Nuclear and Particle Physics
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Module – 04
Radioactive Decays
Lecture – 04
Beta decay- continued

Today we will continue our discussion on Beta Decays. We will first consider the selection rules based on the angular momentum conservation.

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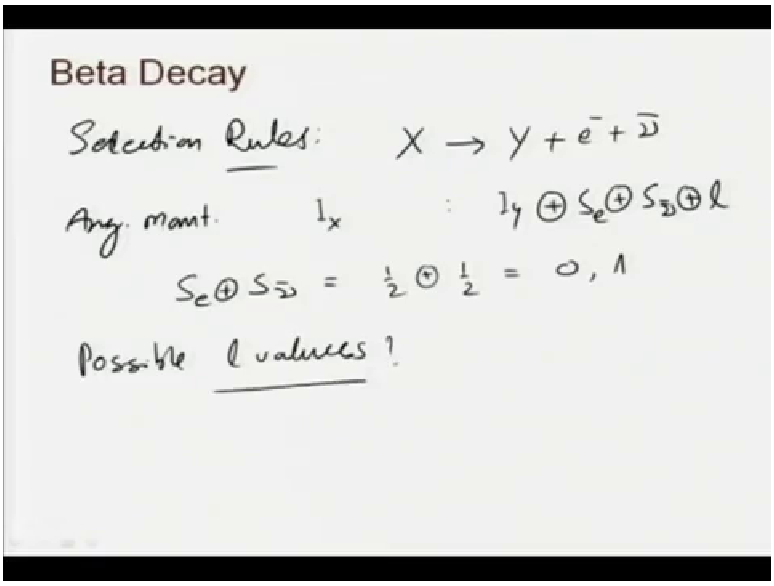

So, the process we are considering is beta decay which converts an X type of neutron nucleus to some Y nucleus with emission of electron and antineutrino as discussed in the last lecture.

So, let us consider the spins of these particles the nuclear spin spins or the angular momentum, let us say angular momentum let me denote it by I X, I Y then the spin of electron denoted by S e and S neutrino plus there could be some orbital angular momentum relative or will angular momentum here, for the electron and neutrino system.

First let us consider these plus S nu both of these are spin half particles. So, half plus half is equal to either 0 or 1 as per the angular momentum addition rules. l values the orbital angular momentum of the e nu system with respect to the Y Z.

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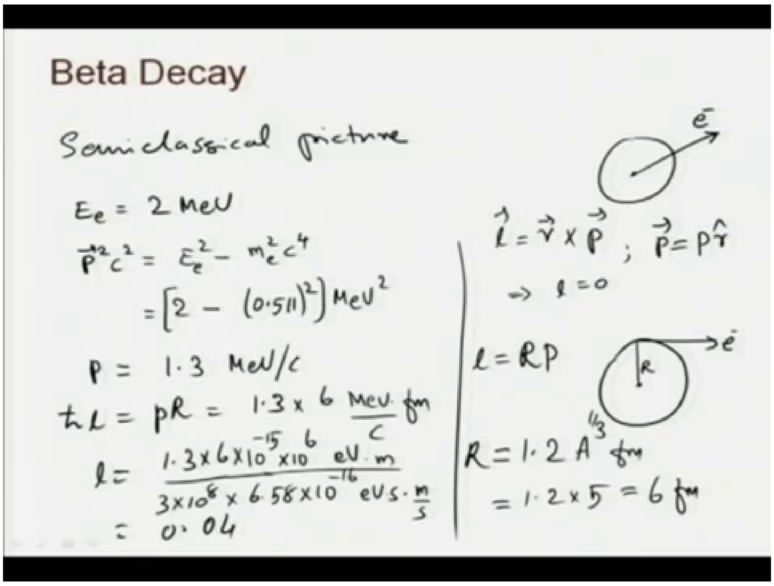

So, to get an idea of what kind of l values are allowed or possible, let us consider some kind of a semi classical picture where we have to consider the nucleus as some kind of a spherical ball and let us consider the case if the electron is originating at the center of this and moving directly along the radius then it does not have any angular momentum l because l is r cross p in the coordinate system. When you consider the relative angular momentum of electron we are talking about the r the position coordinate of the electron with respect to, the position coordinate of the electron.

Let us say this is the case you consider this to be the center and electrons are emitted from the nucleus like this in that case r of electron is along the radius of this part of the nucleus and l s r cross p and p is now along the r cap direction because it is moving along the r cap direction. So, that will give you l equal to 0 kind of things. But if it is not the case if it is actually moving say tangential to this r that is the case when r cross p will be maximum and even the magnitude is maximum the r perpendicular direction the distance between the sender and the direction of p, if that is also maximum which is possible if the electron is emitted from somewhere at the periphery of the nucleus. So, in that clay case r is l is equal to basically R, R is now capital R the size of the nucleus times P the momentum of this that is the maximum kind of possibility, classically just in a classical picture. So, it is actually giving you some kind of an estimate not an exact picture or exact values quantum mechanically we will have slightly different distance. But to get a feel of this thing we will take this kind of picture.

