Nuclear and Particle Physics

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Module – 04
Radioactive Decays
Lecture – 05
Gamma decay

Today we will discuss some aspects of Gamma decay.

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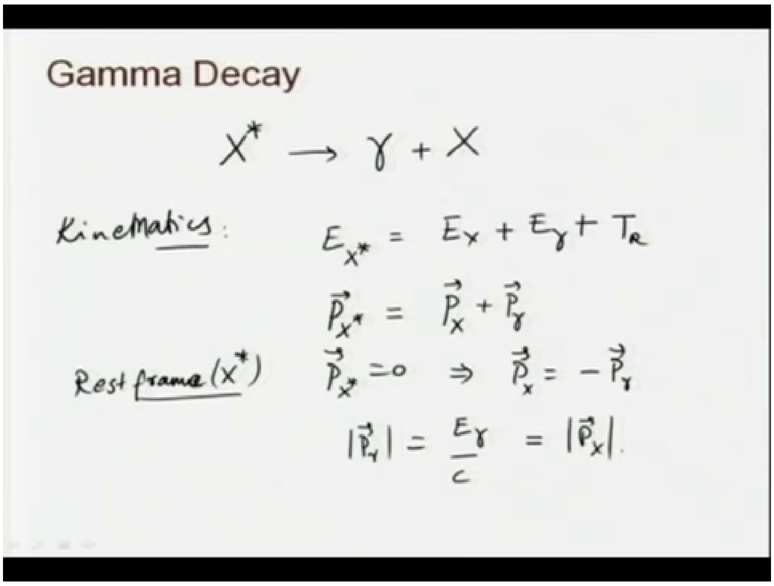

An excited nucleus can emit gamma rays and come down to lower energy states, can say the ground state for example. So, let us first look at the kinematics as usual. So, one is the energy conservation, so let us consider E x star as the energy of the initial state and E x as the energy of the final nucleus energy state of the final nucleus, and you have the energy of the photon gamma particle gamma ray emitted and when the photon is emitted you will also have the recoil kinetic energy for the resultant product nucleus. Let us denote that by T recoil, T r the kinetic energy corresponding to that this is one thing. And therefore, the energy conservation equation is initial energy E x star is equal to sum of the final state energies E x plus E gamma which is the energy of the photon emitted plus the kinetic energy of the recoiling nucleus for the atom.

Now, momentum conservation will tell us that the three momentum of the initial nucleus P x star should be equal to the two particles the sum of the energy momentum of the two particles in the final state P x and P of photon. Let us consider as usual the rest frame of the final sorry the rest frame of the initial nucleus. In that case we have P x star equal to 0 which will lead to P x equal to minus P gamma and magnitude of the photon its momentum is equal to the energy of the photon divided by the speed C because it is a massless particle the only energy is the kinetic energy and therefore, momentum related to this motion is energy divided by C which is in magnitude also equal to P x momentum energy the magnitude of the momentum of the daughter nucleus.

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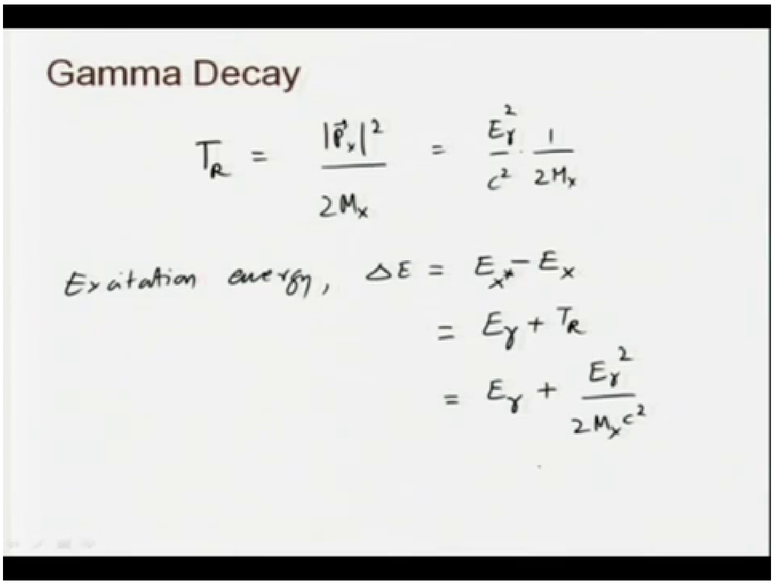

Kinetic energy T r is we will use the non relativistic expression which is enough here P x square divided by twice mass of the daughter nucleus. This is equal to E gamma square over C square which is P x square in magnitude into 1 over 2 M x.

