

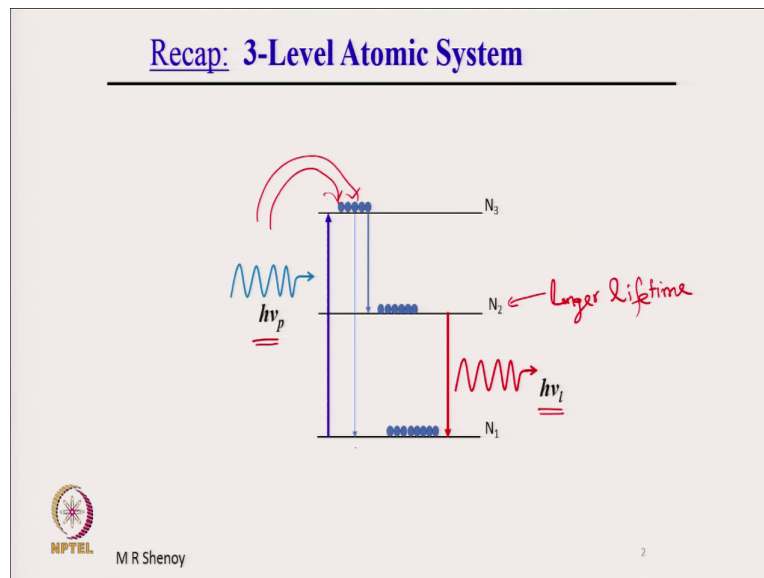
Introduction to LASER
Prof. M. R. Shenoy
Department of Physics
Indian Institute of Technology, Delhi

Lecture - 09
Laser Rate Equations: 3-Level System

Welcome to this MOOC on Lasers. In this lecture, today we will see Laser Rate Equations in a 3 Level System. A very quick recap that in the last class, we discussed about 2 level, 3 level and 4 level systems. In particular in 2 level systems, we saw that population inversion cannot be achieved in steady state even by external pumping.

And then we saw that the 2 level system would in general act like an absorber and we also discussed about saturable absorbers. So, today we will take the 3 level systems, write down the laser rate equations and then find out the condition under which amplification by stimulated emission is possible,.

(Refer Slide Time: 01:17)

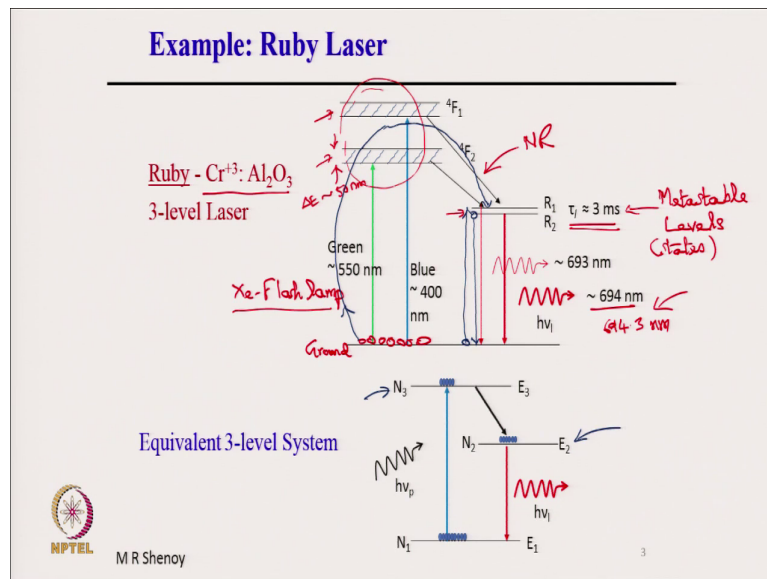


So, a very quick recap of what is the 3 level atomic system. So, basically the 3 levels which participate in the interaction are the ground state and two excited states are the upper levels. N_1 , N_2 , N_3 are the population in these levels and the possible transitions we have shown that, from ground state, we can have absorption go to the third level and then the atoms can come down directly to the level 1 or they could also come down to level 2 and then come down to level to the ground level.

Any other transition we have neglected in this. Now, $h\nu_p$ is the energy of the pump photons and $h\nu_l$ is the energy of photons corresponding to the transition from 2 to level 2 to level 1. So, what we have discussed is, it is possible to have population inversion between level 2 and level 3 if we pump sufficiently fast to this upper level and if the atoms come down rapidly to the level 2 and then they stuck get stuck or wait here for a longer time.

In other words if this level 2 has a longer lifetime, if this has a longer time, then it is possible that the atoms will life time. Then the atoms can accumulate in level 2 and soon they can become more than the number of atoms in the ground state N_1 and population inversion can be achieved. So, we will write the rate equations for this system and see what is the method, whether this condition is correct or not, alright.

