

**Nuclear and Particle Physics**  
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**Module – 05**  
**Nuclear Reactions**  
**Lecture – 02**  
**Type of Reactions**

We were discussing the nuclear scattering and last discussion; we had discussed the basics of what is called the scattering cross section say for example.

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**Nuclear Reactions**

$$a + X \rightarrow Y + b$$

differential cross section,

$$\frac{d\sigma}{d\Omega} = \frac{R(\theta, \phi)}{FN}$$

$d\sigma =$  prob. to find "b" in a solid angle  $d\Omega$  around  $(\theta, \phi)$ .

If we are considering a scattering of a light nucleus  $a$ , say alpha particle with the heavy nucleus, it is denoted by  $X$  and producing another heavy nucleus in general something else like  $Y$  plus some light particle  $b$ . So, this could be an alpha particle bombarding on some called nucleus or some other nucleus producing say or scattering is produce and the resultant  $b$  and  $Y$  can be in general different from  $X$  and  $a$ .

But in particular cases like elastic scattering, it could be the same as well. We can also consider neutrons interacting with the nuclei and then producing some final states either one neutron itself or something else and accordingly either the same nucleus or different nucleus different from  $X$  see in such cases we said, we can define what is called the

differential cross section  $d\sigma$  over  $d\Omega$  equal to  $R(\theta, \phi)$  over  $FN$ , we had defined this in the last discussion.

So, please refer to that again basically these  $R(\theta, \phi)$  here is the rate of particles projected or the particle of type  $b$  reaching the or produced or coming out at polar angle  $\theta$  and azimuthal angle  $\phi$  and  $d\Omega$  is the solid angle of the detector if and  $F$  as the flux of the incident beam  $N$  is the number density of the target  $X$  nucleus.

So,  $d\sigma$  actually is then the probability to find  $b$  in a solid angle  $d\Omega$  around  $\theta, \phi$  direction. So, this is what the physical interpretation of this differential cross section is now as I said we can put the detector at angle  $\theta$  and  $\phi$  to see or measure the rate of  $b$  coming out at certain angles or certain positions.

If our detector can also detect what is the energy of this  $b$  particle which are coming out and we will see later that our particle detectors are capable of doing that they are capable of counting the number of particles they are capable of finding the energy of the particle and the momentum of the particle including the direction we can get all this information from the scattering experiment.

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**Nuclear Reactions**

$$d\sigma = \text{prob. to find "b" of energy between } E \text{ and } E + dE, \text{ in solid angle } d\Omega \text{ around } (\theta, \phi)$$

$$= \frac{R(\theta, \phi, E)}{F \cdot N} \cdot d\Omega \cdot dE$$

$$d\sigma = \left[ \frac{R(\theta, \phi, E)}{F \cdot N} \cdot d\Omega \right] dE$$

$$\Rightarrow \frac{d\sigma}{dE}, \quad \frac{d\sigma}{dE \cdot d\Omega}, \quad \frac{d\sigma}{d\Omega}$$

So, suppose we consider such a case and in that case define  $d\sigma$  as the probability to find a  $b$  particle of energy between  $E$  and  $E$  plus  $dE$  experimentally we cannot actually determine the energy precisely right there is always an experimental

uncertainty and let us also consider the angular dimensions along with this. So, probability to find  $b$  of energy between  $E$  and  $E + dE$  some small interval of energy and in solid angle exactly the same thing as before in solid angle  $d\Omega$  between or around  $\theta, \phi$  the solid angle around  $\theta, \phi$  is  $\sin\theta d\theta d\phi$  or this then can be written as  $R(\theta, \phi)$ .