So, here r and p are perpendicular to each other. So, r cross p is just the magnitude magnitude the product of the magnitudes r and p. So, let us consider the energy of the electron say 1 and 1 or 2 Mev. So, let us consider a 2 Mev electron. So, we have the energy momentum relation which says p square or c square is equal to E square minus m e square c power 4 right and we have e to be 2 Mev, m e c is c square is 0.511 Mev. So, square of this is Mev square. So, this is the value of this in Mev square it turns out that.

If you calculate this do the calculation P is equal to I will leave the algebra calculation for it to you P turns out to be 1.3 Mev per c or p is equal to 1.3 square root of this and then that is 1.3 and it turns out so that p is equal to 1.3 Mev per c.

Now, l is equal to or rather the in quantum mechanics it is h cross l which is the momentum which is p and l when it is taken in units of h is equal to p R is equal to 6 by 6. Let us see what is momentum 1one is 1.3 inverse the value of R, R we know is equal to 1.2 in times a power 1 by 3 Fermi. So, if you take something like an, a value around 100 and 125 then you have a 1 by 3 equal to 5. So, let us say it is something like 5 will be approximately a is equal to 6, so 1.2 so that is exactly equal to 6 Fermi. So, we have 6 and units Mev per c divided by c coming from momentum and 6 Fermi is the radius nuclear radius, so Fermi is coming from R.

Now, l is equal to 1.3 into 6 divided by c is 3 into 10 power 8 it convert Fermi into meter it is 10 power minus 15 meters divided by h cross. So, value of h cross is 6.58 say into 10 power minus 16 electron volts second and meter per second coming from c. And in the numerator we have Mev. So, let me write mega electron volt as 10 power 6 electron volt and Fermi we already converted into meter. So, that is actually in millimeter. So, the units are the same for numerator and denominator and therefore as expected l is a dimensionless number so that is the angular momentum in units of h cross. So, this is you do the algebra gain and turns out to be something like 0.04.

So, that is the kind of a maximum expected value of the relative angular momentum of the electron coming from a nucleus they are typical nucleus with an atomic mass number, nuclear mass number a equal to something like 125 which is a much less than 100 times less than 1 almost and therefore, we expect that l value of the electron coming orbital I know the electron coming out is 0, similarly for the neutrinos its similar values 0.

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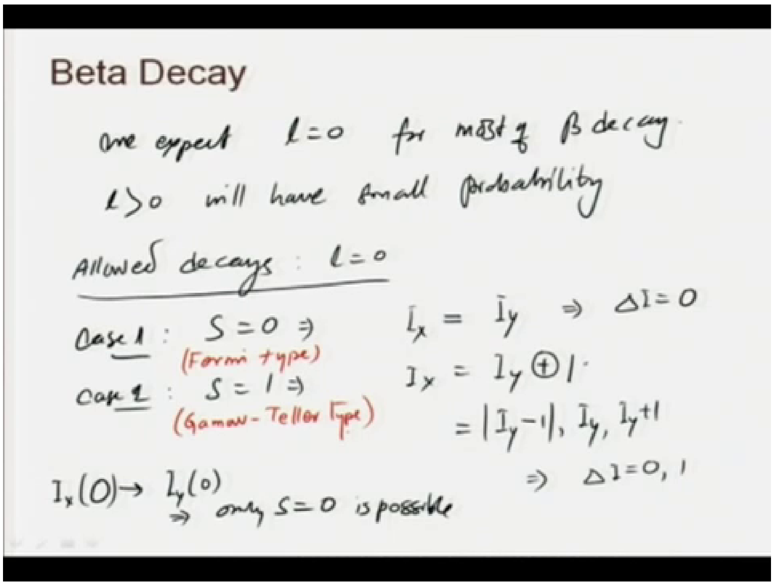

And then therefore, we expect l equal to 0 for most of beta decay. Well, the other way of saying it is that the most probable beta decay we will have l equal to 0. If it is l equal to one although classically it is not possible the way we have discussed quantum mechanically we can actually say that there is very less very less probability for this to happen. So, l other than 0 will be half will have small probability. In fact, the larger they l the less likely it is as expected from this one and in one can actually have more thorough rigorous quantum mechanical calculation and it is also giving similar results all right.

So, now, if we consider l equal to 0 which is incidentally called allowed decays meaning l equal to 0 which is the one which is allowed that is what. And the rest of it l not equal to 0 are called forbidden decays which actually means that they are less likely to happen not completely forward and it is not like no, no, kind of a situation, but it says that that is very unlikely or to happen. There is one situation where these l equal to 0 even if it is energetically possible it will not be possible from the point of view of a angular momentum and parity got sort of considerations and then it is possible to half the only possibility is to have the forbidden kind of a decay. We will come to that directly in a minute.

So, let us consider l equal to 0 and in l equal to 0 as already said S can be 0 or 1. So, case 1, S equal to 0 the total say spin of electron and neutrino combined when S equal to 0 we have the angular momentum balance equation that is S the spin nuclear spin I X the initial nucleus spin I X is equal to I Y plus 0. So, there is nothing to add. So, that actually says that both of these nuclei will then have the same spin or delta I change in the nuclear spins, spins of the nuclear the daughter and the parent nuclei is equal to 0.

