

**Nuclear and Particle Physics**  
**Prof. P Poulse**  
**Department of Physics**  
**Indian Institute of Technology, Guwahati**

**Module – 07**  
**Elementary Particles**  
**Lecture – 02**  
**Quark Model**


We had discussed the elementary particles and then the basic fundamental interactions that they undergo in the last discussion. In this we will further discuss the elementary particles some of their properties and also understand what is the meaning of antiparticle in some sense at some basic level. And further go on to the quark model which explains or gives some understanding of the different compound particles like protons, mesons, pi mesons etcetera that had been discovered discovered in both in cosmic ray is as well as in particle physics experiments.

So, to recap what we had discussed about the elementary particles and the forces in the last lecture. We said we could broadly classify the particles elementary particles into 2 categories, one is called leptons the other is called quarks.

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**Recap:**

**Table of elementary particles as we know them today**

Leptons			Quarks		
Electron Charge: -1 Mass: 0.5 MeV/c <sup>2</sup>	$e$	$e$ Neutrino Charge: 0 Mass: 0 ?	$\nu_e$	Up quark Charge: +2/3 Mass: 2.3 MeV/c <sup>2</sup> ?	Down quark Charge: -1/3 Mass: 4.8 MeV/c <sup>2</sup> ?
Muon Charge: -1 Mass: 106 MeV/c <sup>2</sup>	$\mu$	$\mu$ Neutrino Charge: 0 Mass: 0 ?	$\nu_\mu$	Charm quark Charge: +2/3 Mass: 1275 MeV/c <sup>2</sup> ?	Strange quark Charge: -1/3 Mass: 95 MeV/c <sup>2</sup> ?
Tau lepton Charge: -1 Mass: 1777 MeV/c <sup>2</sup>	$\tau$	$\tau$ Neutrino Charge: 0 Mass: 0 ?	$\nu_\tau$	Top quark Charge: +2/3 Mass: 173.5 GeV/c <sup>2</sup>	Bottom quark Charge: -1/3 Mass: 4.18 GeV/c <sup>2</sup> ?
<b>Force Carriers</b>			<b>Higgs Boson</b>		
Electromagnetic	Photons Mass = 0	$\gamma$	Mass ~ 125 GeV/c <sup>2</sup>		
Weak nuclear	$Z$ Mass ~ 100 GeV/c <sup>2</sup>	$W^\pm$	 Possible candidate Discovered in 2012 CERN - LHC		
Strong nuclear	Gluons Mass = 0	$g$			

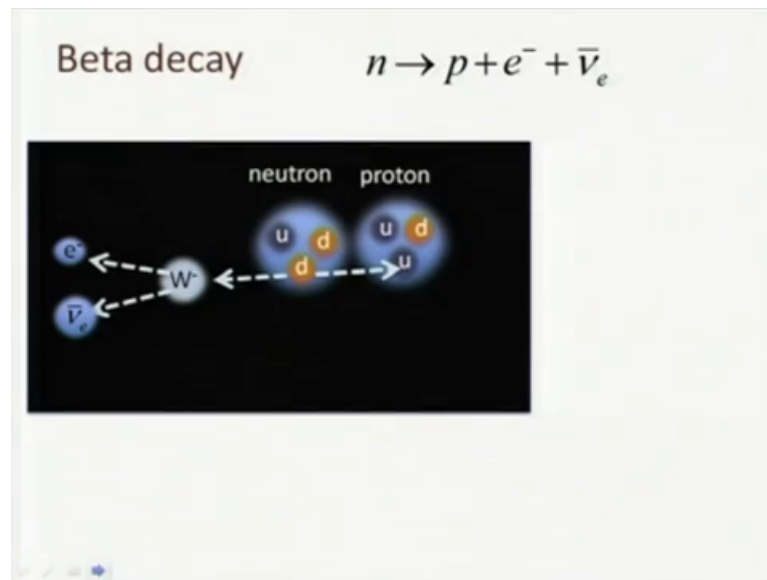
Leptons are the ones which take part in weak interactions as well as electromagnetic interactions if they are charged and quarks are in addition to the weak interactions and

electromagnetic interactions quarks take part in the strong interactions as well, whereas, the leptons do not take part in the weak interaction strong interactions. Further we said the forces these or the in forces the elementary particles experience or the dynamics of these interactions are dictated by what are called the force carriers we can think about elementary particles associated with these interactions. They are called first carriers or we will see later on that they are actually also called a Gauge Bosons of the particular interaction corresponding to the particular interaction.

