Introduction to LASER
Prof. M.R. Shenoy
Department of Physics
Indian Institute of Technology, Delhi

Lecture - 06
Line Broadening Mechanisms

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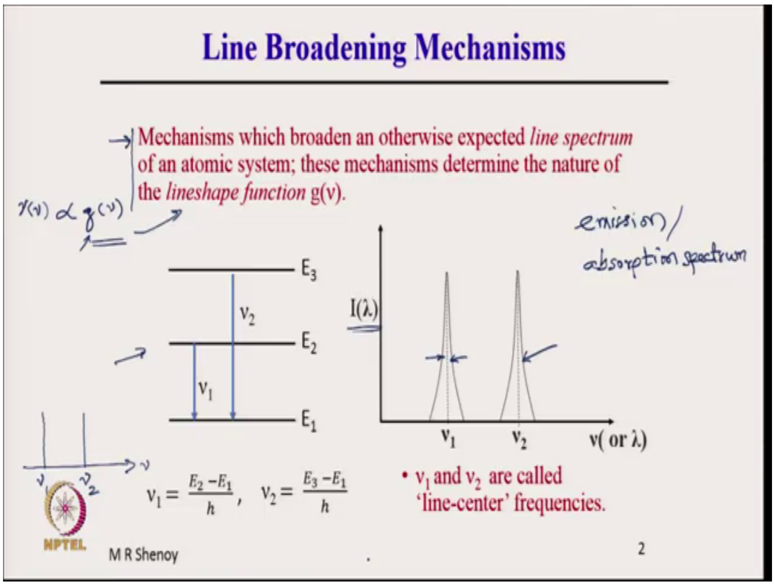

Welcome to this MOOC on Lasers. Today we will discuss Line Broadening Mechanisms. What are these? These are mechanisms which broaden an otherwise expected line spectrum of an atomic system and these mechanisms determine the nature of the line shape function. Please recall that in the last lecture we have seen that the gain coefficient gamma of nu is almost proportional to g of nu.

That is the normalized line shape function which means that the amplification bandwidth and the amplifier performance is essentially determined by g of nu which is the line shape function. And therefore, it is very very important in the study of laser amplifiers that we know the normalized line shape function g nu because that describes the interaction of radiation with matter which leads to a certain bandwidth for the amplifier and therefore, let us look at this.

So, we will try to understand this definition which I have written here that these are mechanisms which broaden an otherwise expected line spectrum of an atomic system. If we are we know that an atomic system is characterized by discrete energy levels E 1, E 2, E 3 etcetera.

Transition between these energy levels would result in a line spectrum which means if you plot nu then at different frequencies corresponding to the transition as shown here nu 1 is equal to E 2 minus E 1 by h and nu 2 is the frequency higher frequency corresponding to the higher energy difference E 3 minus E 1.

And therefore, we expect a line spectrum if ideally E 1, E 2, E 3 where energy levels with a fixed energy value then we should have got a line spectrum like this. But as discussed in the last class when we observe the emission and absorption spectrum when we observe in the emission spectrum emission or absorption spectrum absorption spectrum then we see a finite width over which emission or absorption takes place.

And that is shown here that if you see the spontaneous emission spectrum or the absorption spectrum this is an emission spectrum then you see a finite range of frequencies or wavelengths over which emission takes place or the light intensity is distributed over a range of frequencies centered around the frequencies nu 1 and nu 2. Now, what are the mechanisms which are responsible for this? So, these are the mechanisms which we call as line broadening mechanisms.

So, the definition again is mechanisms which broaden an otherwise expected line spectrum of an atomic system. Now these mechanisms determine the nature of the line shape function gene. So, we will discuss these mechanisms in a little bit more detail.

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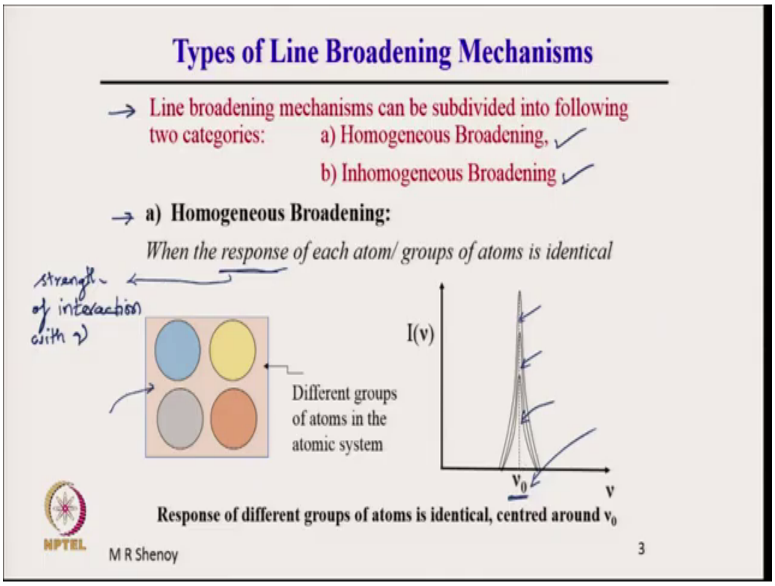

So, types of line broadening mechanisms. So, line broadening mechanisms can be broadly classified into two categories called homogeneous broadening and inhomogeneous broadening. Let us try to understand this, this is a broad category. So, what is homogeneous broadening?

Homogeneous broadening when the response of each atom or groups of atoms in the atomic system is identical then what we have is homogeneous broadening. For example, this is an atomic system. The color is just to indicate the groups. So, this is this is a chamber or a container or a volume in which atoms are there.

But there can be different groups of atoms. Groups could be qualified by different different properties. We will discuss this ah as we go further. At the moment assume that there are different groups of atoms in the atomic system then the response of each atom or each group of atom if it is identical then what we will have is if nu 0 is the line center then the response will be spread around nu 0 for all groups of atoms and therefore, more the number each group of atom contributes to the same spectrum.

And therefore, if more and more number of atoms contribute then the response goes on increasing it becomes stronger and stronger, but all the while centered at one frequency nu 0 that is what we mean by the response is identical, the shape the response. What is this response we are talking of? Response here refers to the strength of interaction. So, strength of interaction is emission or absorption. So, the response here refers to the strength of interaction.

So, strength of interaction with nu interaction with frequency with nu and that is what we call as the response. If the response of each atom is identical; identical means it is centered around the same frequency and the distribution is also same that is what we call as identical. And therefore, more the number of groups of atoms contributing to the response then stronger will be the response, but all centered around the same frequency nu 0 and such broadening mechanisms are called homogeneous broadening.

So, this is called homogeneous broadening this leads to homogeneous broadening. What are its characteristics, we will see shortly. So, the response of different groups of atoms is identical centered around the frequency nu 0.