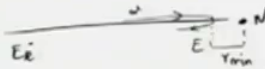
Now, the rate we are actually counting only the particles of type  $b$  the particle be coming out with energy  $E$  right  $E$  divided by  $FN$  that remains the same the tag and the influx of the flux of the beam and the target density number density remains the same and then you had to multiply it by  $d\Omega$  and  $dE$  now if you consider  $R(\theta, \phi) E$  over  $d\Omega$  not over the  $\Omega$  over  $FN d\Omega$  and let us integrate that over  $d\Omega$  or I will put this integration here, oh, but consider  $b$  particle coming with energy around  $E$  in an interval  $dE$ . So, this will give you the number of particles  $b$  coming out with energy around  $E$  anywhere around the nuclear the potential scattering center scattering nucleus. So, we had integrate over the angle then. So, you can actually define  $d\sigma$  over  $dE$  experimentally you can measure this as we explained and we can half also the double distribution  $d\sigma/d\Omega$ . So, the above discussions all these are experimentally we can obtain all these things.

So,  $d\sigma/d\Omega$  etcetera. So, these are very useful quantities I mean all of these will give you some information about the scattering potential. So, that is what the purpose of the scattering experiment that it is to find out what is the dynamics that dynamics under the interaction of these nuclear the projectile and target nucleus. So, now, we will not go into the details of the discussions or the detailed calculations.

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**Nuclear Reactions**

Rutherford Scattering (Coulomb Scattering)

$$\alpha + X \rightarrow X + \alpha$$


Initial velocity =  $v_0$   
Initial K.E =  $\frac{1}{2}mv_0^2$   
Initial P.E = 0

$\Rightarrow$  Total initial energy,  $E_i = \frac{1}{2}mv_0^2$

But let me just mention some qualitative features of Rutherford scattering; Rutherford scattering as you know is an alpha particle on some target nucleus gives you same thing ok.

So, this is actually elastic kind of scattering. So, you do not have different type of particles produce the initial particle is alpha the projectile is alpha target is X say revolved over all whatever and then finally, again there is no change in the things, it is just a kind of a coulomb scattering.

So, it is basically a coulomb scattering there is no change in the nucleus for any suggesting. So, this is one type of reaction that you can think of what happens is that you send alpha particles to the target nucleus. So, if you send it head on if it is actually going to be exactly along the direction of along the line joining the alpha particle and the nucleus target nucleus then this alpha particle will go and then come back in the same direction they will go and come back alright.

So, in such case what is happening you have initially some kinetic energy say  $E_0$  total energy initially or  $E_i$ , let me call it and then you have a  $E_f$ ,  $E_f$ , let me call find out the  $E_r$  or it is actually going and then returning from the  $E$  as it goes towards reaches the nucleus, it feels the nuclear potential and alpha particle these are if you are looking at the coulomb interaction both of these are positively charged particles and the projectile will feel a repulsive force of the nucleus and then eventually the potential will be strong

enough to actually stop this and then push it back. So, that pushing in back will actually give it kinetic energy and then it will be pushed back to this. So, that is what the; what one can one can think of.

So, it is actually in the hands of case of head on relations there is a point at which the velocity is 0 point of return or point of closet approach or so, R minimum. So, this is your nucleus and this is your alpha particle. So, let us say initial velocity is equal to  $V_0$  kinetic energy initial kinetic energy is equal to  $\frac{1}{2} m V_0^2$  where  $m$  is the mass of the projectile initial potential energy let us say it is far away from the target nucleus to start with. So, it is equal to 0 and total energy. So, that will give you total initial energy  $E_i$  to be equal to  $\frac{1}{2} m V_0^2$  only the kinetic energy.

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**Nuclear Reactions**

@  $r_{min} \Rightarrow$  velocity = 0  
 $\Rightarrow K.E = 0$

P. energy =  $\frac{1}{4\pi\epsilon_0} \frac{3Ze^2}{r_{min}}$

Energy Conservation  
 $\frac{1}{2} m V_0^2 = \frac{1}{4\pi\epsilon_0} \frac{3Ze^2}{r_{min}}$

Now, at the point of return as I said at  $r$  minimum where it actually starts tracing the position the path back some kind of a semi classical thought this one is, but it is we can get a qualitative picture of this by this reactor kind of analysis. So, at this point velocity is equal to 0 that will give you kinetic energy equal to 0 potential energy is basically coulombic potential energy  $\frac{1}{4\pi\epsilon_0}$ .

