Nuclear and Particle Physics
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Module – 05
Nuclear Reactions
Lecture – 01
Nuclear Scattering – Preliminaries

We discussed the nuclear decays in the past couple of lectures. Today, we will discuss the nuclear reactions.

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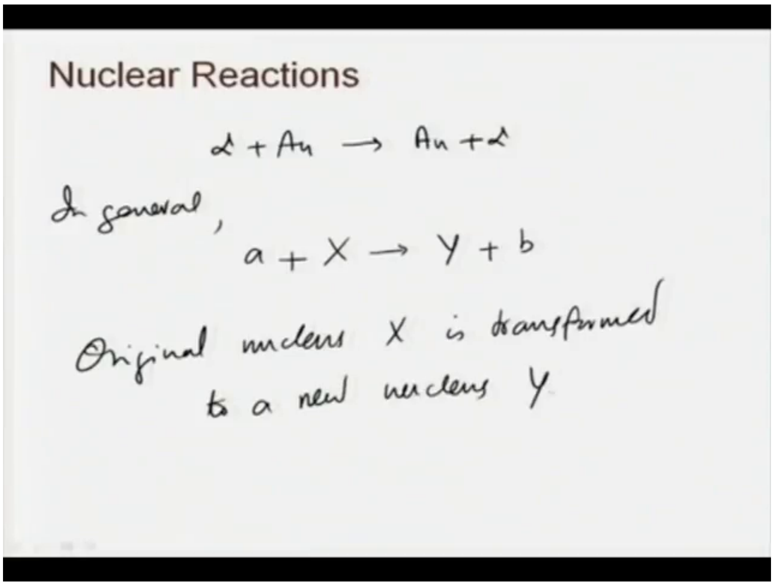

If we send alpha particles to gold nuclei; for example, this alpha particle will see a feel; the nuclear potential of the gold nucleus when it is sufficiently near the nucleus.

And it will scatter off. So, this is a experiment which you are familiar with I believe the the Rutherford's experiments by using alpha particles on gold foil and other elements reveal the fact that nucleus is actually or the positive charged particles are focus concentrated in the nucleus of the atom and various other properties of this nucleus was also understood by such scattering experiments.

In general you can actually consider some a projectile sent to some target material let me call that X or the nucleus of the target as X. And in general, you can expect that the nuclear reaction is slightly more than what is said above in with the case of alpha and A u, it can actually change the nucleus itself and it could emit usually there is something which is emitted; a light particle which will come out and which will be in general different from a. So, in such reactions nucleus or the original nucleus is transformed. X is transformed to a new nucleus Y.

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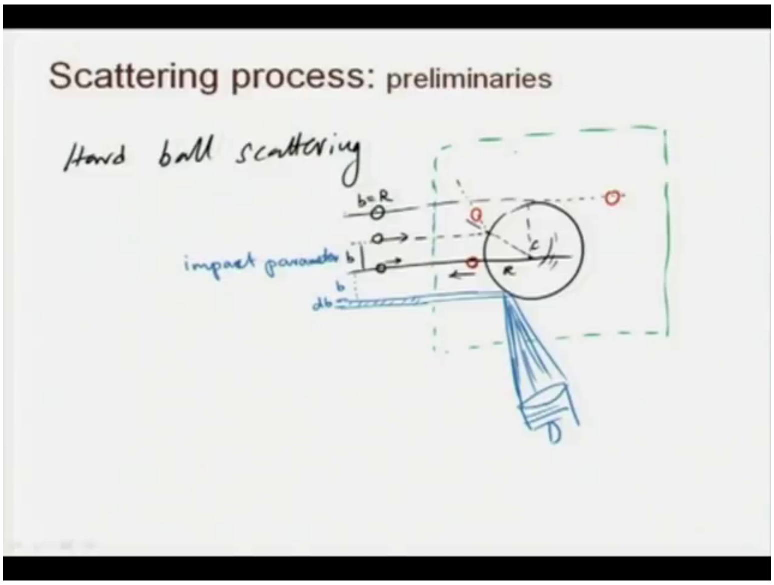

Now, how do we actually understand the properties of this or what are the ways that we can understand this? To start with the let us study the; let us look at the basics of scattering process itself. To simplify matters, we will consider first classical hardboard scattering by which what I mean is the following; if you have a hard sphere take a steel or iron sphere, so, this is your iron ball.