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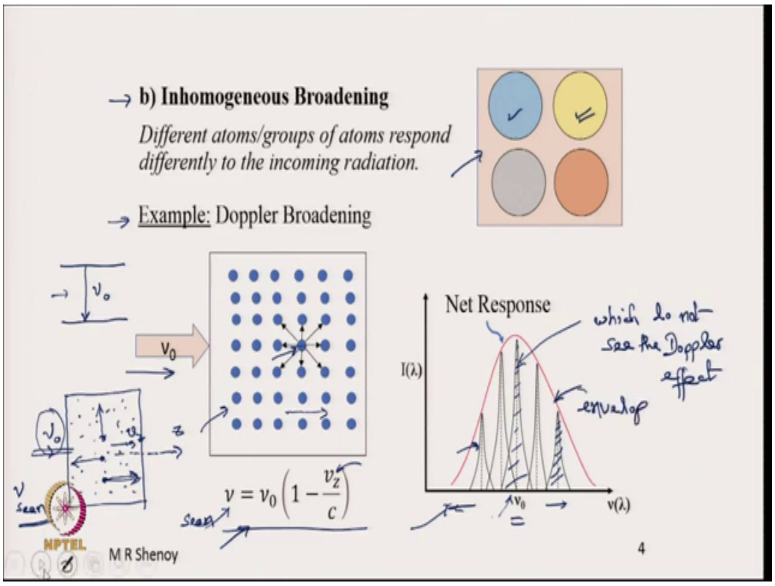

Now, let us see in homogeneous broadening, what is inhomogeneous broadening. Different atoms or groups of atoms respond differently to the incoming radiation. So, the same different groups I have shown here. Now, the response of this group here is different from the response of this group.

So, what is different, it will be clear. So, let me take an example. So, why the response of different groups of atoms in the same atomic system or in the same container should be different. For example, you take Doppler broadening. What is Doppler broadening? So, we have an atomic system these atoms are of course, need not be in a perfect periodic arrangement, but if it is a crystal then it will be in a periodic arrangement.

Now, the atoms are vibrating. For example, if it is a crystal then the atoms are vibrating about their mean position. Now, the vibration for example, the at one particular atom which is shown here this one is vibrating in this direction or it could be vibrating vertically or it would be vibrating in different any possible direction. Now, let us see consider incidence of a radiation nu 0 radiation of frequency nu 0.

It may be easier if we and if we consider a gas. For example, in the container instead of the crystal we will come back to this later. If you consider a gas with atoms groups of atoms travelling in all possible directions randomly traveling in all possible directions and there is an incident radiation here of frequency nu 0, which is incident on the atomic system.

Then atoms which are travelling for example, atoms are in a state of motion. So, atoms which are traveling in this direction and the atoms which are travelling in this direction and atoms which are traveling in this direction will see this incident frequency nu 0 as different.

For example, atoms travelling in this direction towards the radiation will see the frequency nu sean will be higher than the actual frequency because of the Doppler Effect. So, it is given here. So, if we look at this expression here nu here represents the frequency seen by the atom observed by the atom, nu 0 is the actual frequency which is incident atoms which are travelling towards the light because of doppler effect we will see it as a higher frequency.

Therefore, it will see it as nu 0 into 1 minus v z by c. This is higher because please see that we always take the direction of propagation as z. So, v z is the velocity component in the z direction v z and a atom moving in this direction has minus v z that is why 1 minus, minus v z will give a new higher.

And therefore, the frequency seen by atoms here moving in the towards the radiation will see it as a higher frequency, atoms which are travelling in a perpendicular direction will see no change and atoms which are travelling in the direction of the radiation will see it as a lower frequency. And therefore, the same atomic system which has let us say it is a 2-level system characterized by a certain resonance frequency. The actual resonance frequency let us say nu 0 is the actual resonance frequency. Therefore, the incident radiation if it has a resonance frequency nu 0, it should have all of them should have responded to the incident radiation.

That is emission or absorption should have taken place, but because the atoms are moving in different directions they see the incident radiation nu 0 as different frequencies. If thus observed, the frequency seen is different from the actual resonance frequency it will not interact or the strength of interaction will be different for different groups of atoms.

So, what is shown here? Let us come to the graph here. So, nu 0 is the actual atomic resonance frequency. So, that is here atomic resonance frequency and nu 0 is also the incident frequency.

Those atoms which are travelling in a perpendicular direction here will not see the Doppler Effect because v z is 0 and therefore, they will interact strongly with the incident radiation because the incident frequency matches with the atomic resonance and therefore, this group of atoms which are here.

So, this response is because of the group of atoms which are travelling in a direction perpendicular or which do not see the Doppler Effect which do not see the Doppler Effect because they do not see any frequency shift. These ones will see it as a lower frequency.

So, those atoms which are traveling away from the radiation or in the direction which is away from the radiation or in the direction of the radiation will see it as a lower frequency and they would not interact with this radiation or their strength of interaction would be lower. And those groups of atoms which travel towards the radiation will see it as a higher frequency and therefore, the if we have a relatively broadband source different groups of atoms will interact with different frequencies.

Or if we have a near monochromatic radiation nu 0 then different groups of atoms will interact with different strengths. Those centered around nu 0 are the ones which will not see Doppler effect or for which v z is equal to 0. So, it is the same atomic system, but different groups of atoms are travelling in different directions. And therefore, they will see the incoming radiation as different frequencies. Apparent frequency is different and therefore, they will decide whether they would interact with the incoming radiation or not and the strength of interaction for different groups of atoms will be different.

And that is illustrated by this graph here where which is centered with there are responses or different groups of atoms are centered around different frequencies. Now, as you go to higher frequencies or lower frequencies the number of atoms contributing to this for example, here this resonance is much smaller because the number of atoms with the higher velocity or lower velocity are much smaller.

This distribution of course, is given by the Maxwell Boltzmann distribution we will discuss this in detail in the next class line the Doppler Effect will be discussed in detail in the next class.

But today I just wanted to illustrate that how the same atomic system different groups of atoms in the same atomic system can respond to radiation of different frequencies ok. Let us continue.

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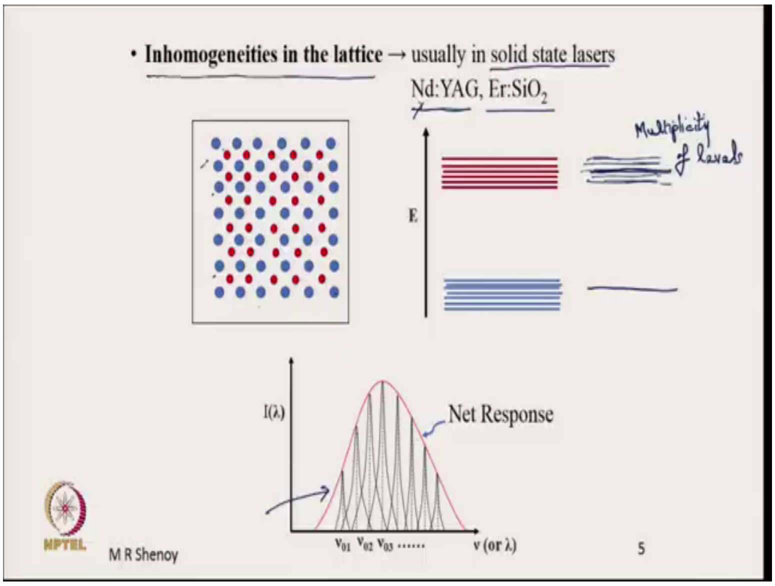

Another very common example is when we have inhomogeneities in the lattice in homogeneities. In homogeneities means, the density of atoms could be different at different places in the atomic system. For example, there could be more clustering in some place. This particularly happens when you have a doped crystals.