Let us; I mean it could be in particular an alpha particle, but let us take as head of and light nucleus with  $z$  number of protons in it let us small  $z$  number as small  $z$ , let me denote the proton number for alpha particles small  $z$  is equal to 4 and let us say the target nucleus has capital  $Z$  number of protons in it right and then in that case the potential is

for our  $\frac{1}{4\pi\epsilon_0}$  small  $z$ , capital  $Z$   $E$  square over whatever is the position and in this particular case of the point of return it is  $R$  minimum.

So, this is the final energy. So, that is also the potential total energy right. So, there is no kinetic energy total energy is equal to this. So, the energy conservation will tell you that initial kind of initial energy half  $mV_0$  square is equal to one over  $4\pi\epsilon_0$   $z$   $z$   $E$  square over  $R$  min. So, it is one relation you can keep in mind or  $R$  min you can find from this what is the closest of projects etcetera.

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The slide is titled "Nuclear Reactions". It contains a diagram on the right showing a particle's trajectory as it approaches a target, with an impact parameter  $b$  and a scattering angle  $\theta$  indicated. To the left of the diagram, the text reads "If not head on ( $b \neq 0$ )". Below this, the differential cross-section is given by the formula:

$$\frac{d\sigma}{d\Omega} = \left( \frac{Zze^2}{4\pi\epsilon_0} \right)^2 \cdot \left( \frac{1}{2mv_0^2} \right)^2 \cdot \frac{1}{\sin^4 \theta/2}$$

Below the formula, it states: "The  $\frac{1}{\sin^4 \theta/2}$  dependence is experimentally confirmed."

If it is not head on if not head on which means impact parameter  $b$  is not equal to 0 we did say what is impact parameter yesterday impact parameter is basically the distance between the direction of momentum.

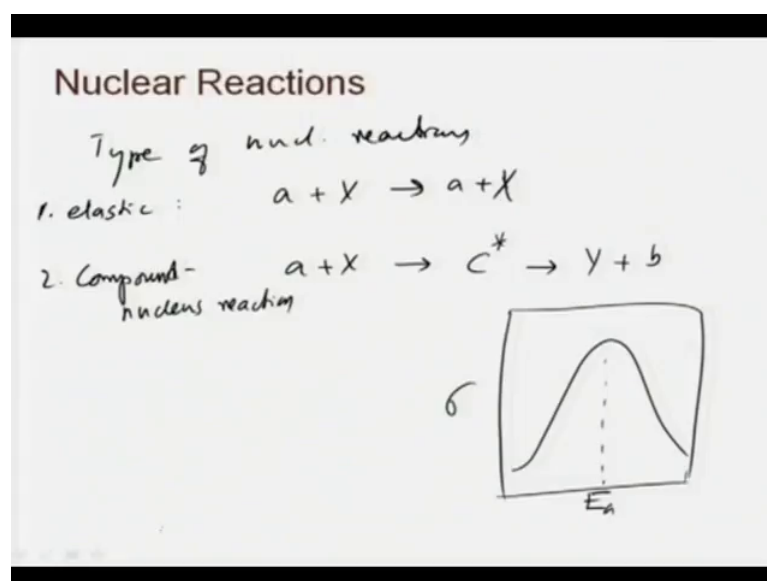
And the direction of the head on collision system of the particle in that case it will scatter at say some angle  $\theta$  depending on what your  $b$  is and what the  $z$  or what the potential that this particle will experience. Now we can make do a computation based on the quantum mechanical carnival based on quantum mechanics and the perturbation theory of quantum mechanics and we will get  $d\sigma/d\Omega$  to be equal to  $Z; Z E$  square over  $4\pi\epsilon_0$   $R$  square  $1$  over  $2 mV_0$  square  $R$  square one over sine square  $\theta$  by  $2$ , this  $\theta$  dependence and on the cross section is an obvious thing now all right since the way we have been discussing the dip in the angle at which the particle will come out will certainly depend on what is the impact parameter.