So, if we consider the define what is called the excitation energy. The energy difference between the lower energy state, state of x and the excited energy state the initial energy state of the nucleus x star then let us denote it by delta E that is equal to E x star minus E x correct. And energy conservation will tell you that this is nothing, but energy of the photon plus the kinetic energy of the recoil energy of the daughter nucleus. This because T r is now written in terms of E gammas E gamma square over twice M x C square we can write delta E as E gamma plus E gamma square over twice M x C square. So, delta E the difference in the excited state energy and the lower state into which the nuclear state is decays is not entirely available to the photon which is emitted because x can have a small recoil it should have a small recoil how small it is because of the momentum conservation. So, the total energy available for the electron is slightly less than the excitation energy.

In fact, we can try to write get E gamma times of delta E and M x by considering this quadratic relation.

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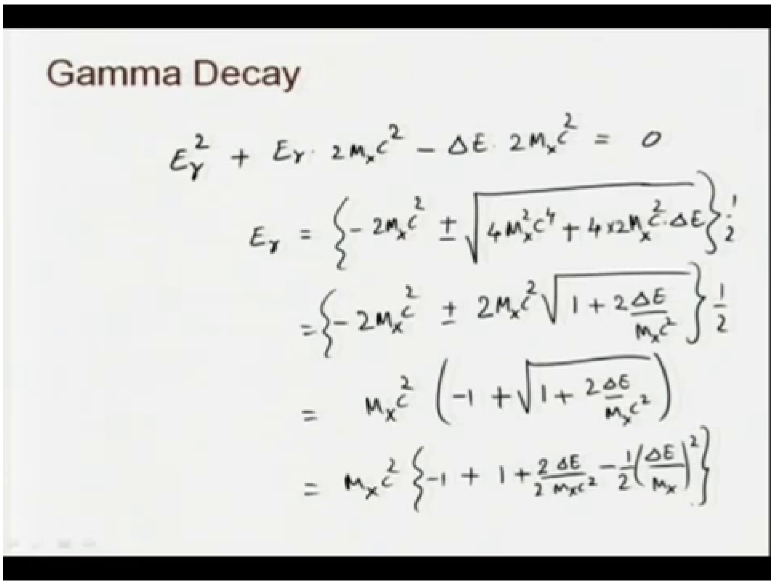

So, let me write the quadratic relation in a slightly different way E gamma square plus let me multiply the expression here by twice M x C square and that will give me E gamma square plus E gamma into twice M x C square minus delta E into twice M x C square is equal to 0. So, this will give me I can use this quadratic equation in E gamma to get E gamma as minus twice M x C square plus or minus under root 4 M x square C for the coefficient of E gamma square of that minus 4 times coefficient of E gamma square which is one.

So, that is 4 times the coefficient of the constant of the constant term the term has a minus sign already there. So, this becomes plus 4 into 2 M x C square times delta E everything under root. Which I can write as minus 2 M x C square plus or minus under root I will take 2 M x C square out and then it is 1 plus 2 delta E divided by M x C square this thing. So, essentially it is if I take M x C square twice there is already an overall factor of 1 over 2.

So, therefore, this becomes M x C square minus 1 if I take the negative root here negative sign in the second term then the total is a negative term because the first term is negative therefore, that is not a valid physical expression for the energy total energy of the photon should be positive. So, the only possible value is plus here and we have an under root 1 plus 2 delta E over M x C square. Expanding this under root I will get M x C square minus 1 plus 1 plus twice delta E over M x C square 1 over 2 of that minus sorry this should be plus here and then there is a minus 1 over 2 square delta E over M x twice.

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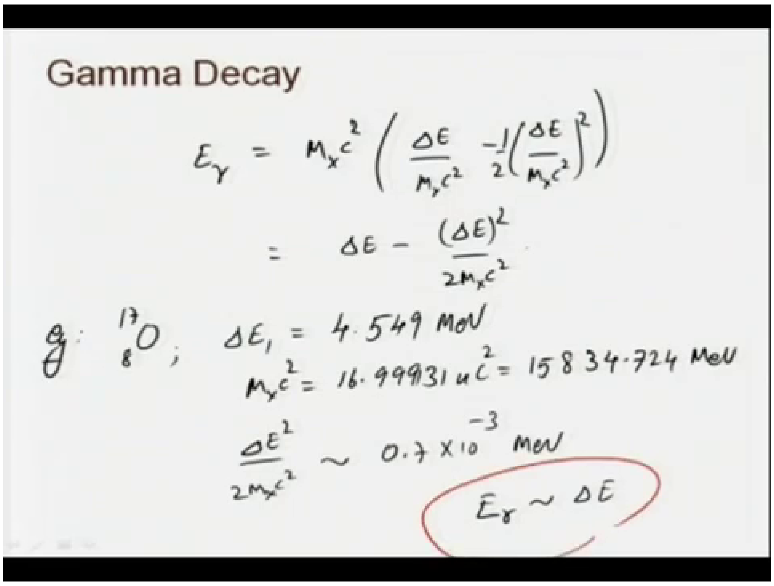