(Refer Slide Time: 03:17)



So, before we write the rate equation; the system that we are discussing is typical of a ruby laser; a ruby laser is a 3 level system. But the actual levels of ruby are shown here. So, this is the ground state. So, ground and there are two bands which are shown here $4F_1$ and $4F_2$ at the excited level, which correspond to wavelength. So, these are approximately 50 nanometer wide; the energy delta E corresponds to delta E is corresponds to; it is not delta E of about 50 nanometer.

So, they are two absorption bands. Therefore, green light around 550 nanometer will be absorbed will excite atoms from this ground state here from the ground state to the $4F_2$ level and blue light around 400 nanometer wavelength will excite atoms to the upper state. These atoms rapidly come down to two intermediate states, which are called R_1 and R_2 ; these are traditional nomenclatures of R_1 and R_2 . The level R_1 and R_2 have a long lifetime, typically about 3 millisecond in the case of ruby.

Ruby is chromium doped alumina; it is a chromium doped alumina crystal, which has levels. So, these levels which have long lifetime are called metastable states; so, metastable, metastable states, levels or states, metastable states. So, whenever you have energy levels which have a long lifetime, atoms can accumulate there. So, what is happening is, atoms are excited to a higher energy bands $4F_1$ and $4F_2$, usually they use a xenon flash lamp.

So, X e flash lamp, flash lamp is used to excite these atoms; we will discuss ruby laser when we discuss about the laser systems towards the end of this course. But right now a very quick outline of this ruby laser, just to show that this is a 3 level system. There are several levels participating, $4F_1$ and $4F_2$ are the excitation bands that is atoms; if they are excited to these bands, they rapidly come down to levels R_1 and R_2 . So, these are rapid non radiative transitions, non radiative transitions, very fast non radiative transitions.

But the transition from level R_1 and R_2 are slow and that is why the lifetime of the level is of the order of 3 milliseconds. And this transition gives down two lines, the 694.3. So, this is 694.3 nanometer is the predominant line of the ruby laser and this is 692.9, which is about 693 nanometer is less dominant. So, this is the ruby laser line which is the dominant laser line.

Now, the important point to note is, the role of these two bands, the role of these two bands is to populate the metastable level through the excitation mechanism. In other words, essentially let me change the color here. So, essentially atoms are excited to this level or this band and then from there, they come down here.

In other words, the band helps in populating this; because if we were to pump atoms directly from here to here, they will come down with the same probability. And therefore, we can never achieve; we have already seen in a 2 level system, we cannot achieve population inversion.

However, if we pump atoms from ground state to a third level or a third band or a multiplicity of levels, from where atoms rapidly come down to an intermediate level; then we can have inversion, population inversion between this intermediate level and the ground state.

And therefore, an equivalent 3 level diagram is shown here. So, the upper excitation here, upper level is actually the combined bands which are shown here. So, there is a third level, atoms are excited to the third level; from where they come down to the second level, where population inversion can take place.

So, right now I have not shown population inversion, but population inversion can take place. And the transitions which are shown are the predominant transitions; in the sense, atoms are absorbed to this higher excited level or multiplicity of levels, from where they come down rapidly to the second level and from there this second level could also be a multiplicity of levels, from there it comes down to the ground state.

And again just to repeat that, if the second level, here intermediate level has a long lifetime or it becomes a meta stable state; then atoms can get accumulated here and create population inversion, which is the necessary condition for amplification by stimulated emission. With this picture in mind, let us write down the rate equations, ok.

(Refer Slide Time: 09:27)

Rate Equations: 3-level System

For a 3-level Atomic System,
in the presence of *pump*,

Rate Equations:

$$\rightarrow \frac{dN_3}{dt} = W_p(N_1 - N_3) - T_{32}N_3 - T_{31}N_3$$

$$\rightarrow \frac{dN_2}{dt} = W_i(N_1 - N_2) + N_3T_{32} - N_2T_{21}$$

$$\rightarrow \frac{dN_1}{dt} = W_p(N_3 - N_1) + W_i(N_2 - N_1) + N_2T_{21} + N_3T_{31}$$

At steady state

$$\left. \begin{aligned} \frac{dN_3}{dt} &= 0 \\ \frac{dN_2}{dt} &= 0 \\ \frac{dN_1}{dt} &= 0 \end{aligned} \right\}$$

4

M R Shenoy

Rate equations for a 3 level system. So, for a 3 level system in the presence of a pump, so this is the pump; a pump which is exciting atoms from the ground state to the third level; the rate equations are, first we are writing dN_3 by dt . So, if N_3 is the number of atoms in level 3, N_2 is the number of atoms here and N_1 is the number of atoms.

Then dN_3 by dt tells us the rate of change of population in level 3 that is N_3 . Now, N_3 would increase because of an absorption W_p times N_1 . So, this is absorption, W_p is the pumping rate per atom, but and therefore, W_p into N_1 will tell us the number of atoms excited to the upper state per unit time per unit volume.

Now, the same W_p will also bring down atoms here by stimulated emission and therefore, we also have. So, W_p into N_1 will increase dN_3 by dt that is N_3 will increase and W_p

into N_3 is the atoms coming down here. And therefore, it is negative; it will reduce N_3 and that is why we have here minus N_3 term, W_p times minus N_3 . What are the transitions?

