Nuclear and Particle Physics
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Module – 04
Radioactive Decays
Lecture – 03
Beta decay

Today we will continue our discussion on Radioactive Decays and discuss the Beta Decay.

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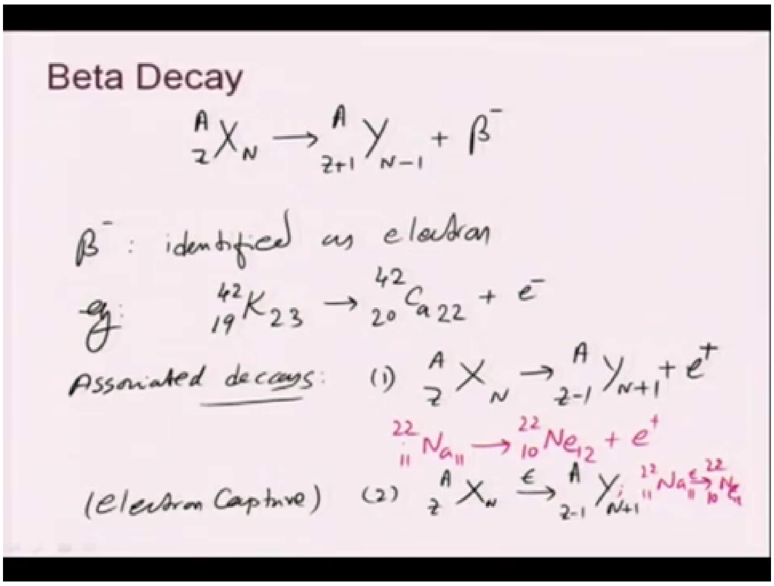

Beta decay is basically nucleus considered a nucleus Z of the nucleus X of atomic number, Z meaning Z number of protons in it and atomic weight A, N neutrons in it. This decays to another nucleus Y with Z number the neutron number, no atomic number equal to Z plus 1 atomic mass remaining the same. Therefore, N is reduced by 1 unit along with the emission of beta particles. These beta particles have one unit of negative electric charge in the unit of electron charge and it is actually identified as electron. Experimentally it is determined that they are nothing, but electrons.

And this electron is not emitted from the atomic electrons, but it is basically coming from the disintegration of the nucleus. So, what we have written here is basically the nuclear transformation not the atomic transformation and a neutron sorry a nucleus with

Z protons and N neutrons change into a nucleus with Z plus 1 protons and N minus 1 neutrons. So, this is a nuclear, these electrons are actually coming from the nucleus not they it is distinct from the atomic electrons the electrons which are moving around the nucleus all right.

An example is the decay of potassium 42, potassium has 19 protons and in this particular isotope of potassium 23 neutrons in it, and it converts itself into calcium with 20 protons and 22 neutrons remain the atomic mass remaining the same along with emission of an electron. Similar to this electron emission there are other decays. So, there is one possibility where instead of proton number increasing the neutron number is increased and proton number is decreased by 1 unit here remaining the same right.

Here the nucleus X has charge Z, Z positive charges Z number of protons in unit or units for the proton charge of course, and Y has Z minus 1 positive charge and therefore, the emitted particle will have 1 unit of positive charge and it is identified as the antiparticle of electron which is called the positron.

Another thing that can happen is again X with Z number of protons and N number of neutrons can actually capture one of the electrons in the atom the nucleus we although we are talking about newt nucleus and the transformation of the nucleus this is all within the atoms. So, in an atom there are electrons around surrounding this nucleus and we can think of one of the low lying electrons being captured. And therefore, then when an electron is captured again the charge electric charge of the nucleus is reduced by 1 unit, reduced by 1 unit. So, we will denote that as Z minus 1 atomic number, and A remains the same and N is increased by 1 unit. And there is no emission of particle it is actually the absorption of electron capture of electron from that thing we call this the electron capture. So, this is basically electron capture and the earlier one is the positron emission. So, an example of positron emission is same if you consider sodium with 11 protons and 11 neutrons each undergoes positron emission to become neon with 10 protons and 12 neutrons along with emission of first row.

And for electron capture again similar same thing you can consider. You can have the N a 11, 22 goes to N e 10 22 12 with absorption of atomic electron. We said that charge of the electron is decreased in the case of positron emission, it is increased by 1 unit, in units of proton charge increased by 1 unit, in the case of beta emission electron emission and then we wrote the atomic numbers corresponding to that as by increasing or decreases by 1 unit. Actually in reality we can actually think of this as happening in terms of neutron and proton reactions.