Finally, this will turn out to be E gamma is equal to M x C square into delta E over M x C square minus delta E by M x C square r square 1 over 2 and that is equal to delta E minus delta E square divided by M x twice M x C square.

So, as I said earlier the energy of the photon is slightly smaller than the energy of the excitation energy delta E. Take an example of oxygen isotope O 17, first excited state of this let me denote it by delta E 1 is equal to the energy corresponding to the first excited state is 4.549 Mev that is the excitation energy of first excitation energy of O 17. And M x is 16.999131 atomic units which corresponds to 15834.724 Mev per C square or if I retake mc square this turns out to be Mev, so much of Mev. So, I think there is no factor of 2 here maybe.

So, if I consider the correction from delta E square over 2 M x C square its approximately 0.7 into 10 power minus 3 Mev about 700 electron volts. So, we can actually neglect this compared to 4 or 4.5 Mev which is this one is something like 4 orders of magnitude smaller than that. So, for all practical purposes we can actually take E gamma to be order of this equal to delta E excitation energy. But if you are considering a solid and gamma emission from the nuclei of the molecules of atoms of the solid then a few 100 soft electron volt is enough to actually eject the nucleus from that. So, the recoil sometimes the recoil is high or even kinetic energy associated with the recoil is about a few 100s of electron volts that is enough to actually make the nucleus come out of the solid this is called radiation damage, of the solid obviously.

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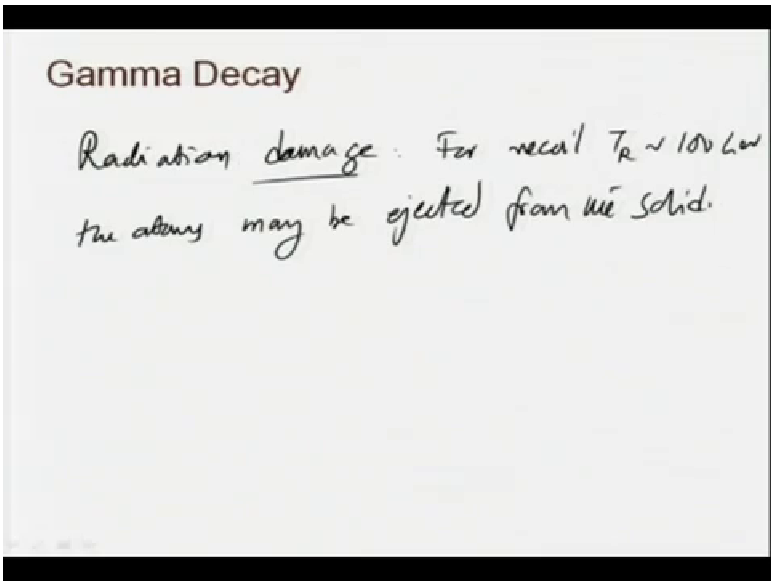

So, basically for recoil kinetic energy of 100 Gev or a few 100 Gevs atoms in the solid maybe ejected, but for lower the photon rays, so loss small excitation energies and this may not happen, but for high energy gamma rays this is possibility. So, that is a side remark that we have all right.

Now, let us consider the probability of the decay of this thing probabilities gamma decay.