For example, in solid state lasers such as Nd:YAG laser, in Nd:YAG neodymium atoms are doped in YAG Yttrium aluminum garnet or erbium silica that is erbium ions are doped in silica matrix. So, in such cases the because of in local in homogeneities the energy level instead of having discrete energy levels which is characteristic of the pure crystal or pure atomic system, it spreads into a large number of layers.

There are different number of energy levels or multiplicity of energy levels. So, this leads to multiplicity of energy levels. So, multiplicity because of in homogeneities when multiplicity of levels. In homogeneities local inhomogeneities cause in homogeneities; if the density fluctuations takes place then the energy levels corresponding energy levels are different at different locations.

And therefore, they respond to different frequencies. The atomic resonances are different at different locations within the same atomic system. And therefore, they respond to a range of frequencies again as illustrated here. And the envelope in both the cases here as well as in the previous case that is Doppler broadening, it is the envelope here. The envelope shows the average or net response. Envelope shows the net response of the atomic system.

So, that is what is shown here. So, this is about inhomogeneously broadened line shape function.

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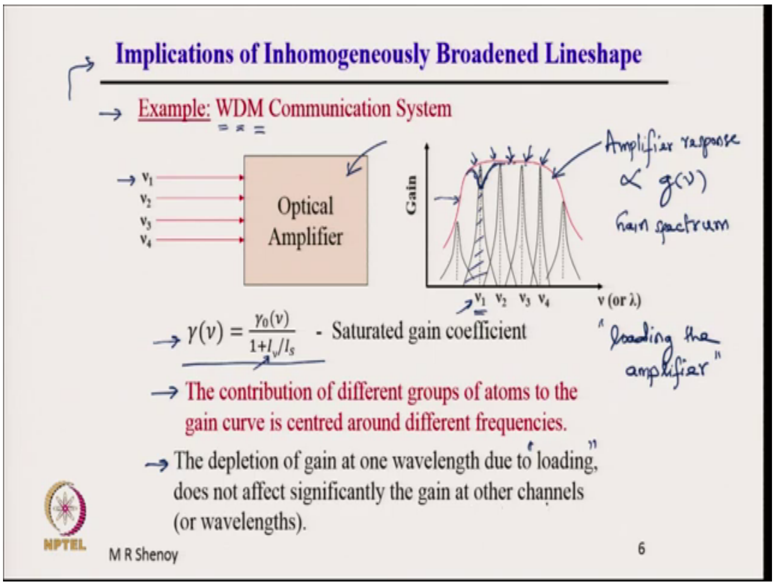

So, what is the implication of these? We will see the implications of both, homogeneously and inhomogeneously broadened line shapes when we discuss about lasers.

But as an amplifier if we consider WDM that is Wavelength Division Multiplexed communication systems which is widely used in optical communication optical fiber communication different wavelengths or different frequencies travel through the same optical fiber and then they pass through the same optical amplifier. We are currently discussing optical amplifier that is laser amplifier.

We will come to the laser itself a little later. Then, if the bandwidth that is the amplification bandwidth. So, this is the net response is net g of nu and therefore, this is amplifier bandwidth. So, this is amplifier response which means gain is provided over a range of frequencies.

Now, if this amplifier response which is directly proportional to g of nu, therefore, you can say that this response is nothing, but g of nu qualitatively it is proportional to g of nu. So, if g of nu has a response a broad response like this it is because there are different groups of atoms contributing to different parts of the gain spectrum.

So, this is the amplifier response. This is the gain spectrum. Gain versus frequency is called the gain spectrum. We will discuss the amplifier again in a little bit more detail later. But right now, one of the important implications of inhomogeneously broadened line shape is that different parts of the gain spectrum are contributed by different groups of atoms.

Therefore, if I draw energy at a particular wavelength lambda 1 or frequency nu 1 which means, I am loading the amplifier that is called loading the amplifier loading the amplifier. Loading the amplifier means, you input to the amplifier a particular frequency and you will load depending on the signal strength. Higher the signal strength the amplifier will be loaded more. So, that is called loading the amplifier.

So, if we load basically loading means we are inputting. So, if we input different frequencies or different wavelengths if you load the amplifier at one frequency nu we will see that because the gain is saturated gain coefficient. Loading means, the signal strength I nu here. The signal strength I nu increases. Therefore, the gain will decrease and therefore, if I put a signal nu 1 then this gain profile will definitely come down at the frequency nu like this.

What I have shown is a dip at nu 1, but I have not shown a dip elsewhere. Suppose in the amplifier please try to understand if I had only one frequency nu 1 at the input of the optical amplifier. This is a the gain spectrum of the amplifier which means, it is a broadband amplifier.

It can amplify all these frequencies as shown here nu 1, nu 2, nu 3, nu 4, but if I input only one frequency nu 1 then the gain at nu 1 will be pulled down. Why it will be pulled down? Because of this expression here, saturated gain coefficient which we discussed in the last class. The gain will be pulled down here, but the gain at other frequencies will not be affected because the gain at other frequencies is contributed by different groups of atoms.

The gain at nu 1 that is this curve here is contributed by a particular group of atoms. So, if the gain has gone down here, it will not affect the gain at other frequencies because gain at other frequencies is contributed by different groups of atoms. So, that is what we have written. Here the contribution of different groups of atoms to the gain curve is centered around different frequencies.

Therefore, the depletion of gain at one wavelength due to loading that is due to an input does not affect significantly the gain at other channels or wavelengths. This is very important in WDM communication system.

Otherwise, if you if it was homogeneously broadened then if you pull down the gain at any frequency the entire gain curve will go down that way it will disturb other frequencies. The gain of other frequencies will also be disturbed. This is very important in communication that other channels the frequencies do not disturb any of the any channel does not disturb the frequency or the gain of other channels. So, inhomogeneously broadened amplifiers are very useful in such situations ok.

