Introduction to LASER
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Lecture - 08
Part-II: Scheme of Light Amplification
Laser Rate Equations: 2-Level System

Welcome to this MOOC on lasers. So, today we will take up part 2 in which, we will discuss the Scheme of Light Amplification. In particular today's lecture we will discuss Laser Rate Equations. So, let me first recap what we have done in part 1.

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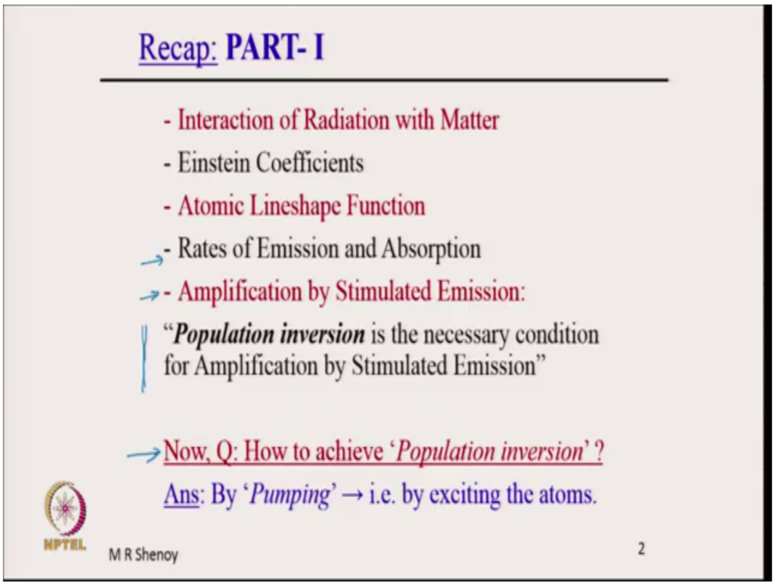

In part 1, we discussed about the Interaction of Radiation with Matter. We saw the Einstein Coefficients, then the we introduced the Atomic Lineshape Function and discussed the Rates of Emission and Absorption. So, rates of emission and absorption and then the condition for

Amplification by Stimulated Emission. And we have seen that Population inversion is the necessary condition for amplification by stimulated emission.

Now, in this part therefore, the next question is how to achieve population inversion? We have seen that the population inversion is the necessary condition and therefore, how to achieve population inversion in atomic system. We have already seen, that at thermal equilibrium, there is no population inversion the number of atoms in the excited state is much much lesser than the number of atoms in the ground state and therefore, how to achieve population inversion.

The answer is by Pumping, that is by exciting the atoms by means of an external pump and that is called Pumping. So, we will discuss this in more detail in this part.

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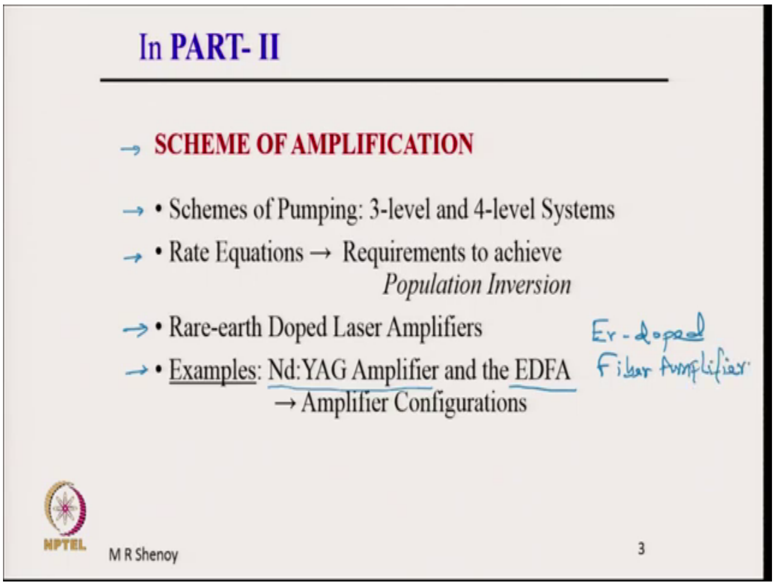

So, in this part that is Scheme of Amplification, we will first see the Scheme of pumping, possible schemes of pumping in 3 level and 4 level atomic systems, we will see what are these 3 level and 4 level systems. And then we will write down the Rate Equations, which describe the rate of change of population of the levels. And they will tell us, they will give us the requirements to achieve Population Inversion, the conditions or the requirements of the atomic system, which can lead to Population Inversion.

And then we will discuss Rare earth Doped Laser Amplifiers with specific examples of Nd YAG Amplifier, Neodymium doped YAG Itanium Aluminum garnet amplifier is a widely used amplifier and laser Nd YAG laser, and the EDFA, Erbium doped fiber amplifier. So, Erbium is a rare earth atom. So, Erbium doped fiber amplifier most widely used amplifier in optical fiber communication systems amplifier. So, EDFA we will see the amplifier configurations and typical characteristics of these amplifiers.