So, for example, the electromagnetic interaction is carried through the exchange of photons. So, photons are called the force carriers of the Gauge Bosons corresponding the electromagnetic interactions. Similarly for weak nuclear interaction there are 3 such a Gauge Bosons one is a neutral Gauge Boson and the another one is and the other 2 are charged Gauge Bosons  $W^+$  minus all of them, all the 3 of them having mass around 100 GeV over  $c^2$  and strong nuclear interactions are carried over through the exchange of what is called gluons there are 8 different types of gluons and they are massless. And then we also further discussed the need of additional an additional particle an extra elementary particle call called the Higgs Boson within the standard model of particle physics.

Of course I should mention as an aside that there are other mechanisms which are proposed from different mechanisms actually which are proposed through which masses of the particles can be generated without or either without any elementary any scalar field or with some composite scalar field. But these models are not tested yet and the standard model also the Higgs mechanism of the standard model is also not tested completely in the sense that we have not proved or shown discovered the elementary scalar particle which is the Higgs Boson we did discuss an element a scalar resonance in 2012 around with the mass around 125 GeV per  $c^2$  and which is a very good candidate for the standard model Higgs Boson.

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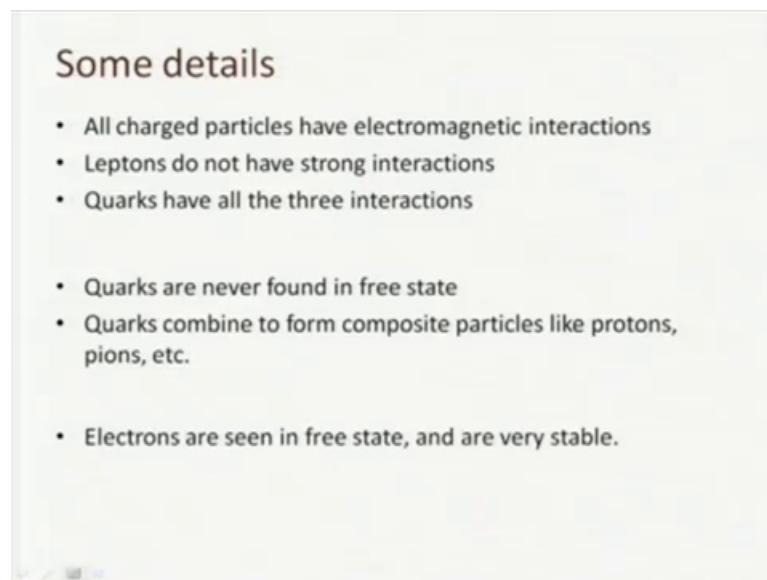


Moving on we will look at what is the meaning of element I mean this antiparticle straight. Let us look at the beta decay. Beta decay at the level of nucleons is basically a neutron changing into a proton with the emission of an electron and a neutral particle which is actually an anti neutrino.

So, although I said this anti neutrino we will come to know what is the meaning of this antiparticle and what is the difference between neutrino and antineutrino etcetera as we go. So, if I consider neutron going to proton the beta minus decay or a negatively charged particle which is identified as the electron very exactly the same exactly similar to the atomic electrons emitted. And we discussed how this process would actually go through the weak interaction, that d quark in a neutron a neutron has 1 u quark 2 d quarks, one of the d quarks convert itself into a u quark and a W minus remember W minus is much more massive compared to the d quark if you look at the mass that we have listed in earth's. But one thing that you can that can happen is that these W Bosons need not to how the mass of the W Boson or general real in W Boson. But you can actually think about a short duration in which this particle live and therefore, energy can be violated and you can think about a particle although it is actually in a real situation real particle should have a mass of about 80 Gev we do not have that much of energy in the d quark.

But it can actually convert itself into virtual  $W^-$  which need not to have the mass or of the real particle, k its energy can be smaller than that. This energy will be or this  $W^-$  particle decays immediately to an electron and an  $\bar{\nu}_e$  note you know. So, this is the electron that comes out of this thing. So, if you look at the charges d quark has minus  $\frac{1}{3}$  charge which decays into a u quark which has a  $\frac{2}{3}$  charge plus and  $W^-$  has minus 1 unit of charge we are talking about charge in units of electronic charge. So, the total charge in the final state  $W^-$  plus u quark together is minus  $\frac{1}{3}$  which agrees with the charge of the initial particle which was only the d quark that is fine. And then when  $W^-$  decays one of them one of the final products is electron having a charge of minus 1 and the other one neutral. So, charge is again preserved no problem all right.