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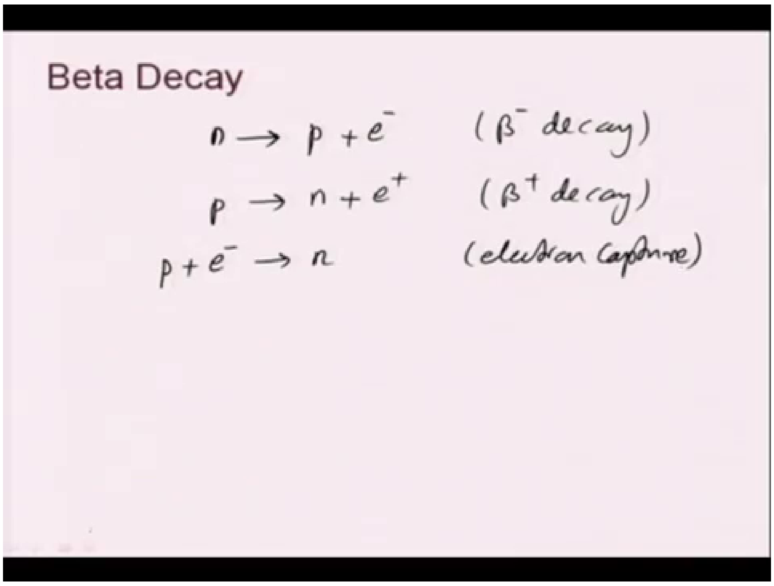

So, we can think about say for example, the beta electron beta minus decay as neutron decaying into a proton plus an electron all right beta decay beta minus decay and the beta plus corresponding to the beta plus it is positron emission with proton converting into a neutron. So, this is beta plus decay and in electron capture we have a proton absorbing an electron or a proton in the nucleus and the electron in the atomic shell combines together to get neutron. So, this is the electron capture.

So, at the nucleon level at the level of the nucleons the beta decay and the associated reactions are in terms of neutron and proton converting into each other right with the emission of electron or absorption of the electron or positron.

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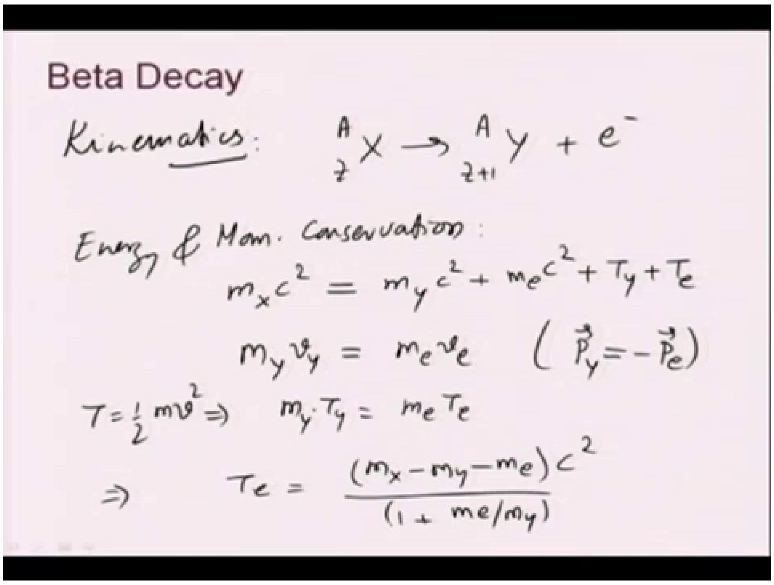

Now, let us come to the kinematics. Let us consider again the beta decay electron emission Z A goes to Y Z plus 1 A plus e minus that is what we know. Now, energy and momentum conservation, energy and momentum conservation, here this is similar to the alpha particle decay that we discussed in the previous lecture. So, we can actually have the energy conservation m x c square let us consider the rest frame of x which means that nucleus x is at rest. So, it is only having the energy equivalent of the mass as its energy no kinetic energy m y c square plus m e c square the mass energies of y nucleus and electron respectively then you have the kinetic energy of y nucleon nucleus and kinetic energy of the electron.