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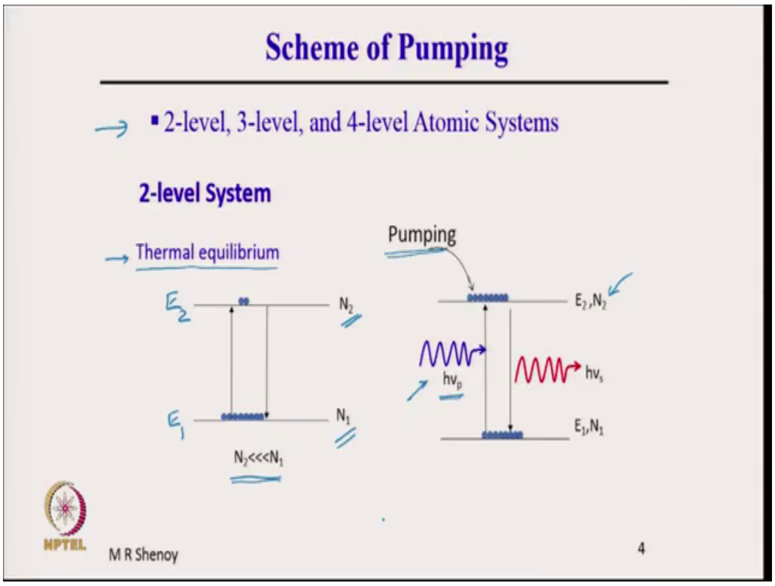

So, first the Scheme of Pumping and let us see first what are 2 level, 3 level and 4 level atomic systems. First 2 level system, what is a 2 level system? A 2 level system is an atomic system; we know that atoms are characterized by large number of discrete energy levels. So, there are not just 2 levels there are many many levels, but a 2 level system when we say we are discussing about a 2 level system, then we are referring to an atom, where two atomic energy levels one is the ground state and an excited state, which is interacting with the radiation.

And in a 2 level system therefore, we are considering the ground state which is here energy E 1 and an excited state usually the first excited state E 2, in the presence of radiation. So, at thermal equilibrium, first at thermal equilibrium here there is no external radiation at thermal equilibrium atoms can get excited to a higher state.

And they would also come down d excite to the ground state. So, there is a dynamic equilibrium atoms keep going up and coming down, and at any instant or at any time there is a dynamic equilibrium with certain number of atoms N 2 in the upper state and N 1 in the ground state, and we have already seen that N 2 is much much less than N 1.

Therefore, there is no question of population inversion, but if you pump that is by some external means for example, I have shown here h nu p, means photons of energy h nu p corresponding to the energy difference E 2 minus E 1, then atoms will go to excited state. It need not be excitation by light, it could be some other means that is why I have shown here pumping by some other means like electric discharge, or any one of the means to excite atoms to the excited upper state.

Then there is radiation emission and radiation which is exciting, there is a dynamic equilibrium now, but in the presence of a pump. And naturally because of the additional excitation mechanism

Thermal equilibrium is the normal excitation mechanism because of the finite temperature of the system, but in the presence of pump it is like an additional pump temperature is one pump, but an external excitation is an additional pump this would always lead to a higher value of N 2, but N 2 may still be not greater than N 1, but N 2 will be more in the presence of an additional pumping mechanism.

And let us we will see the rate equations, which will tell whether it is possible to get N 2 greater than N 1 or not. So, that is a 2 level system.

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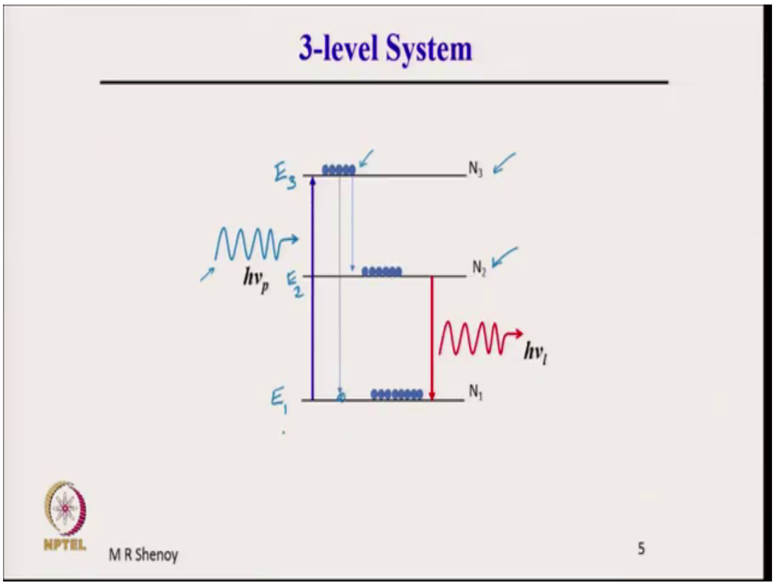

And then let us see, what is a 3 level system? So, here is a 3 level system. Now we are considering three energy levels of the atomic system, this one being the ground state E 1, the upper state could be E 2 or some other higher state also, but the lowest level is the ground state and this could be E 3. Let us say 3 the ground state and 2 excited states.

In this case if we have an external pumping mechanism, which is shown here by the blue sinusoid, h nu p is the energy of the pumping photons, there is the pump photons. They will raise excite atoms from the ground state to an upper state and then from there atoms would make a downward transition to the second state in between and from there they could make a downward transition to the ground state. The atoms in the upper state here could also make a direct transition to the ground state here. So, they could also make a direct transition here to the ground state.

So, depending in such an atomic system that which is called a 3 level atomic system, we will see that it is possible depending on the lifetime of these levels. It is possible to have Population Inversion between the second level here E 2 and E 1, it is possible under certain conditions that N 2 can be greater than N 1. And this we will the conditions we will achieve by writing the rate equations for this 3 level system. So, we will discuss this in detail.