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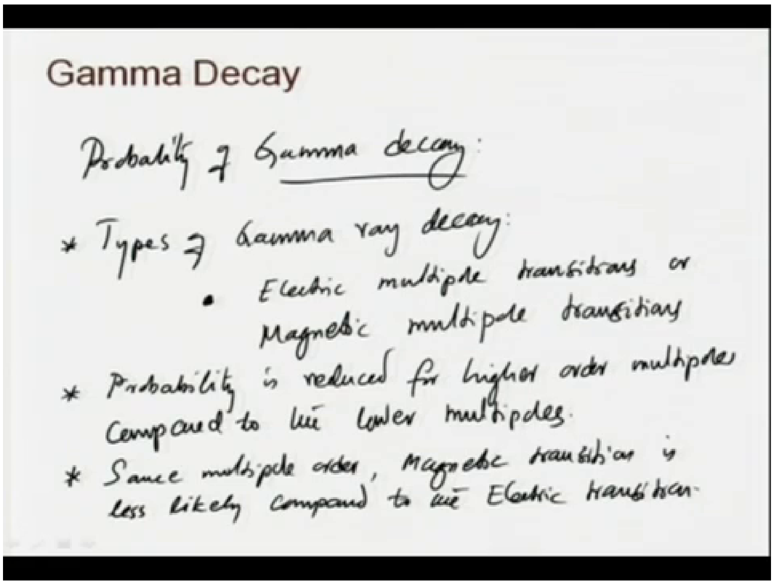

We will not go into again the details of the quantum mechanics exactly similar to the beta and alpha decays we will here, so restrict to some a bit of a qualitative understanding of the gamma ray etcetera. In fact in this particular case of gamma decay we will only discuss just mention some properties of the gamma decay without really going into any calculations of this as far as the probability etcetera is concerned. But one can actually do a full quantum mechanical calculation based on whatever model that you have and electromagnetic model that you have of course. And then you can try to compute all these probabilities for various different gamma emissions.

Here we will restrict ourselves to just simply stating some of or giving some information about this. So, first let us look at what different types of gamma rays are possible whether we can classify this in some fashion. A photon emission is basically due to a multiple oscillations right purely looking at from the point of your electrodynamics. Quantum mechanically also this can be treated in a similar fashion. So, we can actually think about the gamma decay or the photon which are coming out of the gamma decay through the gamma decay as either a due to electric multiple transition or as magnetic multiple transitions. So, that is one of the type of classification that we can have.

The transitions could be either electric multiple transitions or magnetic multiple transitions. If you ask what is the probability for electric type or multiple transitions and magnetic type of multiple transition multiple meaning either dipole quadrupole or higher poles. Probability how does it go as you consider different multiples it like it looks like it says basically probability is reduced for higher multiples higher order multiples compared to the that is compared to the lower multiples which means multiple order one which is actually the dipole transition is more probable if other ways possible compared to the quadrupole which is the 1 or 2 multiple the transition. So, if both of these are possible then the first order which is the dipole transition will be dominating compared to the quadrupole or the order two transition that is one thing.

Another thing is comparing the magnetic and the electric when we compare the magnetic and electric of the same order say same multiple order magnetic electric sorry magnetic transition is less likely compared to the electric transition all right. So, if it is the dipole transition that is we are talking about magnetic dipole transition is less likely compared to the electric dipole transitions. In fact, there is one more aspect to it which is the parity consideration that we will come to in a moment so that being actually needs to also think about understand the angular momentum.

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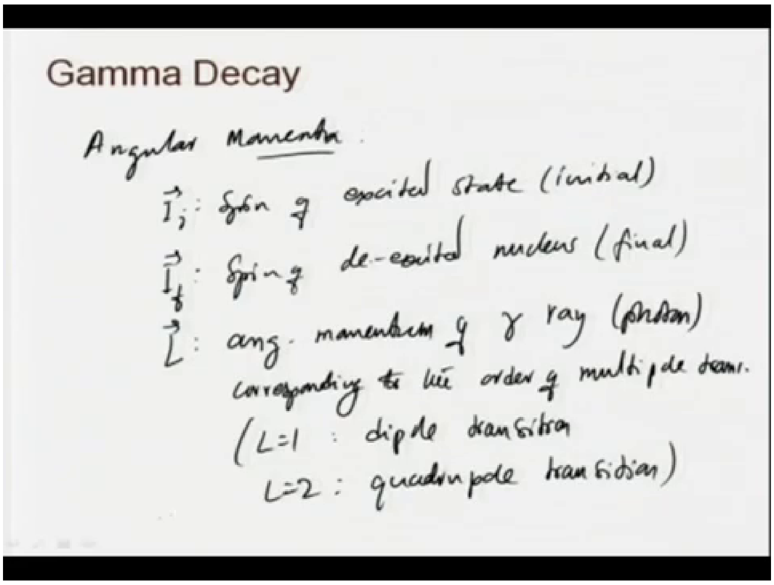

So, let us look at the angular momentum associated with gamma rays decay. There is the spin of initial excited state which is the initial state. Then there is a spin of the final state which is the de-excited states the spin off spin of d excited nucleus that is the final or daughter nucleus and with the multiple emission of the photons there is the photon which carries some angular momentum. In fact, the multiple order is basically the different multiple orders will have a different angular momentum associated to the associated photons.