There is T_{32} into N_3 ; so, T_{32} into N_3 . So, atoms which are spontaneous transitions, in this case it is primarily non radiative transitions. So, n_r here referring to the non radiative transition. So, T_{32} into N_3 and one more term atoms could also come down here, it is written here.

So, T_{31} into N_3 , atoms from here could spontaneously come down here or spontaneously come down here. They may come down by stimulated emission, which is W_p into N_3 . So, these are the only possible transitions as far as level 3 is concerned. So, we have taken care of all the four terms. So, 1, 2, 3 and 4 four transitions.

Now, let us look at dN_2 by dt that is rate of change of N_2 at level 2 here. So, at level 2 you can see there is transition coming from level 3 and from level 2, there is stimulated emission, spontaneous emission and then absorption. And therefore, we have four transitions here as well; therefore we have 4 terms as before W_1 into N_1 , that is this one absorption, the one which is going up here W_1 into N_1 , will increase N_2 , therefore it is positive number.

W_1 into N_2 will come down and therefore, it is a negative minus. And then T_{32} into N_3 will add atoms to level 2 and therefore it is positive, N_3 times T_{32} and minus N_2 times T_{21} ; because from T_{21} , atoms are coming from level 2, the T_{21} transition will bring down atoms, which means there is a loss of atoms at level 2 and therefore, it is a negative sign.

And therefore, this is for this level. And what about the last level N_1 ? So, the rate of change dN_1 by dt will be equal to, you will see that there are six transitions here; therefore six terms will be there, W_p into N_3 atoms coming down from here then atoms coming down by spontaneous emission, stimulated emission; atoms coming down by spontaneous emissions, stimulate.

So, four positive terms spontaneous, spontaneous, stimulated and stimulated and then two negative terms which are absorption, so one going up here and one going up here. So, two

negative terms, there are six terms. So, these are the rate equations, which describe the rate of change of population of levels 1, 2 and 3.

So, that is why these are called rate equations for a 3 level system, alright. So, now, let us pick up the first one here, dN_3 by $d t$ and at steady state, each one of them must be equal to 0. So, at steady state, at steady state, maybe it is written in the next page; at steady state each one of these equal to 0, that is dN_3 by $d t$ equal to 0, dN_2 by $d t$ equal to 0 and dN_1 by $d t$ equal to 0, alright. So, let us use the first equation here, dN_3 by $d t$ equal to 0.

(Refer Slide Time: 14:41)

Rate Equations: 3-Level System (contd.)

$$\frac{dN_3}{dt} = W_p(N_1 - N_3) - T_{32}N_3 - T_{31}N_3 = 0 \quad N_3(\tau_{31} + \tau_{32} + W_p) = W_p N_1$$

At steady state, $\frac{dN_3}{dt} = 0 \Rightarrow \frac{N_3}{N_1} = \frac{W_p}{W_p + T_{31} + T_{32}} \dots\dots\dots (1)$


$$\rightarrow \frac{dN_2}{dt} = W_l(N_1 - N_2) + N_3 T_{32} - N_2 T_{21} = 0$$

$$\Rightarrow \frac{N_2}{N_1} (W_l + T_{21}) = W_l + T_{32} \frac{N_3}{N_1} = W_l + T_{32} \frac{W_p}{W_p + T_{32} + T_{31}}$$

$$\frac{N_2}{N_1} = \frac{W_l}{(W_l + T_{21})} + \frac{W_p T_{32}}{(W_l + T_{21})(W_p + T_{32} + T_{31})} \dots\dots\dots (2)$$

$$\rightarrow N = N_1 + N_2 + N_3 = N_1 \left(1 + \frac{N_2}{N_1} + \frac{N_3}{N_1} \right) \Rightarrow N_1 = \frac{N}{\left(1 + \frac{N_2}{N_1} + \frac{N_3}{N_1} \right)} \dots\dots(3)$$

↳ total no.



M R Shenoy

5

So, this term here is equal to 0. So, if we put that equal to 0; then we get we can simplify this dN_3 equal to 0. So, we take N_3 terms to one side that will leave N_1 only here. So, we have N_3 into; when this goes to the other side, so N_3 into T_{31} plus T_{32} plus W_p . So, will. So,

we have $N_3 = T_{31} + T_{32} + W_p$ equal to W_p into N_1 that is the first term. And therefore, N_3 by N_1 is equal to W_p divided by $W_p + T_{31} + T_{32}$.

Similarly, the second term dN_2 by, second equation dN_2 by dt equal to 0 gives us. So, we can write, take the N_2 terms together and N_3 terms together and then we can write N_2 by N_1 into $W_1 + T_{21}$ is equal to $W_1 + T_{32}$ into this. Now, N_3 by N_1 we have already got the expression here; therefore that is equal to $W_1 + T_{32}$. And for N_3 by N_1 , we substitute one 1, which is W_p divided by this.