And now let us send some smaller beads or balls on to this; what will happen? If you send; so, let me draw a line which actually will pass through the center; center of this ball and if I send a bead along the line joining the bead and the center or the momentum is directed towards the center of the sphere, then what will happen? It will go, I mean, this is simple classical scattering.

So, it will go meet the hardball and the normal a is perpendicular to the tangent at that point and then that set that is along the motion of the normal at the point where the bead meets the hard sphere is along the momentum of the bead in the for it will actually turn back turn around. So, let me; do not confuse you by drawing it in separate positions. So, it will actually go back maybe what I will do is to actually use a different line different color. So, this let us assume that it is actually a straight line. So, now, you have a bead the red bead is the one which will which is bouncing back from the hard sphere.

Now, if you Instead take this original bead and then send it away from or send it in such a way that the momentum is not now directed towards the center if you take the momentum direction and line passing parallel line parallel to that momentum direction beat passing through the center and then there is a small gap let me call that as b.

So, this is just the distance between the line passing through the center parallel to the direction of motion of the beat and the direction or the line the perpendicular direction distance of the bead from that particular line. So, then what will happen is the point where the bead meets the ball the normal to that which is basically the line joining that point and the center for as sphere is the normal.

So, there is an angle between the momentum direction and the normal and therefore, it will actually reflect or bombard this hard sphere and come out not in the same or opposite to the direction as in the case of as in the earlier case again, but will go at some angle making an angle with them. So, this will be the one which goes out. So, red once are the ones which actually are coming out after the scattering now let us say we consider some other point take b to be larger and then b to be equal to. So, let us say this is r. So, b is equal to R case. So, in this case b is equal to R which is the radius of the sphere.

In that case, it just graces right and pass and it and scattered. So, it will; the scattered one is basically not started it moves on in the same direction. So, now, this is what usually happens in the case of a hard ball scattering. So, depending on what the position what is the direction of the momentum and the initial conditions the ball will scatter in different directions.

So, now, let us say we take beats coming out in initially let us send a beam of this beads lot of this beads are put in some particular beam in a particular constraint direction all of them are going in the same direction with the same momentum or slightly different momenta, but let us consider it to be the same momenta say and they are all going in the this one, but they are all not at the same.

B values or b that are d b value is called as the impact parameter. So, the impact parameter for different particles in the beam could be different. So, let us take such a; this one and if I consider as small section of that beam denoted by this d b. Now I am not indicating the individual balls, but many small balls are there in this blue bat and you see that they will reflect as a result of that one and then come out at some angle and the angle with which this will come out called this particles beams beads inside this small band differ differing by.

So, let us again denote the parameter impact parameter as b and small width of this band as d b, then we will see that they will come out at some angle and if I keep my detector d at this position I will receive all this things if the detector is sufficiently large in that way essentially that idea that I want to convey is that depending on the impact parameter the beats will scatter in different with different angled.

And if you consider small some small band of these beam of beads they after scattering will go into a particular cone and if you put a detector in that corn in the interfering that or in the inert put it in the way of the cone then you will receive all this ones.

And if you change the bat parameters suddenly the scattering cones angle will be different. Firstly, secondly, the position of that will also be different. So, this position of this etcetera will depend on what is the impact parameter etcetera not only that if you take a different sphere the scattering sphere the big sphere with a different radius R one instead of R some R prime or 2 R or R by 2, then you will see immediately you can draw that again I do not want to draw it again, but you can do that yourself and you will see immediately that the angle of scattering and the cone angle the width of the cone etcetera will depend on the size of the; so, supposing somebody hides this area from you they put some kind of a screen in front of you right.

So, something like a green screen like this and put it. So, that it will cover your scattering by object that is the big sphere and you send this beam of small beats and you put your detector d there and you receive this d from this one and in fact, if you do change the detector around and then find out what how many particles are coming at different angles etcetera you may be able to reconstruct scattering sphere roughly some idea about this scattering sphere you will be able to obtain without actually looking in the scattering sphere.

But by observing the scattered particles or beads detected by your detector d. So, this is something which is applicable in a very similar fashion in the case of quantum mechanics and especially in the case of nuclear scatterings we will come to that.