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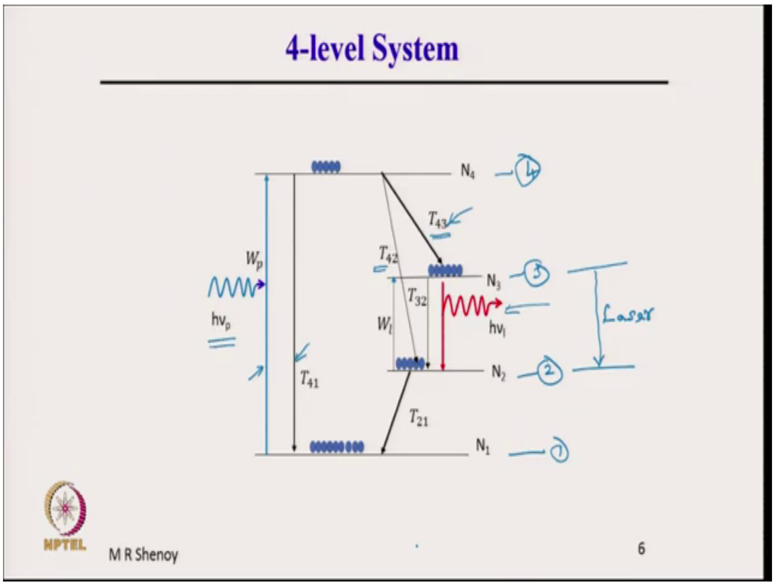

Now let us see what is 4 level system.

So, the 4 level system here refers to, 4 participating levels. So, 1, 2. So, 1, 2, 3 and 4. So, the 4 participating levels, these are not necessarily the 1st excited, 2nd excited, 3rd excited, we will see subsequently that the levels could be different, but there are 4 predominantly participating levels in this interaction.

So, atoms from the ground state are excited to the 4th state here, as I mentioned this 4th could be actually a 8th state, 8th level, 8th excited level, but we call it a 4th state, this will become more clear when we discuss the scheme in more detail.

So, the pump atoms, pump photons, pump photons excite atoms from level 1 to level 4, from there of course, they can make a downward transition as shown here. This transition rate we call as T 41, that is level 4 to level 1, this is excitation upward excitation and T 41 is d excitation from the 4th level to the ground state.

Atoms could also come down to the 3rd level which is represented by T 43, T 43 is the rate at which it would come down, and T 42 is the rate at which atoms would come down from level 4 to level 2. We will see some of the some of the transitions I have shown with bold arrows in a practical 4 level laser system.

We will see that the T 43, which is shown here by the bold arrow is predominant, that is it is a fast transition and therefore, it is possible under certain circumstances if T 43 and T 21 are much faster compared to T 32, then it is possible to have population inversion between this level 3 here and the level 2..

So, between these two levels it is possible to have population inversion. And then we can have laser action as shown here, that the laser action could take place laser action I refer to the generation of coherent laser radiation because of stimulated emission.

So, laser action could take place between level 3 and level 2 and this is called a 4 level system, we will write the rate equations for this 4 level system as well and see and also compare how the 3 level and 4 level lasers work and which one is preferable and why, ok.

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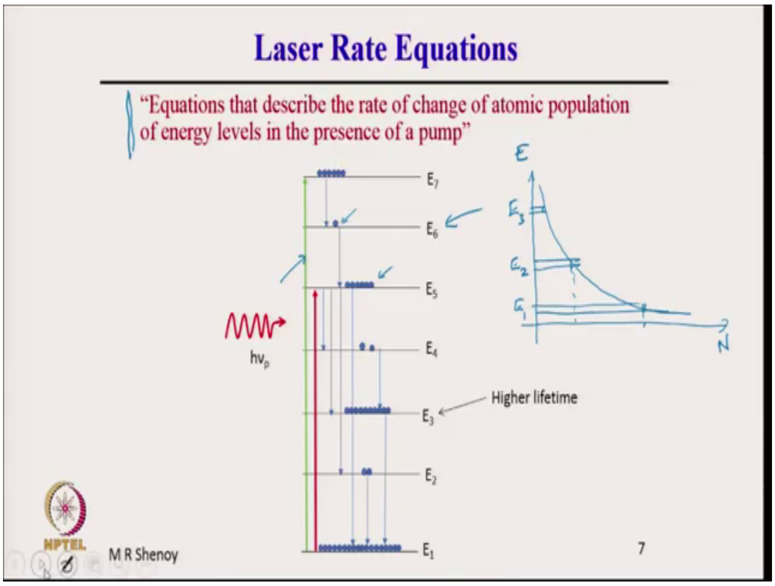

So, I have been mentioning about Laser Rate Equations. So, what are Laser Rate Equations? Laser Rate Equations are equations that describe the rate of change of atomic population of the energy levels in the presence of a pump.

The equations which describe the rate of change of atomic population of the energy levels in the presence of a pump, here pump normally we refer to an external excitation mechanism. So, here a general atomic system with several atomic energy levels of course, they are not equally separated it is just qualitatively shown as E 1, E 2, E 3 I shown here 7 levels.

For example, there is a pump, which is raising a pump may have more than one frequencies. For example, we will see that later on when we see the flash lamps, krypton flash lamp or xenon flash lamp we will see that there are several excitation lines, which can excite atoms from the ground state to an upper state excited state as shown here.