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**Some details**

- All charged particles have electromagnetic interactions
- Leptons do not have strong interactions
- Quarks have all the three interactions
- Quarks are never found in free state
- Quarks combine to form composite particles like protons, pions, etc.
- Electrons are seen in free state, and are very stable.

So, these are some of the things that we said earlier. So, the quarks have all the 3 interactions in a quarks are never found in free stage this is another thing. So, now, we do not see the quarks in free stage, but quarks are always combined to form a composite particle in this. And this electron has a negative charge. Think about a similar particle, but with electronic charge positive.

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### Antiparticles

- “electron” with positive charge: **positron**
- Positrons have the same mass as that of the electrons.
- They are the **anti-particles** of electrons.

Similar to  $\beta^-$  decay (emission of electrons), there are  $\beta^+$  decays with positron emission.

Mass of proton =  $938 \text{ MeV}/c^2$   
Mass of neutron =  $940 \text{ MeV}/c^2$

Free protons cannot undergo beta decay.  
 $\beta^+$  happens only in nucleus (bound states).

neutron

proton

In fact, protons do not decay at all.

And particle which is very similar to the electron, but with electric charge opposite to the electron mass the same as that of the electron is found that is called the positron these positrons are called the anti particles of the electrons. So, I we saw that in beta decay electrons are emitted similar to this in there is what is called the beta plus decay in which you start with the proton and the protons is converted into a neutron and then W plus. Look at the charge again proton has positive charge one unit initially and neutron is neutral W plus has per unit of positive charge. So, that is conserved or if you look at the quark level a you quark from a proton is converted into a d quark.

So, that initially you have u quark to u quarks and one d quark and finally, you have one u quark and t do quark d quarks. So, the u quark initially has a charge of plus 2 by 3 this is converted into a quark d quark of charge minus 1 by 3 and W plus of 1 unit of positive charge. So, total in the initial and the final states the charge electric charges are plus 2 by 3 and this W plus decays to a particle which has the mass same as that of the electron and a neutral particle which is similar to the neutrino and the charge or an electric charge on the electron like particle is now opposite to that of the electron which is positive and this is basically the positron which is busy which is the antiparticle of electron. As although we said this if you look at the energetics mass of the proton is only 938 Mev per c square and mass of the neutron is 940 Mev per c square approximately free protons therefore, cannot undergo beta plus decay because initial energy available if will go to the rest

frame of the proton is only 938 which is less than the mass minimum energy required for the particle to the neutron to be produced.

So, this proton decay happens only within the nucleus. Beta plus decay is observed in nature there are nuclei isotopes which are unstable decaying through the beta plus emission or positron emission. You can think about positron emission tomography in fact, and ask yourself what exactly happens in that case. In fact, the protons do not decay at all it is not that protons do not decay through beta plus emission, but protons are very very stable and the whole construction of the universe the whole construction of everything that is there and all somewhat to this stability of the proton. Again think about what happens if proton would have been would not have been stable as it is and could have decayed fast.

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# Antiparticles

Electron	$e^-$	Positron	$e^+$	Up quark	$u$	Up antiquark	$\bar{u}$
Muon	$\mu^-$	Antimuon	$\mu^+$	Charm quark	$c$	Charm antiquark	$\bar{c}$
Tau lepton	$\tau^-$	Antitau-lepton	$\tau^+$	Top quark	$t$	Top antiquark	$\bar{t}$
Electron neutrino	$\nu_e$	Electron antineutrino	$\bar{\nu}_e$	down quark	$d$	down antiquark	$\bar{d}$
Muon neutrino	$\nu_\mu$	Muon antineutrino	$\bar{\nu}_\mu$	strange quark	$s$	Strange antiquark	$\bar{s}$
Tau neutrino	$\nu_\tau$	Tau antineutrino	$\bar{\nu}_\tau$	Bottom quark	$b$	Bottom antiquark	$\bar{b}$

Of the particle and antiparticle:  
 Mass and spin are the same  
 Electric charge is reversed

All additive quantum numbers are reversed.

Photon and Z are their own antiparticles.  
 Antiparticle of  $W^-$  is  $W^+$   
 Antiparticles of gluons are antigluons.