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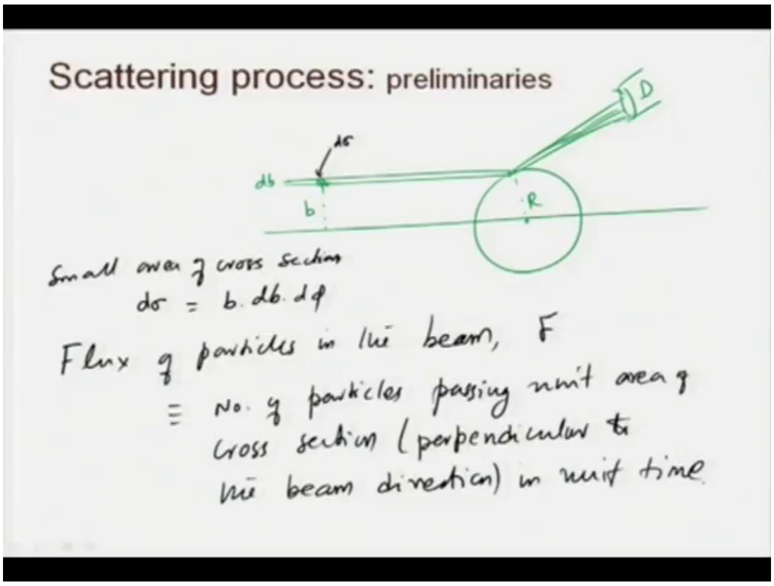

But before that let me define something what is called; what is a very useful and important quantity called the scattering cross section what is that let us again take that hard sphere scattering. So, we already mentioned this told this if we take some small area of cross section.

So, let us lets us consider some small part of the beam with impact parameter p for the inner side of the band and outer side it is b plus d b for the thickness of the band is db and then you will expect that you will get the scattering this will scatter into some kind of an angle which let me denote in this fashion and then it will if you put a detector there you will able to see this particles there correct and now the small a small cross section here let me consider a small cross sectional area there is a if you take cross section or the area of this beam or section of the beam perpendicular to the beam itself.

For the direction of momentum and denote it by d sigma. So, d sigma is some small area there of cross section which means it is perpendicular to the direction of the motion which is in this particular case geometrically it is equal to b d b d pi where pi is the azimuth angle I do not want to complicate the diagram by drawing this, but I hope you understand what is the azimuth angle it is the angle as you go around the beam that you make as you go around the now let us see you have a beam the flux of particles in the beam or the beads in the beam.

let me denote it by F basically that is defined as the number of particles per unit area or passing unit cross sectional area; area of cross section perpendicular to the area of cross section means it is the area perpendicular to the beam direction in unit time. So, let me explicitly write that. So, that is not to confuse you perpendicular to the direction of the beam in unit time.

That is basically what we called the flux. So, just remember that F in our discussion today denotes the number of particles or the beats in it that is passing through a particular through unit area perpendicular to the directional the name in unit time see if I multiply d sigma by F the flux.

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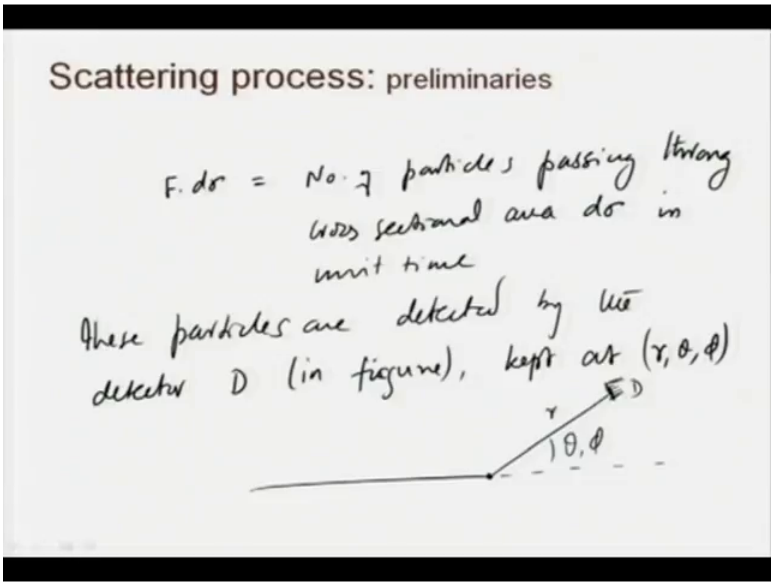

And that will give me will be equal to the number of particles passing through cross sectional area d sigma right in unit time now that is equal to F d sigma and in the hardboard scattering case all of these particles as you can see will scatter into the detector there and this is the these particles are detected by the detector d in figure just refer to the figure right let us say the position of the figure kept at R theta pi. So, in polar coordinates R theta pi is the cross section position of the detector.