So, this I is this delta is equal to 0 and then you can have a S is equal to 1 case and there I X is equal to Y plus S is 1 plus 1. To remind you again and this plus within the circle actually put in to remind you that these are not simple number additions, but rather it is the, it is the additions are is according to the angular momentum addition rules vectorial notations. So, this will tell you that actual number of like addition it is I Y minus when the magnitude by I Y plus 1 these are the possibilities.

Here again delta I is if I X is equal to I Y then delta equal to 0 if delta if I X is equal to I by plus 1 or Y minus 1, 9 root this delta is equal to 0 or 1 now the possibilities. So, it is possible to either have I equal to 0, where I delta equal to 0 or delta is equal to 1 in the case of l equal to 0 a special case is 0 to 0 transition. So, this is I X is equal to 0 to I Y 0 case. So, here delta I is equal to 0. So, it is only possible through S is equal to 0 transition, only S is equal to 0 is possible. All other delta equal to 0 case, it is possible to have either S is equal to 1 transition or S is equal to 0 transitions. So, if it is S equal to 0 transition we will call this Fermi type and if it is equal to one we call this a Gamow-Teller type beta decay.

So, Fermi type beta decay only means that S the total spin of electron and antineutrino is equal to 0. For electron and antineutrino are anti parallel to each other and Gamow-Teller type of reaction indicates that the spins of electron and neutrino are parallel to each other thus giving S equal to 1 for this, all right.

Now, let us take some examples of the, let me write it first as a summary.

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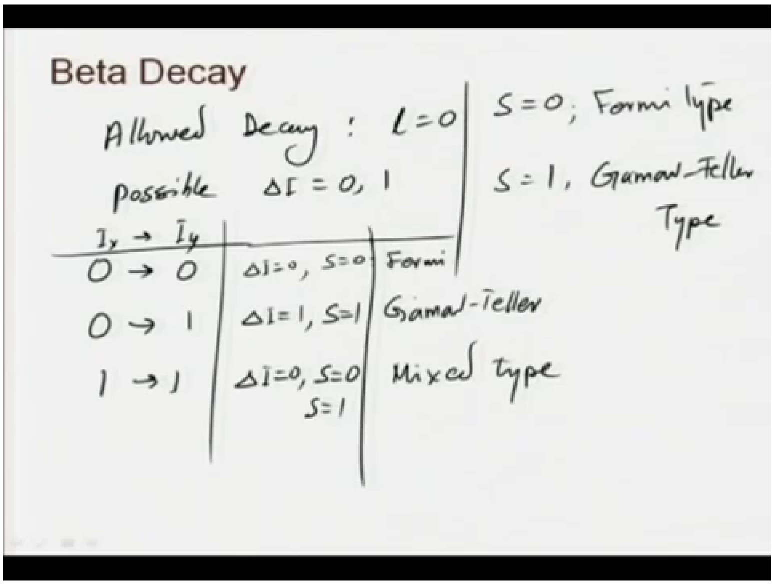

So allowed decay which means l equal to 0 some terminologies and pure Fermi. So, Fermi type possible, possible delta I equal to 0 and 1, that is one thing S is equal to 0 are called Fermi type beta decay S equal to 1 are called Gamow-Teller type, the Gamow-Teller type decay. Hence therefore, as 0 to 0 some cases 0 to 0 transition here I am denoting the, I values of the X. And so I X to I Y 0, 0 tells you that each is delta I equal to 0 and only possibility is S equal to 0. So, it is Fermi type and if it is say 0 to 1 kind of a transition then delta I is equal to 1 S is equal to either 0 or 1. So, it is delta S is equal to and that will be a mixed type then you can have one to one then that is delta equal to 0 still right. So, it is S is equal to 0 or S is equal to 1, so then all right. So, I should correct myself.

This is possible only if S is equal to 1 and therefore, this is equal to a pure Gamow-Teller type. And in the last case I is equal to 1 I X equal to 1 to Y equal to 1 this is delta I equal to 0, but S is equal to 0 or S is equal to 1 both are possible. So, we have actually a both Fermi and Gamow-Teller types or what are called the mixed type all right.

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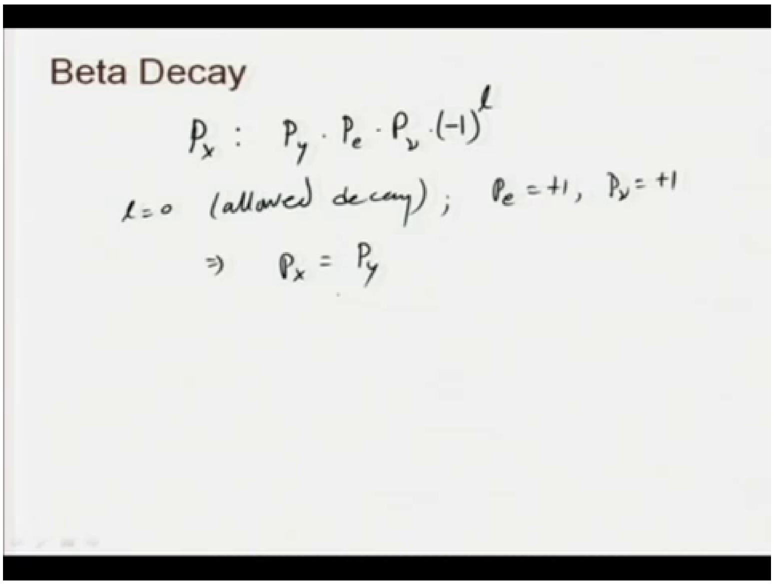