So, this is the angular momentum corresponding of the angular momentum of gamma ray or the photon emitted corresponding to the order of multiple transition. For example, L equal to 1 corresponds to dipole transition as I already said and L equal to 2 corresponds to quadrupole transition all right.

Now, the angular momentum conservation will tell you that you have I i is equal to I f plus L plus is an angular momentum addition.

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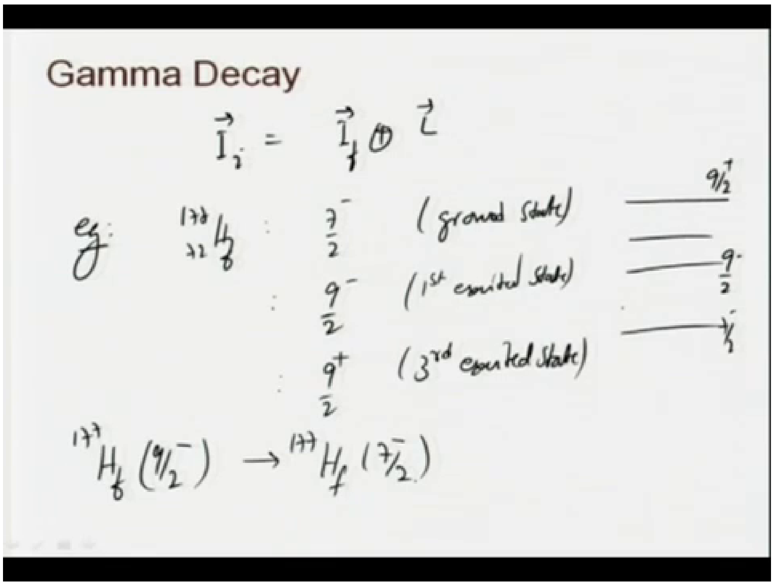

So, let us take an example hafnium 177 has a ground state with spin 7 by 2 and parity negative ground state. It has the first excited state in 9 by 2 minus and it has another excited state in fact, third excited state in 9 by 2 plus. So, it will look like something like this. So, this is 7 by 2 minus and here you have 9 by 2 minus and 9 by 2, say 9 by 2 plus and there is in between something which is actually 11 by 2 minus.

So, now, let us look at the transition from a 9 by 2 minus first excited state of hafnium 177 to the ground state of hafnium 177 which has an excited state sorry which has an angular momentum or the spin state of 7 by 2 and parity minus.

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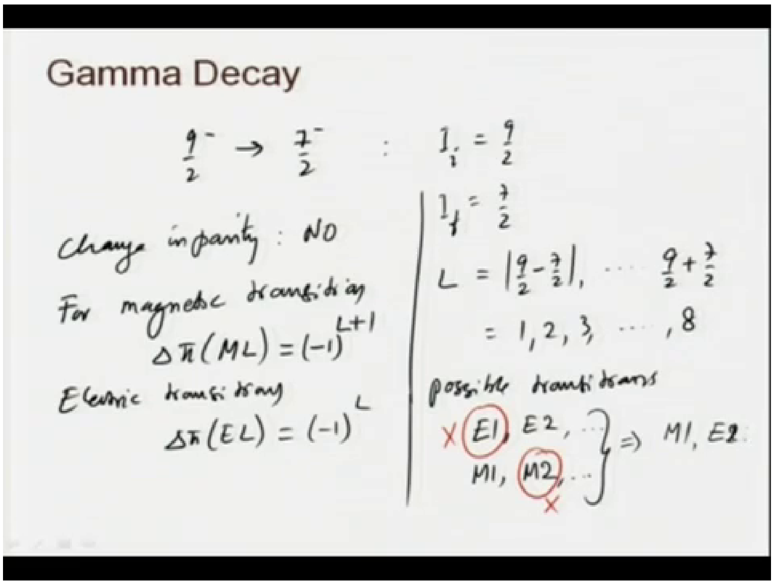

Here the possibilities are 9 by 2 minus 2, 7 by 2 minus, I initial is equal to 9 by 2, I final is equal to 7 by 2. So, possible L values are 9 by 2 minus 7 by 2 the magnitude of that etcetera up to 9 by 2 plus 7 by 2. So, this is 1 2 3 etcetera up to 9 by 2 plus n by 2 is 8.