For example, this photon has sufficient energy to raise atom from E 1 to E 7, from there it makes downward transition and depending on the lifetime of the levels, the steady state number in different levels may be different. As you can see I shown here large number of atoms here very small number of atoms just and again large number of atoms small number.

The number of atoms which are accumulating in any energy level depends on its lifetime. We will discuss this again in detail that the lifetime of the levels will determine the population in the presence of an external pump. And we will exploit this to achieve Population Inversion between two excited levels, not necessarily ground state and an excited level it could be between any two excited levels to get laser action alright.

So, where the atoms are accumulating; that means the transition from for example, the transition from E 3 to ground here must be much slower that is why atoms are getting accumulated. And if you remember in Thermal Equilibrium we have the atomic population dropping exponentially like this.

So, the probability distribution given by Boltzmann distribution, if you remember the population diagram the numbers have to drop like this this is the exponential drop. So, what I have plotted here is energy E versus the number. So, this is the number, number of atoms in level of energy E 1, E 2, E 3 and so on.

But what I have shown in this diagram you see that E 3 has much larger number of atoms than E 2 and similarly E 5 has larger number of atoms than E 4. So, this is possible depending on the lifetime of the levels we will discuss this mathematically writing the rate equation.

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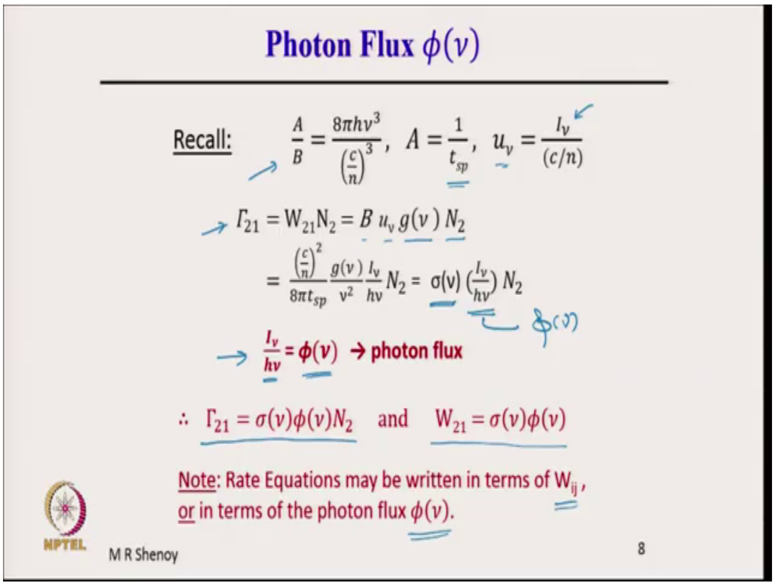

Before we write down the rate equations I would like to bring this concept of photon flux, because several authors you will see in several books that the rate equation is also written in terms of the Photon Flux, phi of nu that is the number of photons per unit time per unit area incident or crossing a play is called the photon flux.

So, recall that we had the relation between Einstein coefficients A by B is equal to 8 pi h nu cube divided by c by n cube. And A is equal to 1 divided by t s p the spontaneous emission lifetime and u nu the energy density at the frequency nu is equal to the intensity I nu divided by c by n.

And we have written this rate of stimulated emission from level 2 to 1 was written as W 2 1 into N 2 where W 2 1 is the rate per atom multiplied by N 2 gives you the total rate of stimulated emission, which was equal to B into u nu into the atomic line shape function into N 2, which is then written in this form. So, we have substituted for B in terms of A.

And therefore, we get this expression here and u nu if you substitute I nu by c by n we get I nu by h nu into N 2, which was written as the cross section sigma of nu. This we have discussed in detail, but I am just recalling into I nu by h nu into N 2, this quantity I nu by h nu is the photon flux.

I nu is the intensity which is power per unit area power is energy per unit time per unit area divided by energy of one photon gives us the number of photons per unit area per unit time, which is called the photon flux, phi nu you. Therefore, gamma 2 1 is equal to sigma of nu here. So, this is the photon flux phi nu. So, phi nu and therefore, gamma 2 1 is equal to this and because our W 2 1 is equal to sigma of nu into phi nu.

Therefore the rate equations maybe return in terms of these W i j that is W 2 1, W 1 2 and 3 1, 4 1 and so on or in terms of the photon flux phi both are related. So, the rate equations may be written in terms of W i j or the photon flux. We will write the rate equations in terms of the transition rates W i j ok.

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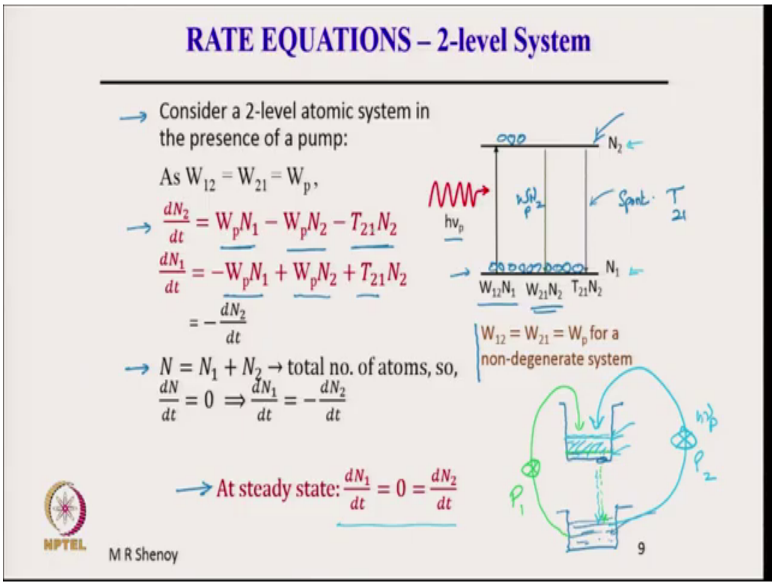

Let us take up the rate equation.