Now, this also says this expression also says that there is no  $\phi$  dependence which is again somewhat expected; it is expected because we have we are talking about a spherically symmetric potential. So, the only direction that we have here is this direction of this particle that is going towards this one and beam is expected to be cylindrically symmetry. So, the cylindrical symmetry will tell you that there is no azimuthal dependency.

So, as long as you have the beam a uniform beam with a with cylindrical symmetry, then we do not expect any azimuthal dependence in  $d\sigma/d\Omega$  the other thing. So, the expression that I wrote was wrong, it is actually sine power 4 theta by 2. So, this particular dependence like  $1/\sin^4(\theta/2)$  is something which you can check what is the angular dependence of this  $d\sigma/d\Omega$  can be checked experimentally  $1/\sin^4(\theta/2)$  dependence is experimentally confirmed in Rutherford scattering experiments all right.

So, we will not go into any further details of this Rutherford scattering nor we will discuss some other aspects of the scattering cross section etcetera and some more detailed a calculation of this when we actually come to the particle physics our scattering experiments elementary particle experiments at the moment, let us go back to the nuclear reactions and then say what kind of reactions are possible etcetera.

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So, one is this type let me call Leslie type of nuclear reactions  $1 + X \rightarrow a + X$  which is actually a Leslie scattering.

Then you can have. So, this is elastic Rutherford type of scattering probably mostly with coulomb potential interaction then if you have enough energy for the projectile what can happen is this projectile can actually go beyond the coulomb barrier it has enough kinetic energy to actually go and then. In fact, interact directly in will enter the nuclear enter the nuclear dimensions go close enough to the nucleons and then they can actually have strong interactions as well there nuclear force can come into picture or will come into picture at that time.

So, coulomb interactions or elastic coulomb scattering like the Rutherford scattering will happen when the projectile has smaller kinetic energy smaller in the sense that it is not large enough to I mean go beyond or overcome the coulomb barrier or columbic repulsion and then it will go and then the reaction will be something similar to what we just now discussed, but if the kinetic energy is much larger than that then we can think about a situation where a plus X goes and interacts with the nucleons resulting in some kind of production or of a realization of some kind of a compound nuclear state which is not a stable one to actually stay there for a long time.

So, the let us say take a new neutron a high energy neutron or large enough energy for the neutron to actually penetrate go near the nucleons in that case this neutron can directly interact with the nucleons through strong nuclear forces and that will actually result in some kind of a compound nucleus is formed then a this compound nucleus is not stable nucleus.

And it can actually therefore, decay now to something else say  $Y + b$  and in one particular case it may be that  $Y$  is equal to  $X$  and  $b$  is equal to  $a$  with say more kind of different kinetic energies compared to a  $X$  could I mean  $Y$  could be an excited state of  $X$  to start with all those possibilities are there it could be a different nuclear. So, such possibilities are generally called a compound nucleus reaction typically.

If you look at the cross section cross section  $\sigma$  and plot this against the kinetic energy of particle  $a$  is. So, energy or throttle energy or kinetic energy of particle the projectile  $a$  then if you are expecting say some kind of a particular excited state of some new intermediate nucleus say that case the cross section will have some kind of large