Now, let me come to the parities considerations. Parity of X is on the right hand or left hand side in the initial state and parity of Y times parity is a multiplicative and a number, you multiply it to get the total resultant then you have the electron and you have the nutrional, this thing and you have the parity due to the orbital angular momentum. Let us say l which is equal to minus 1 power l.

And if we are considering l equal to 0 allowed case then and we also know that P is equal to plus 1, P nu is equal to plus 1 and then therefore, that will give you P x is equal to P y. So, in this discussion in the allowed decay it is always P x is equal to P y. So, l equal to 0 will give you parity of the daughter and parent nuclei to be the same. So, that is one thing.

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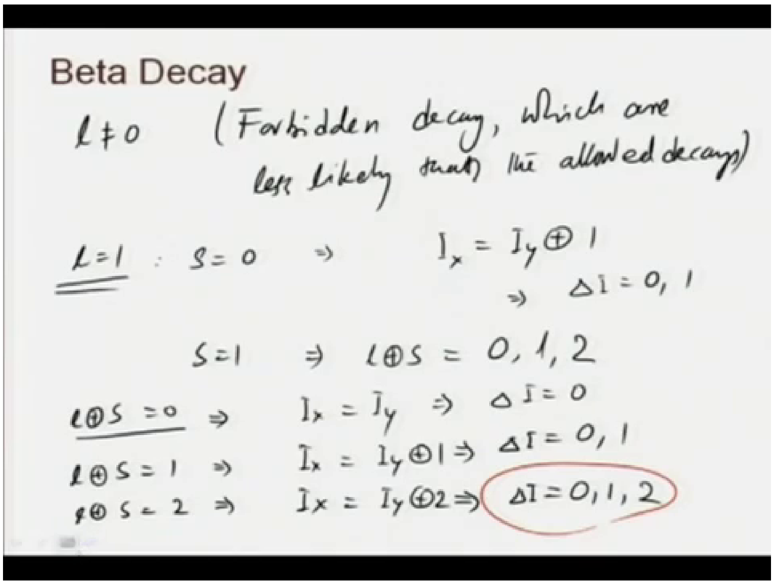

Let us see what happens when l is not equal to 0, l is not equal to 0 are called forbidden decay which are less likely the probability for that to happen is smaller less likely than the allowed once much less than the allowed decays. So, let us say k is 1, l equal to , we consider l equal to 1 and let us consider in that S is equal to 0. So, that will give you delta I is equal to all right.

Let me take the angular momentum relation I X is now equal to I Y plus 1. Again like earlier discussion delta I is equal to 0 or 1, S is equal to 1 will give you l plus S equal to 0 or 1 and each of these case let us consider separately l plus S is equal to 0 will give you I X equal to I Y correct or delta I equal to 0. Now, l plus S is equal to 1 that will give you I X equal to I Y plus 1 actually l plus S can be 0, 1 or 2. I missed the 2 there. When you have l equal to 1 and S equal to 1 you can have l plus S is equal to 0, 1 or 2. So, anyway, l plus S is equal to 1 will give you I X equal to I plus 1 which is again delta I is either 0 or 1 and l plus s is equal to 2 will give you I X is equal to I Y plus 2 that will give you delta I either 0, 1 or 2.

So, in the case of l equal to 1 we can expect, we can expect change in the parity. So, change in the I between the parent and the daughter nuclei to be either 0 or 1 or 2, l equals 0 case we had delta equal to 0 or 1, l equal to 1 case we can have l equal delta equal to 0 delta equal to 1 or delta equal to 2 fine.

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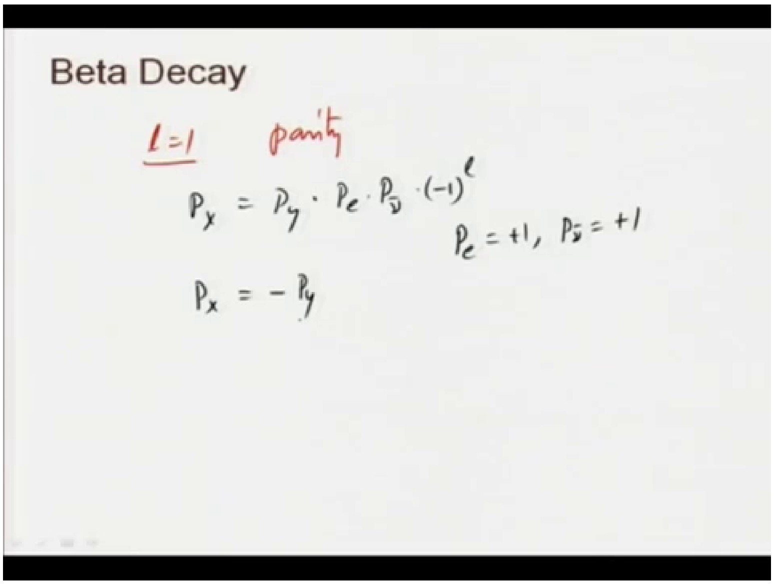