First rate equations for a 2 level system. So, consider a 2 level atomic system in the presence of a pump that is illustrated here. There is an external pump of photon energy h nu p which leads to absorption of atoms from the ground state to the upper state here. And that rate is W 1 2 absorption here stimulated absorption W 1 2 rate multiplied by N 1, the number of atoms in that level and atoms once they reach here.

So, here are the ground state atoms large number of atoms in the ground state. Once the atoms is there they can come down to the lower level by stimulated emission because the same radiation would also stimulate atoms to come down or by spontaneous emission.

So, this is spontaneous emission which is given by the rate T 2 1. So, spontaneous transition rate is T 2 1 and W 1 2 because it is a 2 level system it is a non-degenerate system, for a non-degenerate system here W 1 2 is equal to W 2 1 is equal to W p, the subscript p referring to the pump for a non-degenerate system and therefore, we can write the rate of change of population of level N 2, that is this here, what is the rate of change of population N 2 will increase because of absorption here atoms making upward transition, N 2 will decrease, because of spontaneous emission and stimulated emission, spontaneous emission here and stimulated emission.

So, d N 2 by d t is equal to W p into N 1, please see W p into N 1 is the rate of stimulated absorption that is the number of atoms absorbed per unit time per unit volume that is d N 2 by d t, which leads to a change in the atomic number atomic population of level 2. Hence this is called a rate equation, d N 2 by d t is equal to W p N 1 minus W p N 2 because here W p N 2. So, this is W p N 2 it is written here. So, W p times N 2 and the 3rd term it is also decreasing the number N 2 is decreasing because of spontaneous transitions T 2 1 into N 2.

If you write the rate of rate equation for N 1, then d N 1 by d t equal to N 1 reduces because of this upward transition. So, minus W p times N 1, N 2 was increasing, but N 1 is decreasing because of this transition and N 1 is increasing because of these two because atoms are being added to the lower level and therefore, d N 1 by d t is equal to it is quite obvious.

But I am explaining it because we will write more complicated rate equations for 3 level and 4 level systems and we can see that this is equal to minus d N 2 by d t, because that is true because we have a 2 level system which means the total number of atoms N is shared between the 2 levels. So, we have N 1 plus N 2 total number d N by d t equal to 0, because the total number of atoms is constant therefore, it is 0 which implies d N 1 by d t is equal to minus d N 2 by d t.

Now, at steady state in the presence of pump at steady state d N 1 by d t equal to 0 and it is equal to d. So, this can be imagined very easily like this that you have a bucket, let us say you have a bucket with a hole here and there is another bucket this bucket example that I give is.

So, this is the ground state where there is let us say this is a lower bucket which has plenty of water. So, full of water here and there is very little water here because of the thermal pump that is thermal energy atoms get excited, let me change the color because of the thermal pumps atoms get excited and water is being lifted it is like a pump.

So, we have a pump here. So, pump 1, P 1 atoms get lifted here into the upper level or upper bucket, but the bucket has a small hole because atoms also come down spontaneously. So, atoms will come down from here.

But at any state depending on the pumping rate you will have a certain steady state level water is being pumped water also comes down here, there will be a level is reached at which time the amount of water being pumped is equal to the amount of water which goes down. For example, if this hole is bigger then water will come down much faster the steady state level will be different.

Now, if I use another pump. So, let me change the color again. So, if I use another pump to pump water from here to here , another pump a second pump and this pump is like the external mechanism h nu p we are using a pump. Now, because of a second pump now the nu level will go up like this to a nu height because water is being pumped because of temperature.

Water is being pumped because of the external radiation the second pump. So, this I want to call a second pump , water goes down and now the level has to go higher because a steady state will come when the amount of water going here will be equal to the amount of water this plus this. And therefore, the level will go up, but there will be a steady state.

So, long as the pumping rates are constant we will get a new steady state, this was original steady state now you have a new steady state level in this water and pump scheme. The same thing happens here that because of thermal equilibrium we had a certain distribution of atoms N 2 and N 1, but because of the presence of an additional radiation of energy h nu p the numbers will change now, but there will be a steady state. And at steady state d N 1 that is the rate of change of the numbers in both the levels will be equal to 0, because steady state it is if we put these d N 2 equal to 0 for example, here.