So, which comes out therefore, N_2 by N_1 this term goes to the other side and you have W_1 divided by $W_1 + T_{21}$, this term here plus W_p into T_{32} divided by. So, W_p into T_{32} divided by this term into the other term. So, this is equation number 2. So, we have got expression for N_3 by N_1 and N_2 by N_1 .

Now, recall that, N_1, N is the total number of atoms. So, total number of atoms will be equal to $N_1 + N_2 + N_3$. So, if we take out N_1 . So, N_1 into 1 plus N_2 by N_1 plus N_3 by N_1 ; so, we have expressions for N_2 by N_1 and N_3 by N_1 , equation 1 and 2. And therefore, N_1 is equal to N , total number of atoms divided by the terms in the bracket. So, this is equation 3, very simple algebra. So, let us continue further.

(Refer Slide Time: 17:17)

Rate Equations: 3-Level System (contd.)

Now, $\frac{N_2}{N_1} - 1 = \frac{N_2 - N_1}{N_1}$. Using Eqs. (2) and (3),

$$\Delta N = N_2 - N_1 = \left[\frac{W_i (W_p + T_{32} + T_{31}) + W_p T_{32}}{(W_i + T_{21})(W_p + T_{32} + T_{31})} - 1 \right] \left(\frac{N}{1 + \frac{N_2}{N_1} + \frac{N_3}{N_1}} \right)$$

→ **Typical**: $T_{32} \sim 10^8 - 10^9 \text{ s}^{-1}$ $T_{31} \sim 10^6 \text{ s}^{-1}$ $T_{21} \sim 10^2 - 10^4 \text{ s}^{-1}$

→ e.g. $T_{21} = 1/\tau_1$; Nd:YAG $\tau_1 \sim 200 \mu\text{s}$, Cr:Al₂O₃ $\tau_1 \sim 3 \text{ ms}$, Er: SiO₂ $\tau_1 \sim 10 \text{ ms}$

→ Using the approximation $T_{32} \gg T_{31}$, **SHOW**

$$\frac{N_2 - N_1}{N} = \frac{W_p(T_{32} - T_{21}) - T_{32}T_{21}}{W_i(3W_p + 2T_{32}) + T_{32}(W_p + T_{21}) + 2W_pT_{21}} \dots\dots\dots (4)$$

Thus, for $(N_2 - N_1) > 0$, i. e. to achieve *population inversion*,

$$\underline{W_p(T_{32} - T_{21})} > \underline{T_{32}T_{21}} \quad \Rightarrow \quad \underline{T_{32} > T_{21}}$$

NPTEL
M R Shenoy

6

And therefore, now N_2 by N_1 minus 1 is equal to N_2 minus N_1 by N_1 . Therefore, N_2 minus N_1 ; why are we interested in N_2 minus N_1 ? Because we want to get population inversion; which means N_2 should be greater than N_1 . Therefore, if we call N_2 minus N_1 as ΔN ; then N_2 minus N_1 is equal to N_1 into. So, this is N_1 , the whole thing here is N_1 . So, this is N_1 . So, N_1 into N_2 by N_1 minus 1. So, minus 1 is here and this expression is N_2 by N_1 ; please see the equation, previous equation. So, N_2 by N_1 is this expression here.

So, we can have a common denominator and then we can get this expression. So, this is N_1 . And up to this we have made no approximation at all. Now, we want to see typically, instead of keeping all the terms, some practical approximations can be made. In general in all the subsequent derivations as well, we will try to simplify the expressions by applying practical approximations.

If we have some number which is very big, which is being added by a small number, it makes hardly any difference; but mathematically if you want to do rigorously, you have to keep all the terms, which makes finally at the end of the calculation may be less than one percent difference. And therefore, we will make these practical approximations as and when they are relevant in obtaining simple analytical forms.

So, now let us look at the typical numbers, T_{32} that is that non radiative transitions; if we recall the picture here, the 3 level system. So, you have this pump and this is T_{32} and this is T_{21} . So, this is T_{21} , this is T_{32} . For practical systems, we know that this non radiative transition is very fast; typically T_{32} the numbers are 10 to the power of 8 to 10 power 9 per second. And T_{31} , that is the transition which is coming from level 3 to 1 . So, this is 1 , level 2 , and level 3 .

So, T_{31} is typically 10 to the power of 5 to 10 to the power of 6 per second and T_{21} is the slowest; because for meta stable states, we are looking at practical laser amplifiers, where there is a metastable state which helps us in creating population inversion. So, typically T_{21} is of this order. Let us say, let us take some practical examples; T_{21} is one divided by τ_1 , τ_1 is the lifetime. In the case of Nd:YAG laser τ_1 is approximately equal to 200 or 230 micro second.