Now for l equal to 1, what about parity? Is again P x is equal to P y times P times P nu bar X minus 1 power l and as I said earlier P is taken to be plus 1 P nu bar is taken to be plus 1. So, P x is equal to minus P y. So, l equal to 1 there is parity change in between the nuclear parities of the daughter and the parent nuclei and fine that is one thing.

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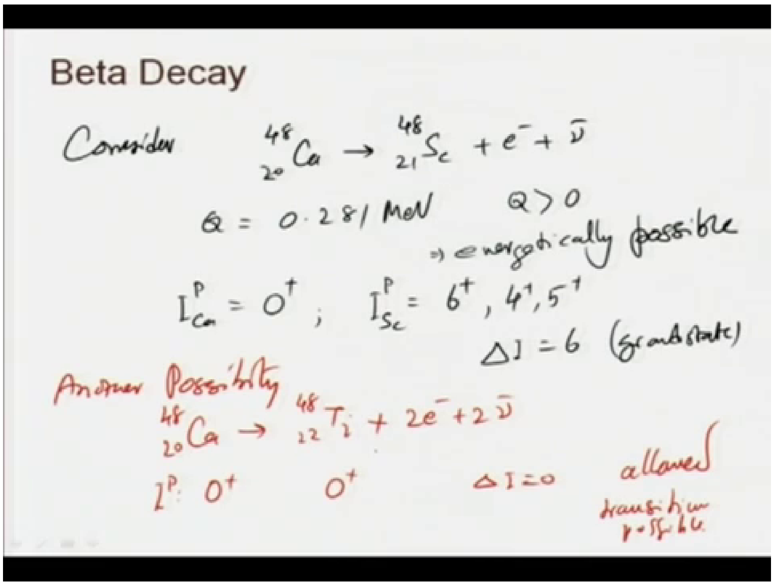

Now, let us consider calcium 20 nu protons take the isotope with 48 that is the mass number if we consider the if we consider the beta decay of this possibility is that it goes to scandium 21 48 plus electron plus antimony say. Compute the Q value of this Q is equal to I will leave it to you to compute it, it is turns out to be 0.281 Mev positive Q value, energetically possible. But I value of calcium I P of calcium 10 48 is equal to 0 plus and I scandium 48 p is 6 plus in the ground state. And if you consider the excited state where still there is possibility, it is possible to actually have positive Q value it is still 4 plus and 5 plus the first few excited states. So, for all of these delta I is 6 ground state 4 for the excited states 4 and 5 for the excited states.

Now, as for this to get this we discussed the l equal to 1 case where we could have get delta I is equal to 2 and to get delta is equal to 6 you need l equal to 5. You can do the algebra exactly the same way as we did earlier. So, now, l equal to 5 it should be very unlikely compared to the case of allowed decay or even l equal to 1 first forbidden decay kind of thing.

Now, let us look at another possibility. Another possibility is that calcium 48 goes to not scandium, but titanium 48 which is having 22 protons in it. So, plus 2 electron plus 2 neutron a side. We are not saying that calcium first goes to scandium and then decays to titanium that is not that is not the case it goes in one step to titanium not as an intermediate tree this one.

In that case if you look at the I P of initial it is 0 plus and T a 28 48 plus 0 plus as the ground state delta I is equal to 0 it is an allowed transition right. It will fall under allowed transitions. Here what let us look at this reaction again what is happening? We have a calcium and X nucleus going to a bi nucleus and 2 electrons and 2 neutrinos. So, this is not single beta decay it is a double beta decay.

So, these are typically called double beta decay explain this ones and we have other examples of double beta decay.

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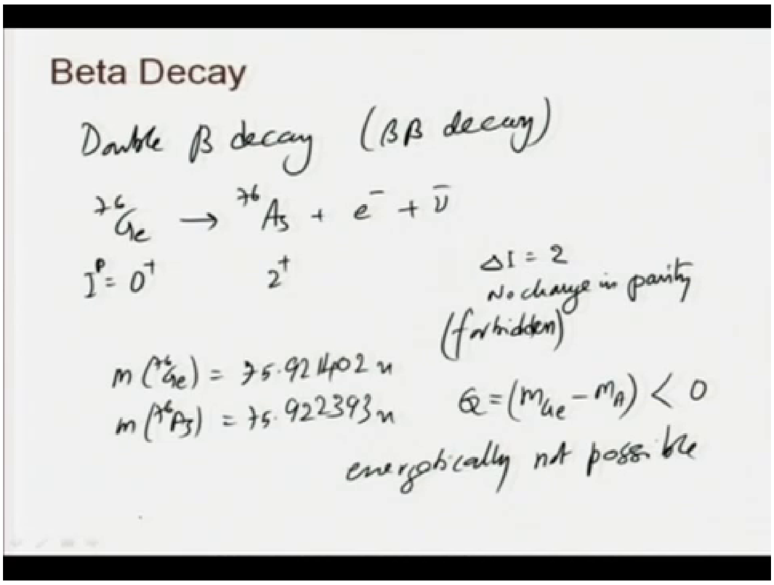