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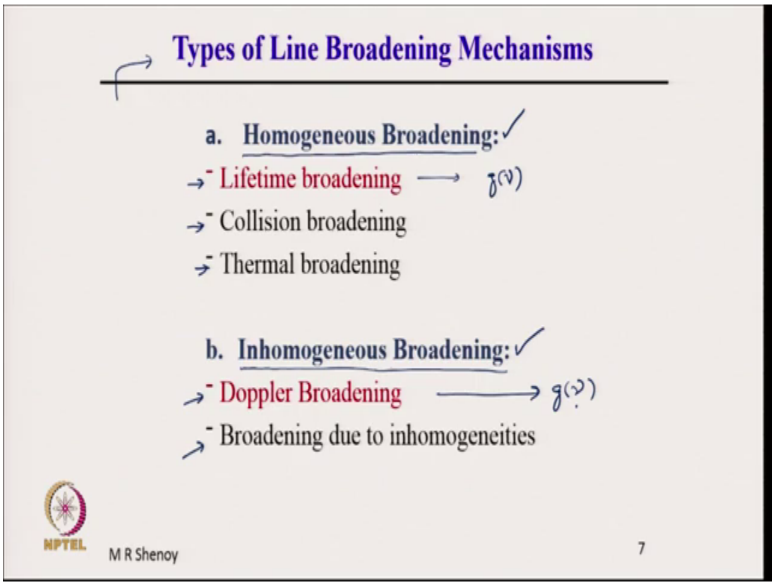

Let us now continue with our discussion; the types of line broadening mechanisms. So, I had discussed what is homogeneous broadening and what is in homo. Within this there are different types of homogeneous broadening can happen due to different mechanisms.

So, what is shown is lifetime broadening, collision broadening, thermal broadening. These all of these mechanisms contribute to a to homogeneous broadening. Whereas, Doppler broadening as i already explained to you qualitatively and broadening due to inhomogeneities.

Again I explain to you qualitatively leads to inhomogeneous broadening. We will discuss the ones which are shown here in red in detail how lifetime broadening leads to a what is the line shape function g nu in the case of lifetime broadening. And similarly I will discuss in the next class in detail Doppler broadening, what kind of g nu function it will give.

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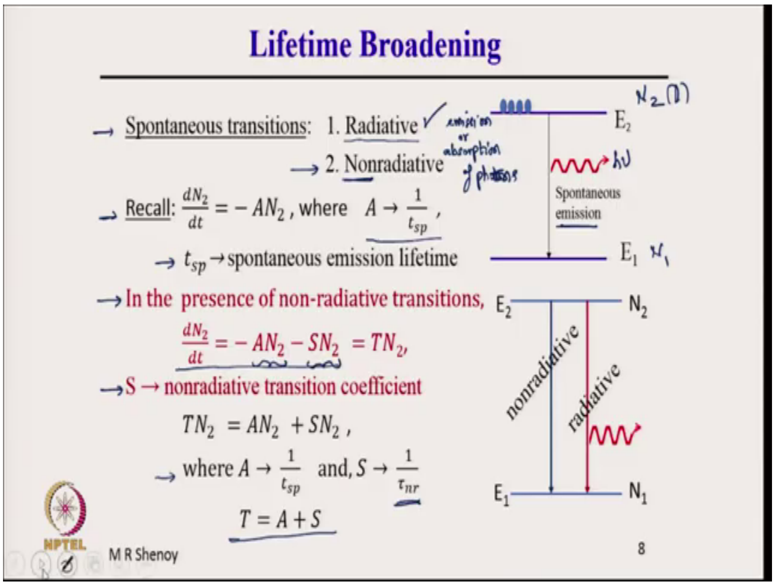

So, let us take up the first one that is lifetime broadening. So, far I discussed qualitatively about the line broadening mechanisms and now let us see with the simple classical approach see how to get the line shape function g nu, which again remember determines the amplifier bandwidth and the response of the amplifier ok.

Spontaneous transitions; transitions when an atom makes a downward transition from an excited level the energy difference E 2 minus E 1 could be given as a photon of radiation given as a radiation of energy h nu or it could this is what we had discussed so far that is radiative transitions.

Radiative transitions are transitions which involve emission or absorption of a photon. Radiative transitions are transitions which involve emission or absorption emission or absorption of a photon that is why the radiation is involved, hence, the name radiative absorption of photons photon.

Non radiative transition as the name indicates does not have involvement of radiation which means, there are no photons involved. A transition from an upper level to a lower level without emission of photons is also possible which is called non radiative.

How does this occur? There are different mechanisms by which non radiative transitions can occur. One of them is for example, an energy an atom colliding with the walls of the container. In a gas particularly when an atom collides with the walls of the container then it could lose energy because of collision. There could be phonon transitions that is a atom can make a downward transition by giving out phonons. Phonons are quanta of lattice vibrations. So, phonon transition means energy is given to the lattice. Let us discuss more about that a little later.

But, so, non radiative means phonon photons are not emitted. Now, recall we had this expression d N 2 by dt that is if we have N 2 number of atoms here and N 1 number of atoms. At t is equal to 0 if N 2 of 0 is the number of atoms then we know that d N 2 by dt that is rate of change of atomic number N 2 is proportional to N 2 and A is the proportionality constant and we have already seen that this A is 1 by tsp. We have considered spontaneous emission. Emission means, emission of a photon.

And therefore, tsp is the spontaneous emission lifetime. However, in the presence of non radiative transitions which means radiative plus non radiative the equation will be modified like this. It is minus A times N 2 this is the radiative part the first one and minus S times N 2, where S is the non radiative transition coefficient.

So, if we write that this is equal to T times N 2, where T is equal to a plus S then we have TN 2 is equal to AN 2 plus SN 2. We have already shown that A is equal to one by tsp, where tsp is the spontaneous emission lifetime.

A corresponding lifetime for non radiative transition if we define it as tau nr, nr standing for non radiative then S will be equal to 1 by tau nr. Please note that A and S must have the same dimensions. A is 1 over time, therefore, S is also 1 over time a non radiative life time which we designate as tau nr. Then we have 1 by t is equal to 1 by tsp plus 1 by t nr tau nr.

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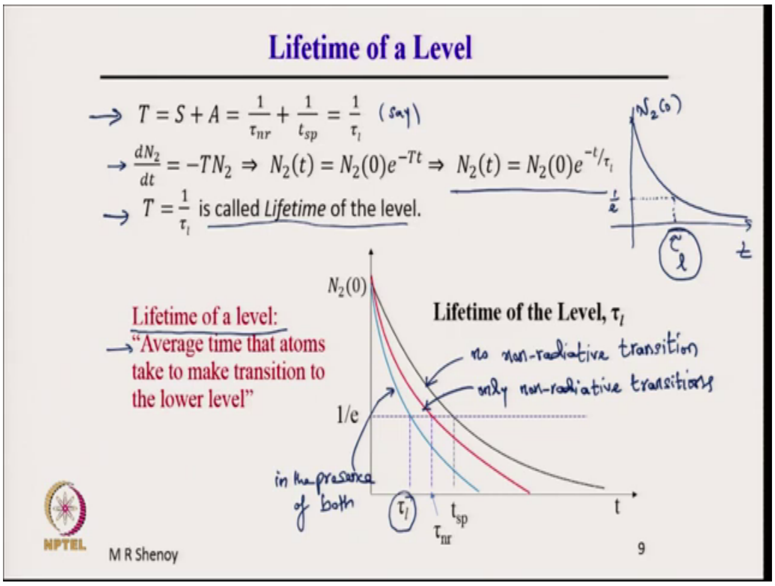

So, let us see here. So, T is equal to therefore, S plus A is equal to 1 over tau nr plus 1 over tau sp which we designate as 1 over tau l say where tau l; T is equal to 1 over tau l is called the lifetime of the level.