Now if you if you are clever enough and if you look at the figure closely then you will say that look the if I extend this cone then actually the position or if I look at the cone and the detector position the R theta pi is not really kind of thing that is the distance from the center of the sphere it is this the different definition of this R theta etcetera you can actually how from either the point p or from o right.

So, the way we have actually discussed it is actually from the point p that we have to discuss this the R theta phi, but in a practical case what happens is that the detector is kept far away from far away from the scattering center far away in the sense that large distance compared to the size of the sphere itself in a classical scattering we can take a football size sphere and keep a detector close to it also say in very close to the surface itself or something like fifty centimeters from the surface that is not very far compared to our large distance compare to the size of the sphere or the scattering object.

But in the case of nuclear scattering think about Rutherford's scattering what is happening is there is an alpha particle which is send us the projectile. So, your bead beam of this beats will be replaced by the beam of alpha particles and you have a scattering object as the nucleus as the scattering objects nucleus of gold or some other material as the scattering object which is the extension of that is which are some kind of a order of Fermi 10 power minus 15 meters where do you keep the detector a detector is usually kept slightly far from this foil say at least half a meter or 1 meter like that.

So, 1 meter is very very very large 10 power 15 times large compared to Fermi refer if you Fermi. So, that kind of distances are much much; very very large. So, for those on practical purposes in such cases we can actually think about the scattering in potential to be very small point like that and you take you half your detector kept at some point somewhere here d and this angle theta and say azimuthal angle, we consider it as phi and a distance half. So, this can be measured from the scattering point or in the case of the hard sphere scattering from the center of the sphere.

So, we will not distinguish we will not mean that the difference between those 2 are going to be very small. So, we will not to worry about that now that we have actually define this identified that in such cases particles passing through d sigma cross section is going to go to a detector which is kept at some distance R at an angle theta at an azimuthal angle pi.

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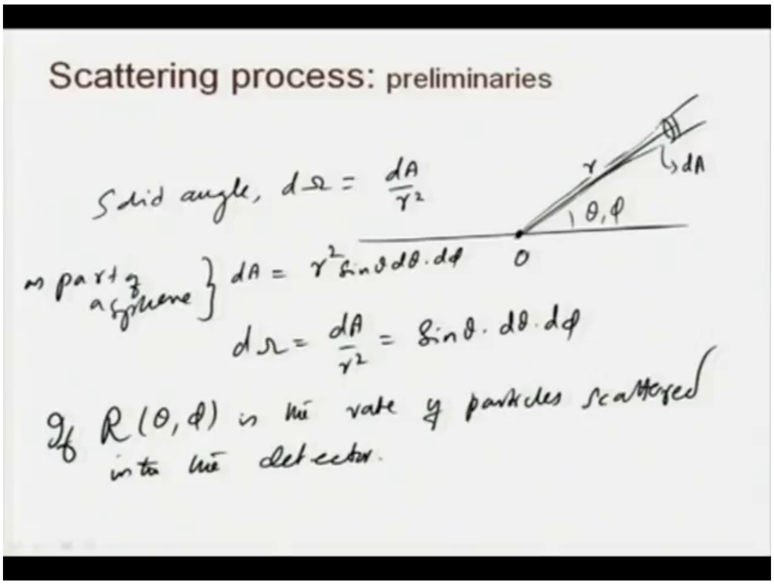

Now, so, this is theta and y let me denote it the way and then you have the detector here your detector has some area of cross section let me denote the area of cross section of the detector as d A and a naught a r is the distance of the detector at from the scattering; send this detector is a 3 dimensional object and then that makes an angle what is called as solid angle; solid angle, let as denote it by d sigma; d omega which is designed as d A divided by d A divided by r square. It is like this, you take the section of the sphere as approximate the area d A as far of this sphere with center o the scattering point and distance or the radius R if d A is the radial and then r square is the this R is the distance or the radius of that sphere then d A itself.

Approximate part of sphere for as part of a sphere d A is equal to r square d R d sin theta d theta d phi. So, solid angle c d omega is d A is r square d A is equal to r square sin theta d theta d phi. So, d omega the solid angle is d A over r square which is essentially sin theta d theta define. Now suppose for convenience I consider R theta phi as the is the rate of particle scattered into the detector.