Now, that we said what is an antiparticle we will not go into the details of this. There are proper ways to hand understand the antiparticles what we can do is to compare the or to understand it through the wave functions in quantum mechanically, the wave functions of the antiparticles and then see asked question is how one goes from or one actually differentiates this particle and antiparticle wave functions etcetera.

So, but here we will not go into that rather we will just lists the particles that we know of elementary particles and their antiparticles. For example, electron is antiparticle is called positron which is symbolized as e plus instead of e minus for the electron. Similarly

muon  $\mu^-$  as an anti muon as the antiparticle which is denoted by  $\mu^+$  and tau minus has an anti part and the particle anti tau electron tau plus. Neutrinos electrons and neutron has an electronic anti neutron muon and neutron has a muon antineutrino and tau neutron then tau anti neutron. Up quark and the up quark, so up anti quark; charm quark, charm anti quark; tau quark, tau anti quark; down quark, down anti quark; strange quark, strange anti quark; bottom quark, bottom anti quark and usually the notation is that for quarks you put the letters u c d b d b's d s b and for the anti quarks you put a bar on top of the letters denoting this corresponding flavor type of the part quark.

Photon and Z particles are their own antiparticles. All these are consistent interpretations and consistent picture that a quantum mechanically you can work out and then you could leave it. But then anti particle W minus and W plus can be thought of as anti particles of each other. Gluons of anti particle quark called anti gluons there are differences thing. Of the particle and any particle we can say there as we mentioned like in the case of positron and electron mass remains, the same spin remains, the same electric charge is reversed there are other quantum numbers also which are reversed additive anonymous any of those otherwise.

We will come to a little later why in beta decay and antineutrino is produced not a neutrino is produced. If you remember the discussion where we actually necessitated the presence of an anti neutrino or a neutral particle either and so far as whatever we have discussed or if we consider in all the aspects of particle particles and their interactions that we considered so far there is no need to actually have and d neutral to be put in there or it could have been neutron or an anti neutron. But for reasons that we will explain in a little while it is an anti neutrino that comes there. And experimentally we have actually shown that it is not the neutron, but the anti neutrino that is there and there is a very definite way of experimentally distinguishing these two particles although the charge of these 2 particles are the electric charge are the same mass is 0 almost over mass is anyway the same for the same. There should be some other quantum number which distinguishes between these two we will come to or that quantum number is later.

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### Bound states of quarks (hadrons)

Quarks and antiquarks form bound states called **mesons**.

$$\pi^+(u\bar{d}), \pi^-(\bar{u}d), K^+(u\bar{s}), K^-(\bar{u}s)$$
$$D^+(c\bar{d}), D^-(\bar{c}d), B^+(u\bar{b}), B_s^0(b\bar{s})$$

Three quarks form bound states called **baryons**.

$$p( uud ), n( udd ), \Lambda( uds ), \Lambda^{++}( uuu ), \Omega^-( sss ), \Lambda_b( udb )$$

No bound state of top quark is found. The top quark (mass = 173.5 GeV/c<sup>2</sup>) produced in high energy scattering experiments decay before they could form bound states (**hadronize**).

So, let us come back to what we mentioned in the second slide, quarks are not free, but they are always found in bound states. Quarks and anti quarks form a bound states called form bound states called a mesons for example, we have pi plus positively charged pi on which is made of a u quark and an anti d quark or qua d anti quark. Pi minus on the other hand is made of a u type anti quark and the d type quark.

So, if you look at the quark contents of pi plus and pi minus you will see that basically instead of u quark it is a chip is there instead of anti particle anti quark d you have a d quark there or d bar use the antiparticle of d in that sense. So, now, therefore, pi plus is antiparticle of pi minus. So, one of them can be thought of as the particle if you consider one as particle the other is an antiparticle of it. These are actually mutually I mean there is nothing strange about antiparticles otherwise. So, therefore, no need to actually say which of this is the actual particle there those kind of questions do not make sense. It is only that one is an antiparticle of the other. But there are other aspects when it comes to normal matter we have seen only particles in the normal matter, you know we do not have particles normal might have made of anti particles at all we will come to this aspect again later.