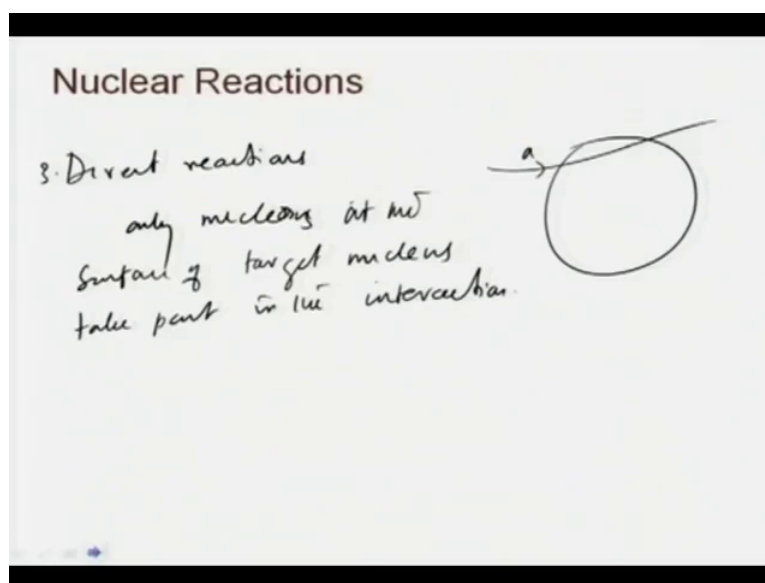


value around the energy of Easter or around the you know for some values of  $E_i$  that will give  $b$  in combination with the energy and the mass energy of the  $X$  whatever is the excited energy of  $c$ .

So, there will be some kind of a large cross section and then when it deviates from that or becomes the energy of  $a$  is larger than this from the value around which the cross section peaks again it will decrease the cross section will decrease because that energy is now not the one which matches with the energy of the excited state ok.

So, this is kind of a picture like there is actually kind of a large peak observed in the case of  $N$  is a indication of compound nucleus formation in these reactions.

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Another possibility is that the nucleus that you are sending it to sending to the light nucleus the projectile that you are sending to the target interacts not with the neon nucleus, but with some nucleons at the periphery on the surface such reactions are called direct reactions.

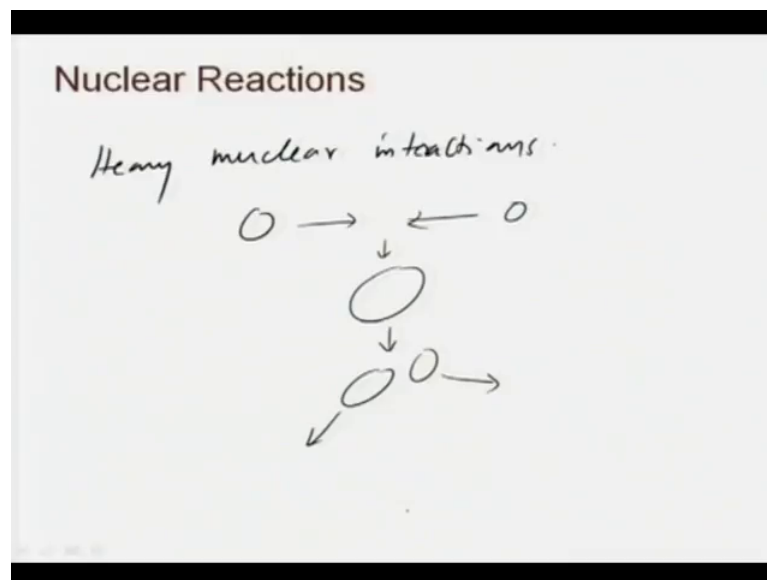
So, in direct reaction what happens is that the nucleons that you are sending the light nucleus that you are sending in neutron or alpha particle goes beyond the potential cumbic potential barrier and interacts with the nucleus, but it does not interact with the entire neuron the entire nucleus is not really taking part in that reaction for most in the case of compound nucleus formation its most of the nuclei are in a kind of a formation

take part in this thing there is an exchange of energy etcetera that the projectile spends enough time with this etcetera.

Well here in the case of direct reactions, they can come interact with nucleons at the periphery the neutrons or protons and the surface and then just go away kind of this. So, such reactions are called direct reactions were the only nucleons at the surface of target nucleus take part in the interaction.

So, this is another possibility in some cases it is possible that we have some intermediate state between these 2 that is also possible and. In fact, in some reactions, it is possible that you have kind of direct reactions as well as the compound kind of interactions etcetera and typically the way we can look at the direct reaction in the compound of deformation is that for the combo nuclear formation the projectile will spend some time with the target nucleus and therefore, the reaction time was typically much larger compared to the direct reaction where it actually comes and there interacts to clean with the neutrons or protons at the periphery and the surface and then goes away.