In the case of ruby, the lifetime is of the order of 3 millisecond; in the case of erbium doped silica, we will see this later, erbium doped silica optical fiber amplifiers, where the lifetime τ_1 of level 2 is of the order of 10 milliseconds, 10 milliseconds is 10 to the power of minus 2 seconds. And therefore, $1/\tau_1$ is equal to 10 to the power of 2 second inverse.

So, that is what we have written here, T_{21} is of the order of 10 power 2 or to 10 power 4 second inverse. The point to note is T_{21} is much smaller compared to T_{32} and T_{31} . So, therefore, using the approximation, T_{32} is much greater than T_{31} ; for example, here there are two orders of difference typically is much greater than T_{31} , which means we are neglecting this term T_{31} here and here. In comparison to T_{32} , we neglect T_{31} .

Then the expression simplifies to this expression here, $W_p N_2 - N_1$ by N ; please see there is a minus 1 here from where these negative signs have come. So, W_p into $T_{32} - T_{21}$, this is the only. So, we have neglected T_{31} comparison to T_{32} . And then you simply simplify show this that, $N_2 - N_1$ by N is equal to this expression, expression number 4.

From this we see that, for $N_2 - N_1$ to be greater than 0; that is to achieve population inversion. Please see N_2 is the number of atoms here, N_1 is the number of atoms here, and N_3 is the number of atoms. So, N_2 greater than N_1 means, there is population inversion; to achieve population inversion, the numerator must be greater than 0.

So, this numerator here must be greater than 0 for population inversion. So, let me write as P I. So, that is what is written here, W_p into $T_{32} - T_{21}$ is much greater than this. This implies first condition T_{32} must be greater than T_{21} . What was this T_{32} and T_{21} ? Look at this, this is T_{21} , rate at which atoms coming and T_{32} ; T_{32} has to be greater than T_{21} .

So, that is what we discussed even before we wrote the mathematics. So, when we discussed here, even before we wrote the mathematics; we said that if this rate is slower compared to this rate, then accumulation can take place at this level. So, now, mathematically we have got this expression that T_{32} must be greater than T_{21} . So, naturally T_{21} should be as small as possible; which means $1/T_{21}$, which is τ_1 lifetime must be as large as possible or we need a metastable state.

(Refer Slide Time: 24:07)

Rate Equations: 3-Level System (contd.)

→ Since $T_{32} \gg T_{21}$ in practice, we can write Eq.(4) as -

$$\frac{N_2 - N_1}{N} = \frac{(W_p - T_{21})T_{32}}{W_i(3W_p + 2T_{32}) + T_{32}(W_p + T_{21}) + 2W_p T_{21}} \dots\dots\dots(5)$$

→ ∴ To have gain, we need $N_2 > N_1 \Rightarrow W_p > T_{21}$

→ $N_2 - N_1 = 0$ is the threshold for Amplification, i.e. for $W_p = T_{21}$

⇒ $W_{pt} = T_{21}$, is the "threshold pumping rate"

→ Scheme of 3-level Amplification

$N_2 > N_1$
or Pop. Inv.

7

MPTEL M R Shenoy

Since T_{32} is much greater, the requirement is T_{32} should be greater than T_{21} ; but we know that T_{32} is much much greater than T_{21} . Again go back to see the practical numbers, T_{32} is of the order of 10^8 to 10^9 and T_{21} is of the order of this in practical laser systems.

And therefore, we have T_{32} much greater than T_{21} ; then we can neglect. So, in this expression look back, if T_{32} is much greater than T_{21} ; so T_{21} can be neglected here and then we have W_p into T_{32} minus T_{32} into T_{21} . So, T_{32} comes out and we have W_p minus T_{21} into T_{32} .

So, that is what is written here; W_p minus T_{21} into T_{32} in the numerator and denominator is written here. Now, to therefore to have gain, we need N_2 greater than N_1 ; which implies

W_p must be greater than T_{21} . What is W_p ? W_p is the pumping rate per atom and when W_p equal to T_{21} , we have N_2 equal to N_1 or $N_2 - N_1$ equal to 0.

So, $N_2 - N_1$ equal to 0 is called the threshold for amplification, it is the threshold for amplification. Why is it called threshold? Because $N_2 - N_1$ equal to 0, implies there is no population inversion; but a any additional atoms N_2 in the level will make $N_2 - N_1$ greater than 0 and suddenly amplification will start. And therefore, the value W_p equal to T_{21} is called the threshold pumping rate.

So, W and we add the small subscript t to indicate that it is the threshold value; the threshold pumping rate is simply equal to T_{21} . What is T_{21} ? One by lifetime and lifetime τ_{21} is a measurable quantity; τ_{21} can be measured in practice and therefore, you know the value of T_{21} . If you know the value of T_{21} , you know what is the threshold pumping rate which is required before amplification starts; so, you know the pumping rate required to start amplification.

Now, once we know the pumping rate, then we can calculate what is the power; because in practice we would require to know the pumping power and we will see how to calculate the practical pumping power in watts or kilowatts or whatever, so units that what is that number, rather than just writing W_p , alright.