So, similar to the pi on's pi plus and pi minus you also have a pi 0 we will mention that later. There is another meson called k plus positively charged k minus on or k on which is made of u quark and an s bar anti quark of type s. Similarly there is a K minus with

made of an which is made of  $u$  bar and then  $s$  and again  $K$  plus and  $K$  minus  $r$  antiparticles for each other.  $D$  plus is another mess on which is observed. So, contents or  $c$  quark a charm quark and a  $D$  bar  $D$  minus opposite to this and a  $c$  bar and the  $D$ ,  $B$  plus  $u$  and  $b$  bar,  $B^0$  is  $b$   $s$  bar.

3 quarks from bound states called variants. For example,  $p$  quark  $p$  is denotes proton which has a quark content of  $uud$ , then you have a neutron which has a quark content of  $udd$ ,  $\lambda$  has a quark content of  $uds$  another barrier. Then there are  $W$  charged baryons  $\Delta^{++}$ ,  $\Delta^+$ ,  $\Delta^0$ ,  $\Delta^-$ ,  $\Delta^{--}$ ,  $\Omega^-$ ,  $\Lambda^0$ ,  $\Lambda^+$ ,  $\Lambda^0$ ,  $\Lambda^-$ ,  $\Lambda^{--}$ . You notice that in none of these examples we have a top quark appearing. It is not because I just randomly chose the examples like this and that group did not have the top quark, but it is because the top quark cannot form bound states or does not have any bound state.

There are no two bound states of talk of top quark found. All other this think quarks  $u$   $d$   $s$   $c$   $b$  all of these, all of these 5 quarks found are found to form bound states of different masses, different properties, etcetera.

The reason for talk quark not forming bound states is the following. The mass of this is very large. So, it can be produced only in very high energy collisions high energy experiments and once these quarks are formed since they cannot be they cannot appear they cannot be live as a free particle, just to mention this particular property is called confinement, that quarks are confined to bound states, this is something which is slightly somewhat you will understood even today. So, the quarks cannot form bound states, top quark the quarks usually pick up other quarks from the vacuum or the other form bound states with other quarks available.

Top quark is produced and then it could also look for other quarks to form pairs or to form bound states of quarks or quarks and antiquarks, but before it gets this time to look around and then pick up this particle it decays, exactly a very heuristic way of saying in. It does not really have enough time to form bound states technically this is called hadronization that formation of the bound states. So, top quark decay time is so small that it does not have enough time to hadronize. So, that is the reason why we do not find top bound states of top quark.

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## Classification of hadrons

Many hadrons are discovered in high energy particle collisions, as well as cosmic rays.

Classification with the help of symmetry groups.

**Isospin:**  
Two quantum numbers,  $I$  and  $I_3$  (similar to the ordinary spin angular momentum and its projection along the quantization direction), can be assigned to the hadrons, so that these are conserved in reactions involving hadrons.

For example,

$$\pi^+, \pi^0, \pi^- : I=1, I_3=+1, 0, -1$$
$$K^+, K^0 : I=\frac{1}{2}, I_3=+\frac{1}{2}, -\frac{1}{2}$$

Now, there are lot of hadrons that we have discovered over the years in high energy collision experiments as well as in cosmic rays. What are cosmic rays? Cosmic rays are particles which are actually coming from outside the earth's atmosphere could be partly from the solar system the sun itself, from the nuclear reactions of sun there are particles which are coming out. And similarly from other stars or from other industrial are objects or galaxy connecting objects or any other objects astrophysical objects or any other source that is outside the; different types of this cosmic rays come in and then these mentioned some time earlier some of these particles pie owns etcetera are found in this cosmic rays positrons electron and many anti particles and particles.

And in high energy physics are scattering experiments we could produce many of these particles again which are other ways not naturally occurring in normal matter, like pions. We do not have atoms with pions inside that in some form, but we have atoms with protons. We do not have atoms with lambda inside that see we are not see in kaons in nature in abnormal matter. But this can be produced and then we have seen them discovered them in experiments scattering experiments and we also seen them in cosmic rays.

So, the first step when you have many such as a say in such particles and for the resonances is to understand them by classifying them. Actually now we will go one step before and start looking at the particles as historically they are seen. Before they were

identified as quark bound states what was the status. We knew we had a lot of protons I mean we have protons and neutrons in normal matter. We knew there are particles called pions, kaons etcetera sets, but we did not know exactly what they are whether they are elementary or whether they have some composites etcetera this information came at a later stage a experimental ever of in 60s.