Let us denote those by t y and t respectively and momentum conservation will tell that m y we have m x sorry we have the x nucleus at rest. So, there is no momentum and the final momentum will be like m v y, m y v y the magnitude of the momentum is equal to m e v e magnitude of the other momentum or actually you can write the vector momentum p e is equal to minus the p y is equal to minus p e and the magnitudes are the same all right. And this in turn can be written in terms of the kinetic energies the second the momentum conservation will tell you that T is equal to half m v square. So, m v y square you can square this and then there will be 1 by m y and t y is equal to m e t e. Putting this together all right in coming the energy equation conservation equation and the momentum conservation in terms of the kinetic energy now gives you together t e equal to m x minus my minus m e c square divided by 1 plus m, m e over m y similar to an expression that we had in the case of alpha decay.

So, this actually tells you that if it was only the product daughter nucleus and the electron which are in the final state which are the products of this decay then we should be able to this, if this is a one particle going to 2 particles in decay similar to the alpha decay then we should be able to compute the kinetic energy of the electron coming out of this d k just as a function of the masses of the particle involved. And the masses of the particles are fixed for these things and then therefore, the kinetic energy is a fixed quantity for a particular reaction a particular nucleus going down to another nucleus plus an electron all right.

So, now, let us see what is actually happening in an experimental observation.

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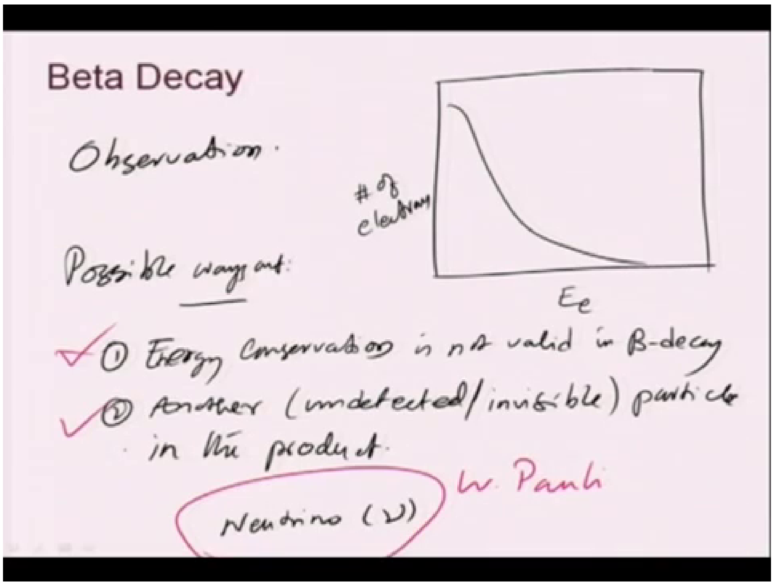

So, observation we should expect electrons to be coming out with one particular energy which means in a sample which is decaying into it through our beta decay we expect that there are electrons coming out all of the electrons having the same kinetic energy. But what is seen is different from this there is a spectrum of energy for the electron that is if you look at measure the energy of the electron. There are electrons with very small energy and there are also electrons some of the electrons having large kinetic energy right.

So, if you consider the number of plot the number of electrons versus the energy typically you get this kind of a spectrum which means the energy of the electron coming out from the beta decay he is not fixed. So, it is happening. What are the possible explanations? Possible ways out one forget about energy conservation, energy conservation because we got that fixed energy of the electron by looking at energy and momentum conservation. So, energy conservation is not valid in beta decay could be one of your conclusions. And another conclusion is as Wolfgang Pauli broadly proposed he said that we do not really have to throw away the energy conservation we can actually live with that if we add one more particle in the final state another undetected or invisible particle in the product right.

So, it turns out that at the end of the day this is not correct and the second one second proposal by Wolfgang Pauli turns out to be the right solution to this problem. And later on this particular particle the third particle which is involved in the beta decay are called neutrinos and usually we denote it by the Greek letter nu. So, beta decay actually predicts also cons analysis of the beta decay actually predicts the existence of a new particle which was not or nor until then fine. And we had to actually checked these proposals and study this new particle and then there are lot of studies that have done on neutrinos and it has become a big topic of research field of research in itself the study of neutrinos. We will come to that sometime towards the end of the this lectures, at least we will touch upon the neutrinos and its properties some time.

Now, if we have 3 particles then how will this kinematics and kilometers look like?