So, let me call this is the double beta decay or sometimes also called beta beta decay. Let us take another example say germanium 76 it has I P is equal to 0 plus goes to arsenic 76 let us say through a beta decay that is a possibility. Possibility meaning that was that is if we consider the beta decay of the germanium single beta decay of germanium then this is what will happen, 2 plus is the I P value of arsenic. So, delta I is equal to 2, no change in parity.

It is actually a then forbidden. It is not possible even to have l equal to once decay because l equal to 1 decay corresponds to a change in the parity in this case there is no change in relation that is one thing. Even if you take the mass of germanium 76 which is equal to 75.921402 units atomic units and mass of arsenic 76 which is 75.922393 units you see immediately that Q is equal to m germanium minus m arsenic and germanium is smaller than m arsenic and therefore, is smaller than 0 or negative.

So, you have a daughter with larger mass which is energetically not possible you cannot have a particle to decay into a particle with larger mass must larger than the parent particle energetically not possible, not possible at all. So, from either of the accounts anyway energetically not possible we will not even consider me to discuss other things, but even if you consider the other whether it is angular momentum and suggestions it is very very unlikely to happen.

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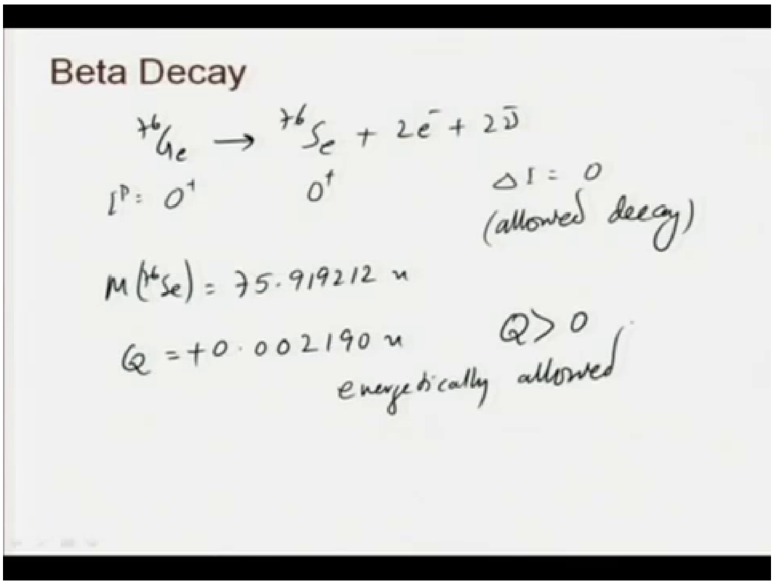

But we can again consider the same germanium 76 and ask the question what about the double beta decay possibility sorry S e 76 and 2 e plus 2 anti neutrinos. Here I P is equal to 0 plus and 0 plus well that is and there is no change in parity, so allowed decay. What about the masses? M S e 76 is equal to 75.919212 units and Q turns out to be equal to plus 0.002190 u whatever is the corresponding Mev is we are not worried about the magnitude as such. But we want to check whether it is possible energetically or not. So, it means that Q should be larger than 0 which is the case here. So, energetically end out no problem. The energetically allowed double beta decay with delta I equal to 0 which means there are no change in parity and therefore, it is and allowed, it is possible to have allowed to decay.

Again we have an example where it is not possible to have a single beta decay, but double beta decay is possible all right, very good.

Now, there is another piece of topic that we would like to discuss under beta decay which is basically the parity violation, of parity violation in beta decay.

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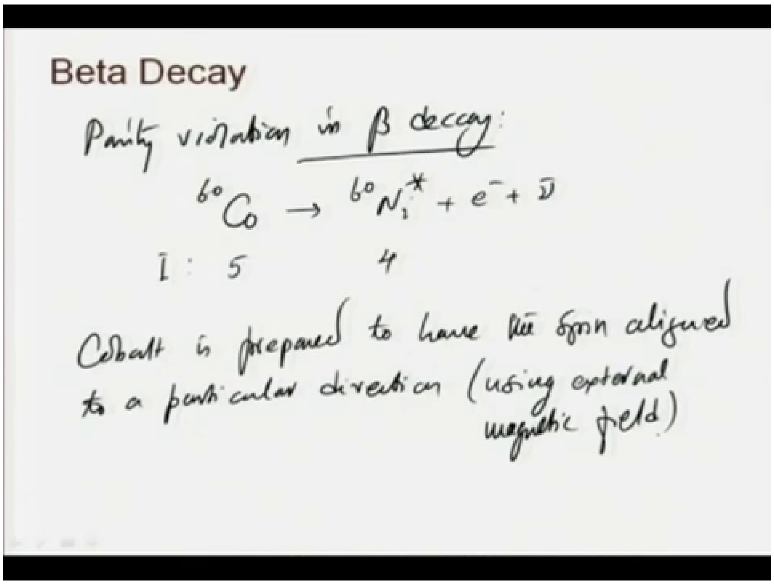