So, as we said higher order poles are less probable compared to lower order. So, maximum possibility is for 1 compared to 2 3 4 etcetera. And if one is possible then it will happen what type electric or magnetic, that will also depend on change in parity now. In this case no change in parity right. And for magnetic transitions change in parity let me denote it by delta pi for magnetic transitions of type I mean of order L see if it is magnetic dipole then L equal to 1 etcetera, this is equal to minus 1 power L plus 1 this is what I am stating without actually showing any proof for this we will not to give any arguments even to suggest how to get this. I will leave it to you to go look for look at this in our textbook or any other book that deals with gamma rays and for electric transitions delta pi, let me denote it by EL electric transition of order L it is minus 1 power L.

So, between magnetic and electric of the same order L, if one has change in parity the other will not have change in parity. So, here just purely looking at the L values possible L values possible transitions are electric dipole, let me denote it by E one electric quadripole etcetera the higher poles let us do not worry about because they are less likely and it is possible that M magnetic dipole magnetic quadrupole etcetera.

With parity consideration E 1 is associated with a change in parity, E 2 is associated with no change in parity because L is equal to 2 and delta pi for electric transition is minus 1 power L if L equal to 2 it is plus 1. So, even is ruled out in the case of in the present example 9 by 2 minus going to 7 by 2 minus because there is no change in parity whereas, even should have a change in parity. So, even is not possible E 2 is possible how about the magnetic transitions in the case of magnetic transitions M 1 is associated with no change in parity L equal to 1 case. So, L plus 1 is 2. So, minus 1 power L plus 1 is minus 1 power 2 is positive whereas, M 2 is associated with change in parity so that is not possible.

So, we can say that it is of type E 2, but it is also possible that it is M 1. Well, we already said that the higher poles are less likely compared to the lower poles that is there is a magnetic dipole transition possibility and electric quadrupole transition possibility. And then we also said that the properties of a gamma decay is if we L if we analyze it tells us that magnetic transitions are less probable compared to the transition. Since the magnetic transition allowed here is one order smaller compared to the electric transitions we can expect that this particular case the transition will have both magnetic type and electric dipole quadrupole type. So, it will be a kind of a mixture of magnetic dipole and electric quadrupole type.

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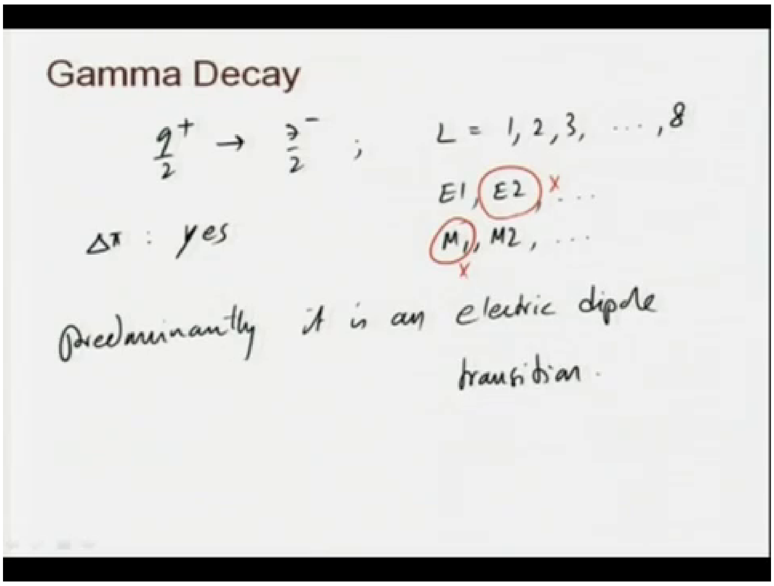

Now, let us consider the transition from 9 by 2 plus excited state to the 7 by 2 minus ground state. Here again possible L values are 1 2 3 etcetera up to 8 and by the possibilities therefore, maybe are E 1, E 2, M 1, M 2, let us stop at this and higher order multiples, but they are less likely like in the earlier case.

Now, change in parity delta pi is yes. So, the initial parity is positive final parity is negative for the nucleus therefore, the gamma ray should be associated with the multiple which has with change in parity. As we said earlier E 1 gives associated with parity change in parity and therefore, that is possible, but E 2 is not possible because E 2 corresponds to no change in parity similarly for the case of magnetic transition M 1 is not possible, but M 2 is possible. And now if you compare E 1 and M 2 as we said electric type of transition is more likely compared to magnetic type and again higher order pole is less likely that is the quadrupole transition is less likely compared to the dipole transition. So, M 2 has double disadvantage here compared to the E 1 therefore, we will say that predominantly it is an electric dipole transition all right.