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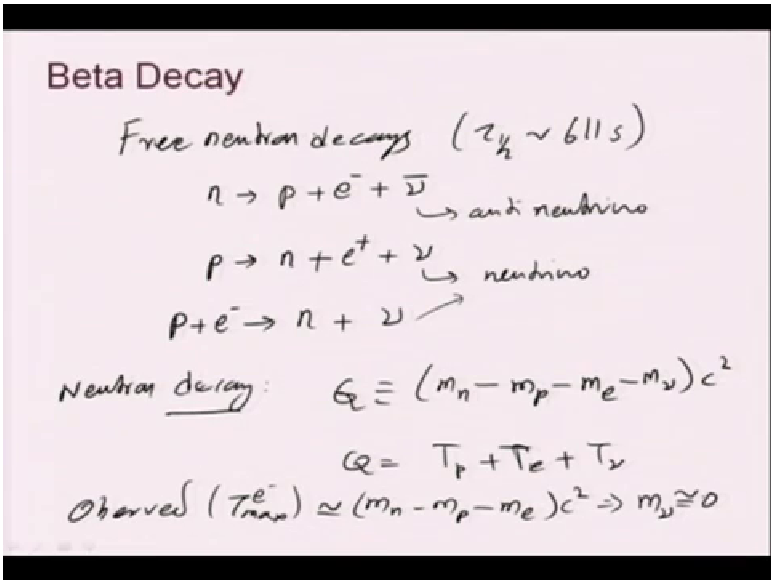

Let us take the free neutron free neutron is not stable these free neutrons actually decays. In fact, the half life of free neutron is about 611 seconds. If you take a few free neutron if the neutrons are within the nucleus, then there are only beta decays and then this beta decay itself happens only for certain isotopes not all of them.

So, in bound states many of the bound states neutrons are stable, but the free neutrons are not and when beta decay in particular we will be discussing the decay of the neutrons within the nucleus. But let us look at the free neutron decayed just for the kinematics n goes to p plus e minus and neutrino. Actually for reasons that will be explained in the later discussions this is actually an anti neutrino that will come out along with electron. So, this is the anti neutrino and in the beta plus decay it is a proton which will decay to convert itself into neutron plus e plus plus neutrino. So, this is a neutrino which is coming out of this all right.

Proton decay is not observed free protons are not observed to decay. So, proton stability is an important aspect for our own existence. So, proton should not be just decaying just like that anyway in the particle physics model building also this proton stability is one of the key ingredients. Then we have the electron capture electron capture is p plus e minus going to neutron plus (Refer Time: 20:11) in neutrino. So, this is also a neutrino along with this one. So, this is the, these are the basic decays.

So, let us look at the neutron decay first. So, the first one in the case of neutron decay we have the Q factor m neutron minus proton number proton mass minus mass of the electron minus mass of the neutrino c square which should be equal to T p plus T e plus T neutrino. And for all practical purposes we can neglect the kinetic energy of proton because proton is about 1800 plus a 1900 times heavier than an electron. Electron is 0.5 Mev and proton is 940 Mev, approximately 938. And therefore, the proton is about 1900 times heavier than the electron and therefore, it will only take very little kinetic energy with it. So, we will neglect that and then Q becomes T e plus T nu and you can ask the question, what is the maximum kinetic energy that we observe for the electron? So, it turns out that observed maximum kinetic energy of electron is approximately m n minus mp minus m e c square and this indicates that m nu is actually equal to or within the experimental uncertainties it is equal to 0. So, approximately equal to 0.

So, we will consider for our practical purposes in our discussions neutrino as massless particle. In fact, in the standard model of particle physics the neutrino is strictly massless, mass is exactly equal to 0 by choice that is what this going is an input to the standard model. Whereas, in reality we know now in the last few years 5 to 10 years, we know that neutrinos have some little mass neutrinos are not completely massless despite the fact that it is very very small. So, the neutrino has some little mass which deviates from the standard model itself is actually a topic of research how to generate this mass d which is not presented by the standard model. That set aside we will not go into those details at least for now. We may mention about it towards the end of the course all right.