Let us consider cobalt 60 which decays to nickel 60. In fact, it decays to one of its excited states plus e minus nu bar. I of cobalt is 5 and I of nickel is 4. Now, let us consider the situation where cobalt is prepared for the experiment. So, you take a large sample of cobalt and prepare it to be to have the spins nuclear spins this has a large nuclear spin I equal to 5 spin aligned to a particular direction right, perhaps most of the time this is the way it is done using external magnetic field.

So, if you do not prepare it in that fashion then what happens is that if you take a sample of cobalt and then it is pinned although it has large spin, then if you have 10,000 or such or much more than that even a 1 mole of this 10 power 23 of these cobalt atoms and nuclei in it and those all will be condemned randomly arranged and therefore, the total spin of the sample is equal to 0.

But you can actually align this family the cobalt spin to be in the same direction by putting them or placing them in the sample in magnetic fields which has some direction. So, the spins tend to align along the magnetic field and then therefore, you get some direction for the magnetic field. So, that is what is happening. So, it is somebody like in a polarized sample.

So, let us look at. So, that is what happens. So, we can actually arrange it and keep that in mind. Now, let me come to what happens under parity for that let us see what is spin.

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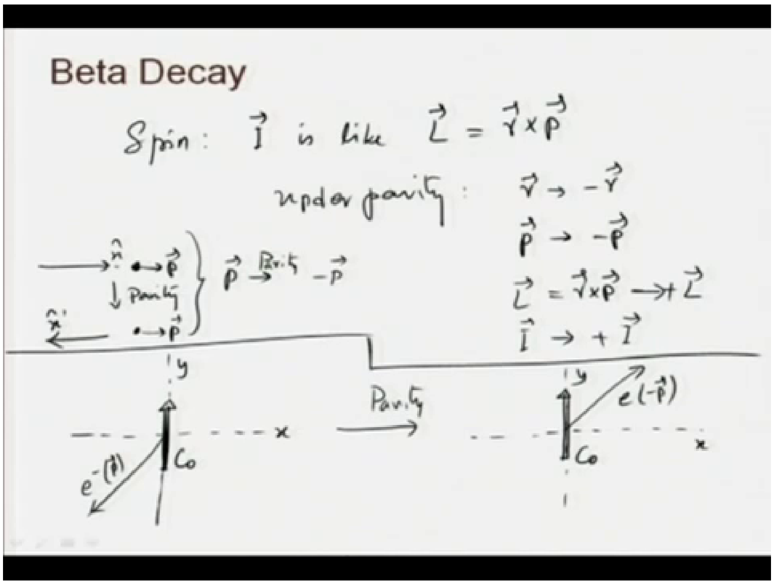

Say spin, I we denote the nuclear spin by I. So, let me consider it as a vector I. This is like the angular momentum orbital angular momentum L, that is what we know and write L as r cross p classically right. So, let us look at that, I will need those properties remain the same random mechanics as spin.

So, what happens under parity, what is parity? By the way parity is the operation of inversion of space coordinates. So, in that case the vector r goes to minus r, vector p momentum goes to minus p which means that if it is moving along the positive x axis and parity operation reverses the direction of x axis then with respect to the nu x axis it is moving now in the negative x axis. So, this is your x and something is moving in the in this direction ok. And under parity x will be reversed now this is your nu x direction let me call it x prime and p is let us say as far as that is concerned this way of doing it, it is going in the same direction. So, effectively you can say p goes to under parity minus p that is one way of looking at it r goes to minus r. But L is equal to r cross p both r and p reverse the sign therefore, this is going to be the same positive L you are right. So, I is this also I also goes to plus I.

Now, let us look at the cobalt 60 decay. Let us consider our x and y directions all right, x and y directions in the experiment let us keep the cobalt. So, that the spin of the cobalt is aligned in the positive y direction all right and let us say there is an electron which is coming out in this direction in the third quadrant. Now after the beta decay of the cobalt it goes down. So, then nickel is there nickel is also aligned in same direction. So, electron is coming out in this direction anyway. So, this let us see what would happen if we apply parity. Spin that is not change under parity, we have actually under parity we will have x plus y inverted or effectively in an active picture we will consider it the parity acting on the vectors itself.

So, cobalt spin remains the same in the same y direction, but electron has a momentum p say the momentum is reverse which means that the direction of emission of the electron is reverse now which is the other direction, which is the direction which falls in the first quadrature, e with, e is the electron with minus p now because of the if this is p first one is p when a nu parity reversed electron will have minus b direct momentum.