But before that we heard all these particles they are not many of them and then we had to understand them. So, classifying them was one task to start with. One way people thought about it is by using some kind of a symmetry principle. So, this symmetry principles and the way to understand particle properties through symmetries have been are used extensively in particle physics we will see that. One particular this thing very useful in classifying the particles this change as assigning them a quantum number called isospin. This is very similar to the ordinary spin in the sense that when we do the algebra when we actually read the mathematically or when we look at the symmetry groups or the mathematical form of the symmetry it is exactly similar to the spin. So, what people found is that successfully consistently one can assign isospin to these hadrons which are observed. So, any of the bound states of the particle quarks we will called hydrants mesons and baryons together are called hadrons. Mesons are hadrons with a quark and an antiquark, baryons are hydrants with 3 quarks met.

Coming to the isospin as an example you consider the group of 3 particles. You have seen  $\pi^+$ ,  $\pi^-$  and  $\pi^0$  and it is found that they all have similar mass around 140, it may be per c square and therefore, people group them under one single isospin group with isospin equal to 1 and  $I_3$  equal to plus 1 0 and minus 1 points.

Similarly the kaons  $K^+$  and  $K^0$  see when I put this symbol. So, then it look natural to call the many group, but do you think about an experiment where you actually discover some particle positively charged particle some particle which are negatively charged particle, some particles which are neutral. So, to classify them you have to look at other properties like you were to collect them in with those having similar mass etcetera. For example,  $K^+$  is not grouped together with  $\pi^-$  because they have different masses  $K$ , kaon. So, have masses around 500 a maybe whereas,  $\pi^-$  mass around 140 GeV. So, it was therefore, other consideration not just the charge that is that will dictate these things.

So, there were only 2 particles of this kind with 500 MeV per c square mass and positively charged and 0 charge that is not exactly true. There was also a K minus which was found later on, but and also another neutral  $K^0$  bar which is anti particle of  $K^0$ . So, there have been many different types of iterations etcetera that has gone through and models were proposed and then this is what it is something like the end researcher that we are discussing. So, there is a K plus and  $K^0$  which can be grouped and  $I$  equal to half,  $I_3$  equal to plus and minus halves and all other mesons could also be classified under this isospin grouping.

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### Classification of hadrons

Baryons could also be classified in a similar way.

$p, n$	: $I = \frac{1}{2}, I_3 = +\frac{1}{2}, -\frac{1}{2}$
$\Sigma^+, \Sigma^0, \Sigma^-$	: $I = 1, I_3 = +1, 0, -1$
$\Lambda$	: $I = 0, I_3 = 0$

Other hadrons could also be classified according to their isospin.

Now, similar to this baryons could also be classified under this or grouped through as isospin groups. For example a proton and a neutron together can be thought of belonging to an isospin group of isospin half and sigma plus, sigma 0, sigma minus another baryon group of baryons we gained with similar mass can be grouped to isospin, to a group of isospin 1. And lambda there was nothing to club that with the same mass. So, this is a particle with isospin equal to 0 standing on its own in a single group in a group of single particle. So, other mesons and baryons could also be classified exactly in an exactly similar way.

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### Quark model

An explanation for the (isospin) grouping of the hadrons could be given considering the quark contents with

Quark	$I$	$I_3$	Antiquark	$I$	$I_3$
$u$	$\frac{1}{2}$	$+\frac{1}{2}$	$\bar{d}$	$\frac{1}{2}$	$+\frac{1}{2}$
$d$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\bar{u}$	$\frac{1}{2}$	$-\frac{1}{2}$
$s$	$0$	$0$	$\bar{s}$	$0$	$0$

The relative sign between in the  $\bar{d}$  and  $-\bar{u}$  states is necessary considering the particle-antiparticle transformation.  
*Introduction to High Energy Physics, D.H. Perkins, Cambridge U.P.*

Similar to the ordinary spin (angular momentum) addition, we can combine different isospin states.

And now you collect them in this fashion and we did not know about the structure of this or quarks. But then if we imagine this particle that made of sub structures called quarks or, you can actually give some explanation for this kind of grouping, why there are 3 pions with in the isospin equal to 1 group, why there are 2 kaons in isospin equal to half group etcetera or why they are protons and neutrons together in isospin half etcetera, for that you need to look at the quark and end of this thing.

So, now we know that already we mentioned that there are u quarks, d quarks etcetera all these quarks were actually discussed. So, let us revisit that again along with the other properties. We know how to assign what is the isospin of this quarks. So, if you look at the u quark and d equal put them in and isospin 1 group and  $I_3$  equal to plus out for u quark and minus half four d quark. Put s separately with isospin equal to 0. Now, in a similar ring you can look at the anti quarks a d bar and a u bar minus k and an s bar in  $I$  equal to half u 1 and u 1, and  $I_3$  equal to plus half minus half; s bar is again isospin 0 group.