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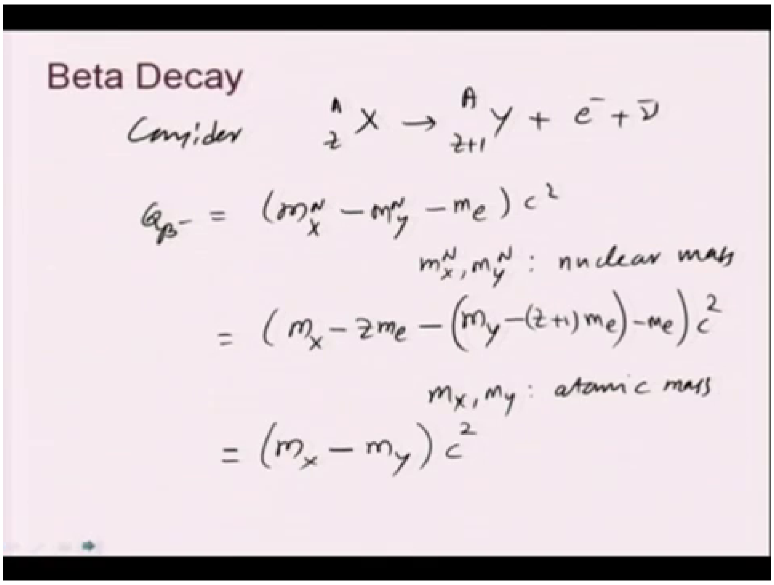

Now let us consider X nucleus with Z protons and A mass number going into Y with mass number A and proton number Z plus 1 and an electron and neutrino and a neutrino.

Now, let me look at Q beta how to write this in this case. So, let me denote the Q factor Q beta minus in this case. So, it is m x minus my minus me c square. These are nuclear masses. So, let me denote this by nuclear masses m n m y n. If I want to write it in terms of, if I want to write it in terms of the atomic masses it will be m x the atomic mass minus x has Z protons in it therefore, the electrons in the atom. So, Z m e right minus m y the atomic mass of y atom by type of atom minus minus of minus. So, it is in brackets minus Z plus 1 me plus 1 protons in the atom therefore, Z plus 1 electrons in the atom minus me c square, so m x n, m y n nuclear masses; m x and m y without the superscript n atomic masses.

We want to write it in terms of the atomic masses because usually that is the one which is provided when measured and provided. So, in terms of atomic atomic masses it is m x minus m y, Z m e minus of minus which is plus Z m e is gone and then m. So, there is 1 plus m e coming from here and there is a minus m e coming from here from the electron and therefore, they actually cancel each other there is no electron mass which will come in its only the atomic masses of the 2 nucleus nuclei the parent and the daughter nuclei. The difference of that is the Q factor for this all right.

Now, we did not consider the binding energies the atomic binding energies that the energy that is needed for the electron to be bound to this nucleus because that is negligible especially it will be only a 1 electron because there are Z electrons in the X atom and Z electrons in the Y atom cancelling each other their contributions. And therefore, and the atomic binding energy is very small compared to the other energies that we are considering here in the Q factor itself, is in of the order of Mev and the atomic binding energy is in terms of kev, if you kev all right.

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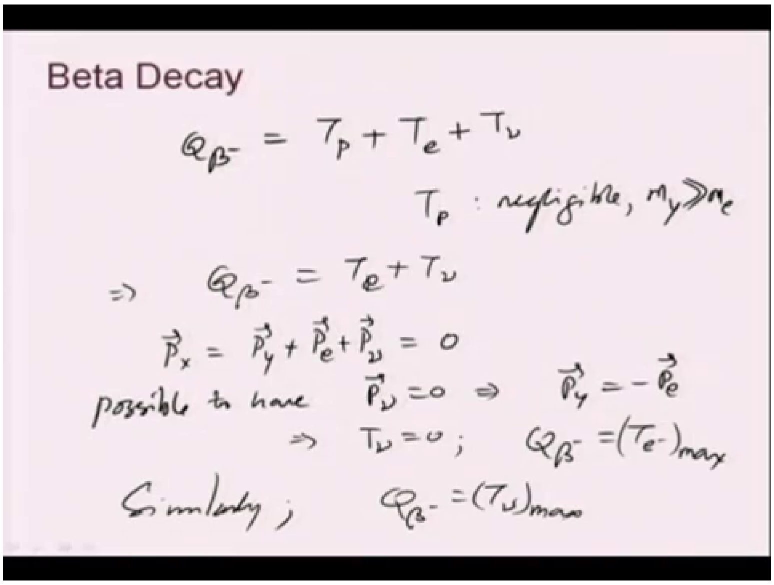

So, now, let me write Q beta minus as kinetic energies. So, I said it is equal to T p plus T e plus T nu and T p is negligible as my is much much larger than m e and that tells me that Q beta minus is equal to T e minus plus T nu. And let us look at the momentum the relation P x is equal to P y plus P e plus P e a nu and in our reference frame this is equal to 0, P x is equal to 0 in the rest frame of x is equal to 0 and it is possible that there are three momenta and one of those momenta any one of them can be 0. So, we can say that it is possible to have p nu equal to 0 and that will tell you that P y is equal to minus P e.