Why is it called as the lifetime of the level? Because d N 2 by d t here is equal to minus t times N 2 that implies N 2 of t is equal to N 2 of 0 into e power minus t capital T into small t. So, capital T is the spontaneous transition rate, it is the coefficient for spontaneous transition rate and small t is time and therefore, N 2 of t is equal to N 2 of 0 is equal into e to the power minus t by tau l because T here is equal to 1 over tau l.

What does this mean? This means at time t is equal to 0, if there are N 2. So, this is time. If there are N 2 of 0 number of atoms here in the excited state then with time it will drop down exponentially where it drops down to 1 by e of its value is called the lifetime tau l. This is 1 by e of N 2 of 0. So, this is called the lifetime. So, tau l is a parameter which is characterizing the lifetime of the upper level. So, it is illustrated more clearly now here. Please see that N 2 of 0 if there was if there were no non radiative transitions.

All of them are only radiative transitions, which means all of them are only spontaneous emissions. Then we would have had the first curve here. So, if we had only spontaneous emission then N 2 of 0 will decay with time as N 2 of 0 into e power minus t by tsp and tsp is defined as where it where the number drops to 1 by e of its original number at t is equal to 0. This we have already seen, the first graph. In this we had assumed that no non radiative transitions no non radiative. We did not talk of radiate non radiative transitions at that time.

We said that every spontaneous emission brings down one atom and gives out one photon transition. If there was no radiative transition and all of it is only due to non radiative transition only due to. So, then we would have had again atoms coming down decaying by non radiative transitions; so, only non radiative transitions only non radiative transition.

Then we would have had the second curve. And if both of them were present then; obviously, the rate will be faster because atoms will decay because of radiative transition and because of non radiative transition. And then we have this the blue curve which is shown here in the presence of both.

So, in the presence of both of both type of transitions and the lifetime in the presence of both is called the lifetime of the level. So, lifetime of a level is the average time that an atom takes that atoms take to make transition to the lower level or it is an average time that atom spend in the excited state that is called the lifetime of a level. So, the contribution to the lifetime of the level comes from both spontaneous emission which is radiative emission and non radiative emissions ok.

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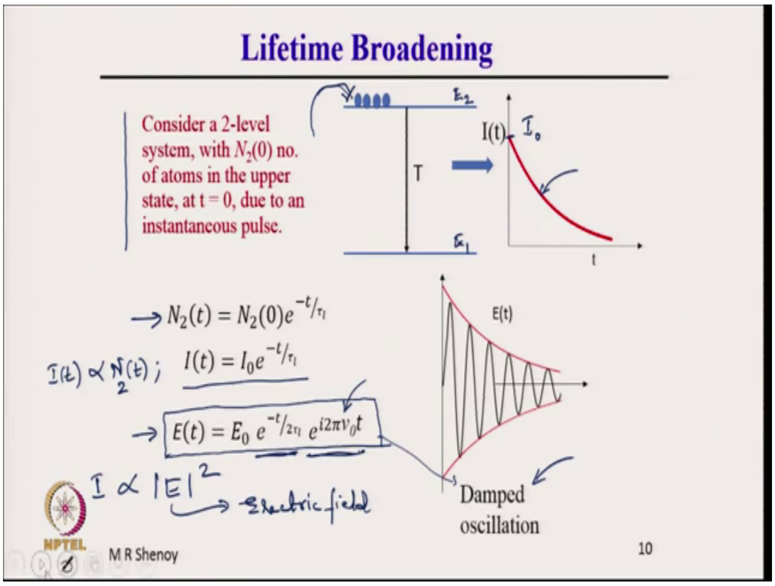

So, let us take up life once. Now that we understand what is lifetime. So, let us take up lifetime broadening. Consider a 2-level system with N 2 of 0 number of atoms in the upper state at t is equal to 0. This is due to some instantaneous pulse. We had a instantaneous burst or a pulse which had lifted let us say one million atoms or one billion atoms to the excited state here E 2.

Instantaneously so many atoms were put there and then there is no more pulse or no more burst and therefore, the atoms will start decaying. They will start coming down to the lower level to the ground state and therefore, the number of atoms will decrease. N 2 with time will be N 2 of t is equal to N 2 of 0 into e power minus t by tau l.

Every atom which is coming down here also gives out radiation and non radiation. Maybe both are present, but the intensity of radiation therefore, coming out will be proportional to the number of atoms.

So, I of t I of t the intensity of radiation which is coming out at any instant is proportional to N 2 of t. And therefore, I of t is equal to I 0; some intensity initial intensity at N 2. If I call this as I 0 then this curve will be I of t is equal to I 0 into e power minus t by tau l that is what is shown here, the intensity. This is observable and measurable. Intensity of radiation coming out of an atomic system of an excited atomic system.

An instantaneously excited atomic system will be exponentially decaying as I of t is equal to I 0. If this is the case we use a heuristic idea that we know that I of t the intensity is proportional to mod E square. So, we know that where E is the electric field, this is the electric field. And therefore, we can write the electric field as E of t is equal to E 0 into e to the power of minus t by twice tau l.

Why twice tau l? Because if we take mod square this term would go, this term is the phase term because with time there is a phase e to the power of i omega t or i 2 pi nu 0 into t.

And this is the exponentially decaying envelope because when you take mod square this will become e to the power of t by tau l and mod E 0 square is I 0. So, if we say mod E 0 square as I 0 then we can write E of t is equal to this fashion, where E of t represents the electric field associated with the radiation which is coming out of the atomic system. So, it looks like a damped oscillation. This if you plot it would look like a damped oscillation. The envelope is given by E to the power of minus t by twice tau l and oscillation. So, that is why oscillation with frequency nu 0.

But we know that whenever the, so, this is amplitude modulated electric field which is amplitude modulated and therefore, there must be a finite spectrum associated with an amplitude modulated carrier. If nu 0 was the carrier frequency and if it is amplitude modulated there would be a finite spectrum associated with this. And how to find out the spectrum associated with this?