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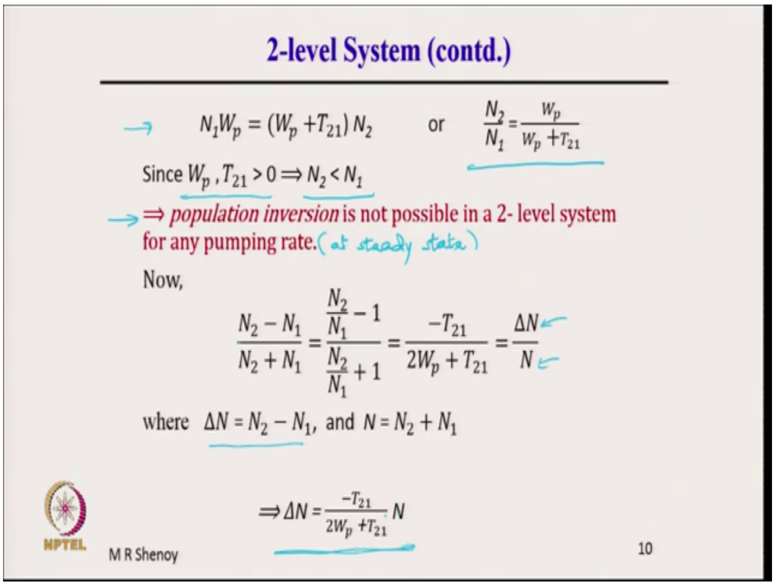

Then we have N 1 into W p. So, this is d N 2 is by d t equal to 0 gives you N 1 W p is equal to this or N 1 by N 2 is equal to W p divided by W p plus T 2 1.

Note that, W p is a pumping rate and T 2 1 is a transition rate, both are positive since W p and T 2 1 are greater than 0, N 2 must be always less than N 1, whatever be the pumping rate unless W p becomes infinity. Only when W p becomes infinity then N 2 by N 1 will become 1. N 2 could become equal to 1 at best, but otherwise N 2 is always less than N 1.

In other words it means population inversion is not possible in a 2 level system for any pumping rate at steady state. I want to add this at steady state because in transient mode it is possible to momentarily achieve population inversion, but you cannot get a steady state population inversion in a 2 level system.

So; obviously, we will have to go to 3 level and 4 level system, but before going let us discuss a little bit more. So, here therefore, N 2 by N 1. So, if you do N 2 by N 1 minus 1 divided by N 2 by N 1 plus 1 , then that will be N 2 minus N 1 divided by N 2 plus N 1 which will be equal to minus T 2 1.

So, here you simply do minus one divided by plus one we get minus T 2 1 divided by 2 p 2 W p plus T 2 1. Now N 2 minus N 1 we call as delta N. So, it is shown here delta N is equal to N 2 minus N 1 and N 2 plus N 1 is the total number of atoms that is N. So, we have therefore, we have delta N is equal to T minus T 2 1 divided by 2 W p plus T 2 1 into N. Now what are we reaching it let us see next.

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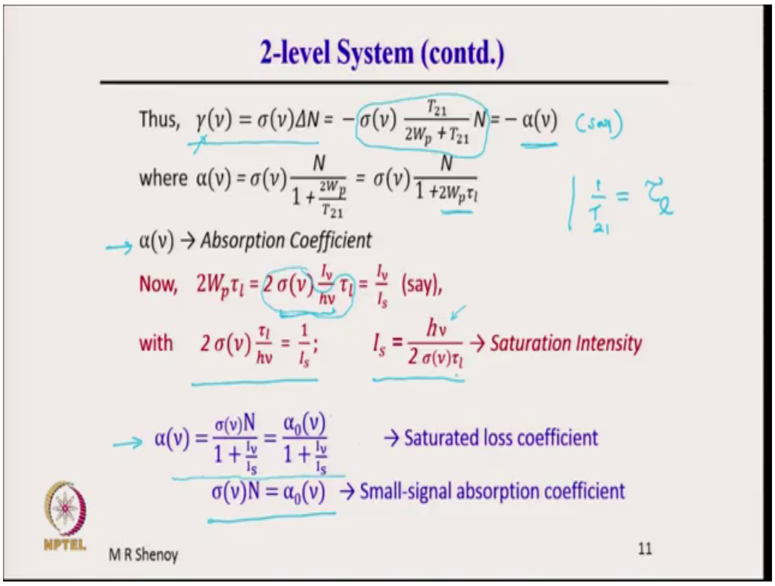

The gain coefficient we have derived this earlier that, the gain coefficient gamma of nu is given by sigma of nu into delta N, where sigma of nu was the cross section interaction cross section therefore, for delta N if we substitute from the previous expression here. So, if we substitute for delta N from this expression in here, then we get gamma of u is equal to minus sigma of nu into T 2 1 divided by twice W p plus T 2 1 into N, which we call the keeping this minus sign here minus alpha nu. So, say where alpha nu is all the rest. So, everything which contains to this 1 is alpha of nu.

So, alpha nu is equal to sigma nu into N. So, we have divided by T 2 1 to the denominator. So, we get 1 plus twice W p by T 2 1. And 1 by T 2 1 we have already seen this is nothing, but equal to lifetime of that level tau l.

And therefore, we write alpha of nu is equal to sigma of nu cross section into N total number of atoms divided by 1 plus W p pumping rate per atom into tau l, where alpha nu is called the absorption coefficient. So, gamma nu is minus alpha nu gamma nu is the gain coefficient. Therefore, alpha nu if it is negative then we call it alpha nu and then that represents the absorption coefficient.

So, absorption coefficient of a 2 level atomic system. Now twice W p into tau l is equal to we have substituted for W p sigma of nu into I nu by h nu into tau l. So, this is W p and this if we call this as I nu by is where I s is all of this. So, all of these if we call as I s that is 1 by I s, then we have 1 by I s.

So, here with 1 by I s equal to this then we have twice W p into tau l is equal to I nu by I s twice W p into tau l here this term is equal to I nu by I s. And therefore, we can write alpha of nu is equal to sigma of nu into N divided by 1 plus I nu by I s, which again we write as alpha 0 of nu that is this term sigma of nu into N in the numerator we call as alpha 0 of nu.