And this will tell you that kinetic energy of the neutrino is equal to 0 and since kinetic energy corresponding to this although P y is large the kinetic energy itself is small because there is a it is P y square divided by m y and that is very small, m v is very large. So, we have Q beta almost equal to T e minus that is the maximum value of this. Similarly we can say maximum of neutrino kinetic energy is also Q beta minus 1. So, that is why earlier we saw that the kinetic and energy spectrum of the electron spreads starts from 0 to almost maximum Q value. We did not release compare this with the Q value at that time, but the maximum corresponds to the Q value of the reaction.

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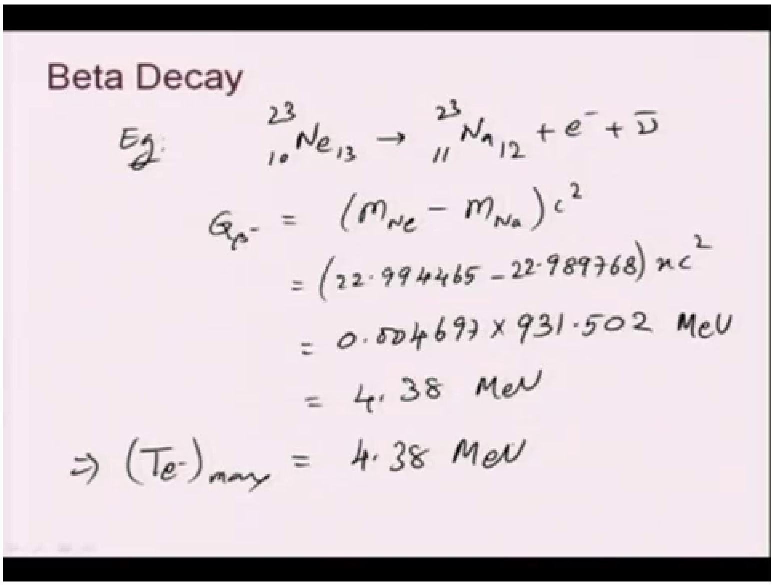

Say in an example, let say consider neon with 10 electrons, 10 protons and 3 neutrons is radioactive it decays to sodium with 11 electrons, 11 neutron, 11 protons and 12 neutrons and an electron and an antineutrino and the Q of this reaction is mass of N e minus mass of N a corresponding I saw tops that we are considering. And in terms of the atomic mass units these are 22.994465 units and 22.989768 units atomic units. So, the energy and this is basically 004697 u c square and this basically you can convert it into Mev and the multiplicative factor to convert is conversion factor is 931.502 to convert u c square into Mev which is equal to 4.38 Mev. So, that tells you that T electro electron maximum in this case is 4.38 Mev.

Experimentally you can check in neon radioactive neon 23 radioactive or beta decay what is the maximum kind of energy of the electrons coming out and then it is experimentally verified where it is (Refer Time: 33:03) equal to 4.38 as expected.

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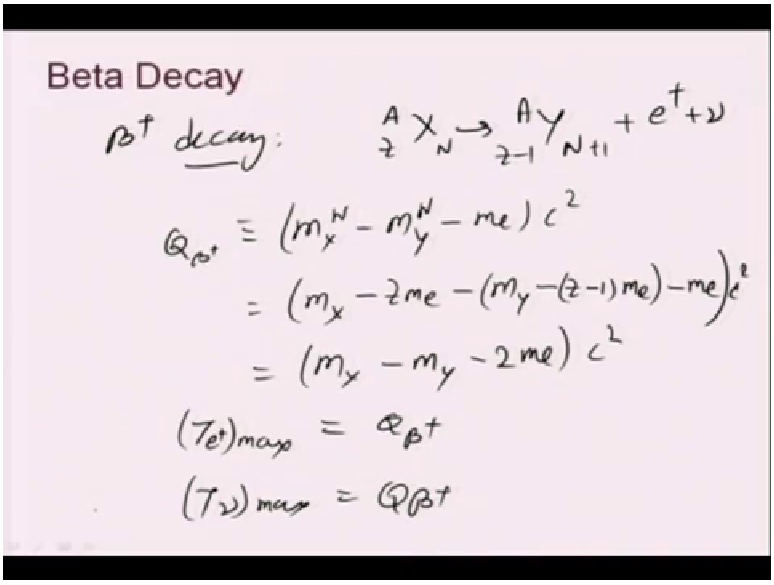