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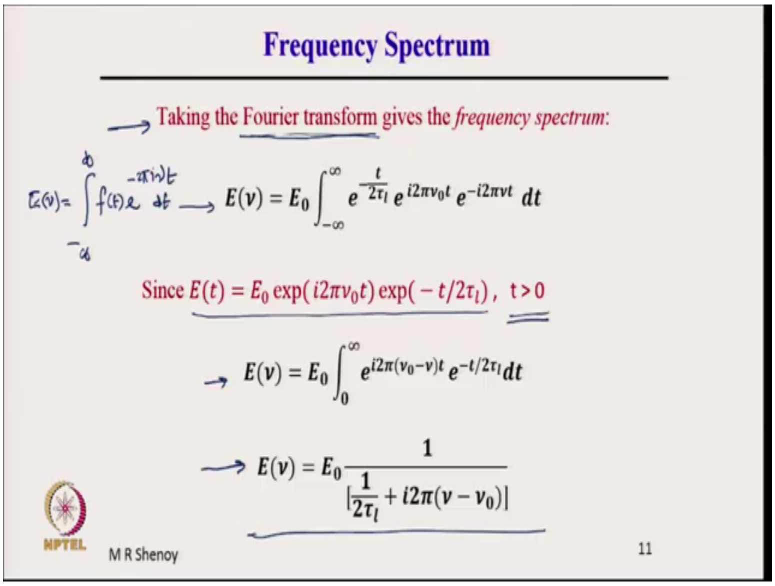

We take the Fourier transform. So, Fourier transform gives us the frequency spectrum. If we have a signal f of t then if we take Fourier transform then we will get the frequency spectrum associated with the signal. And we are making use of this classical concept to determine what is the frequency spectrum associated with this spontaneous transition.

So, the frequency spectrum therefore, here is given by E of nu is equal to E 0. So, this is the function. E of t is this function here and you can see the same function is here. E 0 is a constant amplitude which is taken out; e power minus t by twice tau l into the oscillatory function e power i 2 pi nu 0 into t into e power minus I 2 pi nu t d t from minus infinity to infinity gives you the Fourier transform.

This is Fourier transform of f of t is E of nu is equal to minus infinity to infinity f of t e to the power minus 2 pi i nu into d t i nu t into d t. So, that is what we have written here and therefore, E of nu is equal to E 0 because this the electric field only starts at t greater than 0. Therefore, the integration limit is from 0 to infinity. E nu is equal to E 0 into 0 to infinity e power now we have combined this 2 pi into nu 0 minus nu into t into this.

So, we can simply integrate this and you see that it is a definite integral which can be integrated to get the frequency spectrum E nu given by this expression.

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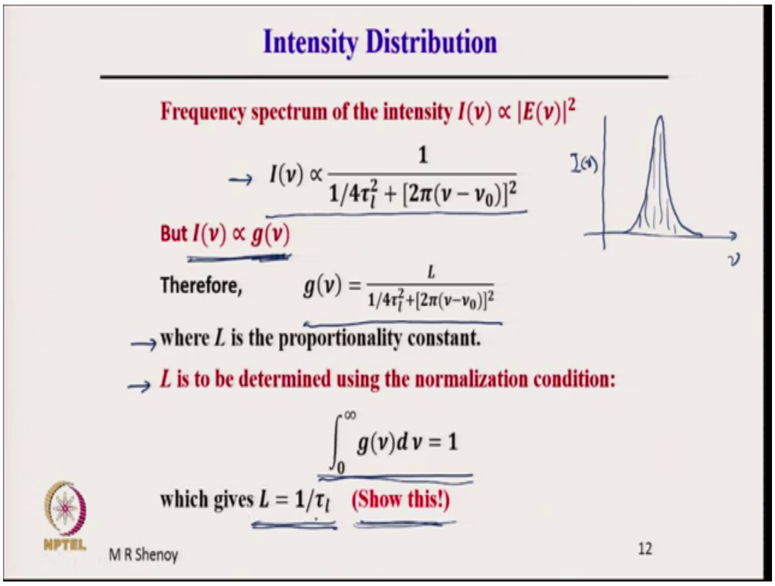

Now, the intensity distribution the intensity frequency spectrum of the intensity is I nu is proportional to mod E nu square. That is the intensity spectrum is proportional to mod E nu square which means that I nu is proportional to. So, we have taken simply mod of this. So, you can see this expression. We simply have taken mod square. When we take mod square E 0 here would become I 0. We can mod E 0 square if we designate we can designate it as I 0, a constant.

But more importantly I nu is proportional to 1 divided by 4 tau l square into 2 pi into nu minus nu 0 mod square. I nu is proportional. What is I nu? Intensity spectrum. You remember we had plotted this I of lambda or I of nu for any source or any transition which.

So, this is the intensity spectrum and this intensity distribution is because tells us the strength of interaction at different frequencies strength of interaction. Interaction refers to emission and absorption and therefore, I nu is proportional to g nu because strength of interaction is given by g nu. g nu gives us the strength of interaction. The intensity spectrum is proportional to the strength of interaction.

And therefore, g nu must be proportional to I nu or g nu is equal to some constant l into the term which is here, where L is the proportionality constant. Now, how to determine L? We can determine L by using the definition of normalized line function. L is to be determined using the normalization condition because the normalized line shape function is defined by this equation; 0 to infinity g nu d nu is equal to 1. That is the strength of interaction if we integrate over all the frequencies is 1 or probability of interaction over all the frequency is 1.

So, if we simply integrate this that is substitute this expression here for g nu and integrate then we will get an expression for L. L is equal to 1 divided by tau L. So, the slide says show this, please work out this. Simply substitute in this expression. The expression g nu in the integral and integrate equate it to one you will get the proportionality constant L as 1 divided by tau l.

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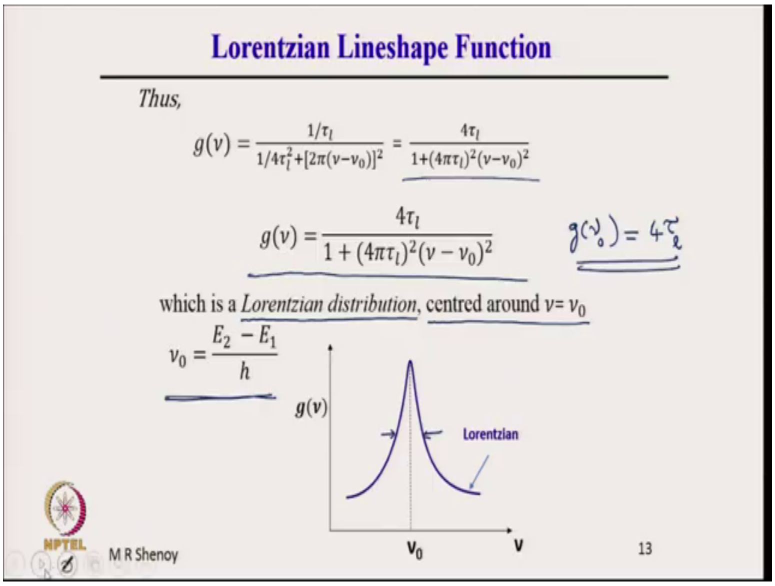

So, what do we get? If we substitute one by tau l for L, so, we have this expression here or it can be simplified to this expression here g nu. So, that is written here. g nu is equal to 4 times tau l into 1 plus 4 pi tau l the whole square into nu minus nu 0 the whole square.

A very simple mathematics and this distribution is called a Lorentzian distribution. A Lorentzian distribution as you can see from the expression is a symmetric distribution centered around nu is equal to nu 0 because you can see in the denominator all are positive quantities.