So, if parity is conserved what will happen if parity is conserved we expect what for every particle electron coming out in the negative Y direction as in the case of first figure we expect an electron also coming out in the positive Y direction as in the case of second figure. So, finally, after some time when we take to say something like a 10,000 such found some 10,000 of electrons coming or large number of electrons coming out of such cobalt decay with the spin of the cobalt aligned in particular direction using the experiment external magnetic field in an experiment.

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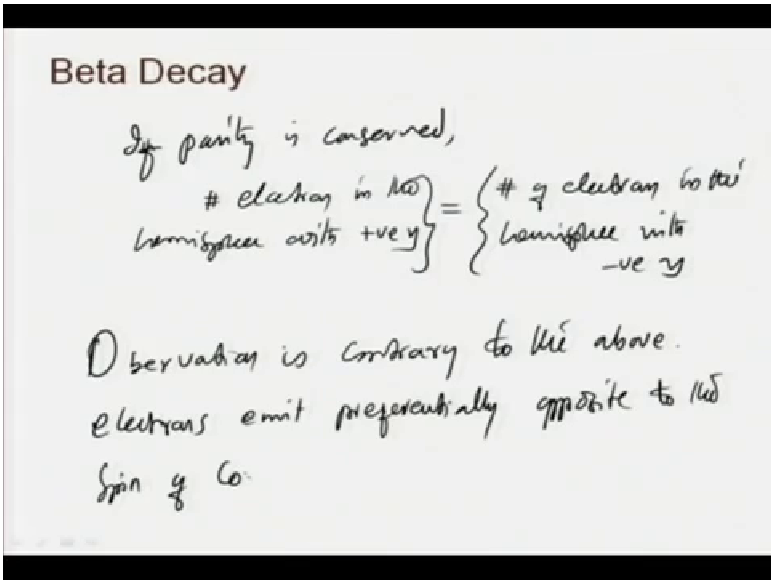

We expect us if parity is conserved, we expect number of electrons in the hemisphere with positive y coordinate is equal to number of electrons detected in the hemisphere with negative y coordinate. This is what we expect if parity is conserved in this particular kind of setup hilarious. In fact, this experiment is performed. The experiment was performed to specifically look at this situation and the experiment is performed by hue and her collaborators the result is that observation is that, it does not happen, which means if you consider the number of electrons coming in the positive y direction and the number of electrons coming in the negative y direction and then this is a violation I mean when this is not the same. So, this says that parity is not conserved in such decays.

In fact, parity is could not conserved there is a preferential electrons found to emitted preferentially opposite to the spin of cobalt. That is the preferred direction for electron. It is not exactly or was it at the distribution if you take the y positive I am going to buy this one it will happen.

It is found that most of it is in that negative y direction. So, this is a parody and in fact, it is found that beta decay is confirmed to be a parity violating process that is not the only process that is that violates parity and there are other processes in fact, beta decay is a weak nuclear process.

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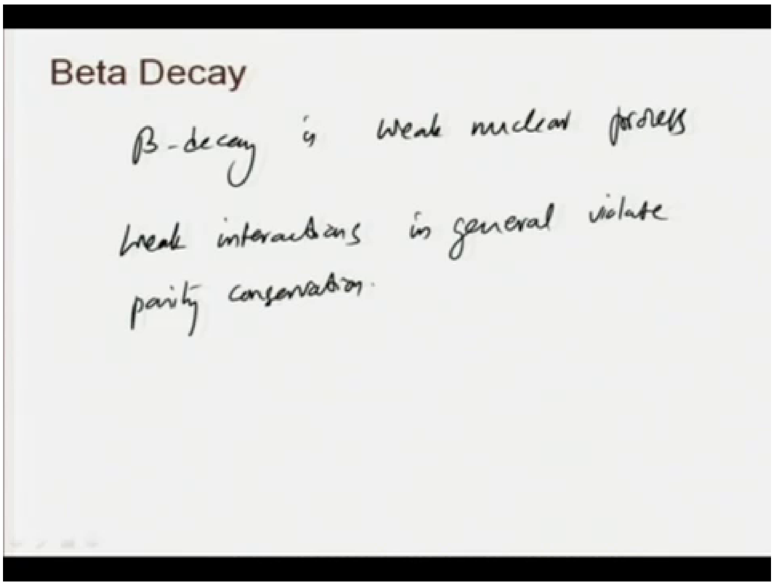

And this is a nu type of interaction the weak interaction and the weak interaction is a nuclear force nuclear reaction nuclear interaction like the strong nuclear interaction, but they are much weaker than the nuclear in strong nuclear interaction. Not only that they all of this weak interaction including beta decay and other different type of weak interactions or violate parity. We will come to more about weak interactions. Let me just write that weak interactions in general violates parity, parity conservation is not possible under the weak interactions.

We will stop our discussion of beta decay for the time being here, but we will come back to beta decay and the weak interaction in slightly more detail later when we still discuss the particle physics part of the course.