Let us consider the beta plus decay it is X Z A going to Y Z minus 1 N plus 1 A plus positron and neutrino. And in a similar way we can define the Q factor as m x the nuclear mass, m y the nuclear mass minus m e. But now when you actually look at in terms of the atomic masses it is m x minus Z m e minus m y minus Z minus 1 m e and m e c square. This turns out to be m x minus my minus 2 m e, now the masses of the electron do not cancel each other.

So, we have in terms of the atomic masses we have a m x minus my minus 2 m e c square as this one and in exactly the same way as earlier beta minus decay we can say that T e plus max is equal to Q beta plus that similarly for the neutrinos because you here also the a kinetic energy carried by the Y nucleus is very very negligible.

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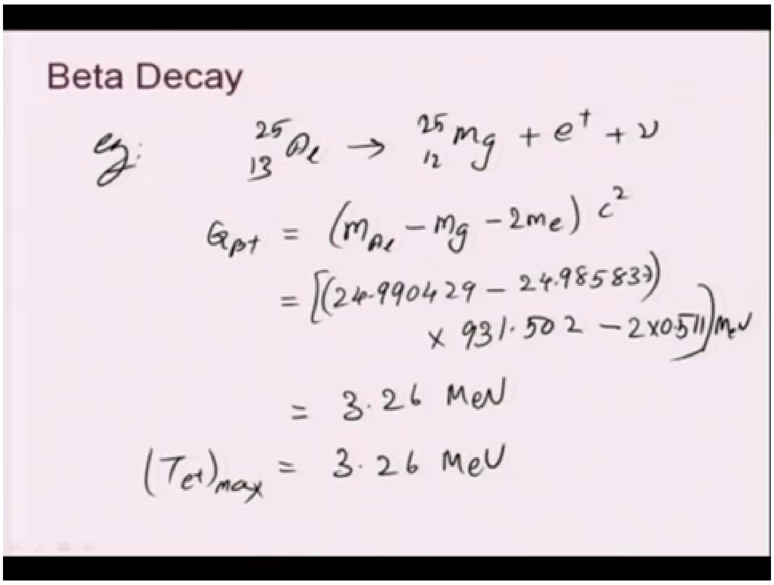

An example is aluminum having 13 protons and 25, let us take aluminum 25, it decays to magnesium 25 plus positron plus neutrino. And Q beta plus is m Al minus m g minus 2 m e c square in terms of the atomic masses of aluminum and magnesium. This is equal to 24.990429 minus 24.985837, these 2 masses into the factor to convert it into Mev is 931.502 minus twice 0.511 which is the electron mass in Mev which you can do the algebra and then it is equal to 3.26 Mev and it is observed that maximum kinetic energy of the positron coming out is indeed equal to 3.26. Neutrons are, neutral particles which I forgot to mention earlier, but neutrinos you must have already guessed we did the charge balancing with neither electron or a positron and the nuclei and the rest of the particles here the neutrino or the anti neutrino do not have any electric charge.

So, they are neutral electrically neutral. They do not interact normally, they interact very very weakly actually we will come to when we discuss the weak nuclear interactions later. Beta decay is an example of the beta weak nuclear interaction we will come to some more details of these weak interaction later, otherwise the neutrinos do not interact. So, normally you do not detect it.

Now, the last bit of today’s discussion is on electron capture.

