of the nucleus given

what the given the value of A. So, here it is a very simple question just to demonstrate this, again take the nickel with mass number 64 and atomic number Z equal to 28. In this and in the lectures that follows in all our discussions in this course, we will denote an atomic nucleus in this fashion. We will take the symbol that usually represents the element here it is nickel for example, and put the atomic number as a subscript on the left side and the mass number as a superscript on the left side. So, that will that is how we are going to represent the nucleus.

So, find out what is the value of the value of the radius that you could estimate for this particular nucleus and then we again discussed the binding energy in detail. So, here is a question related to that, you take 2 nuclei with mass numbers the same A equal to 9 in both these the cases, but Z number is 4 and 5.

One is boron other is beryllium which one is more stable again you could find out what is the binding energy of this, and what is the using the semi empirical mass relation or find out what is the mass of that mass of these objects using the semi empirical mass relation, and find out which one has larger binding energy or smaller mass number the mass value see that will be the most stable one.

And now we have some other question related to again these binding energy, you take an element with mass number 23 or take many elements with mass number 23 with different Z values and n values, and find out which one is the most stable configuration what is the value of Z, which will give you the maximum value for binding energy B. I already told you how to find that out from the expression of the binding energy and if you find this Z to be equal to some fractional number or some value which is not a whole number then find out the nearest whole number and find out what is the element corresponding to that. Given the Z value element is fixed.

So, the element is nickel has Z equal to 28 whatever the value of A is. So, here 23 will have different Z values corresponding to different elements, we want to find out what is the most stable element with A equal to 23 all right.

So, in the next session we will discuss other aspects of the nucleus.