You see that I have put a minus sign there. What is a meaning of a putting a minus sign in the symbol? Basically when I put this symbols u d s, u bar, d bar, s bar we can also think about the wave function or the associated wave function representing these at these letters or the symbols representing the wave functions of this party corresponding party. If that is so then we need to put a minus sign relatively minus sign in the d bar u bar case

and this is and the  $\bar{d}$  quark will now have a  $I_3$  equal to plus half and  $\bar{u}$  quark as will have  $I_3$  equal to minus half. On the other hand in the case of quarks  $u$  had the quark  $I_3$  equal to half and  $b$  had  $I_3$  equal to minus half.

This can be done if you understand how to go from one to the other or the other way of saying it is that quantum mechanics for consistent quantum mechanical picture wave functions of  $u$  and  $d$  are if you consider and transform that to anti quark wave functions. And it will go from  $u$  to  $\bar{u}$  and  $d$  to  $\bar{d}$ , and if you look at the isospin of this particle the transformation will tell you that  $u$  goes to a  $\bar{u}$  and the isospin third component will go from corresponding to this will be half in the case of  $u$  and then minus half in the case of  $\bar{u}$ . One could also imagine at the minus sign along with a  $d$  quark  $\bar{d}$  quark and  $\bar{u}$  quark with a plus sign relative to the  $u$  and  $d$ , but then therefore, it is only the relative sign between these  $\bar{d}$  and  $\bar{u}$  which is important.

Some explanation and some elaboration on this is done in introduction to high energy physics by Donald H Perkins. You could also consult other books discussing quark model to understand the technical details of how this minus sign comes etcetera. We will not discuss any further this aspect in our discussions.

But, when we actually combined 2 quarks or a quark and an antiquark the isospin of that combined state will follow an algebra or algebraic from relation very similar to or exactly similar to in fact, that of the angular momentum addition which is what we are familiar with I believe that a from your quantum mechanics courses.

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$$\begin{aligned}
 &\text{Consider } \begin{matrix} u & +\frac{1}{2} \\ d & -\frac{1}{2} \end{matrix} \quad \text{and} \quad \begin{matrix} \bar{d} & +\frac{1}{2} \\ -\bar{u} & -\frac{1}{2} \end{matrix} \\
 &(I_1 = \frac{1}{2}) \oplus (I_2 = \frac{1}{2}) \Rightarrow I = 1, 0 \\
 &\text{Consider } I = 1 \\
 &|I=1, I_3=1\rangle = |I_1=\frac{1}{2}, I_{31}=\frac{1}{2}; I_2=\frac{1}{2}, I_{32}=\frac{1}{2}\rangle \\
 &|\pi^+\rangle = |u\bar{d}\rangle
 \end{aligned}$$

Now, consider the u quark and d quark as we said just now  $I_3$  is equal to half and  $I_3$  equal to plus half for u quark and minus half for d quark. And for the anti quarks  $v$   $z$  we have an d bar and a minus u bar with  $I_3$  plus half. Let us form a combination of this u quark and this u d system quark system and u bar d bar anti quark system. What are the things that we can form? This is similar to combining spin half system.

So, let me denote the  $I_q$ , denote the isospin of the quark system as  $I_q$  which is half we take that combine that with  $I_{q\bar{}}$  which is again half. Angular momentum addition tells us that the resulting combined system can have an isospin either 1 or 0 half minus half of half plus half this is based on the angular momentum addition rules. Consider the case of  $I$  equal to 1.

Again if we think about  $I$  equal to 1,  $I_3$  equal to 1. There is only one combination that can give this in terms of the quarks which is  $I_q$  equal to half of course, there is only thing,  $I_3 q$  equal to plus half and  $I_q$  equal to half  $q\bar{}$  equal to half and  $I_3 q\bar{}$  equal to plus half. So, that  $I_3 q$  and  $I_3 q\bar{}$  adds up to 1, which is the  $I_3 q$  of the  $I_3$  of the combined system. And ask the question do we have a particle like this found in nature. The right hand side will tell you that it is basically u quark which is  $I_q$  half and  $I_3 q$  equal to half and a d bar quark. So, this is basically the system and right hand side.