So, I mean let me say that again we have a beam of this particles sent to the scattering one point and now you keep a detector at R theta phi and you ask the question how many particles are being detected by this detector in unit time. So, that is the number of particles scattered into the detector or the rate of the particles scattered into the detector, all right.

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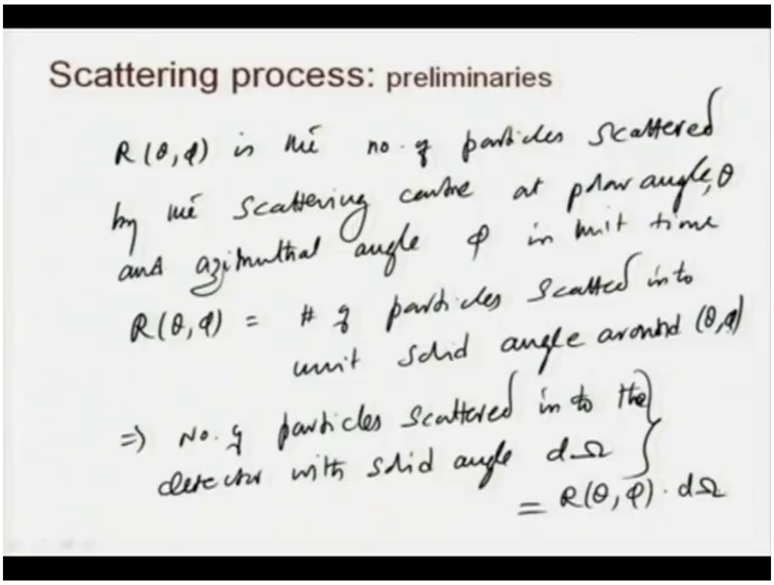

So, this in another way R theta phi is the number of particles scattered by the scattering center in the case of hard sphere scattering the scattering center is basically the sphere hard sphere scattering center to an angle as to an to polar angle theta and azimuthal angle phi polar angle. So, R theta phi is the number of particles scattered by the scattering center at polar angle theta and azimuthal angle phi in unit time.

So, that is basically the right in another way R theta phi is equal to number of particles scattered in to unit solid angle around theta pi fine now we can ask the question what is the rate of particles which are received into angle d omega the d omega let us say is the solid angle defined by the detector.

So, the number of particles scattered into the detector let me call it as detector with solid angle d omega; what I mean here is that the solid the detector when it is there placed at some area of cross section and that area you define divide by the r square the distance square; square of the distance of the detector from the scattering position that is going to be R theta pi which is basically in the number of particles scattered into solid unit solid angle right this is equal to R theta phi times d omega that is a straight forward thing to understand now we already said that if we consider some small d omega d sigma area cross sectional area in the beam and F as the flux then this number is equal to F d sigma.

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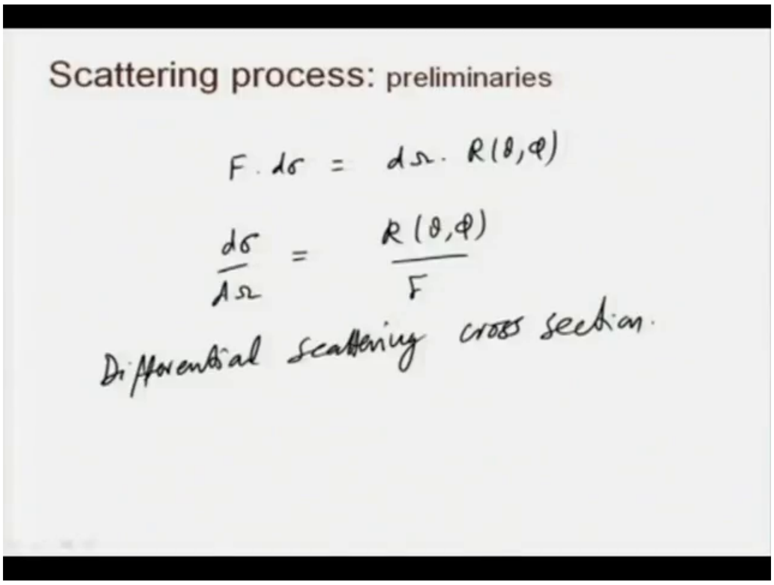

So, we have F d sigma equal to d omega times R theta phi or I can have a quantity which is called d omega over d sigma over d omega s are theta phi over F where F remember is the flux this quantity is called the differential cross section scattering cross section again let us see what is the physical significance of this if you do not know; what is the object that is scattering this suppose let us; now let us replace this hard sphere with some kind of hard surface or object with hard surface made of metals some metal say.