So, that is one of the things that we can understand by just looking at the spin and parity of this one. And in fact, we can do a more involved analysis by looking at the probabilities and then from our theoretical calculations of the dipole transition and see whether that agrees with the observations etcetera. But we will not go into such details in these lectures.

We have another possibility in this case in the case of excited states decaying into ground state or any other low energy state.

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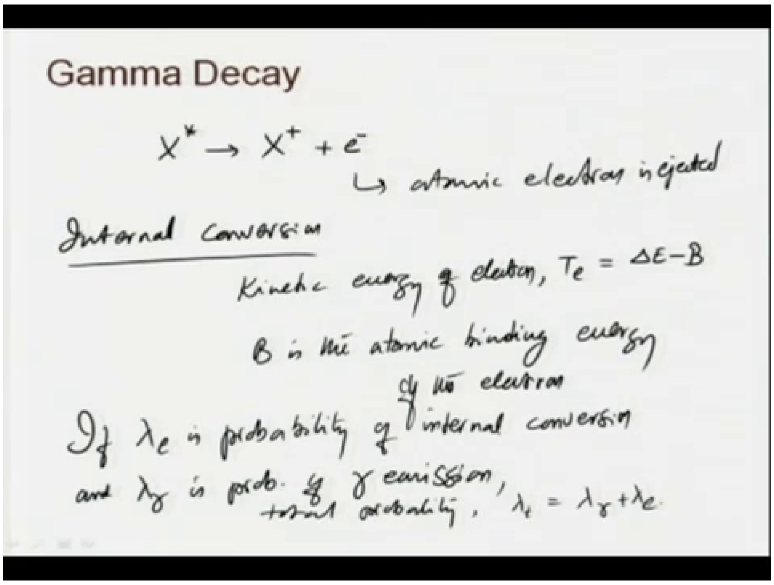

It is possible that instead of emitting a photon the excitation energy of the nucleus is transferred to the atomic electrons, thereby emitting an electron which is basically an atomic electron. So, this is unlike the beta decay. In the case of beta decay a neutron was converted into a proton and an electron was created there which was not present earlier. But in the case of in this particular in this case an atomic electron which already existing in the atom revolving around the nucleus, gets the energy from the excited nucleus the excited nucleus will transfer the energy to the electron atomic electron and such atomic electrons are emitted.

So, e minus is an atomic electron and the atom becomes a positive ion. So, let me just denote by plus X plus just to remind you that it is an atomic electron which is going away from that. This process is called internal conversion. So, if I look at the kinetic energy of electron emitted electron let me denote it by T that is equal to the excitation energy minus the binding energy, to release an electron you had to give some energy to the atom which is equivalent to the binding energy of the electron. So, B is the atomic binding energy, not the nuclear binding energy that we are talking we were talking about in the case of a nucleus, but the electrons atomic binding energy of the electron. It actually depends on in from the which shall the electron is emitted from the k shell from the L shell or whichever shell the binding energy will depend on that all right.

So, from here we actually have can make another statement. Let me denote lambda e has the probability of internal conversion and lambda gamma is the probability of gamma emission from an excited states the total probability of x de-excitation is lambda t equal to lambda gamma plus lambda e. So, this is always there that when asked there will be an a small probability for such internal conversion which is always possible.

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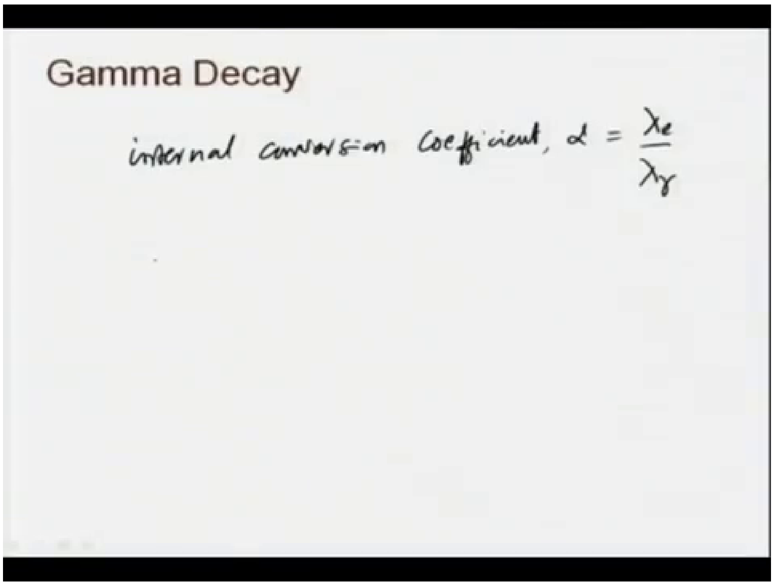

In fact, one can actually define the coefficient of internal conversion or internal conversion coefficient which is basically giving you the relative strength between the probability for emission internal conversion emission of an electron and probability for gamma ray emission. It is usually denoted by alpha. Sometimes with the subscript in fact, you can actually have a subscript for denoting the shell in which the electron is emitted from which the electron is emitted.