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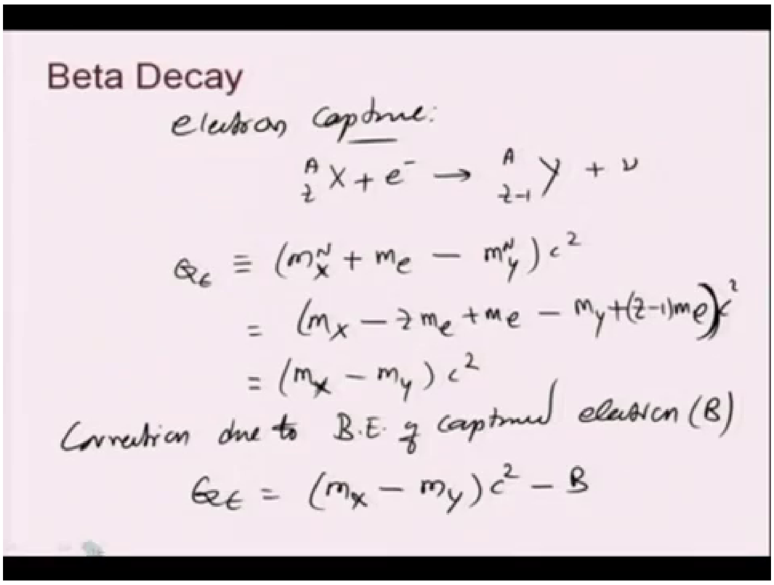

So, in electron capture we have Z A X plus as a nuclear reaction plus e minus going to Y Z minus 1 A plus neutrinos. So, the Q factor in this case let me denote it by epsilon Q epsilon is equal to m x the nuclear mass plus me because there are X nuclear nucleus and electron in the initial state, so they are added minus in the product only my N c square. So, that is equal to m x atomic mass minus Z m e plus m e minus my atomic mass minus Z minus 1 m e, in brackets it is plus whole square, c square. This is equal to m x minus m y the atomic masses c square.

But there is one small correction to it. What is happening in this case is that an atomic electron is captured the atomic electron is not free electron. So, when you add the energy as me c square that is not the energy that actually available, it is only m e c square minus the binding energy, the atomic binding energy which is available. So, Q factor will have to be corrected correction due to binding energy of captured electron. So, let me call it as B. It is then Q epsilon is equal to m x minus m y c square minus the binding energy all right.

So, this is basically, these are the 3 different types of this beta decay and associated processes and one more there is a question about how does this happen or what causes the beta decay.

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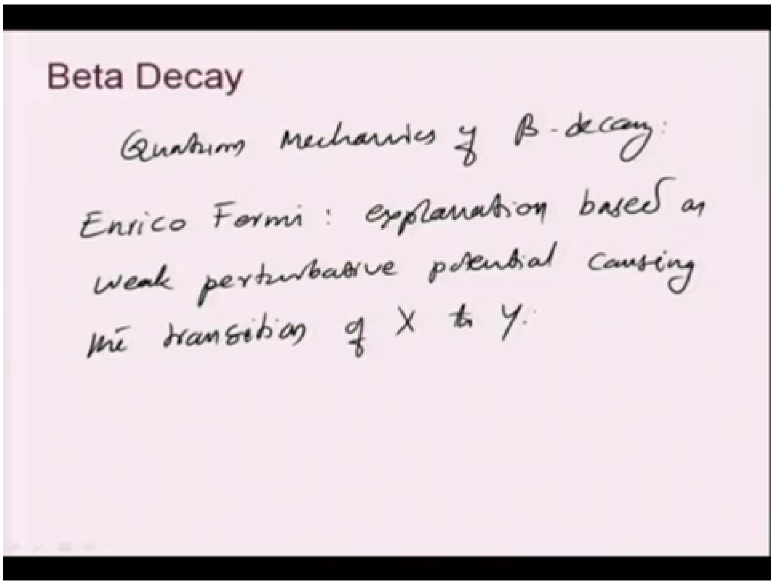

So, we can ask the question what is the theory or what is the quantum mechanics of beta decay. We encounter a new weak force completely different from the strong nuclear force that is needed to explain the beta decay and we will come to that when we discuss the particle physics discussions. For the time being we will just mention the theory proposed by Enrico Fermi. So, the Fermi theory of beta decay. So, he developed a theoretical explanation for beta decay based on the explanation. His explanation was based on weak perturbative potential, some perturbative potential. Since it is weak we can do the perturbation theory and perturbative potential it causes the transition of X to Y.

So, we will discuss the Fermi theory a little more detail when we actually discussed the weak decay of beta pa; I mean in the particle physics section. For the time being we will actually stop here we will not discuss the theory in detail. And there is one bit of the beta decay which is the angular momentum and selection rules concerning the beta decay that we will come to in the next session.