So, when nu is equal to nu 0, the second term here is 0 and therefore, we have g of nu 0. So, we have g of nu 0. nu at nu is equal to nu 0 we have maximum value g of nu 0 is equal to 4 tau l. So, its centered around nu is equal to mu 0. The maximum ats and it drops down a Lorentzian function is of this form.

So, it drops down at nu is equal to around centered at nu equal to nu 0 and on both sides it drops down symmetrically. It is a symmetric function centered at the resonance atomic resonance nu 0 equal to E 2 minus E 1 by h and such a line shape function is called Lorentzian line shape function.

So, the lifetime broadening which is a homogeneous broadening is characterized by a Lorentzian which means, the shape of g of nu is a Lorentzian function. Now, we are interested of course, in finding out the full width at half maximum of the Lorentzian that will give us an idea because the bandwidth is proportional to this. Bandwidth will depend on the full width at half maximum and let us see the full width at half maximum.

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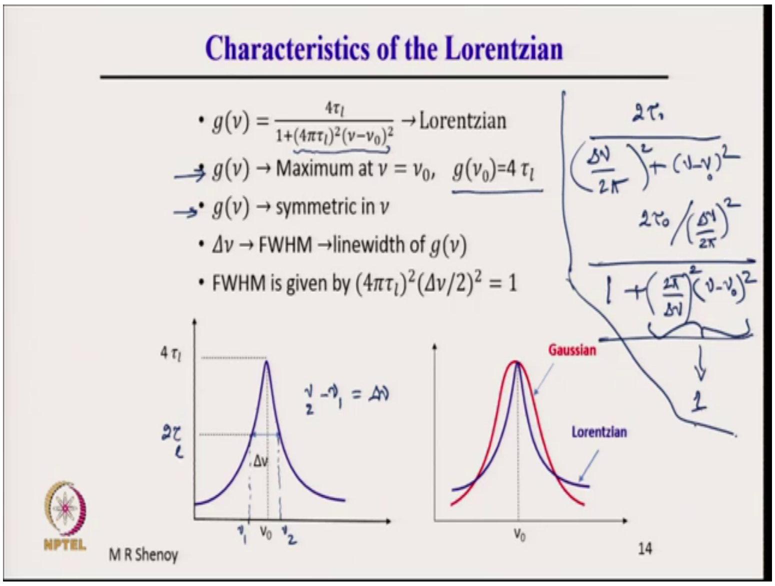

So, what are the characteristic of the Lorentzian? g nu is given by this is the Lorentzian. The maximum value as I have already written.

At nu equal to nu 0 gives us g of nu 0 equal to 4 tau l. It is symmetric a nu and the full width at half maximum is called line width of g of nu. If we designate delta nu as the full width at half maximum then the full width at half maximum is given by this denominator becoming equal to 1. This will become clear for if you have not if it is not clear just let us look at this. Here is the maximum 4 tau l. At full width at half max half of the maximum means this is tau 2 times tau l.

At 2 times tau l if the frequency bandwidth; so, let us say this is nu 1 and nu 2. So, this is nu 1 this is nu 2 then nu 2 minus nu 1 is equal to delta nu. nu 2 minus nu 1 is equal to; maybe it is there in the next slide. Let me show the next slide yeah.

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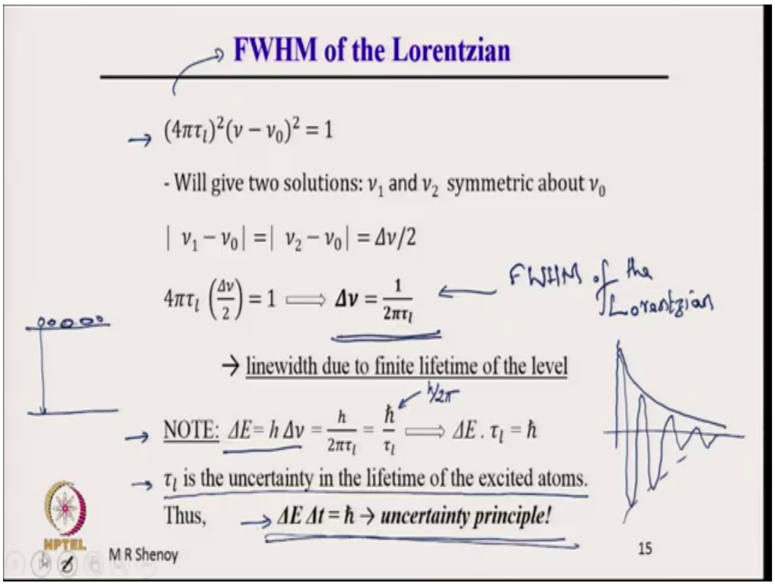

The full width at half maximum of the Lorentzian is given by this term equal to 1. Why is that? Look at this term. If this term becomes 1 this term here becomes 1 then we have in the denominator 2.

So, 4 tau l divided by 2 is half of its value. So, whenever a Lorentzian is given it could be given in different form. For example, it could be given as 2 tau 0 divided by some number here I do not know some number delta nu divided by 2 pi into some number I am writing nu minus nu 0 the whole square. This is also a Lorentzian.

So, what you should do first is put this in this form that is divide by this throughout so that this is just an example. Then you can if you divide then twice tau 0 divided by delta nu by 2 pi here and then in the denominator we will have 1 plus some number that is 2 pi by delta nu into actually this is square because dimensionally 2 pi by delta nu square into nu minus nu 0 the whole square. Now, this must be equal to 1. I have taken an independent example where because the Lorentzian could be described in a general form like this.

But, if you want to find out the full width at half maximum divide the denominator. So, that you write it in the form of 1 plus some quantity here and that some quantity must be equal to 1 at full width to get the full width at half maximum. Because half maximum means whatever when this is 0 at nu is equal to nu 0 the second term is 0.

We see in this example, at nu is equal to nu 0 second term is 0 and the maximum is given by the numerator because there is only 1 here. And that half of that maximum would come when the entire denominator is 2, which means the second term here is 1 that is what is mentioned in this slide here.

So, the full width at half maximum is given by 4 pi tau l the whole square minus nu minus nu 0 square equal to 1. This will give two solutions; nu 1 and nu 2 and nu 1 minus nu 0.

So, this is what I was drawing in the previous here. So, this is the Lorentzian and at half maximum if you solve that equal to then we will get two solutions. At half the value there are two solutions; nu 1 and nu 2. nu 1 minus nu 0 is delta nu by 2 because delta nu is the full width at half maximum. nu 2 minus nu 1 is delta nu. So, nu 1 minus nu 0 equal to nu 2 minus nu 0 equal to delta nu by 2 that is why it is written like this.

So, the important point is to see that there are two solutions and delta nu is the separation nu 2 minus nu 1. So, therefore, this is equal to delta nu by 2 and therefore, delta for nu minus nu 0 if we now substitute this then we get delta nu is equal to 1 divided by 2 pi tau l. This is the full width at half maximum; FWHM of the Lorentzian of the Lorentzian. So, line width what is this, why this delta nu has come? This spectrum has come because of the finite lifetime of the level.