Because this one the expression which we have here for alpha of nu is called the saturated loss coefficient and the quantity I s here is known as saturation intensity. We can see the dimensions for example, h nu is here energy and sigma of nu is area and t tau l is time. So, energy per unit time per unit area is intensity and the dimensionally that is why we have put it as I s, we called it as I s and that term represents the saturation intensity, we will see the importance what is the saturation intensity.

So, the absorption coefficient can be written as sigma of nu into N. So, this is the cross section total number of atoms divided by 1 plus I nu by I s, I s is a characteristic parameter of the material, which is the saturation intensity for a given material at a given wavelength we have a saturation intensity given by h nu divided by twice sigma of nu into tau l.

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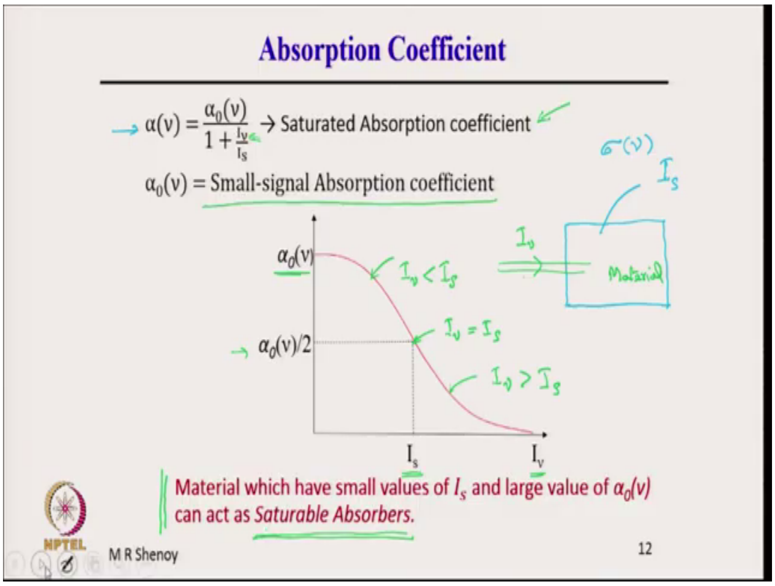

So, let us see this a little bit more. So, alpha of nu this absorption coefficient is a saturated absorption coefficient with the numerator called small signal absorption coefficient. What is the meaning of this? And what is it is importance? if you consider a material, then this material is characterized by a sigma of nu at any at a given frequency it has a cross section and a saturation intensity I s.

If we have radiation of some intensity I nu. So, I nu is passing through this material in the absence of any pumping the material will absorb this radiation, but the absorption by this material will depend on the intensity I nu, that is what is being discussed here.

So, what we have plotted is I nu intensity of the radiation as versus the absorption coefficient, when I nu is very small compared to I s, when I nu in the denominator is very small, then we can neglect it compared to I s. So, this is 0. So, we simply have the last coefficient equal to alpha 0 of nu and that is why. So, what is alpha 0 of nu? Alpha 0 of nu is the absorption coefficient of this given material this is a material for an atomic system. This material when the intensity of the radiation is very small hence the name small signal absorption coefficient.

If the intensity of the radiation I nu increases, then in the denominator I nu becomes more and therefore, as I nu becomes comparable for example, when I nu equal to is then we have alpha of nu is equal to alpha 0 by 2 that is half. So, this also is the definition of is the saturation intensity is the intensity of the radiation at which the absorption coefficient drops down to half of it is value. That is the saturation intensity if the intensity increases further that is when. So, this portion here is for I nu greater than I s and this portion is I nu less than I s and this is when I nu equal to I s.

So, this is the absorption coefficient when I nu equal to I s intensity equal to the saturation intensity. And when the intensity of the radiation is very small then we have the small signal absorption coefficient this is important. Similarly, we will get small signal gain coefficient, which leads to saturation. So, that is why this expression is called saturated expression for absorption coefficient, saturated absorption coefficient and alpha 0 of nu is the small signal absorption coefficient.

What is the important? Materials here, materials which have small values of I s and large values of alpha 0 of nu can act as Saturable Absorbers , Saturable Absorbers are widely used in laser physics. So, let us discuss about Saturable Absorbers.

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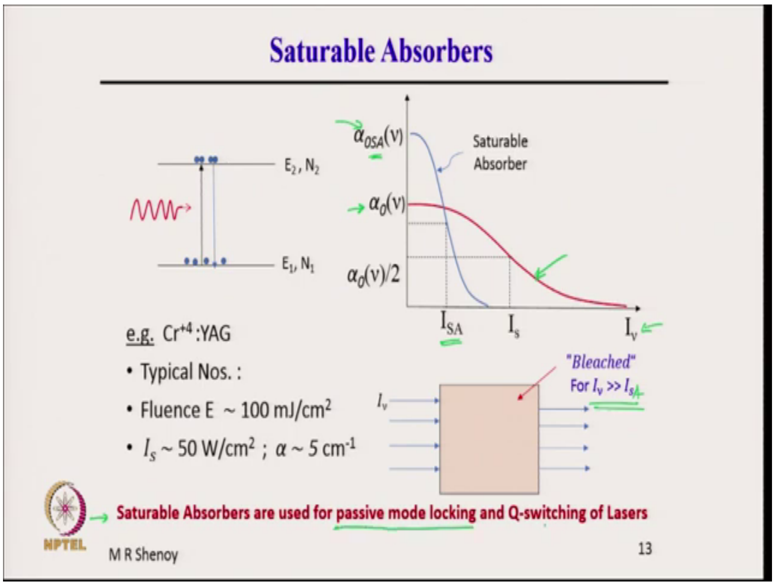

So, if you have a 2 level system then we have the expression for the absorption coefficient and we know the small signal absorption coefficient alpha 0 of nu and the red curve is the curve, which I have plotted in the previous slide. So, we can see the previous slide here. So, this is the red curve the absorption coefficient and discussion of variation of absorption coefficient with the intensity I nu materials in which have small saturation intensity.