So, finally, therefore, the scheme of 3 level amplification is summarized here. You have a pump of energy $h\nu_p$ which is raising, that is which is exciting atoms from the ground state to level 3 and from level 3 they come down rapidly; therefore you can see the steady state number in N_3 is very small and therefore, numbers are the number of atoms shown qualitatively is very small.

And because T_{21} is small, the spontaneous transition rate from here to here is very small compared to T_{32} , atoms will start accumulating here. T_{21} is small means what The lifetime is larger. Lifetime is larger means what? Atoms will accumulate here, atoms take the average

time taken by atoms to make a downward transition or to get d excited is large and therefore, we have accumulation taking place here.

And soon it is possible, if you pump sufficiently hard, sufficiently hard so that your pumping rate is greater than T_{21} ; then it is possible to have N_2 greater than N_1 or population inversion, which implies population inversion. So, when population inversion takes place, it the system can provide amplification by stimulated emission. Amplification to which radiation?

The radiation corresponding to this gap, radiation corresponding to this energy difference; because any incident radiation which we call as $h\nu_l$, l standing for laser subscript, if radiation $h\nu_l$ corresponding to this energy difference will get amplified by stimulated emission, coherent amplification by stimulated emission.

So, this is the pump causing a population inversion which results in amplification of the signal or the laser radiation, which corresponds to the energy difference here. This is the principle of operation of a 3 level amplifier, a scheme of amplification of a 3 level system, ok. Let us discuss something more, an important concept of gain saturation. Now, recall equation 5, it is the same equation I have rewritten; so equation 5 which is here, in this page here. So, this is equation 5. So, it is rewritten here, recall.

(Refer Slide Time: 30:19)

Gain Saturation

Recall Eq.(5),
$$\frac{N_2 - N_1}{N} = \frac{(W_p - T_{21})T_{32}}{T_{32}(W_p + T_{21}) + W_1(3W_p + 2T_{32}) + 2W_p T_{21}}$$

$$\frac{N_2 - N_1}{N} = \frac{\frac{(W_p - T_{21})}{(W_p + T_{21})}}{1 + \frac{W_1(3W_p + 2T_{32})}{T_{32}(W_p + T_{21})} + \frac{2W_p T_{21}}{T_{32}(W_p + T_{21})}}$$


negligible

ζ_s (say)

$$\rightarrow \Delta N \approx \frac{(W_p - T_{21})}{1 + W_1 \tau_s} N \quad \text{where, } \tau_s = \frac{(3W_p + 2T_{32})}{T_{32}(W_p + T_{21})}$$

$\rightarrow W_p = T_{21}$
 $\rightarrow W_p > W_{pt} = T_{21}$
 $W_p \sim T_{21}$

Note: $T_{32} \gg T_{21}$, $T_{21} \sim 10^2 - 10^4 \text{ s}^{-1}$, $T_{31} \sim 10^5 - 10^6 \text{ s}^{-1}$



M R Shenoy

In this equation if you see, if we divide the denominator and the numerator by this term, T_{32} into $W_p + T_{21}$; then what in the numerator of course T_{32} , T_{32} cancels and we have W_p . So, the expression is written here. So, this is the same expression, where we have divided by this first term to all the three terms.

If we look at this last term, T_{32} is 1 by T_{32} is in the denominator and T_{21} is much smaller than T_{32} . W_p is almost of the same order as T_{21} ; because the threshold value is W_{pt} equal to T_{21} and in practical systems, W_p will be slightly greater than W_{pt} if you want to get amplification.

Because this is the threshold value W_{pt} ; so W_p pumping rate must be greater than W_{pt} which is equal to T_{21} for amplification; but in general, W_p is of the order of T_{21} and therefore, W_p is of the order of T_{21} ; but T_{32} is much bigger compared to this number and

therefore, this number is, the third term here in the denominator is negligible. So, we neglect this term.

And then we can write ΔN as what you have in the numerator plus 1 plus W_1 into all of this, W_1 into all of this. So, this we call as τ_s , τ_s ; say τ_s , if we call this as τ_s , τ_s is the unit of time. You can see that it is unit of time, because this is time, inverse time, inverse time, inverse time, inverse time.

So, this cancels, but there is another inverse time; therefore, it goes to the numerator, which must be time and therefore, we call this as τ_s . So, say. So, we say that let this be τ_s ; then we can write ΔN nearly equal to this, we have neglected the third term, that is why this nearly equal to sign. W_p minus T_{21} divided by this, where τ_s is equal to the term which is within the circle.

So, note that, T_{32} is much greater than T_{21} and this we have already discussed, typical numbers are here and therefore, this term was neglected. So, the term last term was neglected and we have an expression for ΔN here; we will take it further and discuss it in the next.