Now let us consider, when we consider a higher excited state of the nucleus decaying down to a lower excited state it is possible that it can either convert to lower excited state by emission of photons and it is possible that we are considering the not the first excited state, but second or third excited state of the nucleus. In that case it can de excite into either the ground state or to the first excited state etcetera whichever is the lower energy state possible.

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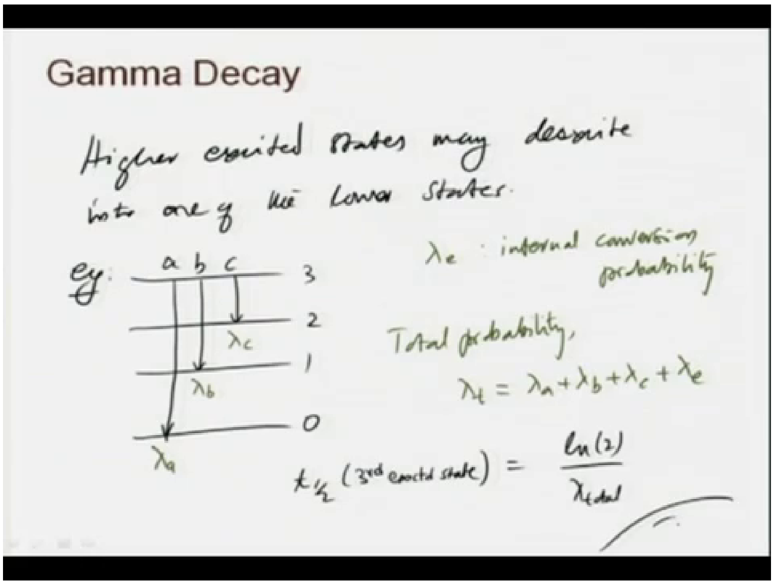

So, in such cases when we consider the probability we like to add all the probabilities of them for this. So, let us say higher excited states may be excite into one of the lower states right that is what just now we said. So, let us consider an example of one ground state like first excited state, second excited state, third excited state. And if an atom is in the third excited state when nucleus is in the third excited state it can emit a photon and come down to ground state. That is one possibility.

Let us denote that as process a another possibility is that it can go down to first excited state by emitting photon again b and third possibility is to go to 2 by emitting a possible missing a photon process 3. Probabilities for these to happen will depend on various things like whether it is an electric transition or a magnetic transition, what kind of dipole, whether it is a dipole transition or a quadrupole transition or higher pole transitions etcetera. Mean time that depends on the spin and parity states of the ground state first excited state, second excited state and the third excited state.

And also depends on the atomic weight the excitation energy itself and the energy gaps between the first and second excited, first and third excited state, second and third excited state etcetera. We did not explicitly write down any of these expressions or the dependence of these quantities on the probability, but I mean they the probability does depend on all those things. So, we will not go into those details. But let me denote the probability for this process a that is the excited de excitation into ground state as lambda a, the one to de excitation to first state as lambda b, the probability for the de-excitation into second excited state as lambda c. And then there is also a probability for internal conversion. In fact, the probability for internal conversion is also many that it is probability for internal conversion by emitting k shell electron emitting an initial electron different electrons in that this thing the cross by electrons. But it could be l shell, k shell or whichever other shells.

So, when we talk about a probability for internal conversion let us say we add all of those probabilities for emission of k shell electron, l shell, l shell electron etcetera call it total as lambda e. So, this is the internal conversion probability. Then the total probability lambda t, let me denote it by lambda t is lambda a plus lambda b plus lambda c plus lambda e, even in the case of lambda a it could be electric and magnetic together like we saw in the example of hafnium.

So, now the half life for example, of decay or de-excitation, so half life t half of third excitation excited state is simply log 2 over lambda total. So, we have to actually compute the partial widths or partial probabilities, probabilities in to decay to different channels and then add all the probabilities and then the total probability will get and the half life will depend on the total probability, so 1 over the x.

So, we will stop here for the time being and then will do other discussion in the next class.