But with some oblige or arbitrary shape or slightly deformed shape and we do not know what the shape is and when we send this particles and then look at the scattered verticals all around this particle this scattering scattered objects and it is expected that we will be then able to get some idea about these scattered particle the scattering particle by looking at by observing the pattern of the scattering.

So, d sigma over d omega will give you some idea about what kind of object is the scattering power particle object is there sitting there and scattering it and you can easily compute this d sigma over d omega not by looking at the beam and beam the cross sectional area as a measuring the cross section area in the beam, but just by looking at the rate of particles which are detected at a detector kept at in a detector kept at solid angle kept at kept at angle theta phi with having a solid angle cross section d omega and knowing the initial flux.

So, this is what idea is and which is actually a very useful thing to understand the scattered potentials in the in the case of nuclear scattering as well as any other cord mechanical scattering particle scattering is included elementary particle scattering included. So, what we do now is to. So, we got some idea of what is this scattering and what are the things that is relevant there and then a particular object called the particular quadratic called cross section or differential scattering cross section.

We will take these ideas into the quantum mechanical system and nuclear scattering say alpha particle and gold nucleus for any other neutron object and some other nucleus and neutron as the projectile and some other nucleus as a target for example, then whatever the system is the nuclear system that we are considering is let us say that this kind of ideas can be observed I mean adopted there.

So, what is the situation there in a quantum mechanical process you cannot actually think about path of the particle like the bead the impact parameter etcetera quantum mechanical process systems as its own uncertainty with this in position measurement and momentum measurements. So, varies things are not very well defined well they are defined, but they do not precisely measured.

So, therefore we cannot actually define quantity is based on particular momentum and particular distance as etcetera in quantum mechanical systems, but still we can send the neutrons we can prepare beam of neutrons we can prepare beam of particles we can send them to target nuclei like gold nucleus and we can keep on over detector around this at various different positions of choices and we will be able to detect this particles and then we will be able to tell how many of this alpha particles are received by these detectors detectors are capable of doing even more, but let us limit our for the discussion.

Let us consider only the number of particles which are detected by the detector this can be done. So, we actually define a similar quantity d sigma over d omega that we just now discussed and defined in the another thing is that the nuclear potential itself it is not a hard sphere potential that we are considering.

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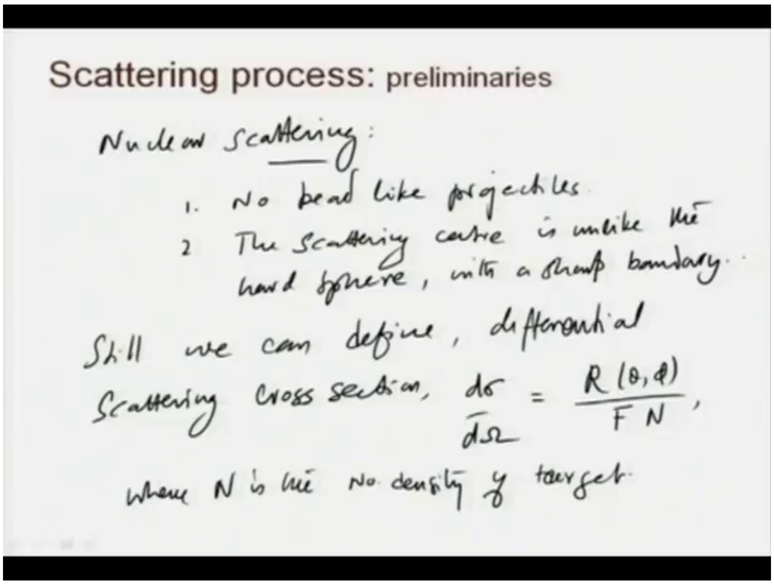

So, in nuclear scatterings we have no bead like classical bead like projectiles what I mean by projectile is the beam of particle which are send to the target and the scattering center is somewhat unlike the hard sphere a hard sphere has a sharp boundary etcetera, but whereas, in the case of nuclear potentials is you know that although the nuclear potential is confined to small regions.