Because of the finite lifetime of the level the intensity was dropping down like this if you recall and then we said therefore, the electric field must be damped oscillation. It must be representing a damped oscillation like this. And when the electric field is a damped oscillation it means you are modulating the electric field. And whenever you modulate the amplitude there will be a corresponding bandwidth here.

So, simple in by classical approach we have seen that this corresponds to a line width and the line width is due to finite lifetime of the level. Note, delta E is equal to h times delta nu because E is equal to h nu.

So, delta E is equal to h delta nu which is h divided by 2 pi tau l because delta nu is 1 by 2 pi tau l which can be written as h cross by tau l. h cross is h by 2 pi. I suppose you are familiar. h cross is h divided by 2 pi and therefore, this implies that delta E into tau l is equal to h cross and this is the uncertainty principle in quantum mechanics.

We have not used anywhere quantum mechanics. We have done only the classical approach and simple frequency spectrum approach associated with modulation, amplitude modulation and we get an expression which is consistent with the uncertainty principle in quantum mechanics, where delta E into delta t is equal to h cross. This comes out because tau l is the uncertainty in the lifetime of the excited atoms.

Recall what is tau l. Tau l is the average time an atom spends in the upper level; average time. Some atoms may come immediately come down some atoms may come after a long time. So, the average time is tau l.

This is the average time that means, for any given atom there is an uncertainty in the decay time that is it may come immediately or it may come after a long time, the average. Therefore, tau l represents the uncertainty and that is consistent with the uncertainty principle delta E into delta tau is equal to h cross.

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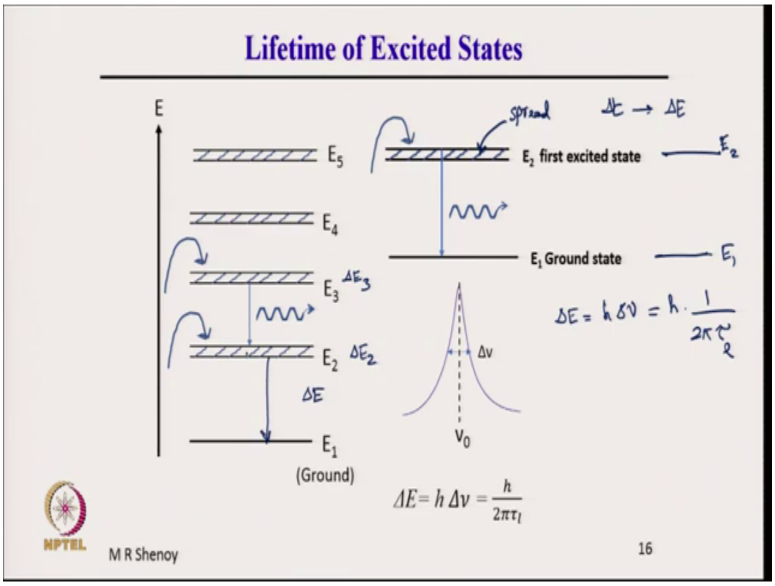

Now, the implications of this is further. Here an atom making a downward transition has an uncertainty delta t associated with it. Whenever there is a delta t then we have a corresponding delta E and what is this delta E? Delta E is this width here. Equivalently we can see that there is a finite width for inters finite spread in energy.

This is spread in energy spread corresponding to an uncertainty delta t. It is not one level, it is not one level like this. We started with discrete energy levels E 2 and E 1, but now we are seeing that an uncertainty in the lifetime of atoms is equivalent to having a finite spread in the energy spectrum or energy associated with any given level.

So, if we are having a transition from here to here then there is a uncertainty associated with this. So, there is a finite delta E. If you are looking at a transition here between 2-levels then there is a delta E 3 here and there is a delta E 2 here. Therefore, the uncertainty is double now. Delta not double that sum of this delta E 3 plus delta E 2.

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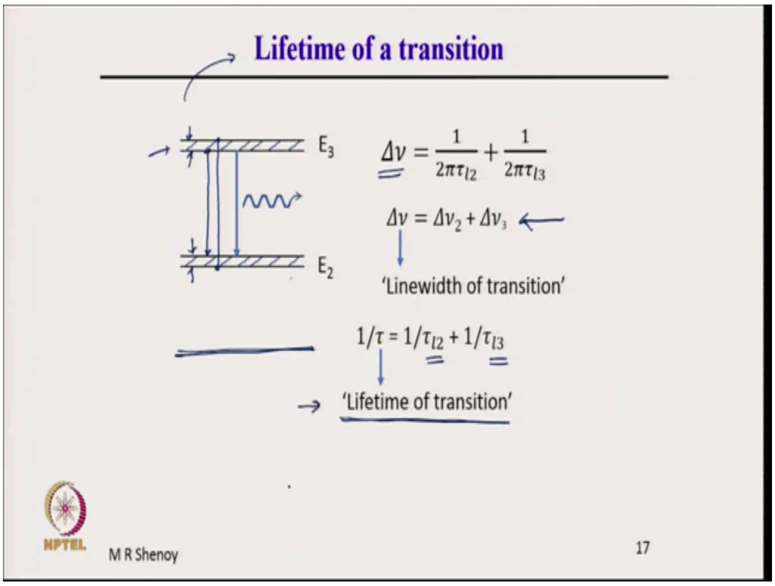

And therefore, I come to the last slide that is lifetime of a transition. We discussed about lifetime of a level which is tau l. So, we had the delta nu or h into delta nu delta E is equal to h into delta nu is 1 by 2 pi into tau l this is delta E, where tau l is the lifetime when you are making transition from an excited state to the ground state. But, if you are looking at a transition between two excited states then there is a finite spread delta E 3 and finite spread delta E 2.

Therefore, there is a spread in the photon spectrum which is coming out in this transition. And therefore, now we have the delta nu is equal to lifetime if I call tau l 2 is the lifetime of the lower level because there is a further level which is here that is the ground state.

So, if tau l 2 is the lifetime of the lower excited state tau l 3 is the lifetime of the upper excited state. Then the lifetime of the transition, this is not lifetime of the level lifetime of the transition is 1 over tau which is the sum of these three because the frequency spread in this transition is delta nu 2 plus delta nu 3.

Delta nu is the spread associated here. Delta nu 2 is the spread associated here. And therefore, please see an atom sitting here can come down to the top. This will give the smallest energy difference and an atom sitting near the top coming down to the bottom here will give the largest energy spread. And therefore, the total spread will be delta E is equal to delta E 3 plus delta E 2 or equivalently delta nu is equal to delta nu 2 plus delta nu 3 and this is called lifetime of the transition.

We will stop here and in the next class we will take up inhomogeneous broadening. So, this is homogeneous broadening. We have seen lifetime broadening and we have also introduced the concept of lifetime of a level and the lifetime of a transition. And in the next lecture we will see inhomogeneous broadening. We will take up the specific example of Doppler broadening and find out what is the kind of line shape we will get in Doppler broadening.

Thank you.