So, S A is the saturation intensity for a saturable absorber. So, saturable absorbers are absorbers which have a small value of saturation intensity and a large value of absorption coefficient. So, this is the small signal absorption coefficient of the saturable absorber, alpha 0 of the saturable absorber and saturation intensity of the saturable absorber.

So, graphically we can show qualitatively, this is a qualitative explanation that saturable absorbers are materials, which have relatively large absorption coefficient at small intensities. And it completely goes down to 0 or we call it it gets bleached for I nu greater much greater than I s for intensities which are much greater than I S A, here I S A it gets completely bleached, bleached means what there is no absorption the absorption completely drops and the material behaves as if it is a transparent material.

At small intensities the absorption coefficient is very high, but at higher intensities it is almost transparent. We call the material gets bleached as the intensity becomes higher and higher such materials are called saturable absorbers.

Saturable absorbers are very important in the study of lasers, because saturable absorbers are used for passive mode locking and Q switching of lasers. We will study both of these in detail at a later stage, but at this point I just wanted to bring in this idea of a saturable absorber from the discussion of a 2 level system.

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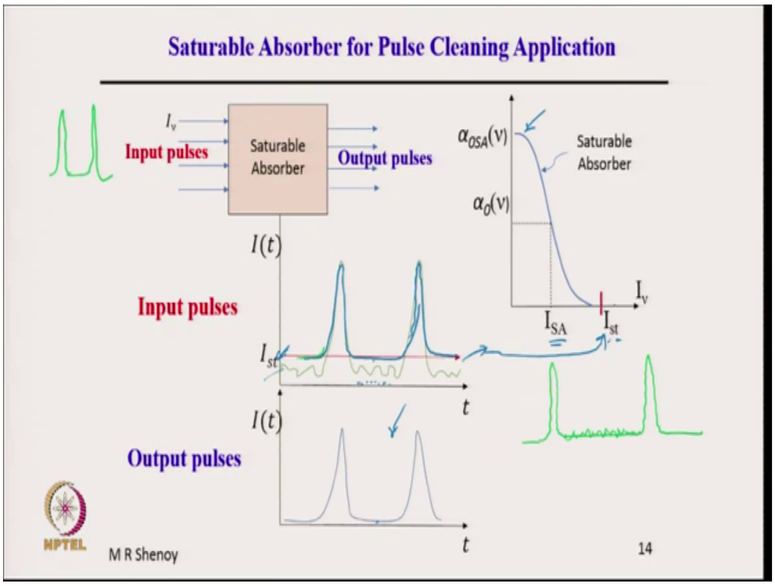

One the, one application of the saturable absorber is for pulse cleaning, which can be easily understood that is why we have introduced this here. So, what you have is input pulses. So, there are pulse. So, this the input pulses we have pulsed input and output pulses here what is shown is the input pulses here, that is the intensity varying as a function of time. So, pulses have high intensity over a small duration of time and then there is no intensity ideally it should be 0.

But usually when pulses are generated there are small noisy fluctuations in the intermediate time period. Ideally we should have had a pulse, which is like this and then completely 0 till the next pulse comes, but usually in the generator pulse generator there are some noisy mechanisms and you have some small intensity fluctuations. So, there are applications where you need clean pulses with 0 fluctuation in between or very little fluctuation in between.

So, one of the ways that can be that can be used to clean the pulses is by passing through the input pulses through the saturable absorber. So, it is illustrated here that beyond a certain intensity it is completely transparent. So, you have all the intensity coming. So, the effectively what has happened is this pulse has become like this. Ok let me show in the blue color.

So, the pulse after if I want to show in the same graph, if I want to show the pulses then it has become like this after sorry, after passing through the saturable absorber that is what is shown in the graph below.

Because the pedestal which contained these low frequency, low intensity variations has been eliminated because at low intensities the absorption coefficient of the saturable absorber is very high, but at higher intensity. So, our laser as the intensity which is more than this. So, the laser intensity if it is here. So, this level that we have set here is the threshold level some threshold level here.

I s t saturable the intensity threshold value, this is the saturation intensity of the saturable absorber and this is the threshold value, that we have set. And you can see beyond this threshold the absorption coefficient is literally 0, which means all the intensity of the pulse, which is above this threshold value here completely passes through because absorption coefficient is 0.

Below that the absorption coefficient is high and therefore, it gets attenuated and therefore, the output would look like this basically we have shifted the baseline here. So, this level has now come to here. So, that is an application of a easily understandable simple application of a saturable absorber.

But more important applications of use of saturable absorbers in mod locking and Q switching will be discussed at a later stage. So, we will stop here with the 2 level system, in the next lecture we will take up 3 level system.

Thank you.