(Refer Slide Time: 33:07)

Gain Saturation (contd.)

$$\Delta N \approx \frac{\frac{(W_p - T_{21})}{(W_p + T_{21})} N}{1 + W_l \tau_s}$$

Using $W_l = \frac{I_\nu \sigma(\nu)}{h\nu}$, $I_s = \frac{h\nu}{\sigma(\nu) \tau_s}$, we get $\Delta N = \frac{\Delta N_0}{1 + I_\nu / I_s}$


Saturation Intensity

$$W_l \tau_s = \frac{I_\nu \sigma(\nu) \tau_s}{h\nu}$$

$$\frac{I_\nu}{I_s} = \frac{I_\nu \sigma(\nu) \tau_s}{h\nu} \cdot \frac{h\nu}{\sigma(\nu) \tau_s}$$

where $\Delta N_0 = \frac{(W_p - T_{21})}{(W_p + T_{21})} N$

Independent of I_ν



M R Shenoy

9

So, here delta N is equal to, this is what we had written there and tau s. Using W l is equal to W l is I nu into sigma of nu, the cross section divided by h nu; this we have derived earlier there in the definition of W l. And I s, if we define I s as h nu divided by sigma of nu into tau s; then we can write delta N is equal to delta N 0. So, we call everything on this numerator as delta N 0 and W l into tau s. So, W l is here, so W l into tau s here. So, maybe I can write here W l into tau s in the denominator is equal to; this is the definition of W l sigma of nu into h nu into tau s here.

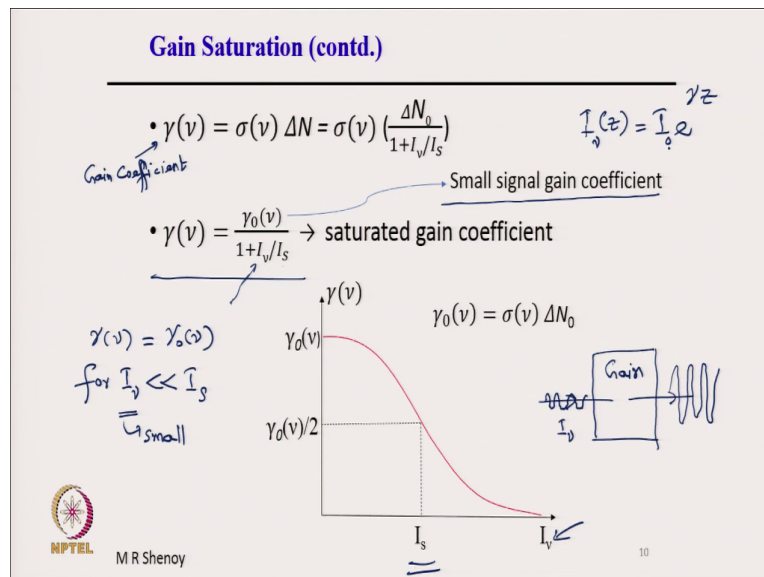
Now, what we are saying is, leaving I nu; if you take these three terms together and call this as 1 by I s or I s is equal to as defined there h nu. So, I s, so this term is 1 by I s, 1 by I s. If we define this as 1 by I s or I s is equal to h nu by sigma of nu into tau s; then we can write this as delta N is equal to delta N 0 into 1 plus I nu by I s. So, we have called this quantity as I s; s

standing for saturation intensity. Why we are calling it as saturation intensity? We will see in a minute.

And what is ΔN_0 ? ΔN_0 is everything which was in the numerator that was here. So, this was ΔN_0 . Note that, ΔN_0 does not contain I_ν or ΔN_0 is independent of I_ν . I_ν is what? I_ν is the intensity of the radiation passing through that medium or atomic system, and I_s is called saturation intensity. Why it is called? We will see in a minute.

So, ΔN is equal to this. We have got an expression, where ΔN is intensity dependent; ΔN_0 is not dependent on the intensity, it depends only on the pumping rate. ΔN_0 depends on W_p , pumping rate; if you change the pumping rate, ΔN_0 changes. T_{21} is the characteristic of the given medium, it is characteristic of the medium; because it is the inverse of life time.

(Refer Slide Time: 35:53)



And therefore, γ of ν , that is the gain coefficient. So, this is the gain coefficient. So, gain coefficient, gain coefficient, γ of ν is equal to σ of ν into ΔN , we have already derived this. And σ N into, now we are substituting for ΔN , ΔN_0 divided by this.

And if we call this numerator σ of ν into ΔN_0 , this independent of intensity as γ_0 of ν ; then γ of ν can be written as γ_0 of ν divided by $1 + I_\nu / I_s$, where I_s is called the saturate, where this expression is called the saturated gain coefficient. I_s is the saturation intensity, but this expression is called the saturated gain coefficient.

Why it is called saturated gain coefficient? We can see that, the gain coefficient is a function of intensity of the radiation. So, here is a plot which is showing the variation of γ of ν with I_ν . As you can see, when I_ν is equal to I_s , which is the saturation intensity the term will become 1 or the gain drops to half of its value, that is the definition of saturation intensity. So, one can measure the saturation intensity by observing that the intensity of the radiation passing through the medium. So, what we are saying is this.