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So, such reactions will have are cuca compared to the other reactions the compound reactions etcetera then we can think about heavy nuclear collisions nucleon heavy nuclear interactions here you can actually think about 2 nucleons of similar type of same kind of energies. So, instead of a light projectile bombarding a heavy target we can think

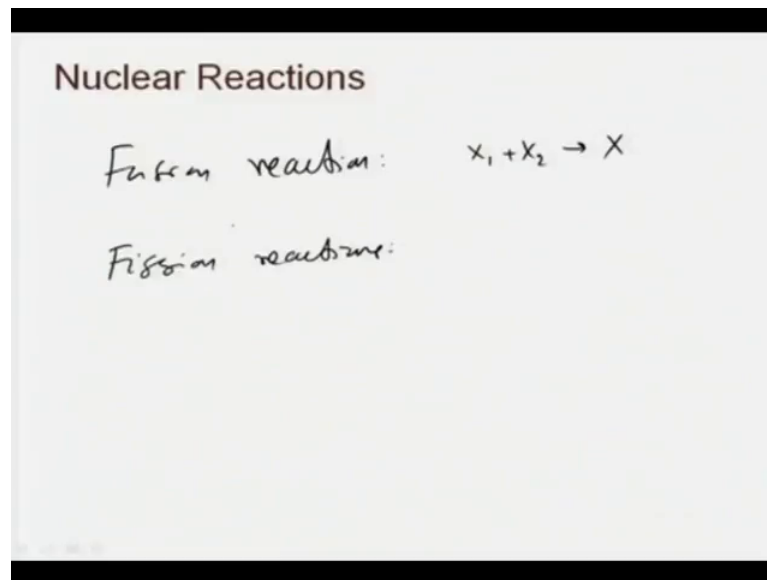
about 2 nuclei of similar atomic mass coming together or heading on to each other colliding with each other and producing nuclei ok.

So, it is possible that they will come together fuse together. So, they will fuse together and then again they will split into 2; 2 or more normally to 2 has a large number of large cross section compared to more than 2. So, this will then be slightly different from the kind of reaction that we have or the compound local information or direct nuclear or direct a nuclear reaction that we just discussed inside the heavy ion collisions heavy nuclear collisions have very are very important.

And then they have a large significance to understand the dynamics of elementary particles even beyond the nucleons actually you can think about very heavy energies the reactions where the energies are so much that when they smash into each other even the protons and neutrons are our smash into each other. So, that they can go through the or in go to the inside of the protons and neutrons and interact directly with the constituents of the protons neutrons we will come to this later what are the constants of neutrons and protons etcetera, but such possibilities are there and in such cases we will also get kind of soup or a plasma state if our understanding of the reactions nuclear reactions is right.

So, there are experiments which are done at very high energies where heavy ions with high energy heavy ions are collided on each other and then you can observe various possibilities there and in what I mean what dynamic what are the resulting finite states and then what are the signatures of this or what are the observations based on these observations we can actually try to understand what are the possibilities with the dynamics of the nuclear reactions there and then we also have nuclear fusion you can have 2 light nuclei coming together and fuse into each other.

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Fusion reaction in which 2 light nuclei say  $X_1$  plus  $X_2$  goes to some heavy nucleus  $X$  fuse together to form it the resultant nucleus may be unstable or it could be I mean usually it is unstable which will go down to something else, but and then you can also have the fission reactions fission reaction can also be like you have a nucleus a heavy nucleus which disintegrates into 2 nucleon or 3 nuclei with somewhat the similar mass now this could also be it could be natural or it could also be induced to by actually sending some high energy neutrons or protons usually neutrons. So, these neutrons will go and then disturb this heavy nucleus and disturb this. So, that it will actually disintegrate into some 2 light nuclei. So, such fission reactions are also possible.

So, these fission and fusion reactions are useful in the sense that they we can extract energy from such reactions. We will stop here and then continue with our discussion later.