Do we have a physical particle or a meson corresponding to this? Charge is equal to plus 1 because you have charge of  $\frac{2}{3}$  plus  $\frac{2}{3}$  and  $b\bar{}$  has charge of plus 1 by 3. This can

be thought of as our pion. You want to see what are the other possibilities when you combine an  $I_q$  equal to half and  $I_q$  bar equal to half.

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$$I_- |I=1, I_3=1\rangle = (\quad) |I=1, I_3=0\rangle$$

General formula:

$$I_- |I, I_3\rangle = \sqrt{(I+I_3)(I-I_3+1)} |I, I_3-1\rangle$$

$$I_- |I=1, I_3=1\rangle = \sqrt{2} |I=1, I_3=0\rangle$$

$$I_- |I_1=\frac{1}{2}, I_{21}=\frac{1}{2}; I_3=\frac{1}{2}, I_{33}=\frac{1}{2}\rangle$$

$$= (1 \cdot 1) |I_1=\frac{1}{2}, I_{21}=\frac{1}{2}; I_3=\frac{1}{2}, I_{33}=\frac{1}{2}\rangle +$$

$$+ |I_1=\frac{1}{2}, I_{21}=\frac{1}{2}; I_3=\frac{1}{2}, I_{33}=-\frac{1}{2}\rangle = |d\bar{d} - u\bar{u}\rangle$$

$$\sqrt{2} |\pi^0\rangle = |d\bar{d} - u\bar{u}\rangle$$

For that you consider this  $I$  equal to 1,  $I_3$  equal to 1, usual way to do it is to actually operate the step down operator on this, operator on this  $I$  equal to 1  $I_3$  equal to 1 by a step down operator and we will get something into  $I$  equal to 1,  $I_3$  is reduced by one unit and equal to 0. What is this something? Something is, in a general case for a step down operation is  $I$  minus  $I$ ,  $I_3$  equal to  $I$  plus  $I_3$ ,  $I$  minus  $I$  plus 1 under root. That is the coefficient that will come with this and  $I$  equal to 1,  $I_3$  equal to 0 oh sorry  $I$ ,  $I_3$  equal to. So, I should write the general form from here. So, it is  $I$ ,  $I_3$  equal to sorry  $I_3$  minus 1.

Then we can actually apply this here in the case of  $I$  equal to 1,  $I_3$  equal to 1 and then we will see that  $I$  minus  $I$  equal to 1  $I_3$  equal to 1 is equal to 1 plus 1 is equal to 2 into 1 minus 1. So, that is root 2 into  $I$  equal to 1,  $I_3$  equal to 0.

Similarly, you can apply this  $I$  minus on  $I$  equal to  $I_q$  equal to 1 by 2, on the right hand side of that equation earlier we had  $I_3 q$  equal to half  $I_q$  bar equal to half  $I_3 q$  bar is equal to plus half this is equal to as per the room formula equation that we had this is equal to half plus half that is equal to 1 into half minus half plus 1. So, that is 1 into 1, so it is actually 1 that is a coefficient. So, first we will apply this  $I$  minus on the  $q$  system and then that will give you  $I_q$  equal to 1 over 2 and  $I_3 q$  equal to minus half and the other one remaining the same bar 1 or 2,  $I_3 q$  bar equal to half.

But then plus  $I_3 q$  equal to half,  $I_3 \bar{q}$  equal to plus half,  $I_3 q$  bar equal to half  $I_3 \bar{q}$  bar is equal to minus half and this if you look at is nothing, but  $I_3 q$  corresponds to a  $d$  and  $I_3 \bar{q}$  bar corresponds to a  $\bar{d}$  plus actually  $I_3 q$  plus that is a  $u$ , but  $I_3 \bar{q}$  bar minus half is a  $\bar{u}$  with a minus sign. So, that is why I put a minus sign there. So, this is the quark content of  $I_3$  equal to  $I$  equal to 1,  $I_3$  equal to 0 which mean. So, we can ask if there is a particle like that and indeed we can identify  $\pi^0$  with this. But you had a root 2 there in as a factor. So, it actually is  $\sqrt{2} \pi^0$  is equal to  $d\bar{d} - u\bar{u}$  or  $\pi^0$  is  $1/\sqrt{2} d\bar{d} - u\bar{u}$ .

We will let look at one more example and then further discuss this quark contents of mesons different other mesons and also that of the baryons the next discussion.