So, you have a medium, which is a gain medium; when light intensity passes through this, you get an amplified output. So, if I can show it as a small sinusoid with small amplitude; then after amplification, you get an amplified output here. Depending on the, this is I_ν , intensity of the radiation which is entering and then of course, inside also it is I_ν ; but I_ν will go on increasing.

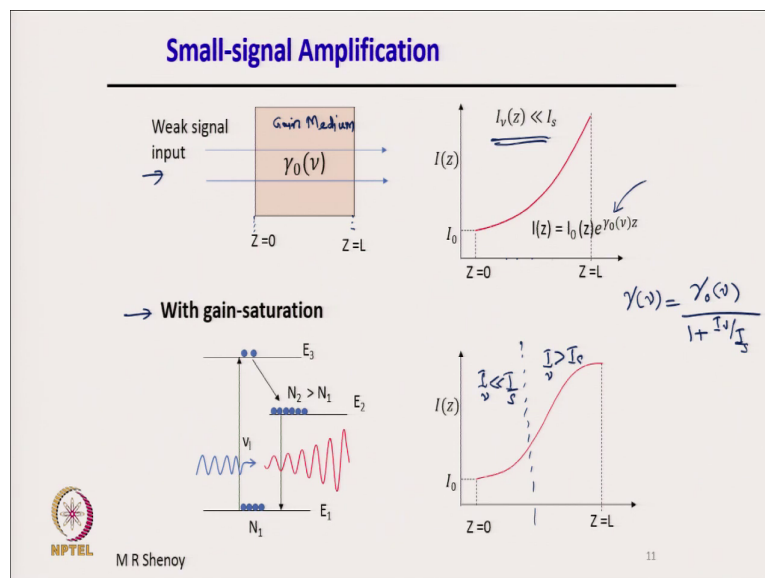
But if I_ν remains small compared to even when it comes out here is small compared to the saturation intensity; then the gain coefficient, if I_ν is much smaller than I_s , then γ of ν is equal to the small signal gain coefficient γ_0 of ν for I_ν much less than I_s .

So, that in the denominator, the second term can be neglected and this is called the small signal gain coefficient; small signal, because the intensity I_ν is small, when this is small, this means this is small. When the intensity is small, then the gain experienced is γ_0 of

nu, which is called the small signal gain coefficient; whereas as the intensity increases, then we will have the saturated gain coefficient. In other words as intensity increases, the gain coefficient drops down as shown in the diagram which is here.

And for large intensity, literally the gain coefficient comes down to 0; gain coefficient comes down to 0 means what? We have I of I nu of z at any intensity is equal to I 0 at the input into e to the power of plus gamma times z; if gamma becomes zero, that means output will be equal to input. Let us discuss this as an application in, it is more clearly illustrated here small signal amplification.

(Refer Slide Time: 39:45)



If we have a weak input signal passing through the gain medium, so this is the gain medium, so the gain which provides a gain coefficient gamma 0 of nu; then the input I 0 will

exponentially get amplified, exponentially it will build up by this expression I of Z , that at any Z here, starting from Z equal to 0, Z equal to 0 is this point input.

And Z equal to L , L because length of the gain medium is L . So, from Z equal to 0 to Z equal to L , the intensity will build up exponentially by this formula, provided the signal intensity for I nu of Z at any value of I Z , I nu is much less than the saturation intensity and this we call small signal amplification.

But what if there is saturation? That is we have a small signal which is incident, after pass as passed through the, as it passes through the medium, it is getting amplified; but if the intensity becomes larger, then the gain coefficient will drop down. And therefore, the intensity will initially build exponentially and then it will slow down and then it will almost saturate. That is why it is called gain saturation; the intensity variation through a medium when, so initially here. So, if I can show, somewhere here I nu is much less than I s.

And here I nu is greater than I s, that is why it starts saturating, the gain drops down; because gain coefficient is given by γ of nu is equal to γ_0 of nu into divided by $1 + I$ nu by I z I s, the saturation end. So, when I nu becomes greater than I s, in the denominator this term continuously increases $1 + 2$, $1 + 3$, $1 + 4$ and so on; that means the gain is continuously dropping down.

If the gain is dropping down means, the intensity, the rate at which the intensity is increasing is slowing down. And when the gain becomes, the γ becomes 0; that is if I nu becomes much larger than I s, then γ would tend to 0, that means the material no more provides any amplification and we say that the gain is saturated.

This is the concept of gain saturation; very important concept of gain saturation, which is fundamental to steady state oscillations of lasers. As we will see later that, this is the fundamental concept; gain saturation is the fundamental concept, which will lead to steady state oscillations of lasers.

We will stop here and in the next lecture, we will take up the 4 level system and we will see what is the advantage of a 4 level system over a 3 level system. So, practical lasers such as Nd:YAG laser and Helium neon laser are 4 level lasers. We will discuss the laser systems towards the later part of the course; but in the next lecture, we will discuss the physics of operation of a 4 level laser.

Thank you.