It is short range potential we saw that its actually not something which actually falls down suddenly to 0, it is at the boundary, it is as it is gradually going to go down to zero. So, there are differences, but even in those cases and then what we do is basically we sent not the beads like particles, but quantum particles towards the scattering potential, but in such cases also still we can define the distribution differential scattering cross section d sigma over d omega which is equal to R theta phi number rate of particles detected by or for rate of particles falling in the unit solid angle around theta and phi divided by F.

Actually in the case of Rutherford's scattering if you can consider Rutherford's scattering, you cannot keep one particular nucleus as your target what you do is to take gold foil and keep it there and gold foil has many nuclei not just one single nuclei. So, the scattering of alpha particle is not by a particular single nucleus, but by some number of nuclei there.

So, the number of articles which has scattered in your particular angle will depend now on what is the number of particles or the number density of the particles to the target particles also. So, we have to consider that as a as well and then we will in along with the flux of the beam we will have to consider the number density where n is the number density of target and F is the flux R is what we have just now defined.

And then that will give you d sigma over d n; n can be computed we know the number density of nucleus in a gold foil or any other target that we will consider in a particular scattering F is what we are preparing the number of alpha particles etcetera or the flux of the in alpha particle or the neutron.

And R is what we are detecting. So, I on this things can be are computer on d sigma over d omega can be computer this is now a computation of this d sigma over d omega from observed and prepared initial quantities and the observed quantities and that should give as we discussed some idea about what is the scattering potential what are the properties of the scattering potential and that is precisely the idea of this scattering experiment, then through the scattering experiment we will be able to actually understand what kind of potential that behalf what are the interaction that the projectile will have with the target, all right.

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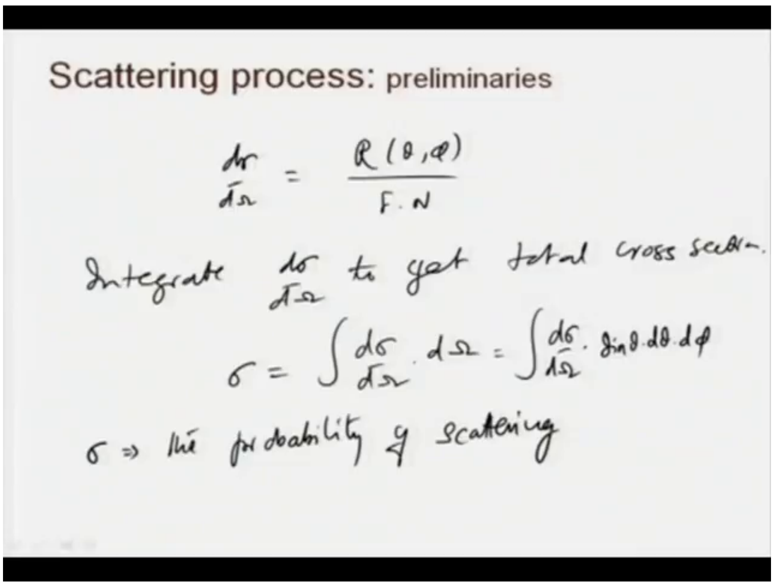

So, we have d sigma over d omega equal to R theta phi over F n correct.

Now we can integrate this what do you mean by integration I mean it mathematically you can integrate this expression, but physically what happens integrate d sigma over d omega to get total cross section right. So, when you say you integrate you take small-small such sections and add them up right.

So, that is what the integration is integration is continues addition in that case of continues variables are continuously changing variables. So, basically you take the detector in an experiment what you can do is to take the detector keep it at some angle take it at some other angle some other angle some other position all this positions and add all these numbers and then that will give you R integral of R theta pi.

When you cover the n their area of the spherical now of the imaginary sphere with radius R which is basically the distance of the detector from the scattering center and center of the sphere is taken as the center of scattering center. So, now, sigma is equal to integral d sigma over d omega which is equal to R by F n into d omega. So, this is what you have to do in that is equal to you can actually I will write it in terms of theta and phi d sigma over d omega sin theta d theta d phi. So, this is the total cross section and what is the meaning of this a; this will give you the probability sigma gives the probability of scattering to take place.

So, this actually will give you the probability for emission or of or probability of detecting particle anyway the total probability it is the probability of detecting the particle anywhere in the around that scattering center now there is one more small piece of this that we want to discuss, but maybe we will discuss that in the next class.